Precalculus

Chapter 11 Review

Part A - Measures of Central Tendency, standard deviation, histograms, & box and whisker plots

A. Answer the following based on the given data. These numbers represent the number of battle victories the gnomes had over the warlocks over a 15 year period:

49 52 59 57 60 60 54 62

59 58 54 48 34 44 56

1. Find the mean, median, mode, and quartiles.

$\bar{x} = 53.733$

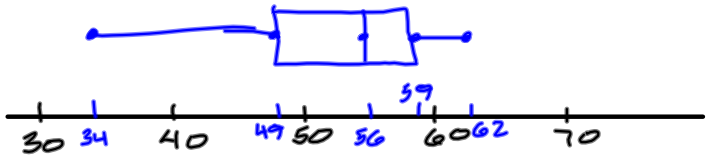
median = 56

mode = 60 & 59

quartiles = $Q_1 = 49$ $Q_3 = 59$

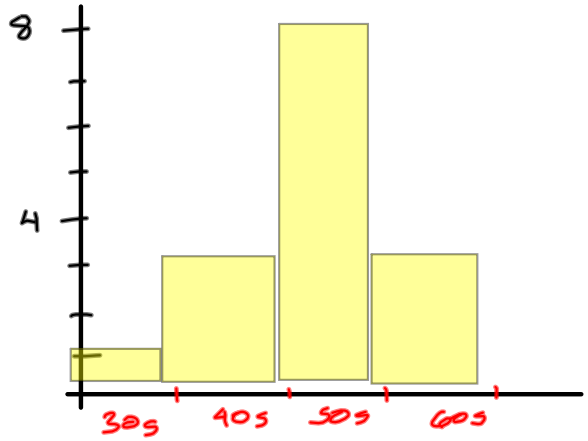
$Q_2 = 56$

2. Construct a box & whisker plot for the data.



30s	1
40s	3
50s	8
60s	3

3. Construct a histogram using 30-39; 40-49; 50-59; 60-69 as categories.



4. Find the standard deviation for the data.

$\sigma = 7.206$

Precalculus

Chapter 11 Review

Section B: Percentiles/cumulative percentages

B. The following frequency table tells the number of students who own *x* amount of garden gnomes. Construct a percentile table for the data:

x	# of students	Cumulative Amt	Percentile
0	5	5	$5/79 = .063 = 6.3$ percentile
1	7	12	$12/79 = .151 = 15.1$ percentile
2	10	22	$22/79 = .278 = 27.8$ percentile
3	11	33	$33/79 = .418 = 41.8$ percentile
4	18	51	$51/79 = .646 = 64.6$ percentile
5	16	67	$67/79 = .848 = 84.8$ percentile
6	12	79	$79/79 = 1.000 = 100$ percentile
	total 79		

Precalculus

Chapter 11 Review

Part C: Probability distributions

C. After polling 100 casual college football fans, the following frequency table shows how many fans attended *x* football games in 2012.

x games	# of fans	$P(x)$	$x \cdot P(x)$	$(x-\mu)^2 \cdot P(x)$
0	9	$9/100 = .09$	$0(.09) = 0$	$(0-2.2)^2(.09) = .4356$
1	14	$14/100 = .14$	$1(.14) = .14$	$(1-2.2)^2(.14) = .2016$
2	37	$37/100 = .37$	$2(.37) = .74$	$(2-2.2)^2(.37) = .0148$
3	28	$28/100 = .28$	$3(.28) = .84$	$(3-2.2)^2(.28) = .1792$
4	12	$12/100 = .12$	$4(.12) = .48$	$(4-2.2)^2(.12) = .3888$
	total 100		sum: 2.2	sum = 1.22

5. Construct a probability distribution for the data.

6. Use the probability distribution to compute the mean for the data.

$\rightarrow \sum x \cdot P(x) = 2.2 = \mu$

7. Use the probability distribution to compute the standard deviation for the data.

$\rightarrow \sigma = \sqrt{\sum (x-\mu)^2 \cdot P(x)} = \sqrt{1.22} = \sigma$

Precalculus

Chapter 11 Review

Part D: Binomial Distributions

D. After extensive research, it is determined that 32% of all Spain Park students have no soul. A thorough physical is conducted on 5 randomly chosen students to determine if they do not have a soul. Construct a binomial distribution for the random variable *X*, which represents the number of students (out of the 5) who do not have a soul. Use the binomial distribution to answer these questions.

success - no soul
$p = .32$
failure - have soul
$q = .68$
$n = 5$

X	P(X)
0	$\binom{5}{0}(.32)^0(.68)^5 = .145 = 14.5\%$
1	$\binom{5}{1}(.32)^1(.68)^4 = .342 = 34.2\%$
2	$\binom{5}{2}(.32)^2(.68)^3 = .322 = 32.2\%$
3	$\binom{5}{3}(.32)^3(.68)^2 = .152 = 15.2\%$
4	$\binom{5}{4}(.32)^4(.68)^1 = .036 = 3.6\%$
5	$\binom{5}{5}(.32)^5(.68)^0 = .003 = 0.3\%$

