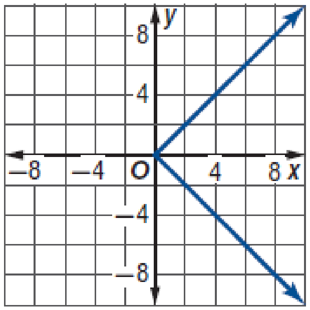
Determine whether each relation represents *y* as a function of *x*.

1. $2y + 5x = 7$

2.



Find each function value.

3. $f(-2)$ if $f(x) = 6 - x^2$

4. $f(3a)$ if $f(x) = \sqrt{x^2 - 4}$

Standardized Test Practice

5. State the domain of $f(x) = \dfrac{1}{\sqrt{x-3}}$.

A $[3, \infty)$

B $(-3, 3)$

C $(3, \infty)$

D $(-\infty, 3) \cup (3, \infty)$

Real-World Example 1 Estimate Function Values

INTERNET **Consider the graph of function *R* shown.**

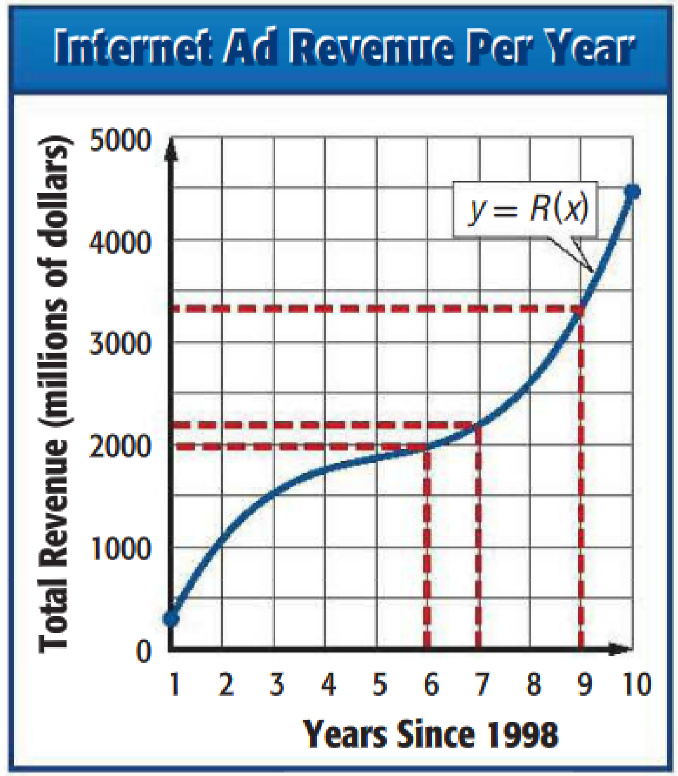
a. Use the graph to estimate total Internet advertising revenue in 2007. Confirm the estimate algebraically.

The year 2007 is 9 years after 1998. The function value at $x = 9$ appears to be about \$3300 million, so the total Internet advertising revenue in 2007 was about \$3.3 billion.

To confirm this estimate algebraically, find $f(9)$.

$$f(9) = 17.7(9)^3 - 269(9)^2 + 1458(9) - 910$$
$$\approx 3326.3 \text{ million or } 3.326 \text{ billion}$$

Therefore, the graphical estimate of \$3.3 billion is reasonable.



b. Use the graph to estimate the year in which total Internet advertising revenue reached $2 billion. Confirm the estimate algebraically.

The value of the function appears to reach $2 billion or $2000 million for x-values between 6 and 7. So, the total revenue was nearly $2 billion in 1998 + 6 or 2004 but had exceeded $2 billion by the end of 1998 + 7 or 2005.

To confirm algebraically, find $f(6)$ and $f(7)$.

