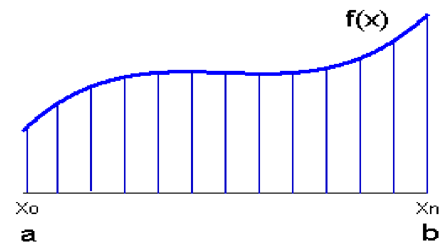
ACCUMULATION

Accumulation with Approximating Area

1. Riemann Sums- Finding the area between the curve and the axis by using the area of rectangles. A= h x w



   A) Right-hand- Find the height of each rectangle by finding the function’s value using the x-value on the right side of each interval.

   B) Left-hand- Find the height of each rectangle by finding the function’s value using the x-value on the left side of each interval.

   C) Midpoint- Find the height of each rectangle by finding the function’s value using the x-value on the in the middle of each interval.

2. Trapezoidal rule- Finding the area between the curve and the axis by using the area of trapezoids. $A=\frac{1}{2}(b_1+b_2)h$ where the h is the width of each interval and b_1 and b_2 are the function’s y values at the left and right hand of each interval.

Accumulating using the Fundamental Theorem of Calculus

It has been proven that if you could make infinitely many rectangles that have infinitely small width and add them together you could find the area between the curve and the axis with very little error.

$$\lim_{n\to\infty}\sum_{i=1}^{n} f\left(a+\frac{b-a}{n}i\right)\frac{b-a}{n} = \int_a^b f(x)\,dx$$

Makes an infinite number of rectangles with really small width (→ $\lim_{n\to\infty}$)

Adding up areas (→ $\sum_{i=1}^{n}$)

Height of rectangles (→ $f\left(a+\frac{b-a}{n}i\right)$)

Width of rectangles (→ $\frac{b-a}{n}$)

According to the Fundamental Theorem of Calculus, if f(x) is a continuous function and $F(x)$ is the antiderivative of f(x), then to find the value of the accumulated area on the interval [a,b] $\int_a^b f(x)\,dx = F(b)-F(a)$.

Accumulating a rate

If you accumulate the area under a rate curve, then you find the net amount of whatever is changing in the problem. For example, if you are given that the rate of cola consumption as $r(t)$, then $\int_0^3 r(t)\,dt$ is the amount of cola consumed over a 3 year period.

Student Study Session Topic: Rate and Accumulation Questions

The integral of a rate of change gives the (net) amount of change. The general form of the equation is $F(x)=F(x_0)+\int_{x_0}^{x} F'(t)\,dt$,where $x=x_0$ is the initial time, and $F(x_0)$ is the initial value. Since this is one of the main interpretations of the definite integral the concept may come up in a variety of situations. In a sense all integrals are accumulators. Particle motion problems can often be approached as accumulation problems.

What should you know how to do?

- Understand the question. It is often not necessary to do as much computation as it seems at first.
- Often these problems contain a lot of writing. Be ready to read, think and apply.
- Understand that "rate" indicates a derivative. The units of measure also indicate derivative (miles per hour, gallons per minute, etc.) since you're working with a derivative, expect to integrate to find the amount.
- Typically the first part of the question asks for an amount, integrate (on your calculator) for the first 2 or three points. 2000 AB4 (a), 2005 AB2 (a), 2006 AB 2(a)
- You may then be asked to write or work with a function that will look like $F(x)=F(x_0)+\int_{x_0}^{x} f(t)\,dt$. If appropriate the derivative is found easily using the Fundamental Theorem of Calculus: $F'(x)=f(x)$.
- You may be asked to explain the meaning of a derivative or definite integral or its value in terms of the context of the problem.
- In-out problems: 2 rates of change work together 2000 AB4, 2005 AB 2
- Max/min and increasing/decreasing analysis 2000 AB 4, 2005 AB 2
- Multiple-choice questions may give the information needed in a graph or in a table.

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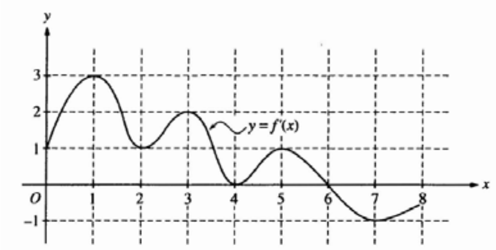
Accumulation Multiple Choice for Student Prep Session

Multiple Choice

Identify the choice that best completes the statement or answers the question.

Questions 1 refers to the graph and the information given below.

