

**1998 AP Calculus AB:  
Section I, Part A**

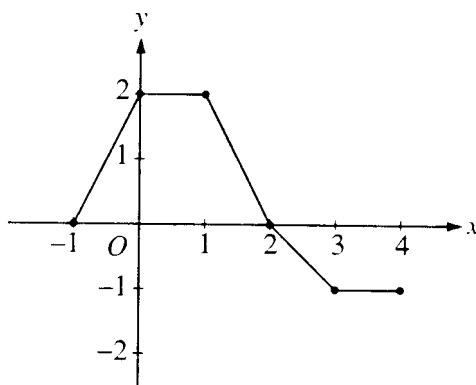
**55 Minutes—No Calculator**

*Note:* Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

1. What is the  $x$ -coordinate of the point of inflection on the graph of  $y = \frac{1}{3}x^3 + 5x^2 + 24$ ?

(A) 5                      (B) 0                      (C)  $-\frac{10}{3}$                       (D) -5                      (E) -10

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2. The graph of a piecewise-linear function  $f$ , for  $-1 \leq x \leq 4$ , is shown above. What is the value of  $\int_{-1}^4 f(x) dx$ ?

(A) 1                      (B) 2.5                      (C) 4                      (D) 5.5                      (E) 8

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3.  $\int_1^2 \frac{1}{x^2} dx =$

(A)  $-\frac{1}{2}$                       (B)  $\frac{7}{24}$                       (C)  $\frac{1}{2}$                       (D) 1                      (E)  $2\ln 2$

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4. If  $f$  is continuous for  $a \leq x \leq b$  and differentiable for  $a < x < b$ , which of the following could be false?

(A)  $f'(c) = \frac{f(b) - f(a)}{b - a}$  for some  $c$  such that  $a < c < b$ .

(B)  $f'(c) = 0$  for some  $c$  such that  $a < c < b$ .

(C)  $f$  has a minimum value on  $a \leq x \leq b$ .

(D)  $f$  has a maximum value on  $a \leq x \leq b$ .

(E)  $\int_a^b f(x) dx$  exists.

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5.  $\int_0^x \sin t \, dt =$

(A)  $\sin x$

(B)  $-\cos x$

(C)  $\cos x$

(D)  $\cos x - 1$

(E)  $1 - \cos x$

---

6. If  $x^2 + xy = 10$ , then when  $x = 2$ ,  $\frac{dy}{dx} =$

(A)  $-\frac{7}{2}$

(B)  $-2$

(C)  $\frac{2}{7}$

(D)  $\frac{3}{2}$

(E)  $\frac{7}{2}$

---

7.  $\int_1^e \left( \frac{x^2 - 1}{x} \right) dx =$

(A)  $e - \frac{1}{e}$

(B)  $e^2 - e$

(C)  $\frac{e^2}{2} - e + \frac{1}{2}$

(D)  $e^2 - 2$

(E)  $\frac{e^2}{2} - \frac{3}{2}$

- 
8. Let  $f$  and  $g$  be differentiable functions with the following properties:

(i)  $g(x) > 0$  for all  $x$

(ii)  $f(0) = 1$

If  $h(x) = f(x)g(x)$  and  $h'(x) = f(x)g'(x)$ , then  $f(x) =$

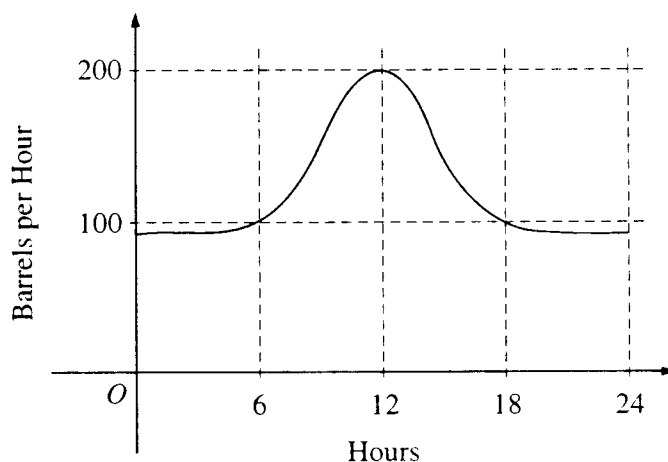
(A)  $f'(x)$

(B)  $g(x)$

(C)  $e^x$

(D)  $0$

(E)  $1$



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500      (B) 600      (C) 2,400      (D) 3,000      (E) 4,800

10. What is the instantaneous rate of change at  $x = 2$  of the function  $f$  given by  $f(x) = \frac{x^2 - 2}{x - 1}$ ?

