NAME ________________________ DATE ____________ PERIOD ____________

4-1 Study Guide and Intervention

Right Triangle Trigonometry

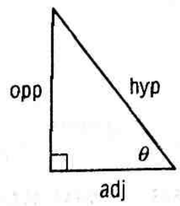
Values of Trigonometric Ratios The side lengths of a right triangle and a reference angle θ can be used to form six **trigonometric ratios** that define the **trigonometric functions** known as **sine, cosine**, and **tangent**. The **cosecant, secant**, and **cotangent** ratios are reciprocals of the sine, cosine, and tangent ratios, respectively. Therefore, they are known as **reciprocal functions**.

Let θ be an acute angle in a right triangle and the abbreviations opp, adj, and hyp refer to the lengths of the side opposite θ, the side adjacent to θ, and the hypotenuse, respectively.

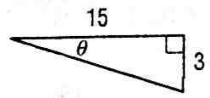
Then the six trigonometric functions of θ are defined as follows.

sine (θ) = $\sin \theta = \frac{\text{opp}}{\text{hyp}}$ cosine (θ) = $\cos \theta = \frac{\text{adj}}{\text{hyp}}$ tangent (θ) = $\tan \theta = \frac{\text{opp}}{\text{adj}}$

cosecant (θ) = $\csc \theta = \frac{\text{hyp}}{\text{opp}}$ secant (θ) = $\sec \theta = \frac{\text{hyp}}{\text{adj}}$ cotangent (θ) = $\cot \theta = \frac{\text{adj}}{\text{opp}}$



Example **Find the exact values of the six trigonometric functions of θ.**



Use the Pythagorean Theorem to determine the length of the hypotenuse.

$15^2 + 3^2 = c^2$ $a = 15, b = 3$

$234 = c^2$ Simplify.

$c = \sqrt{234}$ or $3\sqrt{26}$ Take the positive square root.

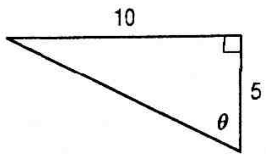
$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{3\sqrt{26}}$ or $\frac{\sqrt{26}}{26}$ $\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{15}{3\sqrt{26}}$ or $\frac{5\sqrt{26}}{26}$ $\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{15}$ or $\frac{1}{5}$

$\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{3\sqrt{26}}{3}$ or $\sqrt{26}$ $\sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{3\sqrt{26}}{15}$ or $\frac{\sqrt{26}}{5}$ $\cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{15}{3}$ or 5

Exercises

Find the exact values of the six trigonometric functions of θ.

1.



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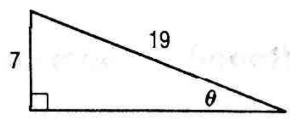

Use the given trigonometric function value of the acute angle θ to find the exact values of the five remaining trigonometric function values of θ.

3. $\sin \theta = \frac{3}{7}$

4. $\sec \theta = \frac{8}{5}$

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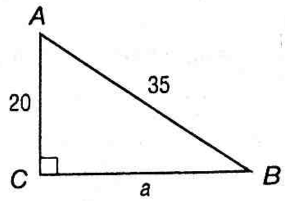
4-1 Study Guide and Intervention *(continued)*

Right Triangle Trigonometry

Solving Right Triangles To solve a right triangle means to find the measures of all of the angles and sides of the triangle. When the trigonometric value of an acute angle is known, the inverse of the trigonometric function can be used to find the measure of the angle.

Trigonometric Function	Inverse Trigonometric Function
$y = \sin x$ $y = \cos x$ $y = \tan x$	$x = \sin^{-1} y$ or $x = \arcsin y$ $x = \cos^{-1} y$ or $x = \arccos y$ $x = \tan^{-1} y$ or $x = \arctan y$

Example **Solve $\triangle ABC$. Round side measures to the nearest tenth and angle measures to the nearest degree.**



Because two lengths are given, you can use the Pythagorean Theorem to find that a is equal to $\sqrt{825}$ or about 28.7. Find the measure of $\angle A$ using the cosine function.

