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Study Guide and Intervention**Using a Problem-Solving Plan****FOUR-STEP PROBLEM-SOLVING PLAN**

When solving problems, it is helpful to have an organized plan to solve the problem. The following four steps can be used to solve any math problem.

- 1 **Explore**—get a general understanding of the problem
- 2 **Plan**—make a plan to solve the problem and estimate the solution
- 3 **Solve**—use your plan to solve the problem
- 4 **Examine**—check the reasonableness of your solution

Example

HEALTH According to a recent study, 1 out of every 10 people is left-handed. If there are 172 people in the eighth grade, predict the number of students who are left-handed.

Explore We know that 1 out of 10 people is left-handed. We also know that there are 172 people in the eighth grade. We need to predict how many of the students are left-handed.

Plan Make a table to organize the information and look for a pattern.

Number of people	10	20	30	40	50
Number who are left-handed	1	2	3	4	5

Solve By extending the pattern, we can predict that 17 students will be left-handed.

Examine For every 10 students in the class, 1 is left-handed. There are 17 groups of 10 in a class of 172 and $17 \times 1 = 17$. The answer is correct.

Exercises

1. **SODA POP** James needs to buy one can of orange soda for every three cans of cola. If James buys 24 cans of cola, how many cans of orange soda should he buy?
2. **VIDEOS** Bob's Video Venue has a membership fee of \$5.00 and tape rentals are \$1.50 each. Video Heaven has no membership fee and tape rentals are \$2.00 each. How many tapes must be rented in order for Bob's Video Venue to be more economical?
3. **COOKIES** A cookie shop offers 6 varieties of cookies and bakes 5 dozen of each kind every day, Monday–Friday. How many cookies are baked in four weeks?
4. **PATTERNS** Find the next term in 2, 6, 18, 54, 162, . . .
5. **TYPING** Jeremy needs to type a 500-word report for science class. He knows he can type about 19 words per minute. About how long will it take Jeremy to type his report?

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Skills Practice**Using a Problem-Solving Plan****Solve each problem.**

- 1. MONEY** Jackie wants to take out an ad in the newspaper for her up-coming garage sale. She can buy a 4-line ad for \$4.35 that will run for three days. If she wants to spend no more than \$15 on advertising, how long can she advertise?
- 2. BASEBALL** Cy Young pitched in 815 games over 22 years. He won 511 games. About how many games did he win per year?
- 3. MONEY** Each month, Kevin's car costs \$59 for insurance, \$42 for maintenance, and \$58 for gas. About how much does it cost Kevin to drive his car for a year?
- 4. MONEY** How many ways can you make change for a dollar using nickels, dimes, and/or quarters?
- 5. FOOD** Friday night Joe decided to order a 1-topping pizza. He had a choice of thin or thick crust and a choice of five toppings (pepperoni, mushrooms, sausage, onions, or peppers). How many different pizzas could he choose from?

Find the next term in each list.

- 6.** 7, 11, 15, 19, 23, ...
- 7.** 2, 4, 8, 16, 32, ...
- 8.** 63, 54, 45, 36, 27, ...
- 9.** 3, 0, 5, 3, 0, 5, ...
- 10.** 0, 4, 8, 12, 16, ...

GEOMETRY Draw the next figure in each pattern.

- 11.** □ ▲ □ □ ▲ ▲ □ □ □ ...

- 12.**

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Practice**Using a Problem-Solving Plan****Solve each problem.**

- RETAIL** At a school bookstore, a ballpoint pen costs \$0.28 and a notepad costs \$0.23. What could you buy and spend for exactly 0.74?
- SOCCER** At soccer practice, each player must kick the ball to every other player present at least once. If there are 17 players at practice, what is the minimum number of kicks required?
- MONEY** Mr. Jasper asked his neighbor, Mark, to feed his cat each day while he went on a two-week vacation. Suppose Mr. Jasper offered Mark two payment options. The first option would pay Mark \$5 per day up front. The second option would pay \$0.01 the first day, then double the pay each day for two weeks. He would pay this option when he returned. Which option should Mark choose?

- NUMBER THEORY** Use the following clues to find the secret number.

I am a 3-digit number.

All of my digits are odd.

I am less than 600.

I am greater than 400.

If you add my digits you get 15.

My last digit is 7.

What am I?

Find the next term in each list.

5. 9, 13, 17, 21, 25, ...

6. 88, 86, 84, 82, 80, ...

7. 0, 7, 14, 21, 28, ...

8. 3, 6, 12, 24, 48, ...

GEOMETRY Draw the next figure in each pattern.

9. ○ □ ▲ ○ ○ □ □ .

10. • •• ••••• .

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Reading to Learn Mathematics***Using a Problem-Solving Plan***

Pre-Activity *Why is it helpful to use a problem-solving plan to solve problems?*

Do the activity at the top of page 6 in your textbook. Write your answers below.

- a. Find a pattern in the costs.
- b. How can you determine the cost to mail a 6-ounce letter?
- c. Suppose you were asked to find the cost of mailing a letter that weighs 8 ounces. What steps would you take to solve the problem?

Reading the Lesson

Write a definition and give an example of each new vocabulary word or phrase.

Vocabulary	Definition	Example
1. conjecture		
2. inductive reasoning		

3. What is the next term: 3, 6, 12, 24 . . .? Explain.
4. Complete this sentence. In the _____ step of the four-step problem-solving plan, you check the reasonableness of your answer.

Helping You Remember

5. Explain why each step of the four-step plan is important.

1-1 Enrichment

Cyclic Numbers

Look closely at the products below. Do you see a pattern?

$$1 \times 142,857 = \underline{142,857}$$

$$2 \times 142,857 = \underline{285,714}$$

$$3 \times 142,857 = \underline{428,571}$$

$$4 \times 142,857 = \underline{571,428}$$

$$5 \times 142,857 = \underline{714,285}$$

$$6 \times 142,857 = \underline{857,142}$$

The same six digits repeat in all of the products. Numbers like 142,857 are called *cyclic numbers*.

- Cyclic numbers are related to prime numbers. A prime number is a number that has exactly two factors, 1 and itself. You can use a calculator and the decimal equivalents of fractions of the form $\frac{1}{p}$, where p is a prime number, to find cyclic numbers. Use a calculator to find the decimal equivalent of each fraction below.
 - $\frac{1}{2}$
 - $\frac{1}{3}$
 - $\frac{1}{5}$
 - $\frac{1}{7}$
- Study the decimal equivalents you found. Do you observe a pattern in any of the digits?
- The cyclic number 142,857 has six digits. The next largest cyclic number has sixteen digits. What fraction do you think might help you find the next cyclic number? Explain.
- Explain why the next largest cyclic number cannot be determined using a scientific calculator.

1-2 Study Guide and Intervention

Numbers and Expressions

Use the **order of operations** to evaluate expressions.

Step 1 Simplify the expressions inside grouping symbols.

Step 2 Do all multiplications and/or divisions from left to right.

Step 3 Do all additions and/or subtractions from left to right.