(A)  $-2$       (B)  $\frac{1}{6}$       (C)  $\frac{1}{2}$       (D)  $2$       (E)  $6$

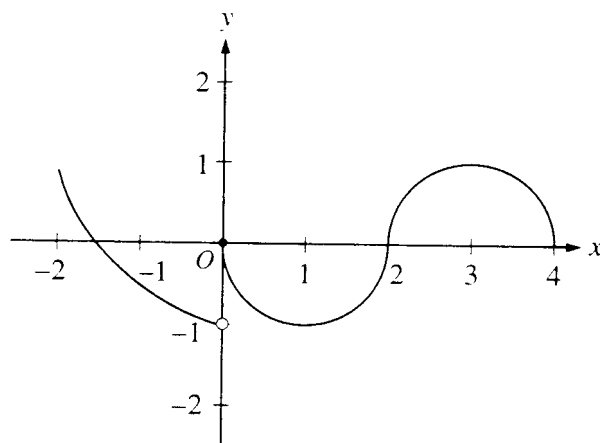
11. If  $f$  is a linear function and  $0 < a < b$ , then  $\int_a^b f''(x) dx =$

(A)  $0$       (B)  $1$       (C)  $\frac{ab}{2}$       (D)  $b - a$       (E)  $\frac{b^2 - a^2}{2}$

12. If  $f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4, \end{cases}$  then  $\lim_{x \rightarrow 2} f(x)$  is

(A)  $\ln 2$       (B)  $\ln 8$       (C)  $\ln 16$       (D)  $4$       (E) nonexistent

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13. The graph of the function  $f$  shown in the figure above has a vertical tangent at the point  $(2, 0)$  and horizontal tangents at the points  $(1, -1)$  and  $(3, 1)$ . For what values of  $x$ ,  $-2 < x < 4$ , is  $f$  not differentiable?

(A) 0 only    (B) 0 and 2 only    (C) 1 and 3 only    (D) 0, 1, and 3 only    (E) 0, 1, 2, and 3

14. A particle moves along the  $x$ -axis so that its position at time  $t$  is given by  $x(t) = t^2 - 6t + 5$ . For what value of  $t$  is the velocity of the particle zero?

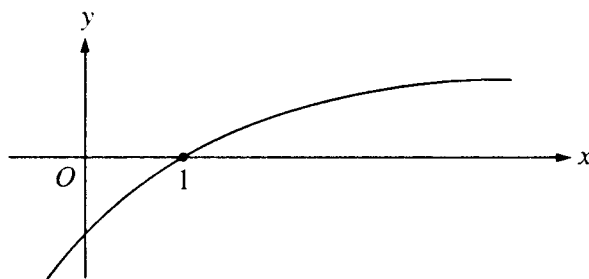
(A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5

15. If  $F(x) = \int_0^x \sqrt{t^3 + 1} \, dt$ , then  $F'(2) =$

(A) -3                      (B) -2                      (C) 2                      (D) 3                      (E) 18

16. If  $f(x) = \sin(e^{-x})$ , then  $f'(x) =$

(A)  $-\cos(e^{-x})$   
 (B)  $\cos(e^{-x}) + e^{-x}$   
 (C)  $\cos(e^{-x}) - e^{-x}$   
 (D)  $e^{-x} \cos(e^{-x})$   
 (E)  $-e^{-x} \cos(e^{-x})$



17. The graph of a twice-differentiable function  $f$  is shown in the figure above. Which of the following is true?

(A)  $f(1) < f'(1) < f''(1)$   
 (B)  $f(1) < f''(1) < f'(1)$   
 (C)  $f'(1) < f(1) < f''(1)$   
 (D)  $f''(1) < f(1) < f'(1)$   
 (E)  $f''(1) < f'(1) < f(1)$

18. An equation of the line tangent to the graph of  $y = x + \cos x$  at the point  $(0, 1)$  is

(A)  $y = 2x + 1$     (B)  $y = x + 1$     (C)  $y = x$     (D)  $y = x - 1$     (E)  $y = 0$

19. If  $f''(x) = x(x+1)(x-2)^2$ , then the graph of  $f$  has inflection points when  $x =$

(A)  $-1$  only    (B)  $2$  only    (C)  $-1$  and  $0$  only    (D)  $-1$  and  $2$  only    (E)  $-1, 0$ , and  $2$  only

20. What are all values of  $k$  for which  $\int_{-3}^k x^2 dx = 0$ ?

(A)  $-3$     (B)  $0$     (C)  $3$     (D)  $-3$  and  $3$     (E)  $-3, 0$ , and  $3$