$\cos\theta = \frac{\text{adj}}{\text{hyp}}$ Cosine function

$\cos A = \frac{20}{35}$ Substitute $b = 20$ and $c = 35$.

$A = \cos^{-1}\frac{20}{35}$ Definition of inverse cosine

$A = 55.15009542$ Use a calculator.

Because A is now known, you can find B by subtracting A from 90°.

$55.15 + B = 90$ Angles A and B are complementary.

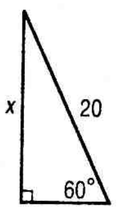
$B = 34.85°$ Subtract.

Therefore, $a \approx 28.7$, $A \approx 55°$, and $B \approx 35°$.

Exercises

Find the value of x. Round to the nearest tenth if necessary.

1.



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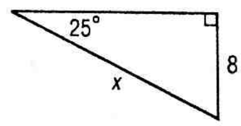
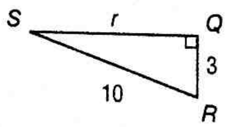


Solve each triangle. Round side measures to the nearest tenth and angle measures to the nearest degree.

3.



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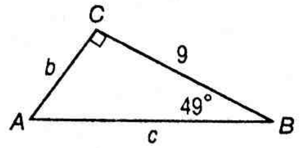


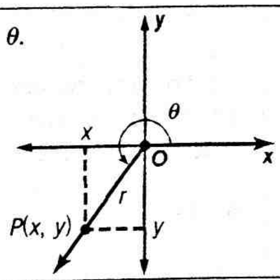
4-3 Study Guide and Intervention

Trigonometric Functions on the Unit Circle

Trigonometric Functions of Any Angle The definitions of the six trigonometric functions may be extended to include any angle as shown below.

Let θ be any angle in standard position and point $P(x, y)$ be a point on the terminal side of θ.
Let r represent the nonzero distance from P to the origin. That is, let $r = \sqrt{x^2 + y^2} \neq 0$.
Then the trigonometric functions of θ are as follows.

$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y},\ y \neq 0$
$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x},\ x \neq 0$
$\tan\theta = \frac{y}{x},\ x \neq 0$	$\cot\theta = \frac{x}{y},\ y \neq 0$



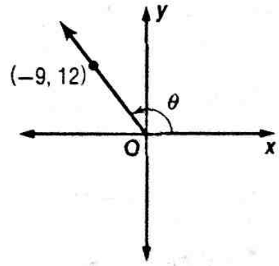
You can use the following steps to find the value of a trigonometric function of any angle θ.

1. Find the reference angle θ'.
2. Find the value of the trigonometric function for θ'.
3. Use the quadrant in which the terminal side of θ lies to determine the sign of the trigonometric function value of θ.

Example **Let (−9, 12) be a point on the terminal side of an angle θ in standard position. Find the exact values of the six trigonometric functions of θ.**

Use the values of x and y to find r.

$r = \sqrt{x^2 + y^2}$	Pythagorean Theorem
$= \sqrt{(-9)^2 + 12^2}$	$x = -9$ and $y = 12$
$= \sqrt{225}$ or 15	Take the positive square root.



Use $x = -9$, $y = 12$, and $r = 15$ to write the six trigonometric ratios.

$\sin\theta = \frac{y}{r} = \frac{12}{15}$ or $\frac{4}{5}$ $\quad$ $\cos\theta = \frac{x}{r} = \frac{-9}{15}$ or $-\frac{3}{5}$ $\quad$ $\tan\theta = \frac{y}{x} = \frac{12}{-9}$ or $-\frac{4}{3}$

$\csc\theta = \frac{r}{y} = \frac{15}{12}$ or $\frac{5}{4}$ $\quad$ $\sec\theta = \frac{r}{x} = \frac{15}{-9}$ or $-\frac{5}{3}$ $\quad$ $\cot\theta = \frac{x}{y} = \frac{-9}{12}$ or $-\frac{3}{4}$

Exercises

The given point lies on the terminal side of an angle θ in standard position. Find the values of the six trigonometric functions of θ.

