# AP Physics - Free Fall

Aristotle (384 – 322 BC), one of your basic ancient Greek philosophers, said that things fall because they want to regain their natural state - earth with earth, water with water, and so on. Thus a rock will fall back to the earth to be with the other rocks. Since a big rock possesses more "earth", it will fall faster than would, say, a feather (which is woefully inadequate in the earth amount category compared with your basic rock). Aristotle's idea appears to be true because a rock certainly falls faster than a feather. In fact it made so much sense, that Aristotle's ideas on the subject were the accepted truth for around 2 000 years until the Renaissance.

The first scientific study of gravity was done by Galileo Galilei (1564 - 1642). He was trained as a mathematician and was a university professor. In the late 1500's Galileo conducted a series of experiments on gravity. He is supposed to have demonstrated that heavy objects and light objects fall at the same speed. The act of doing experiments to find out what would happen – this was a very daring idea.

Here is Galileo's account of the experiment from his book, *Dialogues of two New Sciences*.

> "But I, Simplicio, who have made the test can assure you that a cannon ball weighing one or two hundred pounds or even more, will not reach the ground by as much as a span ahead of a musket ball weighing only half a pound, provided both are dropped from a height of 200 cubits...the larger outstrips the smaller by two finger-breadths, that is, when the larger has reached the ground, the other is short of it by two finger-breadths.

Galileo did not, as is popularly believed, state that the objects would hit the ground at the same time – he understood air resistance. He did understand that without air resistance, the objects ***would*** fall at exactly the same rate.

Galileo wrote about doing the experiment as if he had done it several times, but it is not clear where or when he did it. The story that he dropped cannon balls from the Leaning Tower of Pisa has only one source, his last pupil and biographer, Vincenzo Vivani. He describes a very public event -- the entire university in attendance to witness the thing. But no one at the university ever mentioned witnessing the event. So whether Galileo did or did not do the experiment is sort of up in the air.

Galileo's idea that things fall at the same rate flies in the face of common sense. It seems reasonable that heavy things ought to fall faster than light ones

To study gravity, Galileo found that he had to slow it down. This was because he couldn't measure the time it took an object to fall with the crude instruments of the time. Gravity was "slowed down" by having balls roll down inclined planes (ramps). Gravity still caused the motion, but its effect was decreased to the point where Galileo could gather useful data. Galileo found that the distance that accelerated objects would travel was proportional to the square of the time. More on this later.

***Acceleration of Gravity:*** On the earth, gravity exerts a force on everything with mass. (A ***force*** is a push or pull.) The force makes all objects accelerate downwards, towards the center of the earth. This acceleration varies a tiny little bit depending on where you are - at the North Pole this acceleration is $9.83217 \text{ m/s}^2$ and at the Equator it has a value of $9.78039 \text{ m/s}^2$. This is because the earth is not a perfect sphere. Fortunately we can safely ignore the tiny differences in the acceleration of gravity. The value which is commonly used for this acceleration is 9.80 $m/s^2$. In English units it is 32.0 $ft/s^2$. Gravity's acceleration is kind of special so it is given its very own little symbol, ***g***.

$$g = \quad 9.80 \, \frac{m}{s^2}$$

Drop a rock from the top of a cliff and, in one second, it will reach a speed of 9.80 m/s, after two seconds it will be traveling at 19.6 m/s, in three seconds it's going 29.4 m/s, at four seconds it's speed will be up to 39.2 m/s, and so on. It looks like the rock will keep going faster and faster and faster until it smashes into the earth, and it would, if it were falling in a vacuum. The thing is, see, that the air causes a frictional force that opposes the rock's fall and slows it down. For short drops with dense objects (like rocks) we can reasonably ignore the effects of the air. Oh, the fancy, scientific term for this force exerted by the air is ***drag*** or ***air resistance***, sometimes it is called ***wind resistance***. At high velocities or over long distances, the drag can become significant, especially for objects that are not dense, like feathers or leaves or fluff. In the real world, an object in free fall will accelerate to its ***terminal velocity***. This is the speed at which the force of gravity equals the drag force. The object then stops accelerating and falls at a constant velocity. People jumping out of airplanes experience this. The typical laid out position that sky divers use gives them a terminal velocity of around 100 mph.

