90 Minutes—No Calculator

Notes: (1) In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e).

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. The area of the region between the graph of $y = 4x^3 + 2$ and the x-axis from $x = 1$ to $x = 2$ is

(A) 36 (B) 23 (C) 20 (D) 17 (E) 9

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2. At what values of x does $f(x) = 3x^5 - 5x^3 + 15$ have a relative maximum?

(A) –1 only (B) 0 only (C) 1 only (D) –1 and 1 only (E) –1, 0 and 1

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3. $\int_1^2 \frac{x+1}{x^2+2x}\,dx =$

(A) $\ln 8 - \ln 3$ (B) $\frac{\ln 8 - \ln 3}{2}$ (C) $\ln 8$ (D) $\frac{3\ln 2}{2}$ (E) $\frac{3\ln 2 + 2}{2}$

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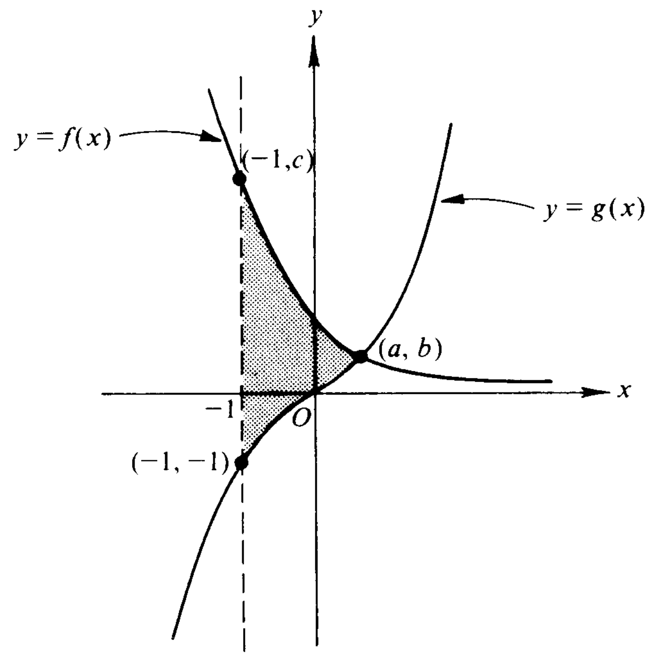
4. A particle moves in the xy-plane so that at any time t its coordinates are $x = t^2 - 1$ and $y = t^4 - 2t^3$. At $t = 1$, its acceleration vector is

(A) $(0,-1)$ (B) $(0,12)$ (C) $(2,-2)$ (D) $(2,0)$ (E) $(2,8)$



5. The curves $y = f(x)$ and $y = g(x)$ shown in the figure above intersect at the point (a, b). The area of the shaded region enclosed by these curves and the line $x = -1$ is given by

(A) $\int_0^a \left(f(x) - g(x)\right) dx + \int_{-1}^0 \left(f(x) + g(x)\right) dx$

(B) $\int_{-1}^b g(x)\,dx + \int_b^c f(x)\,dx$

(C) $\int_{-1}^c \left(f(x) - g(x)\right) dx$

(D) $\int_{-1}^a \left(f(x) - g(x)\right) dx$

(E) $\int_{-1}^a \left(|f(x)| - |g(x)|\right) dx$

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6. If $f(x) = \dfrac{x}{\tan x}$, then $f'\left(\dfrac{\pi}{4}\right) =$

(A) 2 (B) $\dfrac{1}{2}$ (C) $1 + \dfrac{\pi}{2}$ (D) $\dfrac{\pi}{2} - 1$ (E) $1 - \dfrac{\pi}{2}$

7. Which of the following is equal to $\int \frac{1}{\sqrt{25-x^2}}\,dx$?

(A) $\arcsin \frac{x}{5} + C$ (B) $\arcsin x + C$ (C) $\frac{1}{5}\arcsin \frac{x}{5} + C$

(D) $\sqrt{25-x^2} + C$ (E) $2\sqrt{25-x^2} + C$

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8. If f is a function such that $\lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = 0$, which of the following must be true?

(A) The limit of $f(x)$ as x approaches 2 does not exist.

(B) f is not defined at $x = 2$.

(C) The derivative of f at $x = 2$ is 0.

(D) f is continuous at $x = 0$.