1. (2, −5) **2.** (12, 4) **3.** (−3, −8)

Find the exact value of each expression.

4. $\sin\frac{5\pi}{3}$ **5.** csc 210° **6.** cot (−315°)

4-3 Study Guide and Intervention *(continued)*

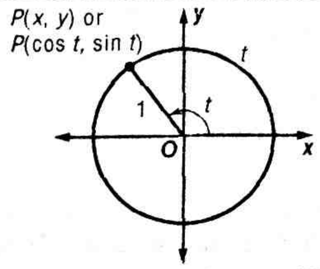
Trigonometric Functions on the Unit Circle

Trigonometric Functions on the Unit Circle You can use the unit circle to find the values of the six trigonometric functions for θ. The relationships between θ and the point $P(x, y)$ on the unit circle are shown below.

Let t be any real number on a number line and let $P(x, y)$ be the point on t when the number line is wrapped onto the unit circle. Then the trigonometric functions of t are as follows.

$\sin t = y$ $\cos t = x$ $\tan t = \frac{y}{x}, x \neq 0$

$\csc t = \frac{1}{y}, y \neq 0$ $\sec t = \frac{1}{x}, x \neq 0$ $\cot t = \frac{x}{y}, y \neq 0$



Therefore, the coordinates of P corresponding to the angle t can be written as $P(\cos t, \sin t)$.

Find the exact value of $\tan \frac{5\pi}{3}$. If undefined, write *undefined*.

$\frac{5\pi}{3}$ corresponds to the point $(x, y) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ on the unit cirle.

$\tan t = \frac{y}{x}$ Definition of $\tan t$

$\tan \frac{5\pi}{3} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $x = \frac{1}{2}$ and $y = -\frac{\sqrt{3}}{2}$ when $t = \frac{5\pi}{3}$

$\tan \frac{5\pi}{3} = -\sqrt{3}$ Simplify.

Exercises

Find the exact value of each expression. If undefined, write *undefined*.

1. $\tan \frac{\pi}{2}$

2. $\sec -\frac{3\pi}{4}$

3. $\cos \frac{7\pi}{6}$

4. $\sin \frac{5\pi}{4}$

5. $\cot \frac{4\pi}{3}$

6. $\csc -\frac{5\pi}{3}$

7. $\tan -60°$

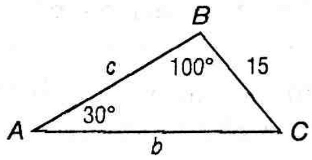
8. $\cot 270°$

The Law of Sines and the Law of Cosines

Solve Oblique Triangles The **Law of Sines** can be used to solve an oblique triangle when given the measures of two angles and a nonincluded side (AAS), two angles and the included side (ASA), or two sides and a nonincluded angle (SSA).

The **Law of Cosines** can be used to solve an oblique triangle when given the measures of three sides (SSS) or the measures of two sides and their included angle (SAS).

Example **Solve $\triangle ABC$. Round side lengths to the nearest tenth and angle measures to the nearest degree.**



Because two angles are given,
$C = 180° - (100° + 30°)$ or $50°$.

Use the Law of Sines to find b and c.

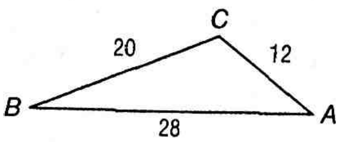
$\frac{\sin A}{a} = \frac{\sin B}{b}$	Law of Sines	$\frac{\sin A}{a} = \frac{\sin C}{c}$
$\frac{\sin 30°}{15} = \frac{\sin 100°}{b}$	Substitution	$\frac{\sin 30°}{15} = \frac{\sin 50°}{c}$
$b \sin 30° = 15 \sin 100°$	Cross products	$c \sin 30° = 15 \sin 50°$
$b = \frac{15 \sin 100°}{\sin 30°}$	Divide each side by sin 30°.	$c = \frac{15 \sin 50°}{\sin 30°}$
$b \approx 29.5$	Use a calculator.	$c \approx 23.0$

Therefore, $b \approx 29.5$, $c \approx 23.0$, and $C = 50°$.