When an object is released and allowed to fall, its motion can be described by the following table (ignoring air resistance):

| Time | Velocity | Distance |
|---|---|---|
| 1 s | 9.8 m/s | 4.6 m |
| 2 s | 19.6 m/s | 19.6 m |
| 3 s | 29.4 m/s | 44.1 m |
| 4s | 39.2 m/s | 78.4 m |
| 5s | 49.0 m/s | 122 m |
| &tc | | |

The kinematic acceleration equations can be used to describe the motion of falling objects.

- A ball is thrown straight upward. If it takes 4.25 seconds to reach the top of its path, what is its initial speed?

Since the ball is traveling upward, and the acceleration is downward, the ball will slow down as it moves up. For the upward part of its motion, its final velocity will be zero – it will then momentarily come to rest and then change direction and begin to accelerate downward. Since we know that for the upward part of its journey the final velocity is zero, we can easily calculate the initial velocity.

$$\cancel{v} = v_0 + at \quad v_0 = -at \quad = -\left(-9.80\frac{m}{s^{\cancel{2}}}\right)(4.25\,\cancel{s}) \quad = \quad \boxed{41.6\frac{m}{s}}$$

The velocity and acceleration have opposite directions, so one of the quantities must be negative. We've chosen "down" as the negative direction for the above solution (but hey, you could choose up to be negative if you like).

- A stone is thrown straight up from top of building with an initial speed of 35.5 m/s. (a) How high does it go from the top of the building? (b) How much time to reach the maximum height? (c) If the building is 45.2 m tall, how much time will it take to hit the ground from when it was initially launched?

(a) $v^2 = v_o^2 + 2ay$ $v$ at top is zero so; $0 = v_o^2 + 2ay$

$$2ay = -v_o^2 \qquad y = \frac{-v_o^2}{2a} \quad = -\left(35.5\frac{m}{\cancel{s}}\right)^2\left(\frac{1}{2\left(-9.80\frac{\cancel{m}}{\cancel{s}^2}\right)}\right) \quad = \quad \boxed{64.3\,m}$$

(b) $v = v_o + at$ $0 = v_o + at$ $t = \frac{-v_o}{a}$

$$t = -35.5\frac{\cancel{m}}{\cancel{s}}\left(\frac{1}{-9.80\frac{\cancel{m}}{s^{\cancel{2}}}}\right) \quad = \quad \boxed{3.62\,s}$$

(c) ***The stone takes 3.62 s to reach the highest point in its path, it then must fall 64.3 m (to the top of the building) and then another 45.2 m to hit the deck below. So figure the problem from the top of the ball's path, where its velocity is zero and just before it begins to fall back down.***

***It's initial velocity is zero, and, since the stone will be falling down, we can, what the heck, assume that down is positive (we can do this! We are in charge!):***

$$y = \frac{1}{2}at^2 \qquad t = \sqrt{\frac{2y}{a}} \qquad t = \sqrt{\frac{2(64.3\,\cancel{m} + 45.2\,\cancel{m})}{9.80\frac{\cancel{m}}{s^2}}} \quad = \quad 4.73\,s$$

***The total time for the ball to be in the air is*** $3.62\,s + 4.73\,s = \boxed{8.35\,s}$

***Negative or Positive:*** You get to select the coordinate system that you use to solve problems. This means you get to decide where the displacement is zero and what direction will be positive or negative. Look at what happens if you have a negative acceleration, such as **– 9.8** $m/s^2$. Does this mean the object is decelerating (slowing) or does it mean that the object is moving along a negative (perhaps the ***y***) axis? It would depend on the problem. For an object moving on the ***x***-axis it would mean decelerating. For an object falling along the ***y***-axis, due to gravity, it means the object is accelerating, but in the downward direction. You choses you your directions for this stuff basically so that the calculations and everything are easiest.