(E) $f(2) = 0$

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9. If $xy^2 + 2xy = 8$, then, at the point $(1, 2)$, y' is

(A) $-\frac{5}{2}$ (B) $-\frac{4}{3}$ (C) -1 (D) $-\frac{1}{2}$ (E) 0

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10. For $-1 < x < 1$ if $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{2n-1}$, then $f'(x) =$

(A) $\sum_{n=1}^{\infty} (-1)^{n+1} x^{2n-2}$

(B) $\sum_{n=1}^{\infty} (-1)^{n} x^{2n-2}$

(C) $\sum_{n=1}^{\infty} (-1)^{2n} x^{2n}$

(D) $\sum_{n=1}^{\infty} (-1)^{n} x^{2n}$

(E) $\sum_{n=1}^{\infty} (-1)^{n+1} x^{2n}$

11. $\dfrac{d}{dx}\ln\left(\dfrac{1}{1-x}\right) =$

(A) $\dfrac{1}{1-x}$ (B) $\dfrac{1}{x-1}$ (C) $1-x$ (D) $x-1$ (E) $(1-x)^2$

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12. $\displaystyle\int \frac{dx}{(x-1)(x+2)} =$

(A) $\dfrac{1}{3}\ln\left|\dfrac{x-1}{x+2}\right| + C$ (B) $\dfrac{1}{3}\ln\left|\dfrac{x+2}{x-1}\right| + C$ (C) $\dfrac{1}{3}\ln\left|(x-1)(x+2)\right| + C$

(D) $\left(\ln|x-1|\right)\left(\ln|x+2|\right) + C$ (E) $\ln\left|(x-1)(x+2)^2\right| + C$

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13. Let f be the function given by $f(x) = x^3 - 3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0,3]$?

(A) 0 only (B) 2 only (C) 3 only (D) 0 and 3 (E) 2 and 3

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14. Which of the following series are convergent?

I. $1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \ldots + \dfrac{1}{n^2} + \ldots$

II. $1 + \dfrac{1}{2} + \dfrac{1}{3} + \ldots + \dfrac{1}{n} + \ldots$

III. $1 - \dfrac{1}{3} + \dfrac{1}{3^2} - \ldots + \dfrac{(-1)^{n+1}}{3^{n-1}} + \ldots$

(A) I only (B) III only (C) I and III only (D) II and III only (E) I, II, and III

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15. If the velocity of a particle moving along the x-axis is $v(t) = 2t - 4$ and if at $t = 0$ its position is 4, then at any time t its position $x(t)$ is

(A) $t^2 - 4t$ (B) $t^2 - 4t - 4$ (C) $t^2 - 4t + 4$ (D) $2t^2 - 4t$ (E) $2t^2 - 4t + 4$

16. Which of the following functions shows that the statement "If a function is continuous at $x = 0$, then it is differentiable at $x = 0$" is false?

(A) $f(x) = x^{-\frac{4}{3}}$ (B) $f(x) = x^{-\frac{1}{3}}$ (C) $f(x) = x^{\frac{1}{3}}$ (D) $f(x) = x^{\frac{4}{3}}$ (E) $f(x) = x^3$

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17. If $f(x) = x\ln\left(x^2\right)$, then $f'(x) =$

(A) $\ln\left(x^2\right)+1$ (B) $\ln\left(x^2\right)+2$ (C) $\ln\left(x^2\right)+\frac{1}{x}$ (D) $\frac{1}{x^2}$ (E) $\frac{1}{x}$