21. If  $\frac{dy}{dt} = ky$  and  $k$  is a nonzero constant, then  $y$  could be

(A)  $2e^{kty}$     (B)  $2e^{kt}$     (C)  $e^{kt} + 3$     (D)  $kty + 5$     (E)  $\frac{1}{2}ky^2 + \frac{1}{2}$

22. The function  $f$  is given by  $f(x) = x^4 + x^2 - 2$ . On which of the following intervals is  $f$  increasing?

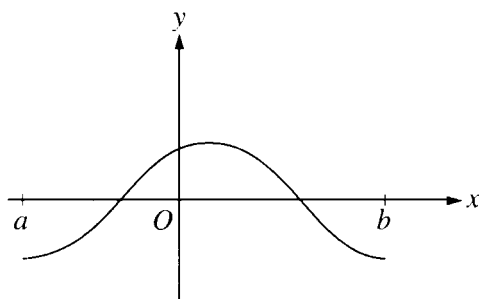
(A)  $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B)  $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

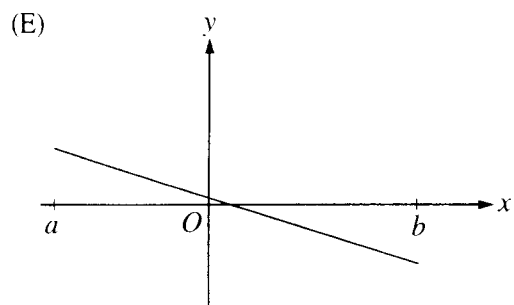
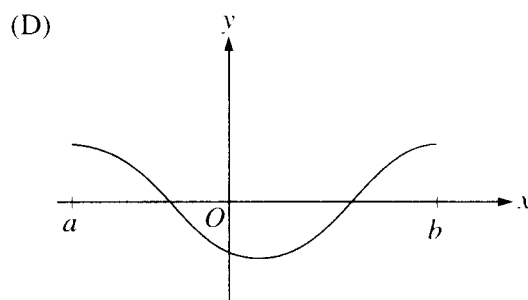
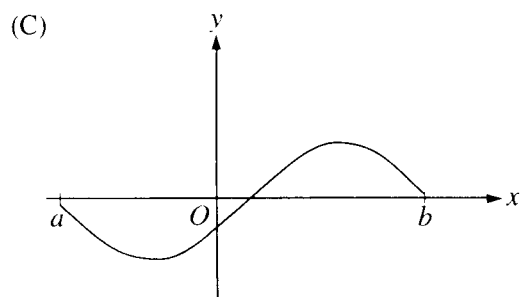
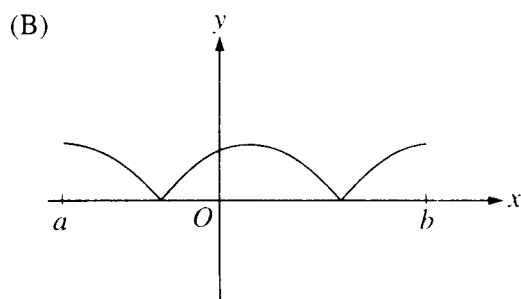
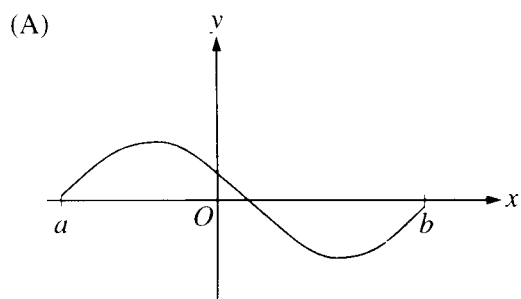
(C)  $(0, \infty)$

(D)  $(-\infty, 0)$

(E)  $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$



23. The graph of  $f$  is shown in the figure above. Which of the following could be the graph of the derivative of  $f$ ?



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24. The maximum acceleration attained on the interval  $0 \leq t \leq 3$  by the particle whose velocity is given by  $v(t) = t^3 - 3t^2 + 12t + 4$  is

(A) 9                      (B) 12                      (C) 14                      (D) 21                      (E) 40

25. What is the area of the region between the graphs of  $y = x^2$  and  $y = -x$  from  $x = 0$  to  $x = 2$ ?

(A)  $\frac{2}{3}$                       (B)  $\frac{8}{3}$                       (C) 4                      (D)  $\frac{14}{3}$                       (E)  $\frac{16}{3}$

|        |   |     |   |
|--------|---|-----|---|
| $x$    | 0 | 1   | 2 |
| $f(x)$ | 1 | $k$ | 2 |

26. The function  $f$  is continuous on the closed interval  $[0, 2]$  and has values that are given in the table above. The equation  $f(x) = \frac{1}{2}$  must have at least two solutions in the interval  $[0, 2]$  if  $k =$