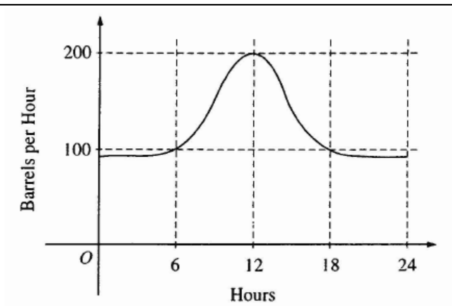


The function f is defined on the closed interval [0, 8]. The graph of its derivative f' is shown above.

1. At what value of x does the absolute minimum of f occur?

A) 0
B) 2
C) 4
D) 6
E) 8

2.



The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

A) 500
B) 600
C) 2,400
D) 3,000
E) 4,800

3.

t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at t =0 is 11 feet per second, the approximate value of the velocity at t = 6, computed using a left-hand Riemann sum with three subintervals of equal length, is

A) 26 ft/sec
B) 30 ft/sec
C) 37 ft/sec
D) 39 ft/sec
E) 41 ft/sec

4. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

A) $\int_{1.572}^{3.514} r(t)dt$

B) $\int_{0}^{8} r(t)dt$

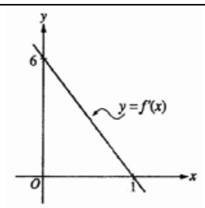
C) $\int_{0}^{2.667} r(t)dt$

D) $\int_{1.572}^{3.514} r'(t)dt$

E) $\int_{0}^{2.667} r'(t)dt$

5. A pizza, heated to a temperature of 350 degrees Fahrenheit (°F), is taken out of an oven and placed in a 75°F room at time t =0 minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time t = 5 minutes?

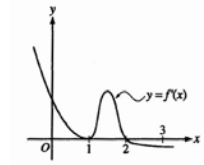
A) 112°F
B) 119°F
C) 147°F
D) 238°F
E) 335°F



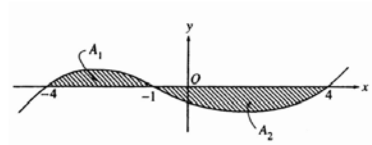
6. The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$. then $f(1)$ =
A) 0
B) 3
C) 6
D) 8
E) 11

7. Insects destroyed a crop at the rate of $\dfrac{100e^{-0.1t}}{2-e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons did the insects destroy during the time interval $7 \le t \le 14$?
A) 125
B) 100
C) 88
D) 50
E) 12

8.



The graph of f', the derivative of the function f, is shown above. If $f(0) = 0$, which of the following must be true?
I. $f(0) > f(1)$
II. $f(2) > f(1)$
III. $f(1) > f(3)$
A) I only
B) II only
C) III only
D) I and II only
E) II and III only



9. The graph of $y = f(x)$ is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2 ,

$$\int_{-4}^{4} f(x)dx - 2\int_{-1}^{4} f(x)dx =$$

A) A_1
B) $A_1 - A_2$
C) $2A_1 - A_2$
D) $A_1 + A_2$
E) $A_1 + 2A_2$

10.

x	2	5	10	14
$f(x)$	12	28	34	30

The function f is continuous on the closed interval [2, 14] and has values as shown in the table above. Using the subintervals [2, 5], [5,10], and [10, 14], what is the approximation of $\int_2^{14} f(x)dx$ found by using a right Riemann sum?
A) 296
B) 312
C) 343
D) 374
E) 390

11. The expression $\frac{1}{50}\left(\sqrt{\frac{1}{50}}+\sqrt{\frac{2}{50}}+\sqrt{\frac{3}{50}}+\ldots+\sqrt{\frac{50}{50}}\right)$ is a Riemann sum approximation for

A) $\int_0^1 \sqrt{\frac{x}{50}}\,dx$

B) $\int_0^1 \sqrt{x}\,dx$

C) $\frac{1}{50}\int_0^1 \sqrt{\frac{x}{50}}\,dx$

D) $\frac{1}{50}\int_0^1 \sqrt{x}\,dx$

E) $\frac{1}{50}\int_0^{50} \sqrt{x}\,dx$

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12. The closed interval $[a, b]$ is partitioned into n equal subintervals, each of width Δx, by the numbers $x_0, x_1, \ldots, x_n$ where $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$. What is $\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{x_i}\,\Delta x$?

