1-Sample Confidence Intervals—Student Notes

A **confidence interval** is an interval that is computed from sample data and provides a range of plausible values for a population parameter.
A **confidence level** is a number that provides information on how much "confidence" we have in the method used to construct a confidence interval estimate. This level specifies the percentage of all possible samples that produce an interval containing the true value of the population parameter.

Constructing a Confidence Interval

The steps below should be followed when asked to calculate a confidence interval.

1. Identify the population of interest and define the parameter of interest being estimated.
2. Identify the appropriate confidence interval by name or formula.
3. Verify any conditions (assumptions) that need to be met for that confidence interval.
4. Calculate the confidence interval
5. Interpret the interval in the context of the situation.

- The general formula for a confidence interval calculation is:

$$statistic \pm (critical\ value)(std\ dev\ of\ statistic)$$

- The critical value is determined by the confidence level.
- The type of statistic is determined by the situation in the problem. There are two types of statistics that we will discuss…means and proportions.

Confidence Interval for Proportions (1-prop z-int)

Step 1: We are finding an interval that describes the population proportion (p).
Step 2: The general formula uses the sample proportion ($\hat{p}$) and the sample size (n).

$$statistic \pm (critical\ value)(std\ dev\ of\ statistic)$$

$$\hat{p} \pm (z^*)(\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Step 3: The conditions that need to be met for this interval are:

1. The sample is a simple random sample.
2. The population is large relative to the sample ($N \geq 10n$).
3. The sampling distribution of the sample proportion is approximately normal ($np \geq 10$ and $n(1-p) \geq 10$).

Step 4: Complete the calculations for the interval.
Step 5: Write the answer in the context of the original problem.

Sample Question for 1-sample proportion Confidence Intervals

The owner of a popular chain of restaurants wishes to know if completed dishes are being delivered to the customer's table within one minute of being completed by the chef. A random sample of 75 completed dishes found that 60 were delivered within one minute of completion. Find the 95% confidence interval for the true population proportion.

Confidence Interval for Means (unknown population standard deviation)(t-int)

Step 1: We are finding an interval that describes the population mean (μ).

Step 2: The general formula uses the sample mean ($\bar{x}$), the sample standard deviation (s), the sample size (n), and the degrees of freedom (n-1).

$$statistic \pm (critical\ value)(std\ dev\ of\ statistic)$$

$$\bar{x} \pm (t^*)(\frac{s}{\sqrt{n}})$$

Step 3: The conditions that need to be met for this interval are:

1. The sample is a simple random sample.
2. The population is large relative to the sample ($N \geq 10n$).
3. The sampling distribution of the sample mean is approximately normal (Central Limit Theorem... $n \geq 30$ or the problem states that the population is normally distributed).

Step 4: Complete the calculations for the interval.

Step 5: Write the answer in the context of the original problem.

Sample Question for 1-sample means Confidence Intervals

A biology student at a major university is writing a report about bird watchers. She has developed a test that will score the abilities of a bird watcher to identify common birds. She collects data from a random sample of people that classify themselves as bird watchers (data shown below). Find a 90% confidence interval for the mean score of the population of bird watchers.

4.5	9.1	8	5.9	7.0	5.2	7.3	7.0	6.6	5.1
7.6	8.2	6.4	4.8	5.8	6.2	8.5	7.3	7.8	7.4

Confidence Interval Type	Formula	Conditions	Calculator Test
1-sample proportions	$\hat{p} \pm (z^*)(\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	1. The sample is a simple random sample. 2. The population is large relative to the sample ($N \geq 10n$). 3. The sampling distribution of the sample proportion is approximately normal ($np \geq 10$ and $n(1-p) \geq 10$).	1-prop z-interval
1-sample means	$\bar{x} \pm (t^*)(\frac{s}{\sqrt{n}})$	1. The sample is a simple random sample. 2. The population is large relative to the sample ($N \geq 10n$). 3. The sampling distribution of the sample mean is approximately normal (Central Limit Theorem, $n \geq 30$ or the problem states that the population is normally distributed).	t-interval

Other Confidence Interval Topics

- Margin of Error (ME)—the product of the critical value and the standard deviation of the statistic (also called the standard error of the statistic). It is the plus or minus part of the confidence interval.
- Some problems might ask you to determine the sample size required to give a specific margin of error. This requires a little algebra to work backward through the equation. The equations are listed in the chart below.

Confidence Interval Type	Formula for finding the sample size within a given ME
1-sample proportion (1-prop z-interval)	$n = \hat{p}(1-\hat{p})(\frac{z^*}{ME})^2$ Note: If $\hat{p}$ is not given in the problem, let $\hat{p} = 0.5$.
1-sample mean (t-interval)	$n = \left(\frac{t^* s}{ME}\right)^2$

- Keep in mind the effects of changing the confidence level. A large confidence level (say 99% as compared to 90%) produces a larger margin of error. To be more confident we must include more values in our range.
- Do not confuse the meaning of a confidence level with the interpretation of a confidence interval. A 95% confidence level means that if we repeated the sampling process many times, the resulting confidence interval would capture the true population parameter 95% of the time.