Example 1

$$6 \cdot 5 - 10 \div 2$$

$$6 \cdot 5 - 10 \div 2 \quad \text{Multiply 6 and 5.}$$

$$= 30 - 10 \div 2 \quad \text{Divide 10 by 2.}$$

$$= 30 - 5 \quad \text{Subtract 5 from 30.}$$

$$= 25$$

Example 2

$$4(3 + 6) + 2 \cdot 11$$

$$4(3 + 6) + 2 \cdot 11 \quad \text{Evaluate } (3 + 6).$$

$$= 4(9) + 2 \cdot 11 \quad \text{Multiply 4 and 9, and 2 and 11.}$$

$$= 36 + 22 \quad \text{Add 36 and 22.}$$

$$= 58$$

Translate verbal phrases into numerical expressions.

Example 3

Write and evaluate a numerical expression for the product of seventeen and three.

Phrase the product of seventeen and three

Key Word product

Expression $17 \cdot 3$

Exercises

Find the value of each expression.

1. $6 + 3 \cdot 9$

2. $7 + 7 \cdot 3$

3. $14 - 6 + 8$

4. $26 - 4 + 9$

5. $10 \div 5 \cdot 3$

6. $22 \div 11 \cdot 6$

7. $2(6 + 2) - 4 \cdot 3$

8. $5(6 + 1) - 3 \cdot 3$

9. $2[(13 - 4) + 2(2)]$

10. $4[(10 - 6) + 6(2)]$

11. $\frac{67 + 13}{34 - 29}$

12. $6(4 - 2) + 8$

13. $3[(2 + 7) \div 9] - 3$

14. $(8 \cdot 7) \div 14 - 1$

15. $\frac{4(18)}{2(9)}$

16. $(9 \cdot 8) - (100 \div 5)$

Write a numerical expression for each verbal phrase.

17. eleven less than twenty

18. twenty-five increased by six

19. sixty-four divided by eight

20. the product of seven and twelve

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Skills Practice***Numbers and Expressions***

Name the operation that should be performed first. Then find the value of each expression.

1. $2 - 3 \cdot 0$

2. $25 \div 5 - 4$

3. $5 + 2 - 3$

4. $2 \cdot 5 + 6$

5. $9 \div 3 \cdot 2 + 1$

6. $5 + 2 \cdot 8 + 2 - 5$

Find the value of each expression.

7. $4 + 2 \cdot 8$

8. $30 - 12 \cdot 2$

9. $5 + 2 \cdot 3 + 4$

10. $10 - 2 \cdot 4 - 1$

11. $15 - 10 \div 2$

12. $25 - 6 \cdot 4 + 9$

13. $(14 + 6) \div 5$

14. $100 + 50 \div 10$

15. $14 - (4 \cdot 2)$

16. $(3 + 4) \cdot (5 + 3)$

17. $6(4 + 5)$

18. $\frac{8 \cdot 9}{3 \cdot 4}$

19. $(2 + 3) \cdot 5 + 1$

20. $24 - 24 \div 8$

21. $56 \div (3 + 4)$

22. $2[(4 + 5) \cdot 3]$

Write a numerical expression for each verbal phrase.

23. the difference of seventeen and three

24. eleven more than six

25. the sum of eight, twenty, and thirty-five

26. the quotient of forty and eight

27. one hundred decreased by twenty-five

28. three more than one dozen

29. the product of twenty and thirty

30. five less than fifty

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Practice

Numbers and Expressions

Find the value of each expression.

1. $4 + 2 \cdot 8$
2. $30 - 12 \cdot 2$
3. $6(6 \div 2) \cdot 9$
4. $6(6) \div 2 \cdot 9$
5. $6(6) \div (2 \cdot 9)$
6. $6(6 \div 2 \cdot 9)$
7. $12 - 2 \cdot 5 + 3$
8. $(4 + 5) \cdot (4 + 5)$
9. $100 \div (16 + 9) \cdot 6$
10. $25 + 30 \div 6 \cdot 5$
11. $16 - 49 \div 7 \cdot 2$
12. $(2 \cdot 11 + 1) - (3 \cdot 6 + 5)$
13. $\frac{4(10 + 2)}{2(24 \div 3)}$
14. $2 + 4 \cdot 6 - 3 \cdot 5 + 6 \cdot 2$
15. $(8 + 4) \cdot (6 - 3)$
16. $\frac{2(6 + 4)}{2(8 - 6)}$
17. $4(8 + 2 \cdot 5 - 6)$
18. $2(105 \div 15 - 6)$
19. $14 \div 2 \cdot 5 + 3$
20. $4(4 + 5) \div 3(10 - 7)$

Write a numerical expression for each verbal phrase.

21. thirty-one increased by fourteen
22. the difference of sixteen and nine
23. the sum of seven, four, and eighteen
24. three times forty
25. the quotient of eighty-one and three
26. four more than the product of seven and eight
27. the cost of three slices of pizza at \$2 each
28. the number of days in six weeks
29. **BOWLING** Alicia rented bowling shoes for \$3 and played 4 games at \$2 each. Write and evaluate an expression for the total cost of bowling.
30. **TICKETS** Adult tickets for a movie cost \$6 and children's tickets cost \$3. If two adults and three children go to the movies, how much will they pay?

1-2

Reading to Learn Mathematics

Numbers and Expressions

Pre-Activity *Why do we need to agree on an order of operations?*

Do the activity at the top of page 12 in your textbook. Write your answers below.

- Study the expressions and their respective values. For each expression, tell the order in which the calculator performed the operations.
- For each expression, does the calculator perform the operations in order from left to right?
- Based on your answer to parts **a** and **b**, find the value of each expression below. Check your answer with a scientific calculator.
 $12 - 3 \times 2$ $16 \div 4 - 2$ $18 + 6 - 8 \div 2 \times 3$
- Make a conjecture** as to the order in which a scientific calculator performs operations.

Reading the Lesson

Write a definition and give an example of each new vocabulary word or phrase.

Vocabulary	Definition	Example
1. numerical expression		
2. evaluate		
3. order of operations		

4. In the boxes below, write three different expressions using two operations that each have a value of 6.

6
 6
 6

Helping You Remember

5. A mnemonic device helps you remember something. Create your own mnemonic device to remember the order of operations. For example, list the operations in order, use the first letter of each operation and create a phrase with words starting with the same letters.

1-2 Enrichment**Operations Search**

This is a fun activity that you can try on your own as well as with your family or classmates.

In each exercise below, you are given some numbers. Insert operations symbols (+, −, ×, ÷) and parentheses so that a true mathematical sentence is formed. Follow the specific instructions for each problem, remembering to observe the order of operations.

Do not change the order of the numbers.

1. 5 4 3 2 1 = 3

2. 5 4 3 2 1 = 0

3. 5 4 3 2 1 = 1

4. 5 4 3 2 1 = 50

Do not change the order of the numbers. You may put two numbers together to form a two-digit number.

5. 1 2 3 4 5 6 7 8 = 90

6. 8 7 6 5 4 3 2 1 = 25

Change the order of the numbers and put numbers together to form a two- or three-digit number, as needed.