A) $\frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

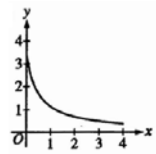
B) $b^{\frac{3}{2}} - a^{\frac{3}{2}}$

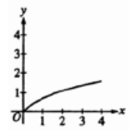
C) $\frac{3}{2}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

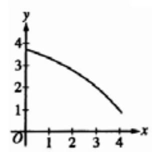
D) $b^{\frac{1}{2}} - a^{\frac{1}{2}}$

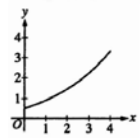
E) $2\left(b^{\frac{1}{2}} - a^{\frac{1}{2}}\right)$

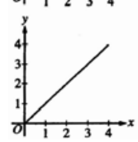
13. If a trapezoidal sum overapproximates $\int_0^4 f(x)dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)dx$ which of the following could be the graph of $y = f(x)$?

A) 

B) 

C) 

D) 

E) 

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x	2	5	7	8
$f(x)$	10	30	40	20

14. The function f is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)dx$?

A) 110
B) 130
C) 160
D) 190
E) 210

Accumulation Multiple Choice for Student Prep Session
Answer Section

MULTIPLE CHOICE

1. ANS: A DIF: Hard NOT: 1997 #9

2. ANS: D

“Barrels per hour” indicates that the graph is the graph of the *rate* at which oil is flowing; the rate is a derivaitve. To find the *amount* of oil calculate the definite integral of the *rate* over the time interval (24 hours). This can be approximated by finding the “area” between the graph and the *x*-axis. The “area” is about 5 rectangles each of which has an “area” of $100\frac{\text{barrels}}{\text{hour}}\cdot 6\,\text{hours} = 600\text{ barrels}$. The amount is $5\cdot 600\text{ barrels} = 3000\text{ barrels}$.

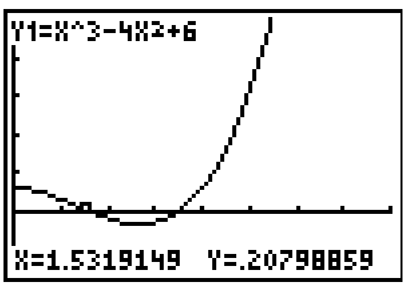
Answer D

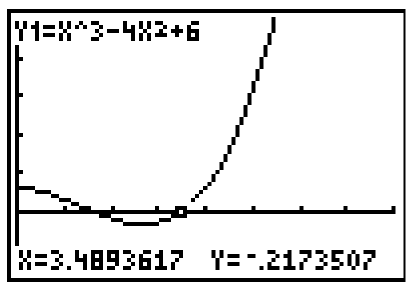
DIF: Easy NOT: 1998 #9

3. ANS: E DIF: Hard NOT: 1998 #91

4. ANS: A

Since $r(t)$ is a rate of change, it is a derivative. The integral of a rate of change gives the net amount of change. Thus A, B and C are possible answers. The balloon is losing altitude when the $r(t)$ is negative. Graph $r(t)$ and find where it is negative.





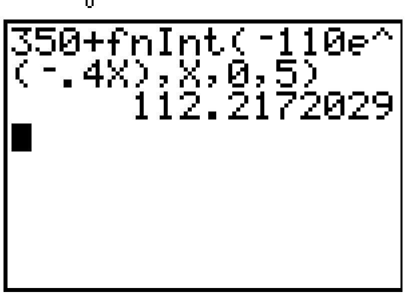
In this case tracing the graph gives answers close enough that we can derermine which choice is correct: Choice A. Note: If three decimal place accuracy is required do NOT use “trace” to find values.

DIF: Hard NOT: 2003 #82

5. ANS: A

The integral of a rate of change is the net amount of change so the temperature is the initial temperature plus the net change:

$$350 + \int_0^5 -110e^{-0.4t}\,dt = 112^\circ$$



DIF: Hard NOT: 2003 #84

6. ANS: D

The line above has the equation $y = -6x + 6$. This is the derivative of *f* so

$$\frac{df}{dx} = -6x + 6$$

$$f(x) = -3x^2 + 6x + C$$

$$f(0) = -3(0) + 6(0) + C = 5$$

$$f(x) = -3x^2 + 6x + 5$$

$$f(1) = -3(1)^2 + 6(1) + 5$$

$$f(1) = 8$$

DIF: Medium NOT: 2003 #22

7. ANS: A DIF: Easy NOT: 2003 #80

8. ANS: B DIF: Medium NOT: 2003 #90

9. ANS: D DIF: Hard NOT: 1997 #19

10. ANS: D

Add the area of the 3 rectangles indicated:

$2(28) + 5(34) + 4(30) = 374$

Answer D

Note that B is the left Riemann sum and C is the trapezoidal rule approximation.