(A) 0                      (B)  $\frac{1}{2}$                       (C) 1                      (D) 2                      (E) 3

27. What is the average value of  $y = x^2\sqrt{x^3 + 1}$  on the interval  $[0, 2]$ ?

(A)  $\frac{26}{9}$                       (B)  $\frac{52}{9}$                       (C)  $\frac{26}{3}$                       (D)  $\frac{52}{3}$                       (E) 24

28. If  $f(x) = \tan(2x)$ , then  $f'\left(\frac{\pi}{6}\right) =$

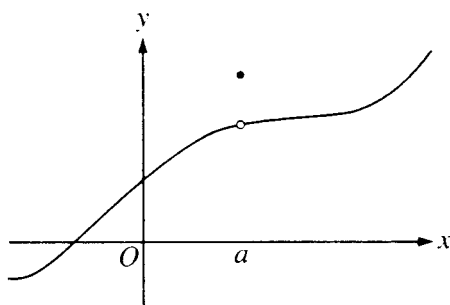
(A)  $\sqrt{3}$                       (B)  $2\sqrt{3}$                       (C) 4                      (D)  $4\sqrt{3}$                       (E) 8



50 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.



76. The graph of a function  $f$  is shown above. Which of the following statements about  $f$  is false?

- (A)  $f$  is continuous at  $x = a$ .
- (B)  $f$  has a relative maximum at  $x = a$ .
- (C)  $x = a$  is in the domain of  $f$ .
- (D)  $\lim_{x \rightarrow a^+} f(x)$  is equal to  $\lim_{x \rightarrow a^-} f(x)$ .
- (E)  $\lim_{x \rightarrow a} f(x)$  exists.

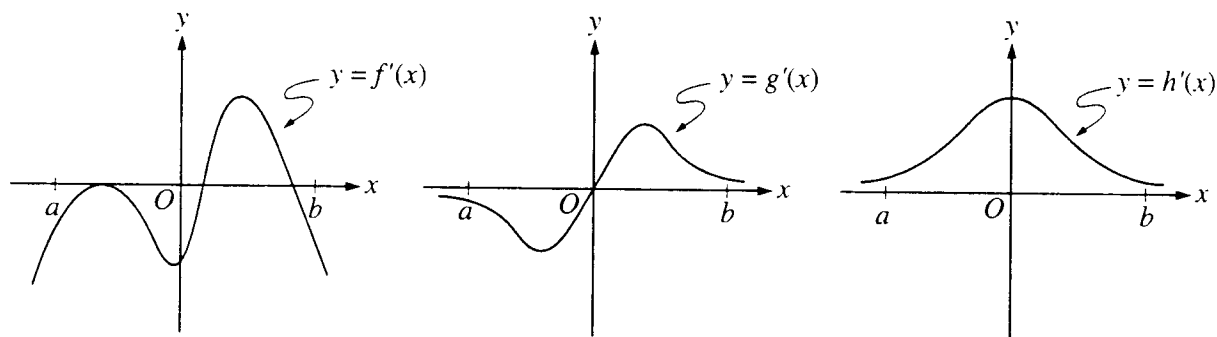
77. Let  $f$  be the function given by  $f(x) = 3e^{2x}$  and let  $g$  be the function given by  $g(x) = 6x^3$ . At what value of  $x$  do the graphs of  $f$  and  $g$  have parallel tangent lines?

- (A)  $-0.701$
- (B)  $-0.567$
- (C)  $-0.391$
- (D)  $-0.302$
- (E)  $-0.258$

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78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference  $C$ , what is the rate of change of the area of the circle, in square centimeters per second?

- (A)  $-(0.2)\pi C$   
 (B)  $-(0.1)C$   
 (C)  $-\frac{(0.1)C}{2\pi}$   
 (D)  $(0.1)^2 C$   
 (E)  $(0.1)^2 \pi C$



79. The graphs of the derivatives of the functions  $f$ ,  $g$ , and  $h$  are shown above. Which of the functions  $f$ ,  $g$ , or  $h$  have a relative maximum on the open interval  $a < x < b$ ?
- (A)  $f$  only  
 (B)  $g$  only  
 (C)  $h$  only  
 (D)  $f$  and  $g$  only  
 (E)  $f$ ,  $g$ , and  $h$

80. The first derivative of the function  $f$  is given by  $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$ . How many critical values does  $f$  have on the open interval  $(0, 10)$ ?

- (A) One  
 (B) Three  
 (C) Four  
 (D) Five  
 (E) Seven

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81. Let  $f$  be the function given by  $f(x) = |x|$ . Which of the following statements about  $f$  are true?