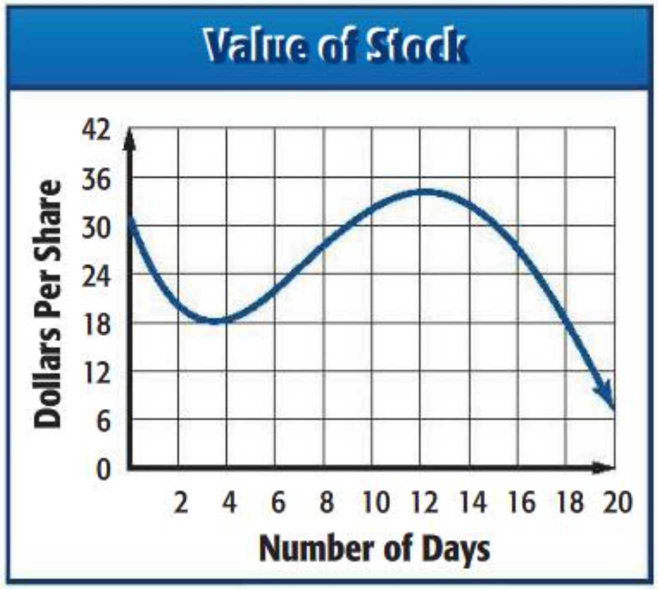
$f(6) = 17.7(6)^3 - 269(6)^2 + 1458(6) - 910$ or about 1977 million

$f(7) = 17.7(7)^3 - 269(7)^2 + 1458(7) - 910$ or about 2186 million

In billions, $f(6) \approx 1.977$ billion and $f(7) \approx 2.186$ billion. Therefore, the graphical estimate that total Internet advertising revenue reached $2 billion in 2005 is reasonable.

Guided Practice

1. STOCKS An investor assessed the average daily value of a share of a certain stock over a 20-day period. The value of the stock can be approximated by $v(d) = 0.002d^4 - 0.11d^3 + 1.77d^2 - 8.6d + 31, 0 \leq d \leq 20$, where d represents the day of the assessment.

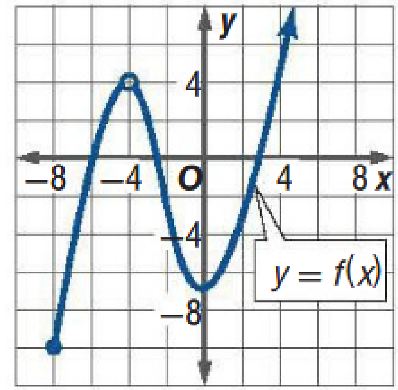


A. Use the graph to estimate the value of the stock on the 10th day. Confirm your estimate algebraically.

B. Use the graph to estimate the days during which the stock was valued at \$30 per share. Confirm your estimate algebraically.

Example 2 Find Domain and Range

Use the graph of f to find the domain and range of the function.



Domain

- The dot at $(-8, -10)$ indicates that the domain of f starts at and includes -8.
- The circle at $(-4, 4)$ indicates that -4 is not part of the domain.
- The arrow on the right side indicates that the graph will continue without bound.

The domain of f is $[-8, -4) \cup (-4, \infty)$. In set-builder notation, the domain is $\{x \mid -8 \leq x, x \neq -4, x \in \mathbb{R}\}$.

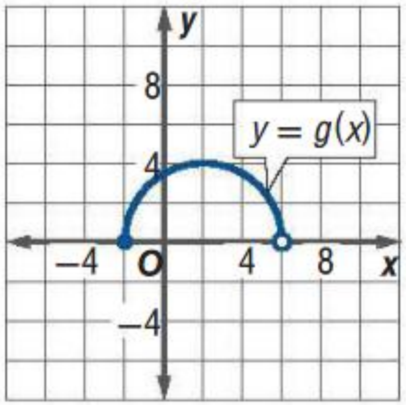
Range

The graph does not extend below $f(-8)$ or -10, but $f(x)$ increases without bound for greater and greater values of x. So, the range of f is $[-10, \infty)$.