DIF: Medium NOT: 2003 #25

11. ANS: B DIF: Hard NOT: 1997 #24

12. ANS: A DIF: Easy NOT: 1997 #25

13. ANS: A

If a trapezoidal sum overapproximates the integral the graph must be concave up (the top of the trapezoid lies above the graph). If a right Riemann sum underapproximates the integral the graph must be decreasing (the right side rectangle lie under the graph). So the graph must be decreasing and concave up -- Choice A.

DIF: Medium NOT: 2003 #85

14. ANS: C

The altitude of each trapezoid lies on the *x*-axis and have lengths of 3, 2 and 1 respectively. The *y*-value given are the bases of the trapezoids. Therefore, the sum of the area of the trapezoids is the approximation of the integral.

$$\int_2^8 f(x)dx \approx \frac{1}{2}(3)(10 + 30) + \frac{1}{2}(2)(30 + 40) + \frac{1}{2}(1)(40 + 20) = 160$$

Answer C

DIF: Medium NOT: 1998 #85

2000 AB 4

4. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

2005 AB 2

The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1+3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

2008 AB 3

3. Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

(b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

(c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

2009 AB B1

At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

$$\frac{dR}{dt} = \frac{1}{16}\left(3 + \sin\left(t^2\right)\right) \text{ centimeters per year}$$

for $0 \le t \le 3$, where time t is measured in years. At time $t = 0$, the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t)$.

(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \le t \le 3$. Use your expression to find $R(3)$.

(b) Find the rate at which the cross-sectional area $A(t)$ is increasing at time $t = 3$ years. Indicate units of measure.

(c) Evaluate $\int_0^3 A'(t)\,dt$. Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

2009 AB B6

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

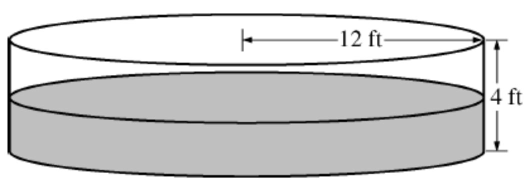
The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

(b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

(d) Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

2010 AB B3

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

(b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

(c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

(a) Method 1: $\int_0^3 \sqrt{t+1}\,dt = \frac{2}{3}(t+1)^{3/2} \Big\vert_0^3 = \frac{14}{3}$	Method 1: 3 { 2 : definite integral; 1 : limits; 1 : integrand; 1 : answer }
– or –	– or –
Method 2: $L(t)$ = gallons leaked in first t minutes $\frac{dL}{dt} = \sqrt{t+1}$; $L(t) = \frac{2}{3}(t+1)^{3/2} + C$ $L(0) = 0$: $C = -\frac{2}{3}$ $L(t) = \frac{2}{3}(t+1)^{3/2} - \frac{2}{3}$; $L(3) = \frac{14}{3}$	Method 2: 3 { 1 : antiderivative with C; 1 : solves for C using $L(0) = 0$; 1 : answer }
(b) $30 + 8 \cdot 3 - \frac{14}{3} = \frac{148}{3}$	1 : answer
(c) Method 1: $A(t) = 30 + \int_0^t (8 - \sqrt{x+1})\,dx$ $= 30 + 8t - \int_0^t \sqrt{x+1}\,dx$	Method 1: 2 { 1 : $30 + 8t$; 1 : $-\int_0^t \sqrt{x+1}\,dx$ }
– or –	– or –
Method 2: $\frac{dA}{dt} = 8 - \sqrt{t+1}$ $A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + C$ $30 = 8(0) - \frac{2}{3}(0+1)^{3/2} + C$; $C = \frac{92}{3}$ $A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + \frac{92}{3}$	Method 2: 2 { 1 : antiderivative with C; 1 : answer }
(d) $A'(t) = 8 - \sqrt{t+1} = 0$ when $t = 63$ $A'(t)$ is positive for $0 < t < 63$ and negative for $63 < t < 120$. Therefore there is a maximum at $t = 63$.	3 { 1 : sets $A'(t) = 0$; 1 : solves for t; 1 : justification }

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2005 AB 2

(a) $\int_0^6 R(t)\,dt = 31.815$ or 31.816 yd^3

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer with units} \end{cases}$

(b) $Y(t) = 2500 + \int_0^t (S(x) - R(x))\,dx$

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$

(c) $Y'(t) = S(t) - R(t)$

$Y'(4) = S(4) - R(4) = -1.908$ or $-1.909\text{ yd}^3/\text{hr}$

1 : answer

(d) $Y'(t) = 0$ when $S(t) - R(t) = 0$.

