AP Calculus BC Course Description and Syllabus

· **Course Objective:** This course is designed to prepare the students for the AP Exam in May. Students will learn to use graphical, numerical, verbal and analytical techniques in solving calculus problems involving limits, derivatives, and integrals, and their applications.

· **Resources:** All of these are made available by the school.

1. **Textbook:** *Calculus of a Single Variable* (9^{th} ed.) – Larson & Edwards (ISBN: 978-0-547-21290-6)
2. **TI Nspire CAS CX:** Perhaps the best calculator on the market for AP Calculus is the TI Nspire CAS CX. We will be using this calculator extensively this year and it is the best, most user-friendly calculator that is allowed on the AP Calculus exam in May. The calculator will not only be used to get answers, but also to interpret results you have on paper as well as supporting your conclusions. I do have a classroom set that you may use during class, but it would be highly advantageous for you to have your own so you can practice with it at home.
3. ***Calculus Calculator Labs Workbook* (by Benita Albert & Phyllis Hillis):** Through these labs students will investigate precalculus and calculus concepts on the graphing calculator. With each lab you should be prepared to explain your findings either in writing or verbally.

· **NO FLIPPED CLASSROOM THIS YEAR:** I'm sorry if you were looking forward to that this year. In my years of using the flipped classroom I've discovered that it makes it extremely easy for lazy students to be even lazier. My class structure this year will be more traditional, with notes taken in class and book work assigned for homework. We will occasionally have a video lesson for homework, but that will not be the norm.

· **Expectations:**

1. Be mature. Don't try to use faulty or cliché logic to escape your responsibilities as a student – such as, "When are we ever going to use this?" The gods of math are watching, it's on my test, and it's likely on the AP Exam in May. That should be good enough motivation to do your best.
2. Do all your homework and come back to class with questions if you're confused.
3. If you have an extreme lack of understanding in a particular area come see me outside of class time. I cannot spend time giving a private tutoring session during class time.
4. Be respectful of other me, my classroom, and other students.

· **Grading Breakdown:** Grades will be given by total points. This means there are no categories and I compute averages the easy way – simply divide the total points you accumulated by the total points possible. Classwork is worth about 10 points per assignment, weekly quizzes are worth anywhere from 30 to 60 points, and the unit tests will typically be 180 points. All major tests will be given on an extended day. They consist first of a calculator portion, then a non-calculator portion. Tests are graded according to AP Exam grading criteria, then scaled to reflect an appropriate percentage for the grade recorded.

· **Classroom Donations:** I am usually in need of paper towels, Kleenex, and hand sanitizer. If you are a kind soul who wants to make a donation these are the things I would like.

· **Social Media:** I will occasionally post announcements and such on Instagram and Twitter. Follow @MathGnome to get these announcements.

I can be reached at pcarboni@hoover.k12.al.us if you ever have any questions or concerns regarding the class.

AP Calculus BC Topics Outline

Unit 1: A review of functions

Upon completing this unit students should be able to do the following:

1. Accurately graph the functions in the library of functions (lines, quadratic, cubic, absolute value, reciprocal, exponential, logarithmic, greatest integer, trig, and signum)
2. Apply transformations to the library of functions.
3. Identify the domain, range, intercepts, symmetry, period, and discontinuities of functions.
4. Graph polar & parametric functions.
5. **Use the graphing calculator for the following:**
    - **a.** Solve equations graphically by finding *x* intercepts or intersections of two graphs.
    - **b.** Explore domain and range of functions graphically.
    - **c.** Store solutions in the calculator for easy retrieval.