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18. $\int \sin(2x+3)\,dx =$

(A) $-2\cos(2x+3)+C$ (B) $-\cos(2x+3)+C$ (C) $-\frac{1}{2}\cos(2x+3)+C$

(D) $\frac{1}{2}\cos(2x+3)+C$ (E) $\cos(2x+3)+C$

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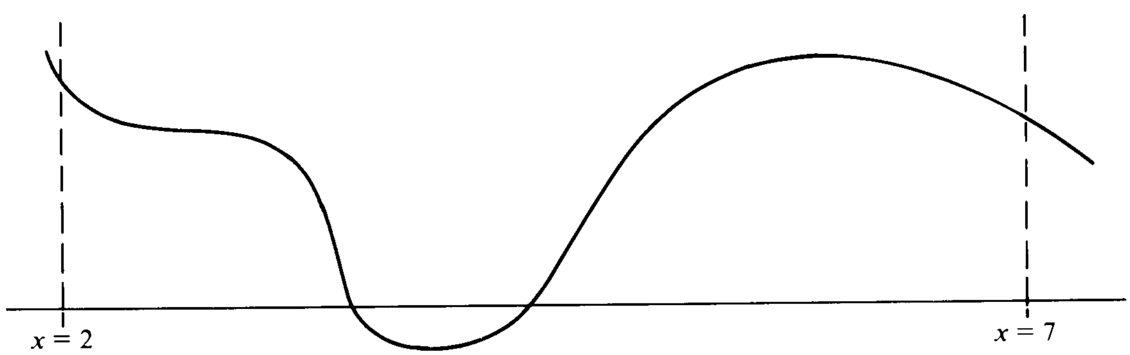
19. If f and g are twice differentiable functions such that $g(x) = e^{f(x)}$ and $g''(x) = h(x)e^{f(x)}$, then $h(x) =$

(A) $f'(x)+f''(x)$ (B) $f'(x)+\left(f''(x)\right)^2$ (C) $\left(f'(x)+f''(x)\right)^2$

(D) $\left(f'(x)\right)^2+f''(x)$ (E) $2f'(x)+f''(x)$



20. The graph of $y = f(x)$ on the closed interval $[2,7]$ is shown above. How many points of inflection does this graph have on this interval?

(A) One (B) Two (C) Three (D) Four (E) Five

21. If $\int f(x)\sin x\,dx = -f(x)\cos x + \int 3x^2 \cos x\,dx$, then $f(x)$ could be

(A) $3x^2$ (B) x^3 (C) $-x^3$ (D) $\sin x$ (E) $\cos x$

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22. The area of a circular region is increasing at a rate of 96π square meters per second. When the area of the region is 64π square meters, how fast, in meters per second, is the radius of the region increasing?

(A) 6 (B) 8 (C) 16 (D) $4\sqrt{3}$ (E) $12\sqrt{3}$

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23. $\lim\limits_{h\to 0} \dfrac{\int_1^{1+h} \sqrt{x^5+8}\,dx}{h}$ is

(A) 0 (B) 1 (C) 3 (D) $2\sqrt{2}$ (E) nonexistent

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24. The area of the region enclosed by the polar curve $r = \sin(2\theta)$ for $0 \le \theta \le \frac{\pi}{2}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\pi}{8}$ (E) $\frac{\pi}{4}$

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25. A particle moves along the x-axis so that at any time t its position is given by $x(t) = te^{-2t}$. For what values of t is the particle at rest?

(A) No values (B) 0 only (C) $\frac{1}{2}$ only (D) 1 only (E) 0 and $\frac{1}{2}$

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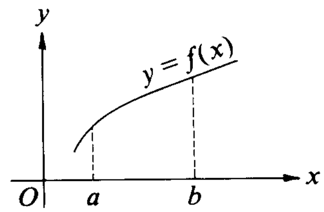
26. For $0 < x < \frac{\pi}{2}$, if $y = (\sin x)^x$, then $\frac{dy}{dx}$ is

(A) $x\ln(\sin x)$ (B) $(\sin x)^x \cot x$ (C) $x(\sin x)^{x-1}(\cos x)$

(D) $(\sin x)^x(x\cos x + \sin x)$ (E) $(\sin x)^x(x\cot x + \ln(\sin x))$



27. If f is the continuous, strictly increasing function on the interval $a \le x \le b$ as shown above, which of the following must be true?

I. $\int_a^b f(x)\,dx < f(b)(b-a)$

II. $\int_a^b f(x)\,dx > f(a)(b-a)$

III. $\int_a^b f(x)\,dx = f(c)(b-a)$ for some number c such that $a < c < b$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

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28. An antiderivative of $f(x) = e^{x+e^x}$ is

(A) $\dfrac{e^{x+e^x}}{1+e^x}$ (B) $\left(1+e^x\right)e^{x+e^x}$ (C) e^{1+e^x} (D) e^{x+e^x} (E) e^{e^x}

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29. $\displaystyle\lim_{x\to\frac{\pi}{4}} \frac{\sin\left(x-\frac{\pi}{4}\right)}{x-\frac{\pi}{4}}$ is

(A) 0 (B) $\dfrac{1}{\sqrt{2}}$ (C) $\dfrac{\pi}{4}$ (D) 1 (E) nonexistent

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30. If $x = t^3 - t$ and $y = \sqrt{3t+1}$, then $\dfrac{dy}{dx}$ at $t = 1$ is

(A) $\dfrac{1}{8}$ (B) $\dfrac{3}{8}$ (C) $\dfrac{3}{4}$ (D) $\dfrac{8}{3}$ (E) 8

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31. What are all values of x for which the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$ converges?