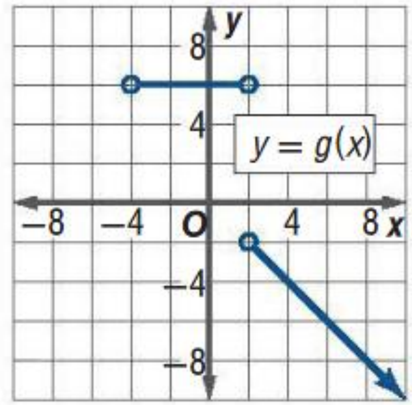
GuidedPractice

Use the graph of g to find the domain and range of each function.

2A.



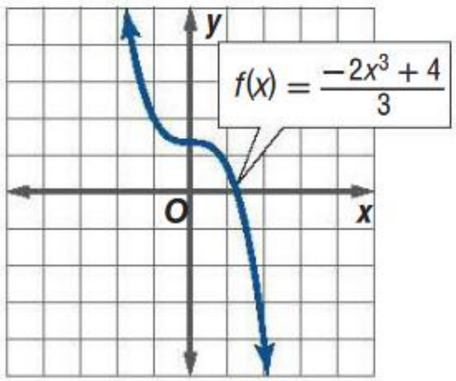
2B.



Example 3 Find y-Intercepts

Use the graph of each function to approximate its y-intercept. Then find the y-intercept algebraically.

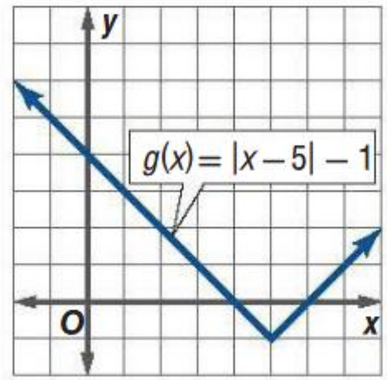
a.



Estimate Graphically

It appears that $f(x)$ intersects the y-axis at approximately $\left(0, 1\frac{1}{3}\right)$, so the y-intercept is about $1\frac{1}{3}$.

b.



Estimate Graphically

It appears that $g(x)$ intersects the y-axis at (0, 4), so the y-intercept is 4.

Solve Algebraically

Find $f(0)$.

$$f(0) = \frac{-2(0)^3 + 4}{3} \text{ or } \frac{4}{3}$$

The y-intercept is $\frac{4}{3}$ or $1\frac{1}{3}$.

Solve Algebraically

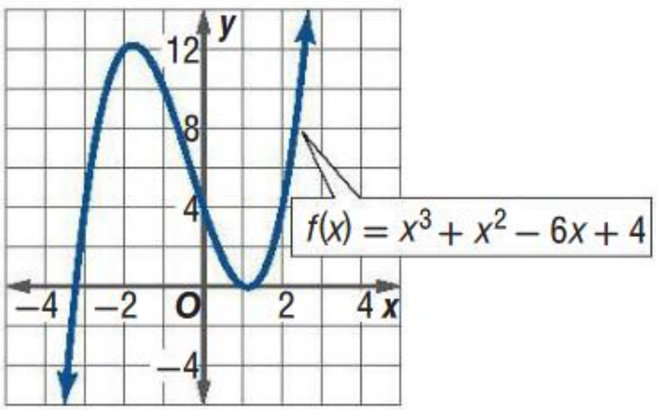
Find $g(0)$.

$$g(0) = |0 - 5| - 1 \text{ or } 4$$

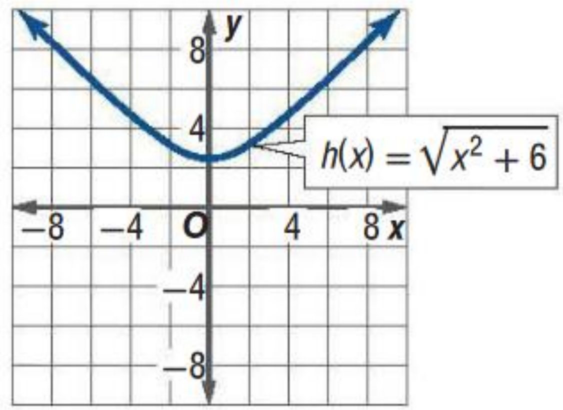
The y-intercept is 4.