Topic	Textbook Section	Activities/Projects
20 Functions to Know & Love	Supplement	**1. Transformations Project:** Reviews transformations, characteristics of functions (domain, range, intercepts, discontinuities, period, and symmetry), and knowing the library of functions. **2. Calculus Calculator Lab 1 - *What Should You See Graphically?*** Students use their calculators to investigate the affects of negatives and absolute values on the function $f(x) = x^x$. **3. Calculus Calculator Lab 2 – *Functional Collages*:** Students explore piece-wise functions and analyze implicit relations by decomposing them into functional pieces.
Discontinuities & asymptotes	Supplement	
Domain & Range	Supplement	
Intermediate Value Theorem	1.4 (P.80, #83-94)	
Parametric Graphs	10.2	
Polar Graphs	10.4	

Unit 2: Limits

Upon completing this unit students should be able to do the following:

1. Solve limits of functions by analyzing their graphs or a table of values.
2. Use algebra to solve limits through substitution, factoring, conjugates, or other algebraic techniques.
3. Evaluate one-sided limits.
4. Use limits to describe asymptotic or unbounded behavior of graph.
5. Use limits to discuss both verbally and in writing the continuity of a function at a given point.
6. Describe the three types of discontinuities (infinite, removable, and jump).
7. Explain both verbally and in writing the relationship between slope and rates of change.
8. Find average rates of change and estimate instantaneous rate of change from a table of values.
9. Use the limit of a difference quotient to compute instantaneous rate of change or general derivatives.
10. Write the equation of a line tangent or normal to a graph at a given *x* coordinate.
11. Sketch the graph of $f'(x)$ given the graph of $f(x)$.
12. Explain both verbally and in writing the relationship between differentiability and continuity.
13. Recognize important limits – most notably the limit definition of a derivative and the limit forms of the constant *e*.
14. Use the graphing calculator to find limits of functions by analyzing the graph or a table of values.

Topic	Textbook Section	Activities/Projects
Evaluate Limits with Graphs	1.2	**1. Calculus Calculator Lab 3 - *Important Limits and their Extensions*:** Students will use their calculator's graphing and table functions to explore the following limits: a. $\lim_{x\to 0}(1+x)^{1/x}$ b. $\lim_{x\to 0}\frac{\sin(x)}{x}$ c. $\lim_{x\to 0}\frac{\cos(x)-1}{x}$ **2. Group Presentation:** Students will present AP problems to the class, explain the solutions, and field questions from their peers.
Evaluate Limits with Tables	1.2	
Evaluate Limits with Algebra a. Plug in b. Factor c. Conjugates	1.3	
One-sided limits	1.4	
Limits involving infinity a. As $x\to\infty$. b. Answers involving ∞ (unbounded behavior)	a. 3.5 b. 1.5	
Rates of Change ➔ Approximate instantaneous rate of change on a table or graph	Supplement	
Continuity (defined by limits)	1.4	
Limit Derivatives with difference quotients a. Slope at a point (instantaneous rate of change) b. General derivative c. Tangent and Normal Lines	2.1	
Relating graphs of *f* and *f'*	Supplement	
Differentiability and continuity	Supplement	

Unit 3: Derivative Shortcuts and Basic Applications

Upon completing this unit students should be able to do the following:

1. Use derivative shortcuts to find derivatives of polynomials, quotients, products, composition of functions, exponential functions, and logarithmic functions.
2. Find derivatives implicitly (used for equations that cannot easily expressed as $y = f\left(x\right)$).
3. Use derivative shortcuts to find equations of tangent and normal lines and find locations of horizontal and vertical tangents.
4. Use properties of logarithms to simplify functions and make differentiation easier.
5. Use implicit differentiation to prove the formula for the derivative of an inverse function.
6. Use properties of $f'(x)$ and $f''(x)$ to describe characteristics of $f(x)$.
7. Use position, velocity, and acceleration to explain particle motion.
8. Find derivatives of polar & parametric curves.
9. Use vectors to describe particle motion.
10. **Students will use their graphing calculator to do the following:**
    - **a.** Compute the value of the derivative at a single point.
    - **b.** Use the graphing capabilities to solve equations and find critical numbers.
    - **c.** Store solutions to solved equations for later use.
    - **d.** Analyze the graph of a function at non-differentiable points and explain why the symmetrical derivative approach in Calculator Lab 4 might yield an incorrect answer.