(A) $-1 \le x < 1$ (B) $-1 \le x \le 1$ (C) $0 < x < 2$ (D) $0 \le x < 2$ (E) $0 \le x \le 2$

32. An equation of the line <u>normal</u> to the graph of $y = x^3 + 3x^2 + 7x - 1$ at the point where $x = -1$ is

(A) $4x + y = -10$ (B) $x - 4y = 23$ (C) $4x - y = 2$ (D) $x + 4y = 25$ (E) $x + 4y = -25$

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33. If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

(A) $-\frac{\ln 2}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{\ln 2}{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) $\ln 2$

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34. Which of the following gives the area of the surface generated by revolving about the y-axis the arc of $x = y^3$ from $y = 0$ to $y = 1$?

(A) $2\pi\int_0^1 y^3\sqrt{1+9y^4}\,dy$

(B) $2\pi\int_0^1 y^3\sqrt{1+y^6}\,dy$

(C) $2\pi\int_0^1 y^3\sqrt{1+3y^2}\,dy$

(D) $2\pi\int_0^1 y\sqrt{1+9y^4}\,dy$

(E) $2\pi\int_0^1 y\sqrt{1+y^6}\,dy$

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35. The region in the first quadrant between the x-axis and the graph of $y = 6x - x^2$ is rotated around the y-axis. The volume of the resulting solid of revolution is given by

(A) $\int_0^6 \pi\left(6x - x^2\right)^2 dx$

(B) $\int_0^6 2\pi x\left(6x - x^2\right)dx$

(C) $\int_0^6 \pi x\left(6x - x^2\right)^2 dx$

(D) $\int_0^6 \pi\left(3 + \sqrt{9 - y}\right)^2 dy$

(E) $\int_0^9 \pi\left(3 + \sqrt{9 - y}\right)^2 dy$

36. $\int_{-1}^{1} \frac{3}{x^2}\,dx$ is

(A) –6 (B) –3 (C) 0 (D) 6 (E) nonexistent

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37. The general solution for the equation $\frac{dy}{dx} + y = xe^{-x}$ is

(A) $y = \frac{x^2}{2}e^{-x} + Ce^{-x}$

(B) $y = \frac{x^2}{2}e^{-x} + e^{-x} + C$

(C) $y = -e^{-x} + \frac{C}{1+x}$

(D) $y = xe^{-x} + Ce^{-x}$

(E) $y = C_1e^{x} + C_2xe^{-x}$

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38. $\lim_{x\to\infty} \left(1+5e^{x}\right)^{\frac{1}{x}}$ is

(A) 0 (B) 1 (C) e (D) e^5 (E) nonexistent

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39. The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the x-axis are squares, then its volume is

(A) $\frac{\left(1-e^{-6}\right)}{2}$ (B) $\frac{1}{2}e^{-6}$ (C) e^{-6} (D) e^{-3} (E) $1-e^{-3}$

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40. If the substitution $u = \frac{x}{2}$ is made, the integral $\int_{2}^{4} \frac{1-\left(\frac{x}{2}\right)^2}{x}\,dx =$

(A) $\int_{1}^{2} \frac{1-u^2}{u}\,du$

(B) $\int_{2}^{4} \frac{1-u^2}{u}\,du$

(C) $\int_{1}^{2} \frac{1-u^2}{2u}\,du$

(D) $\int_{1}^{2} \frac{1-u^2}{4u}\,du$

(E) $\int_{2}^{4} \frac{1-u^2}{2u}\,du$

41. What is the length of the arc of $y = \frac{2}{3}x^{\frac{3}{2}}$ from $x = 0$ to $x = 3$?

(A) $\frac{8}{3}$ (B) 4 (C) $\frac{14}{3}$ (D) $\frac{16}{3}$ (E) 7

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42. The coefficient of x^3 in the Taylor series for e^{3x} about $x = 0$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{2}$ (E) $\frac{9}{2}$