Use these four digits: 4 3 2 1

Make these totals:

7. 1312

8. 2

9. 16

10. 1

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Study Guide and Intervention

Variables and Expressions

An **algebraic expression** is a combination of variables, numbers, and at least one operation. To evaluate an algebraic expression, replace the variable(s) with numbers and follow the order of operations.

Example 1

ALGEBRA Evaluate each expression if $r = 6$ and $s = 2$.

a. $8s - 2r$

$$\begin{aligned} 8s - 2r &= 8 \cdot 2 - 2 \cdot 6 && \text{Replace } r \text{ with 6 and } s \text{ with 2.} \\ &= 16 - 12 && \text{Multiply.} \\ &= 4 && \text{Subtract.} \end{aligned}$$

b. $3(r + s)$

$$\begin{aligned} 3(r + s) &= 3(2 + 6) && \text{Replace } r \text{ with 6 and } s \text{ with 2.} \\ &= 3 \cdot 8 && \text{Evaluate the parentheses.} \\ &= 24 && \text{Multiply.} \end{aligned}$$

Example 2

FOOTBALL Teams earn three points for field goals and six points for touchdowns.

a. Assuming no other points, write an expression for a team's total points.

Words three points for field goals and six points for touchdowns

Variables Let f = number of field goals and t = number of touchdowns.

Expression $3f + 6t$

The total points for the team is $3f + 6t$.

b. Find the total score if a team scored two field goals and three touchdowns.

$$\begin{aligned} 3f + 6t &= 3 \cdot 2 + 6 \cdot 3 && \text{Replace } f \text{ with 2 and } t \text{ with 3.} \\ &= 6 + 18 && \text{Multiply.} \\ &= 24 && \text{Add.} \end{aligned}$$

The team scored a total of 24 points.

Exercises

ALGEBRA Evaluate each expression if $x = 10$, $y = 5$, and $z = 1$.

1. $x + y - z$

2. $\frac{x}{y}$

3. $2x + 4z$

4. $xy + z$

5. $\frac{6y}{10z}$

6. $x(2 + z)$

7. $x - 2y$

8. $\frac{(x + y)}{z}$

Translate each phrase into an algebraic expression.

9. eight inches taller than Mycala's height
10. twelve more than four times a number
11. the difference of sixty and a number
12. three times the number of tickets sold

1-3

Skills Practice***Variables and Expressions***

ALGEBRA Evaluate each expression if $x = 4$, $y = 6$, and $z = 3$.

1. $x + y + z$

2. $3x + y$

3. $x - z$

4. $x + y - 3z$

5. $15z$

6. $3(x + z)$

7. $xz \div y$

8. $yz - x$

ALGEBRA Evaluate each expression if $a = 7$, $b = 9$, $c = 2$, and $d = 5$.

9. $a + b + c$

10. $a + b - (c + d)$

11. $3a + 4d$

12. bcd

13. $(a + b) \cdot (c + d)$

14. $c(4 + d)$

15. $\frac{b}{a + c}$

16. $a + b - 3c$

17. $ab - cd$

18. $\frac{bc}{a - d}$

ALGEBRA Translate each phrase into an algebraic expression.

19. two inches shorter than Kathryn's height

20. the quotient of some number and thirteen

21. some number added to seventeen

22. six centimeters shorter than the length of the pencil

23. three pounds lighter than Adlai's weight

24. the difference of some number and eighteen

25. three dollars more than the cost of a ticket

26. eight more than the product of a number and four

27. half as many pieces of candy

28. twice as long as the length of the string

1-3

Practice

Variables and Expressions

ALGEBRA Evaluate each expression if $x = 12$, $y = 20$, and $z = 4$.

1. $x + y + z$

2. $4x - y$

3. $3x + 2y$

4. $y - 3z$

5. $x + y \div z$

6. $yz + x$

7. $(y - x) + (y - z)$

8. $\frac{y}{z} + \frac{x}{z}$

9. $\frac{5x}{3y}$

10. $z(y - x) + 4z$

ALGEBRA Evaluate each expression if $a = 3$, $b = 6$, $c = 5$, and $d = 9$.

11. $a + b + c + d$

12. $\frac{(a + b + c)}{2}$

13. $ab + bc$

14. $6d - c \cdot c$

15. $3(a + b + c)$

16. $\frac{100}{5c}$

17. abc

18. $10(6c - 3d)$

19. $\frac{2(a + b)}{6(b - c)}$

20. $4[(d - a) + c]$

ALGEBRA Translate each phrase into an algebraic expression.

21. six times a number minus eleven

22. the product of eight hundred and a number

23. the quotient of thirty and ten times a number

24. five times the sum of three and some number

25. half the distance to the school.

26. **RECYCLING** In order to encourage recycling, the city is offering five cents for every pound of newspapers collected, twenty-five cents per pound for cans and ten cents per pound for glass bottles or jars.

a. Write an expression for the total amount earned from recycling.

b. If Chen brings in ten pounds of newspapers, eight pounds of cans, and two pounds of glass, how much will he receive?

1-3

Reading to Learn Mathematics***Variables and Expressions*****Pre-Activity** *How are variables used to show relationships?*

Do the activity at the top of page 17 in your textbook. Write your answers below.

- a. Suppose the baby-sitter worked 10 hours. How much would he or she earn?
- b. What is the relationship between the number of hours and the money earned?
- c. If h represents *any number of hours*, what expression could you write to represent the amount of money earned?

Reading the Lesson

Write a definition and give an example of each new vocabulary word or phrase.

Vocabulary	Definition	Example
1. variable		
2. algebraic expression		
3. defining a variable		

4. Name three things that make an algebraic expression.
5. Why do you think replacing a variable with a number is called the Substitution Property of Equality?

Helping You Remember

6. *Variable* is a word used in everyday English.
 - a. Find the definition of *variable* in the dictionary. Write the definition.
 - b. Explain how the English definition can help you remember how *variable* is used in mathematics.

1-3 Enrichment

Hypatia

Hypatia, pronounced *hi PAY sha*, was the first woman to be mentioned in the history of mathematics. Born about A.D. 370, Hypatia lived in Alexandria and served as a professor at the famous Library of Alexandria. Hypatia wrote important commentaries on the works of mathematician Appollonius and the scientist Ptolemy. She also excelled in the fields of astronomy, medicine, and philosophy.

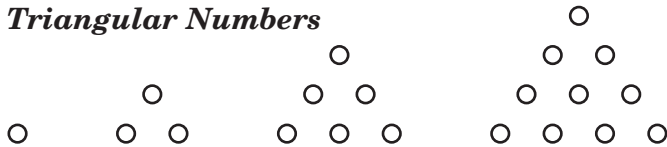
Egypt was in great political turmoil during Hypatia's lifetime. Because of her influence among scholars of the day, Hypatia became the target of criticism from those who equated science with paganism. In A.D. 415, she was murdered by an angry mob. Soon after her death, the

library was destroyed and the Dark Ages began. The serious study of mathematics was limited for the next 500 years.