GuidedPractice

3A.

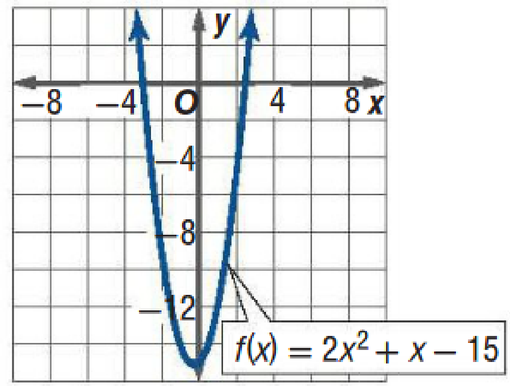


3B.



Example 4 Find Zeros

Use the graph of $f(x) = 2x^2 + x - 15$ to approximate its zero(s). Then find its zero(s) algebraically.



Estimate Graphically

The x-intercepts appear to be at about −3 and 2.5.

Solve Algebraically

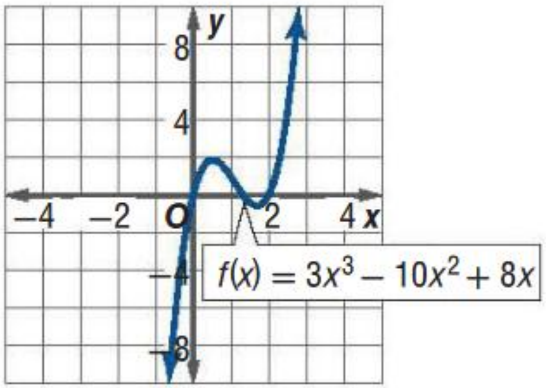
$2x^2 + x - 15 = 0$		Let $f(x) = 0$.
$(2x - 5)(x + 3) = 0$		Factor.
$2x - 5 = 0$ or	$x + 3 = 0$	Zero Product Property
$x = 2.5$	$x = -3$	Solve for x.

The zeros of f are −3 and 2.5.

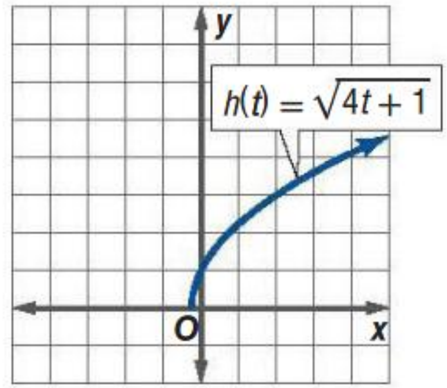
Guided Practice

Use the graph of each function to approximate its zero(s). Then find its zero(s) algebraically.

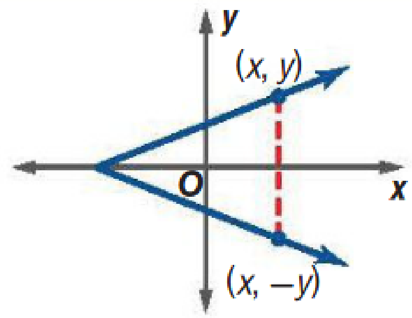
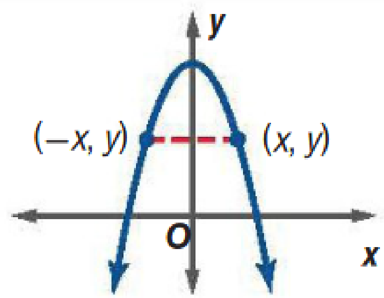
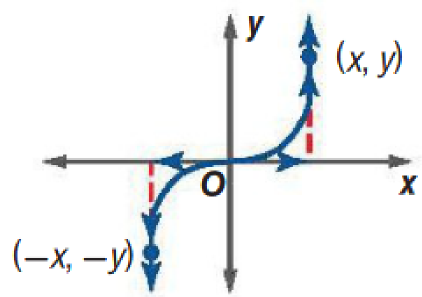
4A.