AP Multiple Choice Questions for Practice and Discussion

1. A random sample of 100 visitors to a popular theme park spent an average of $142 on the trip with a standard deviation of $47.50. Which of the following would represent the 98% confidence interval for the mean amount of money spent by all visitor to the theme park?
   a. ($130.77, $153.23)
   b. ($132.57, $151.43)
   c. ($132.69, $151.31)
   d. ($140.88, $143.12)
   e. ($95.45, $188.55)

2. How large of a random sample is required to insure that the margin of error is 0.08 when estimating the proportion of college professors that read science fiction novels with 95% confidence?
   a. 600
   b. 300
   c. 150
   d. 75
   e. 25

3. A quality control specialist at a plate glass factory must estimate the mean clarity rating of a new batch of glass sheets being produced using a sample of 18 sheets of glass. The actual distribution of this batch is unknown, but preliminary investigations show that a normal approximation is reasonable. The specialist decides to use a t-distribution rather than a z-distribution because
   a. The z-distribution is not appropriate because the sample is too small.
   b. The sample size is large compared to the population size.
   c. The data comes from only one batch.
   d. The variability of the batch is unknown.
   e. The t-distribution results in a narrower confidence interval.

4. The board of directors at a city zoo is considering using commercial fast food restaurants in their zoo rather than the current eateries. They are concerned that major donors to the zoo will not approve of the proposed change. Of the 280 major donors to the zoo, a random sample of 90 is asked "Do you support the zoo's decision to use commercial fast food restaurants in the zoo?" 50 of the donors said no, 38 said yes and 2 had no opinion on the matter. A large sample z-interval, $\hat{p} \pm (z^*)(\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, was constructed from these data to estimate the proportion of the major donors who support using commercial fast food restaurants in the zoo. Which of the following statements is correct for this confidence interval?
   a. This confidence interval is valid because a sample size of more than 30 was used.
   b. This confidence interval is valid because no conditions are required for constructing a large sample confidence interval for proportions.
   c. This confidence interval is not valid because the sample size is too large compared to the population size.
   d. This confidence interval is not valid because the quantity $n\hat{p}$ is too small.
   e. This confidence interval is not valid because "no opinion" was allowed as a response.

5. A research and development engineer is preparing a report for the board of directors on the battery life of a new cell phone they have produced. At a 95% confidence level, he has found that the battery life is 3.2 ± 1.0 days. He wants to adjust his findings so the margin of error is as small as possible. Which of the following will produce the smallest margin of error?
    a. Increase the confidence level to 100%. This will assure that there is no margin of error.
    b. Increase the confidence level to 99%.
    c. Decrease the confidence level to 90%.
    d. Take a new sample from the population using the exact same sample size.
    e. Take a new sample from the population using a smaller sample size.

6. A biologist has taken a random sample of a specific type of fish from a large lake. A 95% confidence interval was calculated to be 6.8 ± 1.2 pounds. Which of the following is true?
    a. 95% of all the fish in the lake weigh between 5.6 and 8 pounds.
    b. In repeated sampling, 95% of the sample proportions will fall within 5.6 and 8 pounds.
    c. In repeated sampling, 95% of the true population mean of fish weights will be equal to 6.8 pounds.
    d. In repeated sampling, 95% of the time the true population mean of fish weights will be captured in the constructed interval.
    e. We are 95% confident that all the fish weigh less than 8 pounds in this lake.

7. A polling company is trying to estimate the percentage of adults that consider themselves happy. A confidence interval based on a sample size of 360 has a larger than desired margin of error. The company wants to conduct another poll and obtain another confidence interval of the same level but reduce the error to 1/3 the size of the original sample. How many adults should they now interview?
    a. 40
    b. 180
    c. 720
    d. 1080
    e. 3240

8. A researcher is interested in determining the mean energy consumption of a new compact florescent light bulb. She takes a random sample of 41 bulbs and determines that the mean consumption is 1.3 watts per hour with a standard deviation of 0.7. When constructing a 97% confidence interval, which would be the most appropriate value of the critical value?
    a. 1.936
    b. 2.072
    c. 2.250
    d. 2.704
    e. 2.807

AP Free Response Questions for Practice and Discussion

Free Response Problem #1

A random sample of 9th grade math students was asked if they prefer working their math problems using a pencil or a pen. Of the 250 students surveyed, 100 preferred pencil and 150 preferred pen.

(a) Using the results of this survey, construct a 95% confidence interval for the proportion of 9th grade students that prefer to work their math problems in pen.

(b) A school newspaper reported on the results of this survey by saying, "Over half of 9th grade math students prefer to use pen on their math assignments." Is this statement supported by your confidence interval? Explain.

2000 #6 (Married Heights)

A random sample of 400 married couples was selected from a large population of married couples.

- Heights of married men are approximately normally distributed with mean 70 inches and standard deviation 3 inches.
- Heights of married women are approximately normally distributed with mean 65 inches and standard deviation 2.5 inches.
- There were 20 couples in which the wife was taller than her husband, and there were 380 couples in which the wife was shorter than her husband.

(a) Find a 95 percent confidence interval for the proportion of married couples in the population for which the wife is taller than her husband. Interpret your interval in the context of this question.