- I.  $f$  is continuous at  $x = 0$ .
- II.  $f$  is differentiable at  $x = 0$ .
- III.  $f$  has an absolute minimum at  $x = 0$ .

(A) I only    (B) II only    (C) III only    (D) I and III only    (E) II and III only

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82. If  $f$  is a continuous function and if  $F'(x) = f(x)$  for all real numbers  $x$ , then  $\int_1^3 f(2x) dx =$

- (A)  $2F(3) - 2F(1)$
  - (B)  $\frac{1}{2}F(3) - \frac{1}{2}F(1)$
  - (C)  $2F(6) - 2F(2)$
  - (D)  $F(6) - F(2)$
  - (E)  $\frac{1}{2}F(6) - \frac{1}{2}F(2)$
- 

83. If  $a \neq 0$ , then  $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x^4 - a^4}$  is

- (A)  $\frac{1}{a^2}$
  - (B)  $\frac{1}{2a^2}$
  - (C)  $\frac{1}{6a^2}$
  - (D) 0
  - (E) nonexistent
- 

84. Population  $y$  grows according to the equation  $\frac{dy}{dt} = ky$ , where  $k$  is a constant and  $t$  is measured in years. If the population doubles every 10 years, then the value of  $k$  is

- (A) 0.069
- (B) 0.200
- (C) 0.301
- (D) 3.322
- (E) 5.000

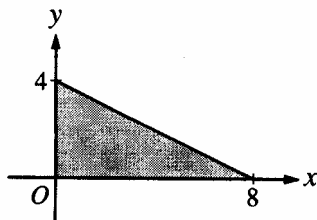
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|        |    |    |    |    |
|--------|----|----|----|----|
| $x$    | 2  | 5  | 7  | 8  |
| $f(x)$ | 10 | 30 | 40 | 20 |

85. The function  $f$  is continuous on the closed interval  $[2, 8]$  and has values that are given in the table above. Using the subintervals  $[2, 5]$ ,  $[5, 7]$ , and  $[7, 8]$ , what is the trapezoidal approximation of

$$\int_2^8 f(x) dx?$$

- (A) 110                      (B) 130                      (C) 160                      (D) 190                      (E) 210



86. The base of a solid is a region in the first quadrant bounded by the  $x$ -axis, the  $y$ -axis, and the line  $x + 2y = 8$ , as shown in the figure above. If cross sections of the solid perpendicular to the  $x$ -axis are semicircles, what is the volume of the solid?

- (A) 12.566                      (B) 14.661                      (C) 16.755                      (D) 67.021                      (E) 134.041

87. Which of the following is an equation of the line tangent to the graph of  $f(x) = x^4 + 2x^2$  at the point where  $f'(x) = 1$ ?

- (A)  $y = 8x - 5$   
 (B)  $y = x + 7$   
 (C)  $y = x + 0.763$   
 (D)  $y = x - 0.122$   
 (E)  $y = x - 2.146$

88. Let  $F(x)$  be an antiderivative of  $\frac{(\ln x)^3}{x}$ . If  $F(1) = 0$ , then  $F(9) =$

- (A) 0.048                      (B) 0.144                      (C) 5.827                      (D) 23.308                      (E) 1,640.250

89. If  $g$  is a differentiable function such that  $g(x) < 0$  for all real numbers  $x$  and if  $f'(x) = (x^2 - 4)g(x)$ , which of the following is true?
- (A)  $f$  has a relative maximum at  $x = -2$  and a relative minimum at  $x = 2$ .  
 (B)  $f$  has a relative minimum at  $x = -2$  and a relative maximum at  $x = 2$ .  
 (C)  $f$  has relative minima at  $x = -2$  and at  $x = 2$ .  
 (D)  $f$  has relative maxima at  $x = -2$  and at  $x = 2$ .  
 (E) It cannot be determined if  $f$  has any relative extrema.
- 
90. If the base  $b$  of a triangle is increasing at a rate of 3 inches per minute while its height  $h$  is decreasing at a rate of 3 inches per minute, which of the following must be true about the area  $A$  of the triangle?
- (A)  $A$  is always increasing.  
 (B)  $A$  is always decreasing.  
 (C)  $A$  is decreasing only when  $b < h$ .  
 (D)  $A$  is decreasing only when  $b > h$ .  
 (E)  $A$  remains constant.
- 
91. Let  $f$  be a function that is differentiable on the open interval  $(1, 10)$ . If  $f(2) = -5$ ,  $f(5) = 5$ , and  $f(9) = -5$ , which of the following must be true?
- I.  $f$  has at least 2 zeros.  
 II. The graph of  $f$  has at least one horizontal tangent.  
 III. For some  $c$ ,  $2 < c < 5$ ,  $f(c) = 3$ .
- (A) None  
 (B) I only  
 (C) I and II only  
 (D) I and III only  
 (E) I, II, and III
- 
92. If  $0 \leq k < \frac{\pi}{2}$  and the area under the curve  $y = \cos x$  from  $x = k$  to  $x = \frac{\pi}{2}$  is 0.1, then  $k =$
- (A) 1.471      (B) 1.414      (C) 1.277      (D) 1.120      (E) 0.436