8. What is the mean and standard deviation for this data?

$\mu = np = 5(.32) = 1.6$

$\sigma = \sqrt{npq} = 1.043$

9. What is the probability that exactly 2 of the 5 students have no soul?

32.2%

10. What is the probability that *at least* 3 students have no soul?

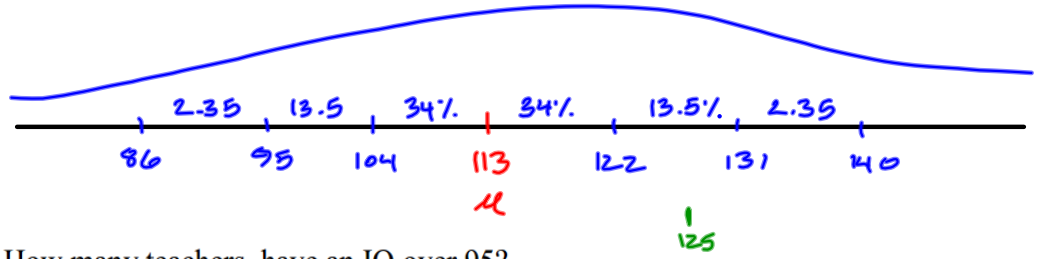
$P(3, 4, \text{or } 5) = 15.2 + 3.6 + 0.3 = 19.1\%$

Precalculus

Chapter 11 Review

Part E: Normal Distributions

E. The IQ of all 132 of the teachers at Spain Park is normally distributed with a mean of 113 and a standard deviation of 9.



11. How many teachers have an IQ over 95?

95 → 113: 47.5%.

over 113: 50 %.

97.5% → .975 (32) = 128.7 teachers

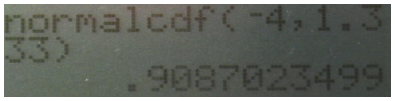
13. How many teachers have an IQ between 104 & 125?

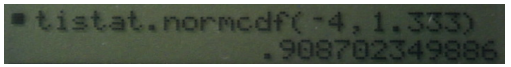
less than 104 : 13.5 + 2.35 = 15.85%.

less than 125: $z = \frac{x-\mu}{\sigma} = \frac{125-113}{9} = 1.333$

T1-84 → normalcdf(-4, 1.333) = .909 → 90.9%.

T1-89 → TISTAT.normcdf(-4, 1.333) = .909

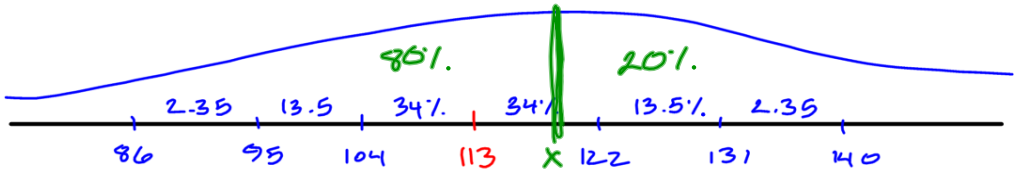




between 104 & 125: 90.9 − 15.85 = 75.05 %.

.7505 (132) = 99.066 teachers

E. The IQ of all 132 of the teachers at Spain Park is normally distributed with a mean of 113 and a standard deviation of 9.



14. Teachers with an IQ in the top 20% get a free Snickers bar. What IQ does a teacher need for this prestigious reward?

Z-score: invnorm(.80) = .842

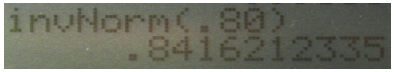
$z = \frac{x-\mu}{\sigma} \rightarrow 9(.842) = \frac{x-113}{9} \cdot 9$

$7.575 = x - 113$

$x = 120.575$

T1-89 →

T1-84 →



sample

15. Thirty teachers are selected at random. What is the probability that the average IQ of these 30 teachers is under 115?

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{9}{\sqrt{30}} = 1.643$ = standard error

$z = \frac{x-\sigma}{\sigma_{\bar{x}}} = \frac{115-113}{1.643} = 1.217$

normalcdf(-4, 1.217) = .888 → 88.8% chance

Precalculus

Chapter 11 Review

Part F: Normal Approximations for Binomial Distributions

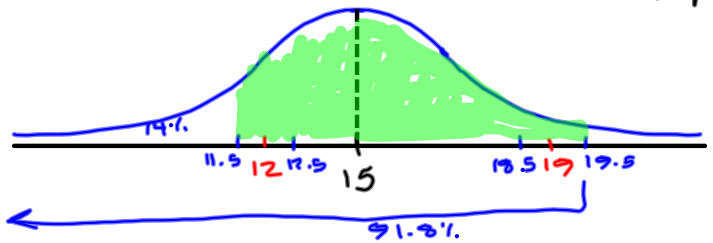
→ success → $p = .3$, $\therefore q = .7$, $n = 50$

F. It is determined that 30% of gnomes have beard implants. Fifty gnomes are selected at random. Use a Normal Approximation of the binomial distribution to answer these questions: $np > 5$ & $nq > 5$

$50(.3) = 15$ $\quad 50(.7) = 35$

$\mu = np = 50(.3) = 15$

$\sigma = \sqrt{npq} = 3.24$



16. What is the probability that more than 17 gnomes have beard implants?

$P(X > 17)$ → use $X = 17.5$ → $z = \frac{17.5 - 15}{3.24} = .772$

normcdf$(-4, .772) = .7799$

77.99%

$100 - 77.99 =$ **22.01%**

17. What is the probability that at least 12 and at most 19 gnomes have beard implants?

$P(12 \le X \le 19)$

↓ 11.5 ↓ 19.5

11.5: $z = \frac{11.5 - 15}{3.24} = -1.080$ → normcdf$(-4, -1.080) = .140 = 14\%$

19.5: $z = \frac{19.5 - 15}{3.24} = 1.389$ → normcdf$(-4, 1.389) = .918 = 91.8\%$

between 11.5 & 19.5: $91.8 - 14 =$ **77.8%**