4B.



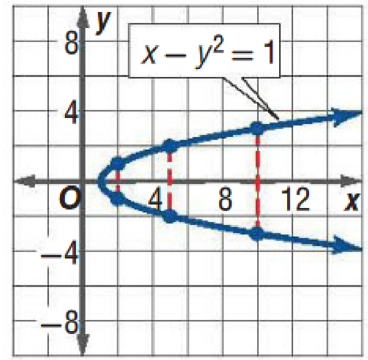
KeyConcept Tests for Symmetry

Graphical Test	Model	Algebraic Test
The graph of a relation is *symmetric with respect to the x-axis* if and only if for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.		Replacing y with $-y$ produces an equivalent equation.
The graph of a relation is *symmetric with respect to the y-axis* if and only if for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.		Replacing x with $-x$ produces an equivalent equation.
The graph of a relation is *symmetric with respect to the origin* if and only if for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.		Replacing x with $-x$ and y with $-y$ produces an equivalent equation.

Example 5 Test for Symmetry

Use the graph of each equation to test for symmetry with respect to the *x*-axis, *y*-axis, and the origin. Support the answer numerically. Then confirm algebraically.

a. $x - y^2 = 1$



Analyze Graphically

The graph appears to be symmetric with respect to the *x*-axis because for every point (x, y) on the graph, there is a point $(x, -y)$.

Support Numerically

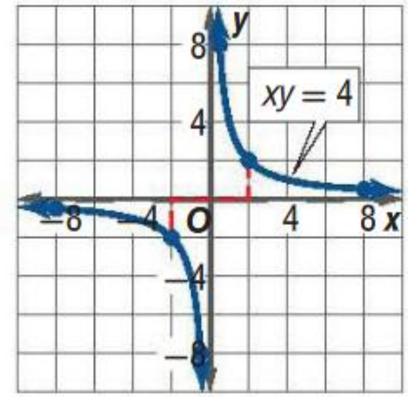
A table of values supports this conjecture.

x	2	2	5	5	10	10
y	1	−1	2	−2	3	−3
(*x*, *y*)	(2, 1)	(2, −1)	(5, 2)	(5, −2)	(10, 3)	(10, −3)

Confirm Algebraically

Because $x - (-y)^2 = 1$ is equivalent to $x - y^2 = 1$, the graph is symmetric with respect to the *x*-axis.

b. $xy = 4$



Analyze Graphically

The graph appears to be symmetric with respect to the origin because for every point (x, y) on the graph, there is a point $(-x, -y)$.

Support Numerically

A table of values supports this conjecture.

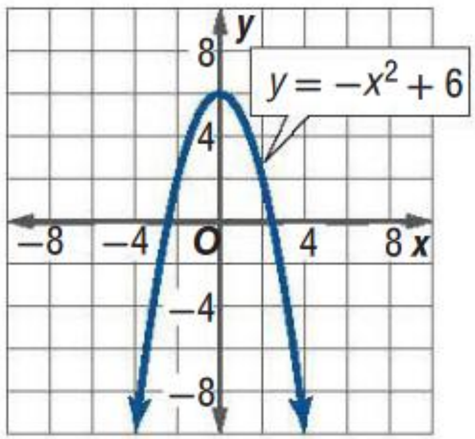
x	−8	−2	−0.5	0.5	2	8
y	−0.5	−2	−8	8	2	0.5
(x, y)	(−8, −0.5)	(−2, −2)	(−0.5, −8)	(0.5, 8)	(2, 2)	(8, 0.5)

Confirm Algebraically

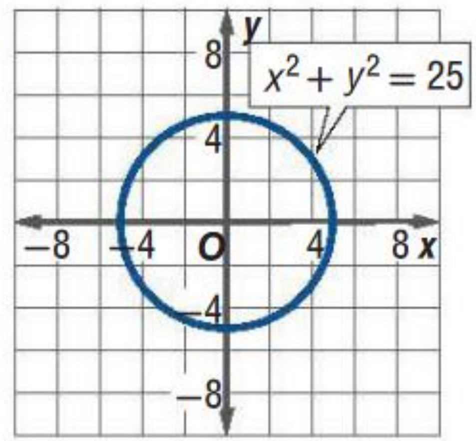
Because $(-x)(-y) = 4$ is equivalent to $xy = 4$, the graph is symmetric with respect to the origin.