## 1998 AB

- |       |       |
|-------|-------|
| 1. D  | 24. D |
| 2. B  | 25. D |
| 3. C  | 26. A |
| 4. B  | 27. A |
| 5. E  | 28. E |
| 6. A  | 76. A |
| 7. E  | 77. C |
| 8. E  | 78. B |
| 9. D  | 79. A |
| 10. D | 80. B |
| 11. A | 81. D |
| 12. E | 82. E |
| 13. B | 83. B |
| 14. C | 84. A |
| 15. D | 85. C |
| 16. E | 86. C |
| 17. D | 87. D |
| 18. B | 88. C |
| 19. C | 89. B |
| 20. A | 90. D |
| 21. B | 91. E |
| 22. C | 92. D |
| 23. A |       |

## 1998 BC

- |       |       |
|-------|-------|
| 1. C  | 24. C |
| 2. A  | 25. C |
| 3. D  | 26. E |
| 4. A  | 27. D |
| 5. A  | 28. C |
| 6. E  | 76. D |
| 7. E  | 77. E |
| 8. B  | 78. B |
| 9. D  | 79. A |
| 10. E | 80. B |
| 11. A | 81. B |
| 12. E | 82. B |
| 13. B | 83. C |
| 14. E | 84. B |
| 15. B | 85. C |
| 16. C | 86. C |
| 17. D | 87. D |
| 18. B | 88. C |
| 19. D | 89. A |
| 20. E | 90. A |
| 21. C | 91. E |
| 22. A | 92. D |
| 23. E |       |

1. D  $y' = x^2 + 10x$ ;  $y'' = 2x + 10$ ;  $y''$  changes sign at  $x = -5$
2. B  $\int_{-1}^4 f(x)dx = \int_{-1}^2 f(x)dx + \int_2^4 f(x)dx$   
 $= \text{Area of trapezoid(1)} - \text{Area of trapezoid(2)} = 4 - 1.5 = 2.5$
3. C  $\int_1^2 \frac{1}{x^2} dx = \int_1^2 x^{-2} dx = -x^{-1} \Big|_1^2 = \frac{1}{2}$
4. B This would be false if  $f$  was a linear function with non-zero slope.
5. E  $\int_0^x \sin t dt = -\cos t \Big|_0^x = -\cos x - (-\cos 0) = -\cos x + 1 = 1 - \cos x$
6. A Substitute  $x = 2$  into the equation to find  $y = 3$ . Taking the derivative implicitly gives  
 $\frac{d}{dx}(x^2 + xy) = 2x + xy' + y = 0$ . Substitute for  $x$  and  $y$  and solve for  $y'$ .  
 $4 + 2y' + 3 = 0$ ;  $y' = -\frac{7}{2}$
7. E  $\int_1^e \frac{x^2 - 1}{x} dx = \int_1^e x - \frac{1}{x} dx = \left( \frac{1}{2}x^2 - \ln x \right) \Big|_1^e = \left( \frac{1}{2}e^2 - 1 \right) - \left( \frac{1}{2} - 0 \right) = \frac{1}{2}e^2 - \frac{3}{2}$
8. E  $h(x) = f(x)g(x)$  so,  $h'(x) = f'(x)g(x) + f(x)g'(x)$ . It is given that  $h'(x) = f(x)g'(x)$ .  
Thus,  $f'(x)g(x) = 0$ . Since  $g(x) > 0$  for all  $x$ ,  $f'(x) = 0$ . This means that  $f$  is constant. It is given that  $f(0) = 1$ , therefore  $f(x) = 1$ .
9. D Let  $r(t)$  be the rate of oil flow as given by the graph, where  $t$  is measured in hours. The total number of barrels is given by  $\int_0^{24} r(t)dt$ . This can be approximated by counting the squares below the curve and above the horizontal axis. There are approximately five squares with area 600 barrels. Thus the total is about 3,000 barrels.
10. D  $f'(x) = \frac{(x-1)(2x) - (x^2-2)(1)}{(x-1)^2}$ ;  $f'(2) = \frac{(2-1)(4) - (4-2)(1)}{(2-1)^2} = 2$
11. A Since  $f$  is linear, its second derivative is zero. The integral gives the area of a rectangle with zero height and width  $(b-a)$ . This area is zero.