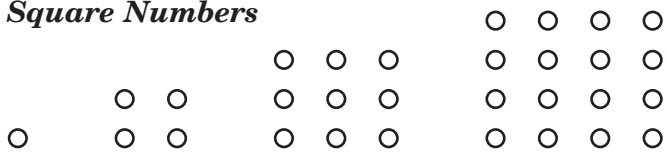
One of the things Hypatia studied was the relationship between number patterns and geometry. Investigate the geometric patterns below.



Triangular Numbers



Square Numbers



1. Draw the fifth and sixth figures in the pattern of triangular numbers. Then write the first six triangular numbers.
2. Draw the fifth and sixth figures in the patterns of square numbers. Then write the first six square numbers.
3. Draw the first four pentagonal and hexagonal numbers.
4. Use counters or drawings to determine if there is a number that is both square and triangular.

1-4

Study Guide and Intervention**Properties**

In algebra, there are certain statements called **properties** that are true for any numbers.

Property	Explanations	Example
Commutative Property of Addition	$a + b = b + a$	$6 + 3 = 3 + 6$ $9 = 9$
Commutative Property of Multiplication	$a \cdot b = b \cdot a$	$4 \cdot 5 = 5 \cdot 4$ $20 = 20$
Associative Property of Addition	$(a + b) + c = a + (b + c)$	$(3 + 4) + 7 = 3 + (4 + 7)$ $14 = 14$
Associative Property of Multiplication	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$(2 \cdot 5) \cdot 8 = 2 \cdot (5 \cdot 8)$ $80 = 80$
Additive Identity	$a + 0 = 0 + a = a$	$10 + 0 = 0 + 10 = 10$
Multiplicative Identity	$a \cdot 1 = 1 \cdot a = a$	$5 \cdot 1 = 1 \cdot 5 = 5$
Multiplicative Property of Zero	$a \cdot 0 = 0 \cdot a = 0$	$15 \cdot 0 = 0 \cdot 15 = 0$

Example**Simplify $3 \cdot (x \cdot 5)$.**

$$\begin{aligned}
 3 \cdot (x \cdot 5) &= 3 \cdot (5 \cdot x) && \text{Commutative Property of Multiplication} \\
 &= (3 \cdot 5) \cdot x && \text{Associative Property of Multiplication} \\
 &= 15 \cdot x && \text{Multiply 3 and 5.}
 \end{aligned}$$

Exercises**Name the property shown by each statement.**

1. $75 + 25 = 25 + 75$

2. $2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4$

3. $14 \cdot 1 = 14$

4. $p \cdot 0 = 0$

5. $6 + (5 + m) = (6 + 5) + m$

6. $2(6) = 6(2)$

Simplify each expression.

7. $24 + (x + 6)$

8. $3 \cdot (4a)$

9. $9 + (12 + c)$

10. $13d \cdot 0$

**1-4 Skills Practice**
Properties

Name the property shown by each statement.

1. $9 \cdot 7 = 7 \cdot 9$

2. $37 \cdot 0 = 0$

3. $1 \cdot 87 = 87$

4. $14 + 6 = 6 + 14$

5. $3(6a) = (3 \cdot 6)a$

6. $2b + 0 = 2b$

Find each sum or product mentally.

7. $4 + 23 + 46$

8. $327 \cdot 6 \cdot 0$

9. $2 \cdot 15 \cdot 10$

10. $5 \cdot 16 \cdot 20$

11. $14 + 24 + 6 + 26$

12. $43 + 38 + 7$

13. $25 \cdot 0 \cdot 8$

14. $11 + 28 + 19$

ALGEBRA Simplify each expression.

15. $(x + 5) + 4$

16. $(6a)10$

17. $38 + (v + 12)$

18. $8(q3)$

19. $16p \cdot 0$

20. $16 + (22 + x)$

21. $8(p9)$

22. $(17 + 33) + x$

23. $3(11k)$

24. $16 + (y + 9)$

25. $m(13 \cdot 5)$

26. $17 + (n + 0)$

1-4 Practice

Properties

Name the property shown by each statement.

- $55 + 6 = 6 + 55$
- $6 \cdot 7 = 7 \cdot 6$
- $(x + 3) + y = x + (3 + y)$
- $1 \cdot mp = mp$
- $9 + (5 + 35) = (9 + 5) + 35$
- $67 + 0 = 67$
- $7x \cdot 0 = 0$
- $4(3 \cdot z) = (4 \cdot 3)z$

Find each sum or product mentally.

- $18 + 17 + 22$
- $12 + 15 + 8 + 5$
- $60 \cdot 4 \cdot 2$
- $49 \cdot 0 \cdot 16$
- $2 \cdot 157 \cdot 5$
- $14 + 25 + 16$

ALGEBRA Simplify each expression.

- $(m + 11) + 19$
- $(9 \cdot b) \cdot 10$
- $19 + (v + 8)$
- $(28 + 12) + x$
- $8s \cdot 0$
- $4 \cdot (r \cdot 5)$

- GEOMETRY** The volume of a box is given by $V = \ell \cdot w \cdot h$ where ℓ = length, w = width, and h = height. Find the volume of a box if length is 25 cm, width is 13 cm, and height is 4 cm.
- SCHOOL** In math class each assignment is worth 20 points. David got 17, 20, 19, and 13 points on his last four assignments. How many points did David score altogether?
- True or false:* Multiplying any number by one produces the original number. Explain.

1-4

Reading to Learn Mathematics

Properties

Pre-Activity *How are real-life situations commutative?*

Do the activity at the top of page 23 in your textbook. Write your answers below.

- Suppose you read the Preamble to the U.S. Constitution first and then the Gettysburg Address. Write an expression for the total number of words read.
- Suppose you read the Gettysburg Address first and then the Preamble to the U.S. Constitution. Write an expression for the total number of words read.
- Find the value of each expression. What do you observe?
- Does it matter in which order you add any two numbers? Why or why not?

Reading the Lesson

Write a definition and give an example of each new vocabulary word or phrase.

Vocabulary	Definition	Example
1. properties		
2. counterexample		
3. simplify		
4. deductive reasoning		

Helping You Remember

- Tell what a counterexample is in your own words. Tell how it is used in mathematics and why it is important.

1-4 Enrichment

Algebraic Proof

Axioms are statements assumed to be true without being proven. They are used in the proofs of theorems. The following properties are examples of algebraic axioms. Abbreviations for these properties are used in the examples below.

Commutative Property of Addition (CPA)	Addition Property of Equality (APE)
Associative Property of Addition (APA)	Substitution Property of Equality (SPE)
Subtraction Property of Equality (SubPE)	Additive Identity Property (AIP)

Example 1 Prove: $a + (b + c) = c + (a + b)$

Statement	Reason
$a + (b + c) = (a + b) + c$	APA
$= c + (a + b)$	CPA

Example 2 Prove: $5 + (x + 2) = 7 + x$

Statement	Reason
$5 + (x + 2) = 5 + (2 + x)$	CPA
$= (5 + 2) + x$	APA
$= 7 + x$	SPE

Write the reason for each statement.