GuidedPractice

5A.



5B.

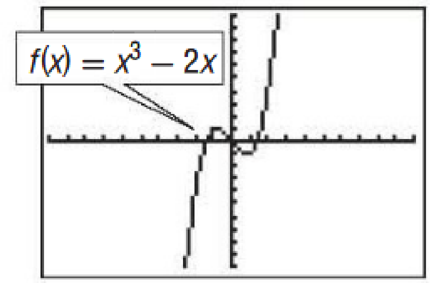


Example 6 Identify Even and Odd Functions

GRAPHING CALCULATOR **Graph each function. Analyze the graph to determine whether each function is *even*, *odd*, or *neither*. Confirm algebraically. If odd or even, describe the symmetry of the graph of the function.**

a. $f(x) = x^3 - 2x$

It appears that the graph of the function is symmetric with respect to the origin. Test this conjecture.



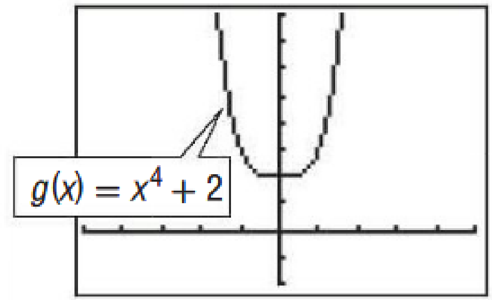
[−10, 10] scl: 1 by [−10, 10] scl: 1

$f(-x) = (-x)^3 - 2(-x)$	Substitute $-x$ for x.
$= -x^3 + 2x$	Simplify.
$= -(x^3 - 2x)$	Distributive Property
$= -f(x)$	Original function $f(x) = x^3 - 2x$

The function is odd because $f(-x) = -f(x)$. Therefore, the function is symmetric with respect to the origin.

b. $g(x) = x^4 + 2$

It appears that the graph of the function is symmetric with respect to the y-axis. Test this conjecture.



[−5, 5] scl: 1 by [−2, 8] scl: 1

$g(-x) = (-x)^4 + 2$ — Substitute $-x$ for x.

$= x^4 + 2$ — Simplify.

$= g(x)$ — Original function $g(x) = x^4 + 2$

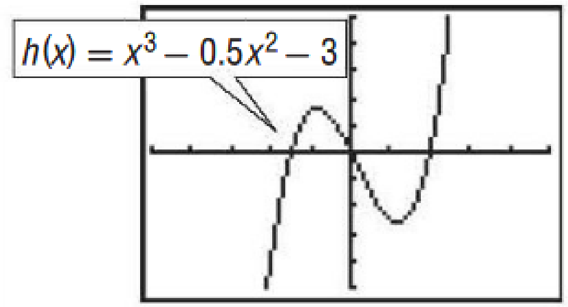
The function is even because $g(-x) = g(x)$. Therefore, the function is symmetric with respect to the y-axis.

c. $h(x) = x^3 - 0.5x^2 - 3x$

It appears that the graph of the function may be symmetric with respect to the origin. Test this conjecture algebraically.

$$h(-x) = (-x)^3 - 0.5(-x)^2 - 3(-x) \quad \text{Substitute } -x \text{ for } x.$$

$$= -x^3 - 0.5x^2 + 3x \quad \text{Simplify.}$$



[–5, 5] scl: 1 by [–5, 5] scl: 1

Because $-h(x) = -x^3 + 0.5x^2 + 3x$, the function is neither even nor odd because $h(-x) \neq h(x)$ and $h(-x) \neq -h(x)$.

GuidedPractice

6A. $f(x) = \frac{2}{x^2}$

6B. $g(x) = 4\sqrt{x}$

6C. $h(x) = x^5 - 2x^3 + x$