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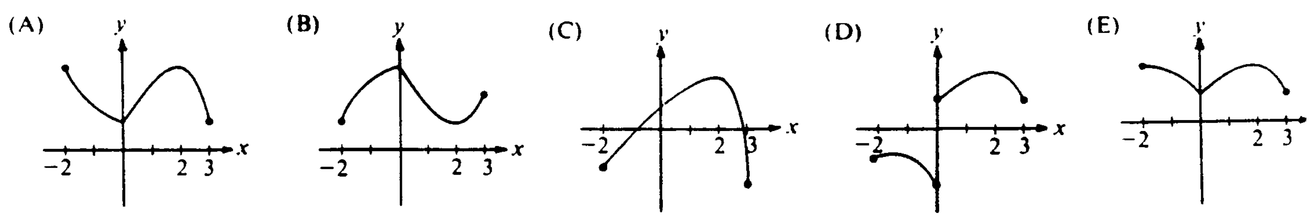
43. Let f be a function that is continuous on the closed interval $[-2,3]$ such that $f'(0)$ does not exist, $f'(2) = 0$, and $f''(x) < 0$ for all x except $x = 0$. Which of the following could be the graph of f?



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44. At each point (x, y) on a certain curve, the slope of the curve is $3x^2y$. If the curve contains the point $(0,8)$, then its equation is

(A) $y = 8e^{x^3}$ (B) $y = x^3 + 8$ (C) $y = e^{x^3} + 7$

(D) $y = \ln(x+1) + 8$ (E) $y^2 = x^3 + 8$

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45. If n is a positive integer, then $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \ldots + \left(\frac{3n}{n}\right)^2\right]$ can be expressed as

(A) $\int_0^1 \frac{1}{x^2}\,dx$ (B) $3\int_0^1 \left(\frac{1}{x}\right)^2 dx$ (C) $\int_0^3 \left(\frac{1}{x}\right)^2 dx$

(D) $\int_0^3 x^2\,dx$ (E) $3\int_0^3 x^2\,dx$

1985 Answer Key

1985 AB

1. D
2. E
3. A
4. C
5. D
6. C
7. E
8. B
9. D
10. D
11. B
12. C
13. A
14. D
15. C
16. B
17. C
18. C
19. B
20. A
21. B
22. A
23. B
24. D
25. E
26. E
27. D
28. C
29. D
30. B
31. C
32. D
33. B
34. A
35. D
36. B
37. D
38. C
39. E
40. D
41. E
42. C
43. B
44. A
45. A

1985 BC

1. D
2. A
3. B
4. D
5. D
6. E
7. A
8. C
9. B
10. A
11. A
12. A
13. B
14. C
15. C
16. C
17. B
18. C
19. D
20. C
21. B
22. A
23. C
24. D
25. C
26. E
27. E
28. E
29. D
30. B
31. D
32. E
33. C
34. A
35. B
36. E
37. A
38. C
39. A
40. A
41. C
42. E
43. E
44. A
45. D

1. D $\int_0^2 (4x^3+2)\,dx = (x^4+2x)\Big|_0^2 = (16+4)-(1+2)=17$

2. A $f'(x)=15x^4-15x^2=15x^2(x^2-1)=15x^2(x-1)(x+1)$, changes sign from positive to negative only at $x=-1$. So f has a relative maximum at $x=-1$ only.

3. B $\int_1^2 \frac{x+1}{x^2+2x}dx = \frac{1}{2}\int_1^2 \frac{(2x+2)\,dx}{x^2+2x} = \frac{1}{2}\ln\left|x^2+2x\right|\Big|_1^2 = \frac{1}{2}(\ln 8-\ln 3)$

4. D $x(t)=t^2-1 \Rightarrow \frac{dx}{dt}=2t$ and $\frac{d^2x}{dt^2}=2$; $y(t)=t^4-2t^3 \Rightarrow \frac{dy}{dt}=4t^3-6t^2$ and $\frac{d^2y}{dt^2}=12t^2-12t$
$a(t)=\left(\frac{d^2x}{dt^2},\frac{d^2y}{dt^2}\right)=(2,\,12t^2-12t) \Rightarrow a(1)=(2,0)$