## 1998 Calculus AB Solutions: Part A

12. E  $\lim_{x \rightarrow 2^-} f(x) = \ln 2 \neq 4 \ln 2 = \lim_{x \rightarrow 2^+} f(x)$ . Therefore the limit does not exist.
13. B At  $x = 0$  and  $x = 2$  only. The graph has a non-vertical tangent line at every other point in the interval and so has a derivative at each of these other  $x$ 's.
14. C  $v(t) = 2t - 6$ ;  $v(t) = 0$  for  $t = 3$
15. D By the Fundamental Theorem of Calculus,  $F'(x) = \sqrt{x^3 + 1}$ , thus  $F'(2) = \sqrt{2^3 + 1} = \sqrt{9} = 3$ .
16. E  $f'(x) = \cos(e^{-x}) \cdot \frac{d}{dx}(e^{-x}) = \cos(e^{-x}) \left( e^{-x} \cdot \frac{d}{dx}(-x) \right) = -e^{-x} \cos(e^{-x})$
17. D From the graph  $f(1) = 0$ . Since  $f'(1)$  represents the slope of the graph at  $x = 1$ ,  $f'(1) > 0$ . Also, since  $f''(1)$  represents the concavity of the graph at  $x = 1$ ,  $f''(1) < 0$ .
18. B  $y' = 1 - \sin x$  so  $y'(0) = 1$  and the line with slope 1 containing the point  $(0, 1)$  is  $y = x + 1$ .
19. C Points of inflection occur where  $f''$  changes sign. This is only at  $x = 0$  and  $x = -1$ . There is no sign change at  $x = 2$ .
20. A  $\int_{-3}^k x^2 dx = \frac{1}{3} x^3 \Big|_{-3}^k = \frac{1}{3} (k^3 - (-3)^3) = \frac{1}{3} (k^3 + 27) = 0$  only when  $k = -3$ .
21. B The solution to this differential equation is known to be of the form  $y = y(0) \cdot e^{kt}$ . Option (B) is the only one of this form. If you do not remember the form of the solution, then separate the variables and antidifferentiate.  

$$\frac{dy}{y} = k dt; \ln |y| = kt + c_1; |y| = e^{kt+c_1} = e^{kt} e^{c_1}; y = ce^{kt}.$$
22. C  $f$  is increasing on any interval where  $f'(x) > 0$ .  $f'(x) = 4x^3 + 2x = 2x(2x^2 + 1) > 0$ . Since  $(x^2 + 1) > 0$  for all  $x$ ,  $f'(x) > 0$  whenever  $x > 0$ .
23. A The graph shows that  $f$  is increasing on an interval  $(a, c)$  and decreasing on the interval  $(c, b)$ , where  $a < c < b$ . This means the graph of the derivative of  $f$  is positive on the interval  $(a, c)$  and negative on the interval  $(c, b)$ , so the answer is (A) or (E). The derivative is not (E), however, since then the graph of  $f$  would be concave down for the entire interval.



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24. D The maximum acceleration will occur when its derivative changes from positive to negative or at an endpoint of the interval.  $a(t) = v'(t) = 3t^2 - 6t + 12 = 3(t^2 - 2t + 4)$  which is always positive. Thus the acceleration is always increasing. The maximum must occur at  $t = 3$  where  $a(3) = 21$
25. D The area is given by  $\int_0^2 x^2 - (-x) dx = \left( \frac{1}{3}x^3 + \frac{1}{2}x^2 \right) \Big|_0^2 = \frac{8}{3} + 2 = \frac{14}{3}$ .
26. A Any value of  $k$  less than  $1/2$  will require the function to assume the value of  $1/2$  at least twice because of the Intermediate Value Theorem on the intervals  $[0,1]$  and  $[1,2]$ . Hence  $k = 0$  is the only option.
27. A  $\frac{1}{2} \int_0^2 x^2 \sqrt{x^3 + 1} dx = \frac{1}{2} \int_0^2 (x^3 + 1)^{\frac{1}{2}} \left( \frac{1}{3} \cdot 3x^2 \right) dx = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{2}{3} (x^3 + 1)^{\frac{3}{2}} \Big|_0^2 = \frac{1}{9} \left( 9^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) = \frac{26}{9}$
28. E  $f'(x) = \sec^2(2x) \cdot \frac{d}{dx}(2x) = 2\sec^2(2x)$ ;  $f'\left(\frac{\pi}{6}\right) = 2\sec^2\left(\frac{\pi}{3}\right) = 2(4) = 8$