Dear Doctor Science,
Why does a Kleenex, when dropped over a waste basket, always end up on the floor instead of the bottom of the basket?
-- Kevin Gustafson, M.D. from Minneapolis, MN

Dr. Science responds:
*Kleenex is a brand name facial tissue, enjoying relatively high status compared to generic brands and lowly toilet paper. So the haughty tissue considers life in a waste basket, even after use, repugnant. Thus, you have two options if you want to avoid picking up tissues from around the basket. Either purchase a lower grade of facial tissue, or get a nicer waste basket. I've seen some copper lined ones with a tooled leather exterior that would attract even the top of the tissue pecking order.*

Dear Cecil:
**Is it true cats always land unharmed on their feet, no matter how far they fall?**
--A D DOO, via America Online

Cecil replies:
I love this question. I love it because (1) it seems completely wild, (2) it nonetheless appears to have some scientific basis, (3) on examination the scientific basis is open to serious question, and--this is the best part--(4) the Teeming Millions figured this all out by themselves. I may be able to retire from this job yet.

Here's the EP version of the story you heard, related to me by AOL user Bmaffitt: "There was a Discovery Channel special on this a while back. The truth is, after a few floors it doesn't really matter [how far the cat falls], as long as the oxygen holds out. Cats have a nonfatal terminal velocity (sounds like a contradiction in terms, but most small animals have this advantage). Once they orient themselves, they spread out like a parachute. There are cats on record that have fallen 20 stories or more without ill effects. As long as the cat doesn't land on something pointy, it's likely to walk away." You're thinking: no freaking way. But the believers trot out a 1987 study from the *Journal of the American Veterinary Medical Association*. Two vets examined 132 cases of cats that had fallen out of high-rise windows and were brought to the Animal Medical Center, a New York veterinary hospital, for treatment. On average the cats fell 5.5 stories, yet 90 percent survived. (Many did suffer serious injuries.)

Well, we know cats have exceptional coordination and balance, so maybe that contributed to the high survival rate. One cat, for example, is known to have survived a 46-story fall. (It apparently bounced off a canopy and into a planter.)

But here's the weird part. When the vets analyzed the data they found that, as one would expect, the number of broken bones and other injuries increased with the number of stories the cat had fallen--up to seven stories. Above seven stories, however, the number of injuries per cat sharply declined. In other words, the farther the cat fell, the better its chances of escaping serious injury.
The authors explained this seemingly miraculous result by saying that after falling five stories or so the cats reached a terminal velocity--that is, maximum downward speed--of 60 miles per hour. Thereafter, they hypothesized, the cats relaxed and spread themselves out like flying squirrels, minimizing injuries. This speculation is now widely accepted as fact.

But there's a potential fatal flaw in this argument, which emerged from a discussion on--I can't suppress a grin--alt.fan.cecil-adams on the Usenet. (In fairness, the objection may have originally been raised on alt.folklore.urban.)

The potential flaw is this: the study was based only on cats that were brought into the hospital. Clearly dead cats, your basic fell-20-stories-and-looks-like-it-came-out-of-a-can-of-Spam cats, go to the Dumpster, not the emergency room. This may skew the statistics and make falls from great distances look safer than they are.

I called the Animal Medical Center to see if this possibility had been considered. The original authors were long gone, so I spoke to Dr. Michael Garvey, head of the medical department and current expert on "high-rise syndrome."

Dr. Garvey was adamant that the omission of nonreported fatalities didn't skew the statistics. He pointed out that cats that had fallen from great heights typically had injuries suggesting they'd landed on their chests, which supports the "flying squirrel" hypothesis.

I suggested this merely meant that a cat landing in this position had a chance of surviving long enough to be brought into the hospital, whereas cats landing in other positions were so manifestly dead that the hospital was never notified. Dr. Garvey didn't buy it, but said this was a matter about which reasonable people might disagree.