5. D Area $=\int_{x_1}^{x_2}(\text{top curve}-\text{bottom curve})\,dx$, $x_1<x_2$; Area $=\int_{-1}^{a}(f(x)-g(x))\,dx$

6. E $f(x)=\frac{x}{\tan x}$, $f'(x)=\frac{\tan x - x\sec^2 x}{\tan^2 x}$, $f'\left(\frac{\pi}{4}\right)=\frac{1-\frac{\pi}{4}\cdot\left(\sqrt{2}\right)^2}{1}=1-\frac{\pi}{2}$

7. A $\int \frac{du}{\sqrt{a^2-u^2}}du = \sin^{-1}\left(\frac{u}{a}\right) \Rightarrow \int \frac{dx}{\sqrt{25-x^2}}dx=\sin^{-1}\left(\frac{x}{5}\right)+C$

8. C $\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=f'(2)$ so the derivative of f at $x=2$ is 0.

9. B Take the derivative of each side of the equation with respect to x.
$2xyy'+y^2+2xy'+2y=0$, substitute the point $(1,2)$
$(1)(4)y'+2^2+(2)(1)y'+(2)(2)=0 \Rightarrow y=-\frac{4}{3}$

10. A Take the derivative of the general term with respect to x: $\sum_{n=1}^{\infty}(-1)^{n+1}x^{2n-2}$

11. A $\frac{d}{dx}\left(\ln\left(\frac{1}{1-x}\right)\right)=\frac{d}{dx}(-\ln(1-x))=-\left(\frac{-1}{1-x}\right)=\frac{1}{1-x}$

12. A Use partial fractions to rewrite $\dfrac{1}{(x-1)(x+2)}$ as $\dfrac{1}{3}\left(\dfrac{1}{x-1}-\dfrac{1}{x+2}\right)$

$$\int \frac{1}{(x-1)(x+2)}dx=\frac{1}{3}\int\left(\frac{1}{x-1}-\frac{1}{x+2}\right)dx=\frac{1}{3}\left(\ln|x-1|-\ln|x+2|\right)+C=\frac{1}{3}\ln\left|\frac{x-1}{x+2}\right|+C$$

13. B $f(0)=0,\ f(3)=0,\ f'(x)=3x^2-6x;$ by the Mean Value Theorem,
$f'(c)=\dfrac{f(3)-f(0)}{3}=0$ for $c\in(0,3)$.
So, $0=3c^2-6c=3c(c-2)$. The only value in the open interval is 2.

14. C I. convergent: p-series with $p=2>1$
II. divergent: Harmonic series which is known to diverge
III. convergent: Geometric with $|r|=\dfrac{1}{3}<1$

15. C $x(t)=4+\int_0^t(2w-4)\,dw=4+(w^2-4w)\Big|_0^t=4+t^2-4t=t^2-4t+4$

or, $x(t)=t^2-4t+C$, $x(0)=4\Rightarrow C=4$ so, $x(t)=t^2-4t+4$

16. C For $f(x)=x^{\frac{1}{3}}$ we have continuity at $x=0$, however, $f'(x)=\dfrac{1}{3}x^{-\frac{2}{3}}$ is not defined at $x=0$.

17. B $f'(x)=(1)\cdot\ln(x^2)+x\cdot\dfrac{\frac{d}{dx}(x^2)}{x^2}=\ln(x^2)+\dfrac{2x^2}{x^2}=\ln(x^2)+2$

18. C $\int\sin(2x+3)\,dx=\dfrac{1}{2}\int\sin(2x+3)(2dx)=-\dfrac{1}{2}\cos(2x+3)+C$

19. D $g(x)=e^{f(x)},\ g'(x)=e^{f(x)}\cdot f'(x),\ g''(x)=e^{f(x)}\cdot f''(x)+f'(x)\cdot e^{f(x)}\cdot f'(x)$
$g''(x)=e^{f(x)}\left(f''(x)+\left(f'(x)^2\right)\right)=h(x)e^{f(x)}\Rightarrow h(x)=f''(x)+\left(f'(x)^2\right)$