76. A From the graph it is clear that  $f$  is not continuous at  $x = a$ . All others are true.
77. C Parallel tangents will occur when the slopes of  $f$  and  $g$  are equal.  $f'(x) = 6e^{2x}$  and  $g'(x) = 18x^2$ . The graphs of these derivatives reveal that they are equal only at  $x = -0.391$ .
78. B  $A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ . However,  $C = 2\pi r$  and  $\frac{dr}{dt} = -0.1$ . Thus  $\frac{dA}{dt} = -0.1C$ .
79. A The graph of the derivative would have to change from positive to negative. This is only true for the graph of  $f'$ .
80. B Look at the graph of  $f'(x)$  on the interval  $(0,10)$  and count the number of  $x$ -intercepts in the interval.
81. D Only II is false since the graph of the absolute value function has a sharp corner at  $x = 0$ .
82. E Since  $F$  is an antiderivative of  $f$ ,  $\int_1^3 f(2x) dx = \frac{1}{2} F(2x) \Big|_1^3 = \frac{1}{2} (F(6) - F(2))$
83. B  $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x^4 - a^4} = \lim_{x \rightarrow a} \frac{x^2 - a^2}{(x^2 - a^2)(x^2 + a^2)} = \lim_{x \rightarrow a} \frac{1}{x^2 + a^2} = \frac{1}{2a^2}$
84. A A known solution to this differential equation is  $y(t) = y(0)e^{kt}$ . Use the fact that the population is  $2y(0)$  when  $t = 10$ . Then  $2y(0) = y(0)e^{k(10)} \Rightarrow e^{10k} = 2 \Rightarrow k = (0.1) \ln 2 = 0.069$
85. C There are 3 trapezoids.  $\frac{1}{2} \cdot 3(f(2) + f(5)) + \frac{1}{2} \cdot 2(f(5) + f(7)) + \frac{1}{2} \cdot 1(f(7) + f(8))$
86. C Each cross section is a semicircle with a diameter of  $y$ . The volume would be given by  $\int_0^8 \frac{1}{2} \pi \left( \frac{y}{2} \right)^2 dx = \frac{\pi}{8} \int_0^8 \left( \frac{8-x}{2} \right)^2 dx = 16.755$
87. D Find the  $x$  for which  $f'(x) = 1$ .  $f'(x) = 4x^3 + 4x = 1$  only for  $x = 0.237$ . Then  $f(0.237) = 0.115$ . So the equation is  $y - 0.115 = x - 0.237$ . This is equivalent to option (D).

88. C  $F(9) - F(1) = \int_1^9 \frac{(\ln t)^3}{t} dt = 5.827$  using a calculator. Since  $F(1) = 0$ ,  $F(9) = 5.827$ .

Or solve the differential equation with an initial condition by finding an antiderivative for  $\frac{(\ln x)^3}{x}$ . This is of the form  $u^3 du$  where  $u = \ln x$ . Hence  $F(x) = \frac{1}{4}(\ln x)^4 + C$  and since

$$F(1) = 0, C = 0. \text{ Therefore } F(9) = \frac{1}{4}(\ln 9)^4 = 5.827$$

89. B The graph of  $y = x^2 - 4$  is a parabola that changes from positive to negative at  $x = -2$  and from negative to positive at  $x = 2$ . Since  $g$  is always negative,  $f'$  changes sign opposite to the way  $y = x^2 - 4$  does. Thus  $f$  has a relative minimum at  $x = -2$  and a relative maximum at  $x = 2$ .

90. D The area of a triangle is given by  $A = \frac{1}{2}bh$ . Taking the derivative with respect to  $t$  of both sides of the equation yields  $\frac{dA}{dt} = \frac{1}{2}\left(\frac{db}{dt} \cdot h + b \cdot \frac{dh}{dt}\right)$ . Substitute the given rates to get  $\frac{dA}{dt} = \frac{1}{2}(3h - 3b) = \frac{3}{2}(h - b)$ . The area will be decreasing whenever  $\frac{dA}{dt} < 0$ . This is true whenever  $b > h$ .

91. E I. True. Apply the Intermediate Value Theorem to each of the intervals  $[2, 5]$  and  $[5, 9]$ .

II. True. Apply the Mean Value Theorem to the interval  $[2, 9]$ .

III. True. Apply the Intermediate Value Theorem to the interval  $[2, 5]$ .

92. D  $\int_k^{\frac{\pi}{2}} \cos x \, dx = 0.1 \Rightarrow \sin\left(\frac{\pi}{2}\right) - \sin k = 0.1 \Rightarrow \sin k = 0.9$ . Therefore  $k = \sin^{-1}(0.9) = 1.120$ .