Topic	Textbook Section	Activities/Projects
Power Rule (include expanding & STDs)	2.2	**1. Group Presentation:** Students will present AP problems to the class, explain the solutions, and field questions from their peers. **2. Calculus Calculator Lab 4 – *Numerical Derivatives?*** Students explore the calculator's "symmetric derivative" algorithm for calculating numerical derivatives. **3. Calculus Calculator Lab 5 – *Derivative of the Inverse Function*:** Students use their calculators to establish the relationship between the derivative of a function and its inverse graphically, numerically, and symbolically. **4. Calculator Exploration:** Students will graph functions and their derivatives on the same screen to investigate how specific characteristics of derivatives (namely extrema and *x*-intercepts) relate to the shape of the function.
Product Rule	2.3	
Quotient Rule	2.3	
Chain Rule	2.4	
Implicit Differentiation	2.5	
Trig Derivatives	2.3	
Exponential Derivatives	5.4 & 5.5	
Logarithmic Derivatives ➔ Include $y = f\left(x\right)^{g(x)}$	5.1 & 5.5	
Inverse Derivatives (with implicit)	Supplement	
Inverse Trig Derivatives	5.6	
Relationships among f, f', and f''	Supplement	
Position, Velocity, & Acceleration a. Direction of Travel b. Total distance vs. displacement c. Speeding up and slowing down	Supplement	
Parametric Derivatives ➔ Include 2nd derivatives	10.3	
Polar Derivatives	10.3 (convert to parametric)	
Vectors (differentiation only) ➔ As related to position, velocity, and acceleration	Supplement	

Unit 4: Applications of Derivatives

Upon completing this unit students should be able to do the following:

1. Apply the Extreme Value, Mean Value, and Rolle's Theorem to functions.
2. Use the first and second derivative to accurately graph functions.
3. Use their knowledge of first and second derivatives to justify locations of extrema and inflection points and intervals of increase or decrease. Justifications must be in written sentences.
4. Use linear approximation to estimate function values, and determine if those estimates are over or under approximations.
5. Solve related rates problems.
6. Find critical numbers and use them to solve optimization problems.
7. Use L'Hopital's Rule to solve limits.
8. **Given a function and a point *P* that is *not* on the curve, students will do the following:**
    a. Find the minimum distance from point *P* to the function.
    b. Find the equation of the line tangent to the function that passes through point *P*.
9. Students will use their graphing calculators to confirm locations of extreme and inflections points by analyzing the graphs of the first and second derivatives.

Topic	Textbook Section	Activities/Projects
Extreme Value Theorem	3.1	**Note:** When supporting your conclusions in writing, students *must* express their justifications in complete sentences from this point forward. **1. Group Presentation:** Students will present AP problems to the class, explain the solutions, and field questions from their peers. Explanations will include thorough explanations. **2. Calculus Calculator Lab 6 – *Getting to Know "Cow" Culus Differentially*:** Students use their calculators to follow a cow traveling along quadratic and cubic graphs. Students "stand" on the coordinate plane and find when the cow is closest to them and when they can "tangentially swat" a fly off the cow. **3. Calculus Calculator Lab 7 – *Mattie's Mean Value Adventure*:** Students use the graphing calculator to interpret the Mean Value Theorem graphically, numerically, and analytically. **4. Justifying Answers:** Students, in groups and individually, will use their knowledge of theorems and curve sketching to thoroughly explain solutions. Explanations will be both oral and written.
Mean Value Theorem & Rolle's Theorem	3.2	
Curve Sketching **a.** Intercepts **b.** Discontinuities **c.** Asymptotes **d.** Increase/Decrease intervals **e.** Local Extrema **f.** Concavity **g.** Inflection Points	3.6 (summary) **a.** supplement **b.** supplement **c.** supplement **d.** 3.3 **e.** 3.3 **f.** 3.4 **g.** 3.4	
Linear Approximation	Supplement	
Optimization	3.7	
Related Rates	2.6	
L'Hopital's Rule **a.** Product & Quotient Indeterminate **b.** Power indeterminate forms	8.7	