1. Prove: $(9 + 6) + (x + 3) = x + 18$

Statement	Reason
$(9 + 6) + (x + 3) = 15 + (x + 3)$	a. _____
$= (15 + x) + 3$	b. _____
$= (x + 15) + 3$	c. _____
$= x + (15 + 3)$	d. _____
$= x + 18$	e. _____

Prove each of the following. Identify a reason for each statement.

2. $9 + (x + 4) = 13 + x$

3. $5 + (x + 11) = x + (10 + 6)$

1-5

Study Guide and Intervention

Variables and Equations

An equation that contains a variable is called an **open sentence**. When the variable is replaced with a number, you can determine whether the sentence is true or false. A value that makes the sentence true is called a **solution**.

Example 1 ALGEBRA Find the solution of $27 - p = 14$. Is it 11, 13, or 15?

Value for p	$27 - p = 14$	True or False?
11	$27 - 11 \neq 15$	false
13	$27 - 13 \neq 14$	true
15	$27 - 15 \neq 14$	false

Verbal sentences can be translated into equations and then solved.

Example 2

ALGEBRA The sum of a number and six is twenty-one. Find the number.

Let n = the number.

Words The sum of a number and six is twenty-one.

Variables Let n = the number.

Equation $n + 6 = 21$ Write the equation.

$15 + 6 = 21$ Think: What number added to 6 is 21?

$n = 15$ The solution is 15.

Exercises

ALGEBRA Find the solution of each equation from the list given.

- $b + 11 = 29$; 16, 18, 20
- $h + 7 = 42$; 35, 37, 39
- $37 - x = 24$; 9, 11, 13
- $26 - m = 18$; 6, 8, 10
- $v - 6 = 5$; 7, 9, 11
- $6r = 48$; 6, 8, 10
- $\frac{63}{a} = 9$; 7, 9, 11
- $k - 16 = 15$; 31, 33, 35
- $121 = 11p$; 9, 11, 13
- $\frac{x}{5} = 15$; 70, 75, 80
- $2n + 1 = 7$; 3, 4, 5
- $11 = 3y - 25$; 10, 11, 12

ALGEBRA Define the variable. Then write the equation and solve.

- The product of seven and a number is fifty-six.
- The quotient of eighty-two and a number is two.
- The difference between a number and four is twelve.

1-5

Skills Practice***Variables and Equations***

ALGEBRA Find the solution of each equation from the list given.

1. $u + 11 = 42$; 29, 31, 33

2. $23 + w = 30$; 7, 8, 9

3. $18 + 17 = g$; 33, 34, 35

4. $s - 16 = 4$; 18, 20, 22

5. $17 - x = 2$; 13, 15, 17

6. $27 - 6 = d$; 17, 19, 21

7. $8r = 24$; 3, 4, 5

8. $16 = 4v$; 2, 3, 4

9. $\frac{42}{x} = 7$; 6, 8, 10

10. $\frac{x}{11} = 7$; 73, 75, 77

ALGEBRA Solve each equation mentally.

11. $c + 9 = 11$

12. $20 = f + 5$

13. $m - 6 = 3$

14. $24 - u = 18$

15. $6r = 36$

16. $8h = 40$

17. $p + 9 = 25$

18. $6 = \frac{30}{z}$

ALGEBRA Define a variable. Then write an equation and solve.

19. The product of 5 and a number is 50.

20. A number decreased by 3 is 9.

21. Eleven more than a number is 33.

22. Forty divided by a number is 8.

23. A number times 9 is 36.

24. A number divided by 7 is 7.

25. Thirty less than 40 is a number.

26. The difference between 20 and 8 is a number.

27. The sum of 4 and a number is 20.

28. The quotient of 50 and a number is 2.

1-5

Practice

Variables and Equations

ALGEBRA Find the solution of each equation from the list given.

1. $w + 16 = 31$; 13, 15, 17

2. $z + 31 = 72$; 37, 39, 41

3. $25 - p = 0$; 21, 23, 25

4. $s - 14 = 2$; 12, 14, 16

5. $19 = t - 21$; 40, 42, 44

6. $b = 15 - 3$; 12, 14, 16

7. $9q = 72$; 6, 8, 10

8. $35 = 5m$; 7, 9, 11

9. $\frac{75}{n} = 15$; 5, 7, 9

10. $\frac{p}{8} = 10$; 80, 84, 88

ALGEBRA Solve each equation mentally.

11. $g + 19 = 29$

12. $26 + h = 35$

13. $n - 6 = 12$

14. $36 \div a = 12$

15. $\frac{90}{45} = u$

16. $3t = 39$

17. $15 + r = 30$

18. $34 - v = 20$

ALGEBRA Define a variable. Then write an equation and solve.

19. The sum of 3, 5, and a number is 15.

20. The difference of a number and 16 is 5.

21. The quotient of 56 and a number is 7.

22. A number increased by 30 is 63.

23. Eight times a number is 32.

24. A number decreased by 4 is 41.

25. **WEATHER** During the month of July, meteorologists recorded 5 inches of rainfall. This is 6 inches below average. Define a variable and write an equation that can be used to determine the average rainfall for July. Find the average rainfall for July.

26. **FOOD** Junot and Lisa ordered a pizza and cut it into six slices. If Junot ate one slice and Lisa ate one slice, how many slices are left?

1-5

Reading to Learn Mathematics***Variables and Equations***

Pre-Activity *How is solving an open sentence similar to evaluating an expression?*

Do the activity at the top of page 28 in your textbook. Write your answers below.

- If Rebecca is x years old, what expression represents Emilio's age?
- What two expressions are equal?
- If Emilio is 19, how old is Rebecca?

Reading the Lesson

Write a definition and give an example of each new vocabulary word or phrase.

Vocabulary	Definition	Example
1. equation		
2. open sentence		
3. solution		
4. solving the equation		

- Complete this sentence. When the _____ in an open sentence is replaced with a number, you can determine whether the sentence is true or false.
- Consider $x - 4 = 6$. Find a value for x that makes the sentence true and another value that makes it false.

Helping You Remember

- Explain how an open sentence is different from an algebraic expression.

1-5 Enrichment**Solution Sets**

Consider the following open sentence.

It is a robot that starred in STAR WARS.

You know that a replacement for the word *It* must be found in order to determine if the sentence is true or false. If *It* is replaced by either *R2D2* or *C3PO*, the sentence is true.

The set $\{R2D2, C3PO\}$ can be thought of as the *solution set* of the open sentence given above. This set includes all replacements for the word that make the sentence true.

Write the solution set of each open sentence.

1. It is the name of a state beginning with the letter A.
2. It is a primary color.
3. Its capital is Harrisburg.
4. It is a New England state.
5. He was one of the Seven Dwarfs.
6. It is the name of a month that contains the letter q.
7. During the 1990s, she was the wife of a U.S. President.
8. It is an even number between 1 and 13.
9. $x + 4 = 10$
10. $31 = 72 - k$
11. It is the square of 2, 3, or 4.

Write a description of each set.

