AP CALCULUS
Stuff you **MUST** Know Cold

Curve sketching and analysis

$y = f(x)$ must be continuous at each:

critical point: $\frac{dy}{dx} = 0$ or <u>undefined</u>.

local minimum :

$\frac{dy}{dx}$ goes (-,0,+) or (-,und,+)

or $\frac{d^2y}{dx^2} > 0$.

local maximum :

$\frac{dy}{dx}$ goes (+,0,-) or (+,und,-)

or $\frac{d^2y}{dx^2} < 0$.

pt of inflection : concavity changes.

$\frac{d^2y}{dx^2}$ goes (+,0,-),(-,0,+), (+,und,-), or (-,und,+)

Basic Derivatives

$$\frac{d}{dx}(x^n) = nx^{n-1}$$
$$\frac{d}{dx}(\sin x) = \cos x$$
$$\frac{d}{dx}(\cos x) = -\sin x$$
$$\frac{d}{dx}(\tan x) = \sec^2 x$$
$$\frac{d}{dx}(\cot x) = -\csc^2 x$$
$$\frac{d}{dx}(\sec x) = \sec x \tan x$$
$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$
$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$
$$\frac{d}{dx}(e^x) = e^x$$

More Derivatives

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$
$$\frac{d}{dx}\left(\cot^{-1} x\right) = \frac{-1}{1+x^2}$$
$$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}\left(\csc^{-1} x\right) = \frac{-1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}(a^x) = a^x \ln a$$
$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

Differentiation Rules

Chain Rule

$$\frac{d}{dx}[f(u)] = f'(u)\frac{du}{dx}$$
$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Product Rule

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$$

"PLUS A CONSTANT"

The Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

where $F'(x) = f(x)$.

Corollary to FTC

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)dt = f\left(b(x)\right)b'(x) - f\left(a(x)\right)a'(x)$$

Intermediate Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, then for any number c between $f(a)$ and $f(b)$, there exists a number d in the open interval (a, b) such that $f(d) = c$.

Rolle's Theorem

If the function $f(x)$ is continuous on $[a, b]$, the first derivative exist on the interval (a, b), and $f(a) = f(b)$; then there exists a number $x = c$ on (a, b) such that

$$f'(c) = 0.$$

Mean Value Theorem

If the function $f(x)$ is continuous on $[a, b]$, and the first derivative exists on the interval (a, b), then there exists a number $x = c$ on (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Theorem of the Mean Value

If the function $f(x)$ is continuous on $[a, b]$ and the first derivative exist on the interval (a, b), then there exists a number $x = c$ on (a, b) such that

$$f(c) = \frac{\int_a^b f(x)dx}{(b-a)}.$$

This value $f(c)$ is the "average value" of the function on the interval $[a, b]$.

Trapezoidal Rule

$$\int_a^b f(x)dx = \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

Solids of Revolution and friends

<u>Disk Method</u>

$$V = \pi \int_a^b [R(x)]^2\, dx$$

<u>Washer Method</u>

$$V = \pi \int_a^b \left([R(x)]^2 - [r(x)]^2\right) dx$$

<u>Shell Method</u>(no longer on AP)

$$V = 2\pi \int_a^b r(x)h(x)dx$$

<u>ArcLength</u>

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}dx$$

<u>Surface of revolution</u> (No longer on AP)

$$S = 2\pi \int_a^b r(x)\sqrt{1 + [f'(x)]^2}dx$$

Distance, velocity and acceleration

velocity $= \frac{d}{dt}$ (position).

acceleration $= \frac{d}{dt}$ (velocity).

velocity vector $= \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$.

speed $= |v| = \sqrt{(x')^2 + (y')^2}$.

$$\text{Distance} = \int_{\text{initial time}}^{\text{final time}} |v|dt = \int_{t_0}^{t_f} \sqrt{(x')^2 + (y')^2}dt$$

average velocity $= \dfrac{\text{final position} - \text{initial position}}{\text{total time}}$.

Integration by Parts

$$\int u dv = uv - \int v du$$

Integral of Log

$$\int \ln x dx = x \ln x - x + \text{C}.$$

Taylor Series

If the function f is "smooth" at $x = a$, then it can be approximated by the n^{th} degree polynomial

$$\begin{aligned} f(x) \approx f(a) &+ f'(a)(x-a) \\ &+ \frac{f''(a)}{2!}(x-a)^2 + \cdots \\ &+ \frac{f^{(n)}(a)}{n!}(x-a)^n. \end{aligned}$$

Maclaurin Series

A Taylor Series about $x = 0$ is called Maclaurin.

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \cdots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

Lagrange Error Bound

If $P_n(x)$ is the n_{th} degree Taylor polynomial of $f(x)$ about c and $\left|f^{(n+1)}(t)\right| \leq M$ for all t between x and c, then

$$|f(x) - P_n(x)| \leq \frac{M}{(n+1)!}|x-c|^{n+1}$$

Alternating Series Error Bound

If $S_N = \sum_{k=1}^{N}(-1)^n a_n$ is the N^{th} partial sum of a convergent alternating series, then

$$|S_\infty - S_N| \leq |a_{N+1}|$$

Euler's Method

If given that $\frac{dy}{dx} = f(x,y)$ and that the solution passes through (x_0, y_0),

$$y(x_0) = y_0$$

$$\vdots$$

$$y(x_n) = y(x_{n-1}) + f(x_{n-1}, y_{n-1}) \cdot \Delta x$$

In other words:

$$x_{\text{new}} = x_{\text{old}} + \Delta x$$

$$y_{\text{new}} = y_{\text{old}} + \left.\frac{dy}{dx}\right|_{(x_{\text{old}}, y_{\text{old}})} \cdot \Delta x$$

Ratio Test

The series $\displaystyle\sum_{k=0}^{\infty} a_k$ converges if

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| < 1.$$

If limit equals 1, you know nothing.

Polar Curves

For a polar curve $r(\theta)$, the **Area** inside a "leaf" is

$$\int_{\theta 1}^{\theta 2} \frac{1}{2}[r(\theta)]^2 d\theta,$$

where $\theta 1$ and $\theta 2$ are the "first" two times that $r = 0$.

The **slope** of $r(\theta)$ at a given θ is

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy/d\theta}{dx/d\theta} \\ &= \frac{\frac{d}{d\theta}[r(\theta)\sin\theta]}{\frac{d}{d\theta}[r(\theta)\cos\theta]} \end{aligned}$$

l'Hopital's Rule

If $\dfrac{f(a)}{g(a)} = \dfrac{0}{0}$ or $= \dfrac{\infty}{\infty}$,

then $\displaystyle\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$.