Unit 5: Integration and Basic Applications of Integration

Upon completing this unit students should be able to do the following:

1. Use area approximation techniques (Riemann Sums and Trapezoidal Rule) to estimate area under a curve.
2. Estimate areas with Riemann sums or trapezoidal rule when given a table of values or a graph.
3. Use areas to measure accumulation over an interval when given a rate of change.
4. Use geometry to measure exact area under linear functions and semicircles.
5. Use infinite limits with Riemann sums to investigate the connection between Riemann Sums and definite integrals.
6. Find antiderivatives through recognition, power rule, u-substitution, parts, trig substitution, and partial fractions.
7. **Understand and apply the both parts of the Fundamental Theorem of Calculus:**
    a. Differentiation of integral defined functions.
    b. Using antiderivatives to compute exact area under a curve.
8. Determine the convergence or divergence of improper integrals.
9. **Students will use their graphing calculators to do the following:**
    a. Evaluate definite integrals.
    b. Compute areas with the graphing features.

Topic	Textbook Section	Activities/Projects
Polynomial & recognition antiderivatives	4.1	**1. Group Presentation:** Students will present AP problems to the class, explain the solutions, and field questions from their peers. Explanations will include thorough explanations. **2. Calculus Calculator Lab 10 – *The Integral Function*:** Students extend the concept of a definite integral by graphing accumulation functions on their calculators. **3. Calculus Calculator Lab 11 – *Want to Market a Calculus Book?*** Students are given a function the represents the sales rate of a calculus book and use antiderivatives to analyze the number of books sold over specified intervals.
Areas: **a.** Riemann Sums (left, right, and midpoints) **b.** Trapezoids **c.** Definite Integral Notation & Properties **d.** As related to position, velocity, and direction of travel **e.** Exact areas with geometry **f.** Exact areas with antiderivatives	**a.** Supplement **b.** 4.6 **c.** 4.3 **d.** Supplement **e.** Supplement **f.** 4.4 (FTC)	
Fundamental Theorem of Calculus	4.4	
Integration with *u*-substitution	4.5 & 8.1	
Integration with trig identities	8.3	
Integration with parts	8.2	
Integration with partial fractions (non-repeating linear factors only)	8.5	
Trig Substitution (not on AP Exam)	8.4	
Using initial values to find specific antiderivatives	Supplement	
Improper Integrals	8.8	

Unit 6: Applications of Integration

Upon completing this unit students should be able to do the following:

1. Use integration to find the area between two curves.
2. **Find the volume of a solid using the following techniques:**
    - **a.** Discs
    - **b.** Washers
    - **c.** Known cross sections
    - **d.** Shells
3. Use an initial condition and a rate of change to compute a final value or amount.
4. Find the length of function over a given interval or the length of a parametric or polar curve.
5. Understand and apply the Average Value Theorem.
6. Find areas of regions enclosed by polar or parametric curves.

Topic	Textbook Section	Activities/Projects
Area between two curves	7.1	**1. Group Presentation:** Students will present AP problems to the class, explain the solutions, and field questions from their peers. Explanations will include thorough explanations. **2. Calculus Calculator Lab 8 – *"Mattie, We've Got You Clocked"*** Students transform velocity data into accumulated distance approximations with Riemann sums and regression analysis **3. Calculus Calculator Lab 12 – *Cars A dn B, Where are You?*** Students use calculators to investigate the relationships between speed, velocity, and acceleration through tables, Riemann sums, and trapezoidal approximations. **4. Calculus Calculator Lab 14 – *Applications of Calculus, Review 1*:** Students review differentiation and area applications of calculus. **5. Calculus Calculator Lab 15 – *Applications of Calculus, Review II*:** Students review the application of integral calculus in regards to areas and volumes.
Volumes of revolutions: **a.** Disks **b.** Washers **c.** Known Cross Sections **d.** Shells (not on AP Exam)	 **a.** 7.2 **b.** 7.2 **c.** 7.2 **d.** 7.3	
Areas as related to position & distance (total change). Include vectors.	4.4 & supplement	
Arc Length	7.4	
Average Value Theorem	4.4	
Area under or enclosed by parametric & polar curves	10.3 & 10.5	