12. $\{A, E, I, O, U\}$
13. $\{1, 3, 5, 7, 9\}$
14. $\{\text{June, July, August}\}$
15. $\{\text{Atlantic, Pacific, Indian, Arctic}\}$

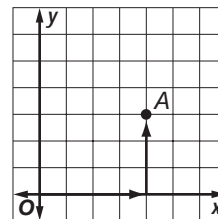
1-6 Study Guide and Intervention

Ordered Pairs and Relations

In mathematics, a **coordinate system** is used to locate points. The horizontal number line is called the **x-axis** and the vertical number line is called the **y-axis**. The point where the two axes intersect is the **origin** (0, 0). An **ordered pair** of numbers is used to locate points in the coordinate plane. The point (4, 3) has an **x-coordinate** of 4 and a **y-coordinate** of 3.

Example 1 Graph A(4, 3) on the coordinate system.

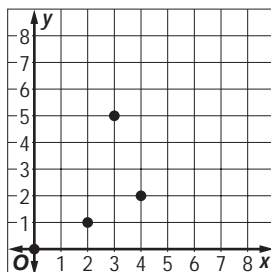
- Step 1** Start at the origin.
- Step 2** Since the x-coordinate is 4, move 4 units to the right.
- Step 3** Since the y-coordinate is 3, move 3 units up. Draw a dot.



A set of ordered pairs is called a **relation**. The set of x-coordinates is called the **domain**. The set of y-coordinates is called the **range**.

Example 2 Express the relation $\{(0, 0), (2, 1), (4, 2), (3, 5)\}$ as a table and as a graph. Then determine the domain and range.

x	y
0	0
2	1
4	2
3	5

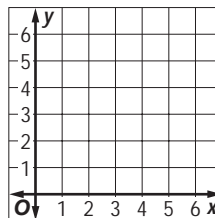


The domain is {0, 2, 4, 3}, and the range is {0, 1, 2, 5}.

Exercises

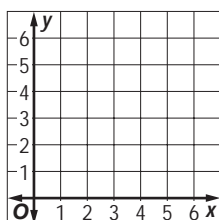
Graph each point on the coordinate system.

- A(4,1)
- B(2,0)
- C(1,3)
- D(5,2)
- E(0,3)
- F(6,4)



7. Express the relation $\{(4,6), (0,3), (1,4)\}$ as a table and a graph. Then determine the domain and range.

x	y



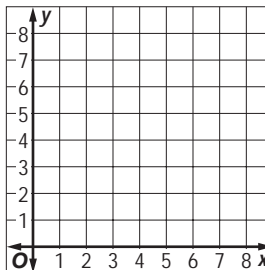
1-6

Practice

Ordered Pairs and Relations

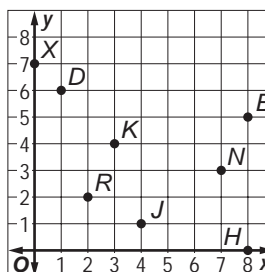
Graph each point on the coordinate system.

- 1. $Q(4, 2)$
- 2. $V(3, 7)$
- 3. $T(0, 3)$
- 4. $B(8, 6)$
- 5. $R(5, 0)$
- 6. $L(4, 4)$



Write the ordered pair that names each point.

- 7. J
- 8. X
- 9. R
- 10. B
- 11. K
- 12. H
- 13. D
- 14. N

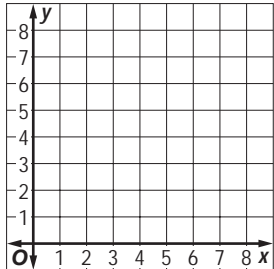


Express each relation as a table and as a graph. Then determine the domain and range.

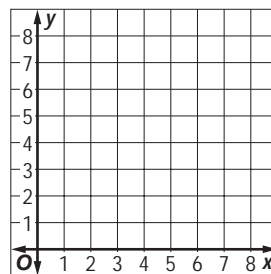
15. $\{(3, 7), (1, 1), (6, 5), (2, 4)\}$

16. $\{(0, 2), (4, 6), (3, 7)\}$

x	y

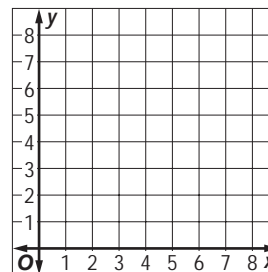


x	y



17. **GEOMETRY** Graph $(2, 1)$, $(2, 4)$, and $(5, 1)$ on the coordinate system.

- a. Connect the points with line segments. What figure is formed?
- b. Multiply each number in the set of ordered pairs by 2. Graph and connect the new ordered pairs. What figure is formed?
- c. Compare the two figures you drew. Write a sentence that tells how the figures are the same and how they are different.



1-6

Reading to Learn Mathematics***Ordered Pairs and Relations*****Pre-Activity** *How are ordered pairs used to graph real-life data?***Do the activity at the top of page 33 in your textbook. Write your answers below.**

- a. Where should Maria place an X now? Explain your reasoning.

- b. Suppose $(1, 2)$ represents 1 over and 2 up. How could you represent 3 over and 2 up?

- c. How are $(5, 1)$ and $(1, 5)$ different?

- d. Where is a good place to put the next O?

- e. Work with a partner to finish the game.

Reading the Lesson**Write a definition and give an example of each new vocabulary word or phrase.**

Vocabulary	Definition	Example
1. coordinate system		
2. y -axis		
3. coordinate plane		
4. origin		
5. x -axis		
6. ordered pair		
7. x -coordinate		
8. y -coordinate		
9. graph		
10. relation		
11. domain		
12. range		

1-6 Enrichment

The Hidden Animal

Graph the following sets of points. Join successive points by a line segment. Begin a new line segment with each numbered set of ordered pairs. When you finish, you will have a picture of an animal.

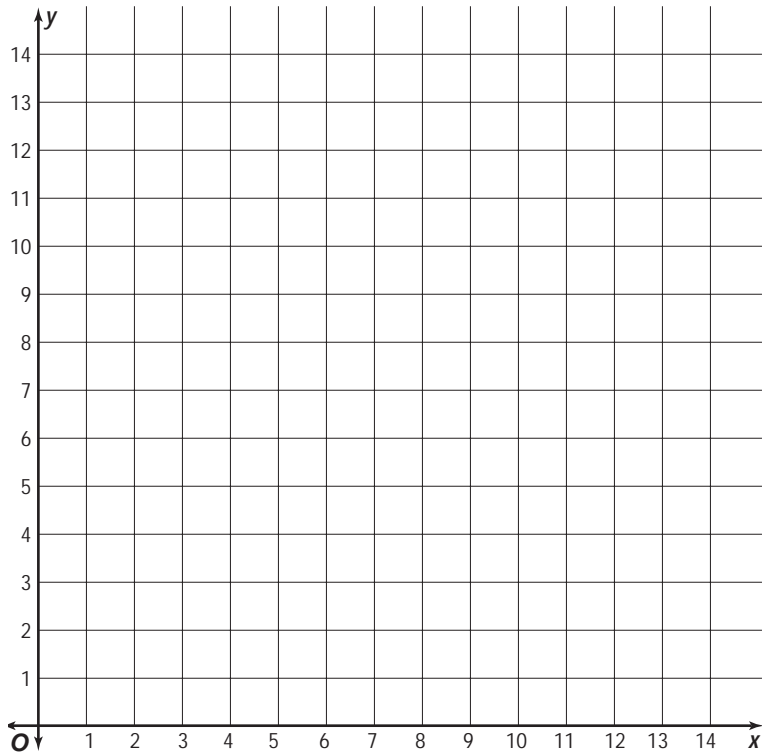
1. $(10, 12), (8, 12), (2, 11\frac{1}{2}), (\frac{1}{2}, 10),$
 $(\frac{1}{2}, 5)$