Exercises

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

1.



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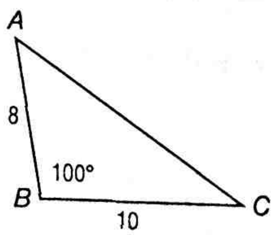

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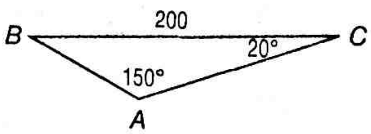

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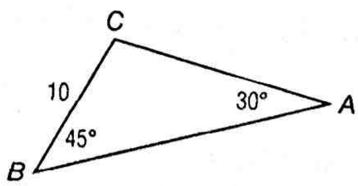


Lesson 4-7

NAME ______________________ DATE ____________ PERIOD ____________

4-7 Study Guide and Intervention *(continued)*

The Law of Sines and the Law of Cosines

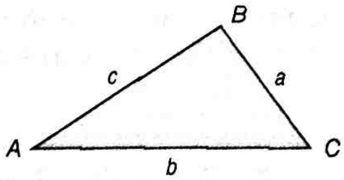
Find Areas of Oblique Triangles When the measures of all three sides of an oblique triangle are known, Heron's Formula can be used to find the area of the triangle.

Heron's Formula

If the measures of the sides of $\triangle ABC$ are a, b, and c, then the area of the triangle is

$\sqrt{s(s-a)(s-b)(s-c)}$,

where $s = \frac{1}{2}(a + b + c)$.



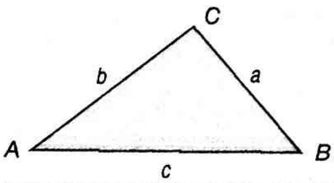
When two sides and the included angle of a triangle are known, the area is one-half the product of the lengths of the two sides and the sine of the included angle.

Area of a Triangle Given SAS

Area $= \frac{1}{2}bc \sin A$

Area $= \frac{1}{2}ac \sin B$

Area $= \frac{1}{2}ab \sin C$



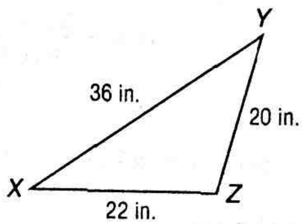
Find the area of $\triangle XYZ$ to the nearest tenth.

The value of s is $\frac{1}{2}(20 + 22 + 36)$ or 39.

Area $= \sqrt{s(s-x)(s-y)(s-z)}$ — Heron's Formula

$= \sqrt{39(39-20)(39-22)(39-36)}$ — $s = 39$, $x = 20$, $y = 22$, and $z = 36$

$= \sqrt{37{,}791}$ or about 194.4 in^2 — Simplify.



Exercises

Use Heron's Formula to find the area of each triangle. Round to the nearest tenth.

1. $\triangle ABC$ if $a = 14$ ft, $b = 9$ ft, $c = 8$ ft

2. $\triangle FGH$ if $f = 8$ in., $g = 9$ in., $h = 3$ in.

3. $\triangle MNP$ if $m = 3$ yd, $n = 4.6$ yd, $p = 5$ yd

4. $\triangle XYZ$ if $x = 8$ cm, $y = 12$ cm, $z = 13$ cm

Find the area of each triangle to the nearest tenth.

5. $\triangle RST$ if $R = 50°$, $s = 12$ yd, $t = 14$ yd

6. $\triangle MNP$ if $n = 14$ ft, $P = 110°$, $N = 25°$

7. $\triangle DEF$ if $d = 15$ ft, $E = 135°$, $f = 18$ ft

8. $\triangle JKL$ if $j = 4.3$ m, $l = 3.9$ m, $K = 82°$