Unit 7: Differential Equations

Upon completing this unit students should be able to do the following:

1. Generate a slope field for a given differential equation.
2. Use a slope field and an initial condition to sketch the solution to a differential equation.
3. Solve separable differential equations.
4. Use Euler's Method to approximate values of the solution to a differential equation.
5. Recognize a logistic differential equation and generate the solution for modeling.
6. Understand and apply their knowledge of isoclines and equilibrium solutions.
7. Students will use their calculators to compute multiple iterations of Euler's Method.

Topics	Textbook Section	Activities/Projects
Slope Fields	6.1	**1. Group Presentation:** Students will present AP problems to the class, explain the solutions, and field questions from their peers. Explanations will include thorough explanations. **2. Calculus Calculator Lab 9 – *A Differential Equation that Models Exponential Behavior*:** Students use their calculators to investigate the family of functions whose derivative is proportional to the function. **3. Calculus Calculator Lab 13 – *Differential Equation Visually and Analytically:*** Students use their calculators to investigate slope fields as a technique for finding solutions to differential equations.
Separable Differential Equations **a.** Initial Value Problems **b.** Exponential Growth ($y' = ky$)	 **a.** 6.3 **b.** 6.2 & 6.3	
Euler's Method	6.1	
Logistic Differential Equations	6.3	

Unit 8: Sequences and Series

Upon completing this unit students should be able to do the following:

1. Understand series as a sequence of partial sums.
2. Understand the properties of the following types of series and use them to determine convergence/divergence of the series: geometric, telescoping, harmonic, *p*-series, alternating.
3. Use direct and limit comparison tests and integral test to determine series convergence.
4. Understand and apply the ratio test for convergence.
5. Explain the difference between absolute and conditional convergence.
6. Express functions as power series through differentiation and integration of known power series.
7. Find the radius and interval of convergence for power series.
8. Use Lagrange and alternating series error bound to estimate series sums.
9. Understand and apply Taylor/Maclaurin series.

Topic	Textbook Section	Activities/Projects
Sequences (converge & diverge)	9.1	**1. Group Presentation:** Students will present AP problems to the class, explain the solutions, and field questions from their peers. Explanations will include thorough explanations. **2.** Upon completion of this unit we should have approximately one month to prepare for the AP Exam. During this time we will have frequent mini-mock exams as well as a full length mock exam. Students will have homework every night featuring released AP Exam questions. Students should be prepared every class to defend answers, either verbally or with written sentences, to the assigned problems the previous night.
Series Defined **a.** Discuss difference between sequences & series Discuss convergence & divergence of series	**a.** 9.2 **b.** 9.2	
Divergence Test	9.2	
Geometric Series	9.2	
Harmonic Series	9.3 (part of *p*-series)	
Integral Test & P-Series Estimating sums with improper integrals	9.3	
Direct Comparison	9.4	
Limit Comparison	9.4	
Alternating Series Test Estimating sums of alternating series	9.5	
Absolute vs. Conditional Convergence	9.5	
Ratio Test	9.6	
Root Test	9.6	
Power Series **a.** Radius & Interval of Convergence **b.** Differentiation and integration of power series	(supplement also) **a.** 9.8 **b.** 9.9	
Taylor & Maclaurin Series **a.** Formula for coefficients **b.** Memorized functions $\left(e^x,\ \cos(x),\ \sin(x),\ \frac{1}{1-x},\ \arctan(x)\right)$ **c.** Lagrange error bound	**a.** 9.10 **b.** 9.10 **c.** 9.7	
Summing a Series **a.** Geometric **b.** Telescoping **c.** Recognized Taylor Series	supplement	