2. $(\frac{1}{2}, 8), (1, 6\frac{1}{2}), (1, 5\frac{1}{2}), (1\frac{1}{2}, 3)$
 $(1\frac{1}{2}, 1), (4, 1), (3\frac{1}{2}, 2), (3\frac{1}{2}, 5\frac{1}{2})$

3. $(8\frac{1}{2}, 6\frac{1}{2}), (8, 6), (8\frac{1}{2}, 4), (8\frac{1}{2}, 1),$
 $(10\frac{1}{2}, 1), (10, 2), (10, 6)$

4. $(10, 5), (12, 3), (12, 1\frac{1}{2}), (10\frac{1}{2}, 1\frac{1}{2}),$
 $(10\frac{1}{2}, 2\frac{1}{2}), (10, 3)$

5. $(8, 6), (6, 5\frac{1}{2}), (3, 5\frac{1}{2}), (3, 2),$
 $(3\frac{1}{2}, 1)$



6. $(12, 12), (11, 12\frac{1}{2}), (9, 11\frac{1}{2}), (8\frac{1}{2}, 10\frac{1}{2}), (8\frac{1}{2}, 9\frac{1}{2}), (9, 8\frac{1}{2}), (10, 8), (11, 9)$

7. $(12, 12), (13, 12), (14, 11), (14, 9), (13\frac{1}{2}, 8), (13\frac{1}{2}, 7), (13, 3), (12\frac{1}{2}, 2), (12, 2\frac{1}{2}),$
 $(12\frac{1}{2}, 3), (12\frac{1}{2}, 6), (12, 7), (11, 7\frac{1}{2}), (10\frac{1}{2}, 7), (10, 6)$

8. Suppose you multiply both coordinates of each ordered pair by 2 and graph the resulting pairs on graph paper using the same scale on the axes as for the drawing above. How would the drawings compare?

1-7

Study Guide and Intervention

Scatter Plots

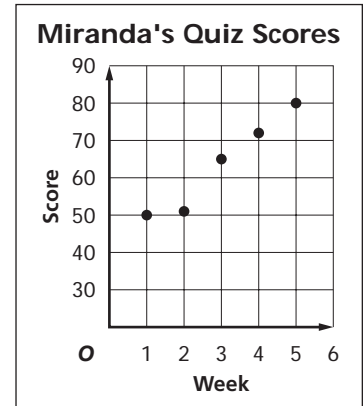
A **scatter plot** is a graph that shows the relationship between two sets of data. In a scatter plot two sets of data are graphed as ordered pairs on a coordinate system. A scatter plot may show a pattern or relationship of the data. The relation may be positive or negative, or there may be no relationship.

Example

SCHOOL The table shows Miranda's math quiz scores for the last five weeks. Make a scatter plot of the data.

Since the points are showing an upward trend from left to right, the data suggest a positive relationship.

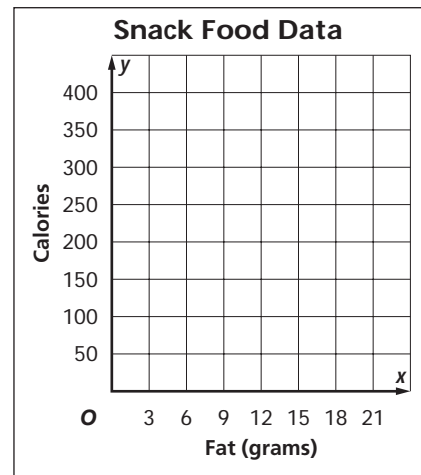
Week	Score
1	50
2	51
3	65
4	72
5	80



Exercises

FOOD For Exercises 1–3, use the table below which shows the fat grams and calories for several snack foods.

Food	Fat grams per serving	Calories per serving
doughnut	13	306
corn chips	13	200
pudding	3	150
cake	13	230
snack crackers	6	140
ice cream (light)	5	130
yogurt	2	70
cheese pizza	18	410

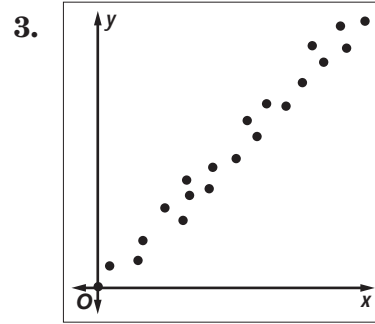
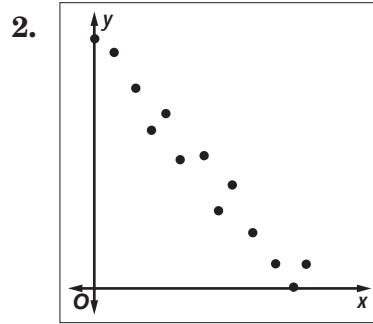
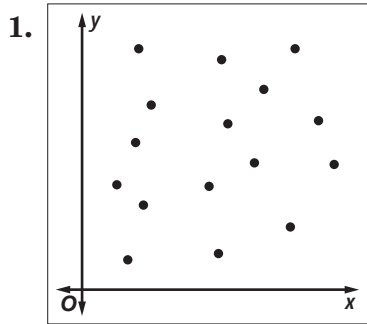


1. Make a scatter plot of the data in the table.
2. What do the x -coordinates represent? y -coordinates?
3. Is there a relationship between fat and calories? Explain.

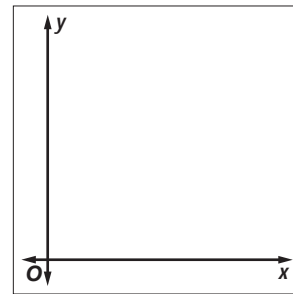
1-7 Skills Practice

Scatter Plots

Tell whether each scatter plot shows a *positive*, *negative*, or *no* relationship.



4. Draw a scatter plot with six ordered pairs that shows a positive relationship. Explain your reasoning.

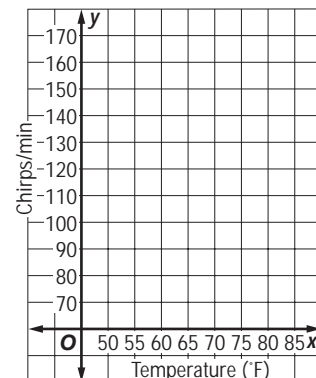


For Exercises 5–8, use the following information:

SCIENCE Scientists have determined that there may be a relationship between temperature and the number of chirps produced by crickets. The table gives the temperature and the number of chirps per minute for several cricket samples.