We await the formation of a committee of New York high-rise doormen to compile truly global statistics on the fate of falling cats. Meanwhile don't believe something just because it was on the Discovery Channel, or for that matter in the Straight Dope.

Dear Cecil:
**Back when I was a kid we used to take the cat up on the roof and toss it off. It was just a one-story house, so the cat didn't have far to fall. That little bugger would spread out his arms and legs and glide on down, just like a flying squirrel. He never seemed to mind it in the least. He'd let us drag him up there again and again. It seems they have a natural ability to protect themselves from falls. Now that's science!**
--Dave, via AOL

Cecil replies:
No, that's stupidity. I got another note telling about some moron who dropped (a) a cat and (b) a chicken out of a Cessna at 800 feet to see what would happen. The cat survived. The chicken didn't. While that might seem to validate the flying-squirrel hypothesis, what it really tells me is that the teenage sadists of the world have gotten the idea that cats are immortal, so anything goes. Nonsense.

Let's review the facts:

1. Nobody says that cats will survive any fall *uninjured.* Of the 132 cats brought to New York's Animal Medical Center after accidental falls, two-thirds required treatment, and half of that number required lifesaving treatment.

2. The flying-squirrel hypothesis may well explain why *some* cats survive extremely long falls. No one has demonstrated that *all* cats will survive long falls. On the contrary, from anecdotal accounts we know that at least some cats are killed--the deaths just aren't reported.

Cecil's assistant Little Ed got into a big online argument with a young fellow who was enamored of the flying-squirrel hypothesis. After Little Ed patiently explained the difference between *some* and *all,* the young fellow conceded Cecil was right to make point number two above. "But so what if Cecil was right?" the young fellow said by way of a parting shot. (I'm paraphrasing here.) "Cecil's point was boring. The flying-squirrel hypothesis is *interesting."*

OK, fine, it's interesting. The ditz pitching the kitty out of the Cessna thought *that* was interesting. Just keep your hands off that cat.
--CECIL ADAMS

Dear sir I write this note to you to tell you of my plight,
For at the time of writing it, I'm not a pretty sight,
My body is all black & blue, my face a deathly grey,
And I write this note to say why I am not at work today.

While working on the 14th floor some bricks I had to clear,
But tossing them down from such a height, was not a good idea,
The foreman wasn't very pleased, he is an awkward sod,
and he said I had to cart them down the ladders in me hod.

Well clearing all these bricks by hand, it was so very slow,
So I hoisted up a barrel and secured a rope below.
But in me haste to do the job, I was too blind to see,
That a barrel full of building bricks was heavier than me.

And so when I untied the rope, the barrel fell like lead,
And clinging tightly to the rope, I started up instead.
I shot up like a rocket, and to my dismay I found
That halfway up I met the bloody barrel coming down.

Well, the barrel broke me shoulder as to the ground it sped,
And when I reached the top, I banged the pulley with me head.
But I clung on tightly, numb with shock, from this almighty blow,
While the barrel spilled out half its bricks some fourteen floors below.

Now when these bricks had fallen from the barrel to the floor,
I then outweighed the barrel & so started down once more.
But I clung on tightly to the rope, me body wracked with pain,
And halfway down I met the bloody barrel once again.

The force of this collision halfway down the office block,
Caused multiple abrasions and a nasty case of shock,
But I clung on tightly to the rope as I fell towards the ground,
And I landed on the broken bricks the barrel had scattered round.

Well as I lay there on the floor I thought I'd passed the worst,
But the barrel hit the pulley wheel & then the bottom burst.
A shower of bricks rained down on me; I didn't have a hope.
As I lay there bleeding on the ground I let go the bloody rope.

The barrel now being heavier, it started down once more.
It landed right across me as I lay there on the floor.
It broke three ribs and my left arm, and I can only say,
"I hope you'll understand why I am not at work today."

---- anonymous