20. C Look for concavity changes, there are 3.

21. B Use the technique of antiderivatives by parts:

$u = f(x) \qquad dv = \sin x\,dx$

$du = f'(x)\,dx \qquad v = -\cos x$

$\int f(x)\sin x\,dx = -f(x)\cos x + \int f'(x)\cos x\,dx$ and we are given that

$\int f(x)\sin x\,dx = -f(x)\cos x + \int 3x^2\cos x\,dx \Rightarrow f'(x) = 3x^2 \Rightarrow f(x) = x^3$

22. A $A = \pi r^2$, $A = 64\pi$ when $r = 8$. Take the derivative with respect to *t*.

$$\frac{dA}{dt} = 2\pi r\cdot\frac{dr}{dt};\ 96\pi = 2\pi(8)\cdot\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = 6$$

23. C $\lim\limits_{h\to 0}\dfrac{\int_1^{1+h}\sqrt{x^5+8}\,dx}{h} = \lim\limits_{h\to 0}\dfrac{F(1+h)-F(1)}{h} = F'(1)$ where $F'(x) = \sqrt{x^5+8}$. $F'(1)=3$

Alternate solution by L'Hôpital's Rule: $\lim\limits_{h\to 0}\dfrac{\int_1^{1+h}\sqrt{x^5+8}\,dx}{h} = \lim\limits_{h\to 0}\dfrac{\sqrt{(1+h)^5+8}}{1} = \sqrt{9} = 3$

24. D $\text{Area} = \dfrac{1}{2}\int_0^{\pi/2}\sin^2(2\theta)\,d\theta = \dfrac{1}{2}\int_0^{\pi/2}\dfrac{1}{2}\left(1-\cos 4\theta\right)d\theta = \dfrac{1}{4}\left(\theta - \dfrac{1}{4}\sin 4\theta\right)\Bigg|_0^{\pi/2} = \dfrac{\pi}{8}$

25. C At rest when $v(t) = 0$. $v(t) = e^{-2t} - 2te^{-2t} = e^{-2t}(1-2t)$, $v(t) = 0$ at $t = \dfrac{1}{2}$ only.

26. E Apply the log function, simplify, and differentiate. $\ln y = \ln(\sin x)^x = x\ln(\sin x)$

$$\frac{y'}{y} = \ln(\sin x) + x\cdot\frac{\cos x}{\sin x} \Rightarrow y' = y\left(\ln(\sin x) + x\cdot\cot x\right) = (\sin x)^x\left(\ln(\sin x) + x\cdot\cot x\right)$$

27. E Each of the right-hand sides represent the area of a rectangle with base length $(b-a)$.

I. Area under the curve is less than the area of the rectangle with height $f(b)$.

II. Area under the curve is more than the area of the rectangle with height $f(a)$.

III. Area under the curve is the same as the area of the rectangle with height $f(c)$, $a < c < b$. Note that this is the Mean Value Theorem for Integrals.

28. E $\int e^{x+e^x}\,dx = \int e^{e^x}(e^x dx)$. This is of the form $\int e^u\,du,\ u = e^x$, so $\int e^{x+e^x}\,dx = e^{e^x} + C$

29. D Let $x-\frac{\pi}{4}=t$. $\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin\left(x-\frac{\pi}{4}\right)}{x-\frac{\pi}{4}}=\lim\limits_{t\to 0}\frac{\sin t}{t}=1$

30. B At t = 1, $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\left.\frac{\frac{3}{2\sqrt{3t+1}}}{3t^2-1}\right|_{t=1}=\frac{\frac{3}{4}}{3-1}=\frac{3}{8}$

31. D The center is $x=1$, so only C, D, or E are possible. Check the endpoints.

At $x=0$: $\sum\limits_{n=1}^{\infty}\frac{(-1)^n}{n}$ converges by alternating series test.

At $x=2$: $\sum\limits_{n=1}^{\infty}\frac{1}{n}$ which is the harmonic series and known to diverge.

32. E $y(-1)=-6$, $y'(-1)=3x^2+6x+7\Big|_{x=-1}=4$, the slope of the normal is $-\frac{1}{4}$ and an equation for the normal is $y+6=-\frac{1}{4}(x+1)\Rightarrow x+4y=-25$.