Temperature (°F)	Chirps/min
71	138
68	97
75	152
80	158
60	81
75	155
84	165

- Make a scatter plot of the data.
- Does there appear to be a relationship between temperature and chirps? Explain.
- Suppose the outside temperature is 65°. About how many chirps per minute would you expect from a cricket?
- Suppose the outside temperature is 55°. About how many chirps per minute would you expect from a cricket?



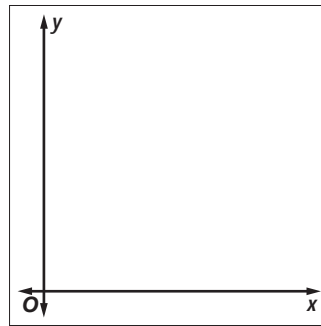
1-7

Practice

Scatter Plots

Determine whether a scatter plot of the data for the following might show a *positive*, *negative*, or *no* relationship.

1. a person's jogging speed and time spent jogging
2. the size of a family and the weekly grocery bill
3. the size of a car and the cost
4. a person's weight and percent body fat
5. time spent playing video games and time spent on outdoor activity
6. Draw a scatter plot with ten ordered pairs that shows a negative relationship.

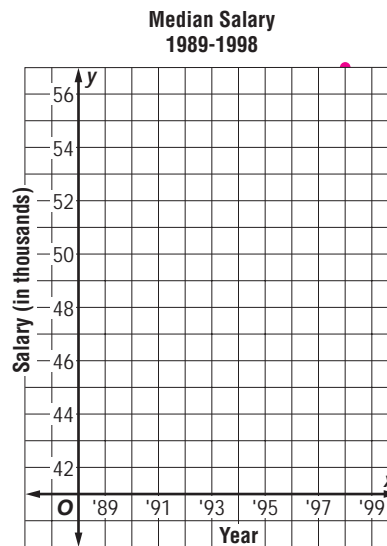


EMPLOYMENT For Exercises 7–9, use the table below, which shows the median salary for women who had completed a college degree for the years 1989–1998.

Year	Salary in Thousands of Dollars
1989	42
1990	43
1991	45
1992	46
1993	48
1994	49
1995	50
1996	51
1997	53
1998	57

Source: *Time Almanac*

7. Make a scatter plot of the data.



8. Does there appear to be a relationship between year and salary?
9. Based on the graph, predict what the median salary was for the year 2000.

1-7

Reading to Learn Mathematics

Scatter Plots

Pre-Activity *How can scatter plots help spot trends?*

Do the activity at the top of page 40 in your textbook. Write your answers below.

- a. What appears to be the trend in sales of movies on videocassette?

- b. Estimate the number of movies on videocassette sold for 2003.

Reading the Lesson

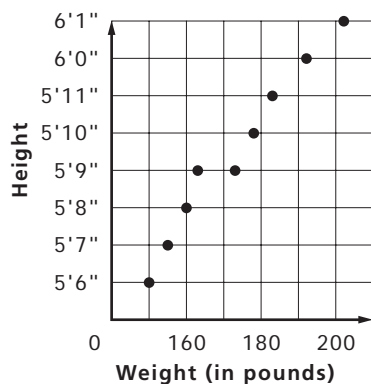
Write a definition and give an example of the new vocabulary term.

	Vocabulary	Definition	Example
1.	scatter plot		

Helping You Remember

2. Scatter plots are used to show relationships.
 - a. For a positive relationship, as x increases, y _____.
 - b. For a negative relationship, as x increases, y _____.
3. The scatter plot compares the weights and heights of the players on a high school football team.

Heights of Football Players



- a. What type of relationship exists, if any?

- b. Based on the scatter plot, predict the weight of a 5' 5" player who decided to join the team.

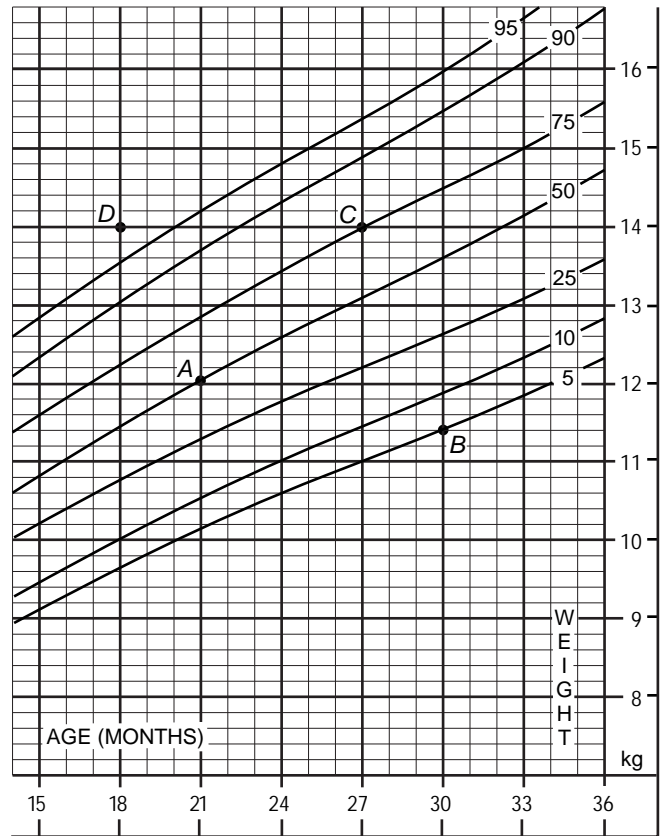
1-7 Enrichment

Growth Charts

Scatter plots are often used by doctors to show parents the growth rates of their children. The horizontal scale of the chart at the right shows the ages from 15 to 36 months. The vertical scale shows weight in kilograms. One kilogram is about 2.2 pounds. The curved lines are used to show how a child's weight compares with others of his or her age.

Look at the point labeled *A*. It represents a 21-month-old who weighs 12 kilograms. It is located on the slanted line labeled 50. This means the child's weight is in the "50th percentile." In other words, 50% of all 21-month-olds weigh more than 12 kilograms and 50% weigh less than 12 kilograms.

The location of Point *B* indicates that a 30-month-old who weighs 11.4 kilograms is in the 5th percentile. Only 5% of 30-month-old children will weigh less than 11.4 kilograms.



Solve.

- Look at the point labeled *C*. How much does the child weigh? How old is he? What percent of children his age will weigh more than he does?
- Look at the point labeled *D*. What is the child's age and weight? What percent of children her age will weigh more than she does?
- What is the 50th percentile weight for a child 27 months old?
- How much weight would child *B* have to gain to be in the 50th percentile?
- If child *D* did not gain any weight for four months, what percentile would he be in?
- How much heavier is a $2\frac{1}{2}$ -year-old in the 90th percentile than one in the 10th percentile?