The only value in $[0, 6]$ to satisfy $S(t) = R(t)$ is $a = 5.117865$.

t	$Y(t)$
0	2500
a	2492.3694
6	2493.2766

The amount of sand is a minimum when $t = 5.117$ or 5.118 hours. The minimum value is 2492.369 cubic yards.

$3: \begin{cases} 1: \text{sets } Y'(t) = 0 \\ 1: \text{critical } t\text{-value} \\ 1: \text{answer with justification} \end{cases}$

2008 AB 3

(a) When $r = 100$ cm and $h = 0.5$ cm, $\frac{dV}{dt} = 2000\text{ cm}^3/\text{min}$ and $\frac{dr}{dt} = 2.5$ cm/min.

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$$

$$2000 = 2\pi(100)(2.5)(0.5) + \pi(100)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.038 \text{ or } 0.039 \text{ cm/min}$$

$4: \begin{cases} 1: \frac{dV}{dt} = 2000 \text{ and } \frac{dr}{dt} = 2.5 \\ 2: \text{expression for } \frac{dV}{dt} \\ 1: \text{answer} \end{cases}$

(b) $\frac{dV}{dt} = 2000 - R(t)$, so $\frac{dV}{dt} = 0$ when $R(t) = 2000$.

This occurs when $t = 25$ minutes.

Since $\frac{dV}{dt} > 0$ for $0 < t < 25$ and $\frac{dV}{dt} < 0$ for $t > 25$, the oil slick reaches its maximum volume 25 minutes after the device begins working.

$3: \begin{cases} 1: R(t) = 2000 \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$

(c) The volume of oil, in cm^3, in the slick at time $t = 25$ minutes is given by $60{,}000 + \int_0^{25} (2000 - R(t))\,dt$.

$2: \begin{cases} 1: \text{limits and initial condition} \\ 1: \text{integrand} \end{cases}$

2009 AB B1

(a) $R(t) = 6 + \int_0^t \frac{1}{16}\left(3 + \sin\left(x^2\right)\right) dx$ $R(3) = 6.610$ or 6.611	$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{expression for } R(t) \\ 1 : R(3) \end{cases}$
(b) $A(t) = \pi(R(t))^2$ $A'(t) = 2\pi R(t) R'(t)$ $A'(3) = 8.858\ \text{cm}^2/\text{year}$	$3 : \begin{cases} 1 : \text{expression for } A(t) \\ 1 : \text{expression for } A'(t) \\ 1 : \text{answer with units} \end{cases}$
(c) $\int_0^3 A'(t)\, dt = A(3) - A(0) = 24.200$ or 24.201 From time $t = 0$ to $t = 3$ years, the cross-sectional area grows by 24.201 square centimeters.	$3 : \begin{cases} 1 : \text{uses Fundamental Theorem of Calculus} \\ 1 : \text{value of } \int_0^3 A'(t)\, dt \\ 1 : \text{meaning of } \int_0^3 A'(t)\, dt \end{cases}$

2009 AB B6

(b) $\int_{20}^{40} v(t)\, dt$ is the particle's change in position in meters from time $t = 20$ seconds to time $t = 40$ seconds. $\int_{20}^{40} v(t)\, dt \approx \frac{v(20) + v(25)}{2} \cdot 5 + \frac{v(25) + v(32)}{2} \cdot 7 + \frac{v(32) + v(40)}{2} \cdot 8$ $= -75$ meters	$3 : \begin{cases} 1 : \text{meaning of } \int_{20}^{40} v(t)\, dt \\ 2 : \text{trapezoidal approximation} \end{cases}$
(d) Since $v'(t) = a(t) > 0$ for $0 < t < 8$, $v(t) \geq 3$ on this interval. Therefore, $x(8) = x(0) + \int_0^8 v(t)\, dt \geq 7 + 8 \cdot 3 > 30$.	$2 : \begin{cases} 1 : v'(t) = a(t) \\ 1 : \text{explanation of } x(8) > 30 \end{cases}$

2010 AB B3

(a) $\int_0^{12} P(t)\, dt \approx 46 \cdot 4 + 57 \cdot 4 + 62 \cdot 4 = 660\ \text{ft}^3$	$2 : \begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{answer} \end{cases}$
(b) $\int_0^{12} R(t)\, dt = 225.594\ \text{ft}^3$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) $1000 + \int_0^{12} P(t)\, dt - \int_0^{12} R(t)\, dt = 1434.406$ At time $t = 12$ hours, the volume of water in the pool is approximately $1434\ \text{ft}^3$.	1 : answer