33. C This is the differential equation for exponential growth.

$y=y(0)e^{-2t}=e^{-2t}$; $\frac{1}{2}=e^{-2t}$; $-2t=\ln\left(\frac{1}{2}\right)\Rightarrow t=-\frac{1}{2}\ln\left(\frac{1}{2}\right)=\frac{1}{2}\ln 2$

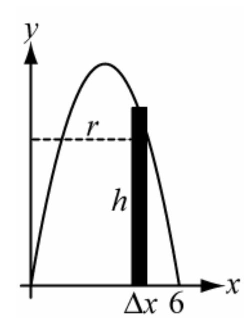
34. A This topic is no longer part of the AP Course Description. $\sum 2\pi\rho\,\Delta s$ where $\rho=x=y^3$

$$\text{Surface Area}=\int_0^1 2\pi y^3\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy=\int_0^1 2\pi y^3\sqrt{1+\left(3y^2\right)^2}\,dy=2\pi\int_0^1 y^3\sqrt{1+9y^4}\,dy$$

35. B Use shells (which is no longer part of the AP Course Description)

$\sum 2\pi rh\,\Delta x$ where $r=x$ and $h=y=6x-x^2$

$$\text{Volume}=2\pi\int_0^6 x\left(6x-x^2\right)dx$$



36. E $\int_{-1}^{1} \frac{3}{x^2}\,dx = 2\int_0^1 \frac{3}{x^2}\,dx = 2\lim_{L\to 0^+}\int_L^1 \frac{3}{x^2}\,dx = 2\lim_{L\to 0^+} -\frac{3}{x}\Big|_L^1$ which does not exist.

37. A This topic is no longer part of the AP Course Description. $y = y_h + y_p$ where $y_h = ce^{-x}$ is the solution to the homogeneous equation $\frac{dy}{dx} + y = 0$ and $y_p = \left(Ax^2 + Bx\right)e^{-x}$ is a particular solution to the given differential equation. Substitute y_p into the differential equation to determine the values of A and B. The answer is $A = \frac{1}{2},\ B = 0$.

38. C $\lim_{x\to\infty}\left(1+5e^x\right)^{1/x} = \lim_{x\to\infty} e^{\ln\left(1+5e^x\right)^{1/x}} = e^{\lim_{x\to\infty}\ln\left(1+5e^x\right)^{1/x}} = e^{\lim_{x\to\infty}\frac{\ln\left(1+5e^x\right)}{x}} = e^{\lim_{x\to\infty}\frac{5e^x}{1+5e^x}} = e$

39. A Square cross sections: $\sum y^2\,\Delta x$ where $y = e^{-x}$. $V = \int_0^3 e^{-2x}\,dx = -\frac{1}{2}e^{-2x}\Big|_0^3 = \frac{1}{2}\left(1-e^{-6}\right)$

40. A $u = \frac{x}{2},\ du = \frac{1}{2}dx$; when $x = 2,\ u = 1$ and when $x = 4,\ u = 2$

$$\int_2^4 \frac{1-\left(\frac{x}{2}\right)^2}{x}\,dx = \int_1^2 \frac{1-u^2}{2u}\cdot 2\,du = \int_1^2 \frac{1-u^2}{u}\,du$$

41. C $y' = x^{\frac{1}{2}},\ L = \int_0^3 \sqrt{1+\left(y'\right)^2}\,dx = \int_0^3 \sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2}\Big|_0^3 = \frac{2}{3}\left(4^{3/2}-1^{3/2}\right) = \frac{2}{3}(8-1) = \frac{14}{3}$

42. E Since $e^u = 1 + u + \frac{u^2}{2!} + \frac{u^3}{3!} + \cdots$, then $e^{3x} = 1 + 3x + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \cdots$

The coefficient we want is $\frac{3^3}{3!} = \frac{9}{2}$

43. E Graphs A and B contradict $f'' < 0$. Graph C contradicts $f'(0)$ does not exist. Graph D contradicts continuity on the interval $[-2,3]$. Graph E meets all given conditions.

44. A $\frac{dy}{dx} = 3x^2 y \Rightarrow \frac{dy}{y} = 3x^2 dx \Rightarrow \ln|y| = x^3 + K;\ y = Ce^{x^3}$ and $y(0) = 8$ so, $y = 8e^{x^3}$

45. D The expression is a Riemann sum with $\Delta x = \frac{1}{n}$ and $f(x) = x^2$.

The evaluation points are: $\frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \cdots, \frac{3n}{n}$

Thus the right Riemann sum is for $x = 0$ to $x = 3$. The limit is equal to $\int_0^3 x^2\, dx$.