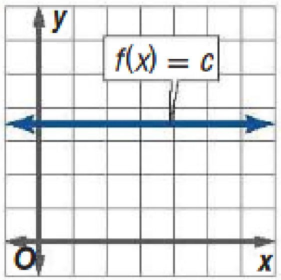
1.5 Parent Functions and Transformations

1 Parent Functions A *family of functions* is a group of functions with graphs that display one or more similar characteristics. A **parent function** is the simplest of the functions in a family. This is the function that is transformed to create other members in a family of functions.

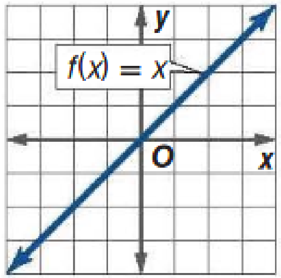
In this lesson, you will study eight of the most commonly used parent functions. You should already be familiar with the graphs of the following linear and polynomial parent functions.

KeyConcept Linear and Polynomial Parent Functions

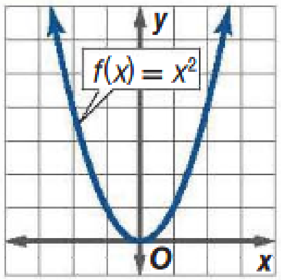
A **constant function** has the form $f(x) = c$, where c is any real number. Its graph is a horizontal line. When $c = 0$, $f(x)$ is the **zero function**.



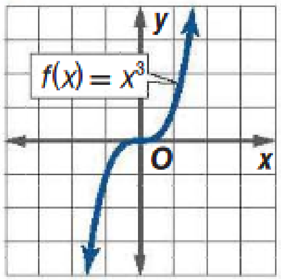
The **identity function** $f(x) = x$ passes through all points with coordinates (a, a).



The **quadratic function** $f(x) = x^2$ has a U-shaped graph.

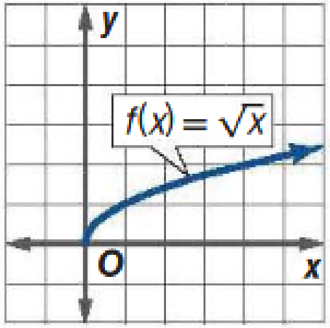


The **cubic function** $f(x) = x^3$ is symmetric about the origin.

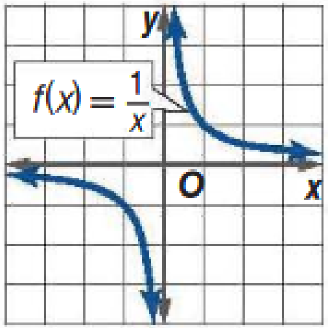


KeyConcept Square Root and Reciprocal Parent Functions

The **square root function** has the form $f(x) = \sqrt{x}$.



The **reciprocal function** has the form $f(x) = \frac{1}{x}$.



KeyConcept Absolute Value Parent Function

Words

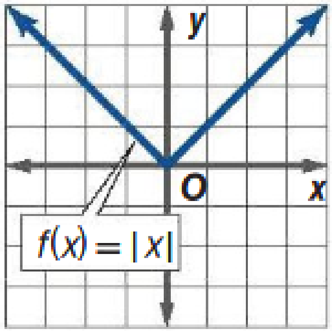
The **absolute value function**, denoted $f(x) = |x|$, is a V-shaped function defined as

$$f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}.$$

Examples

$|-5| = 5$, $|0| = 0$, $|4| = 4$

Model

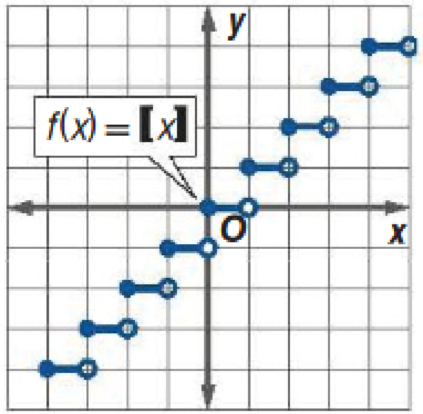


KeyConcept Greatest Integer Parent Function

Words The **greatest integer function**, denoted $f(x) = [\![x]\!]$, is defined as the greatest integer less than or equal to x.

Examples $[\![-4]\!] = -4$, $[\![-1.5]\!] = -2$, $\left[\!\left[\frac{1}{3}\right]\!\right] = 0$

Model



2 Transformations

Transformations of a parent function can affect the appearance of the parent graph. *Rigid transformations* change only the position of the graph, leaving the size and shape unchanged. *Nonrigid transformations* distort the shape of the graph.

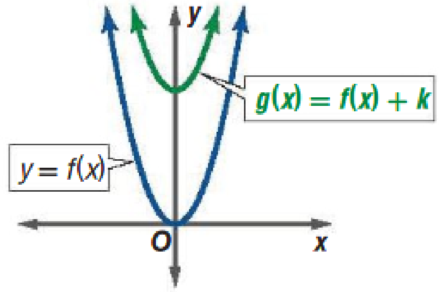
A **translation** is a rigid transformation that has the effect of shifting the graph of a function. A *vertical translation* of a function f shifts the graph of f up or down, while a *horizontal translation* shifts the graph left or right. Horizontal and vertical translations are examples of rigid transformations.

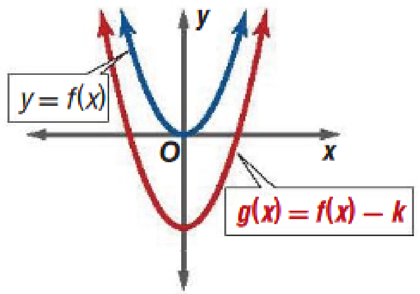
KeyConcept Vertical and Horizontal Translations

Vertical Translations

The graph of $g(x) = f(x) + k$ is the graph of $f(x)$ translated

- **k units up** when $k > 0$, and
- **k units down** when $k < 0$.

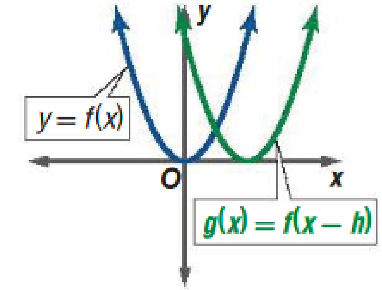


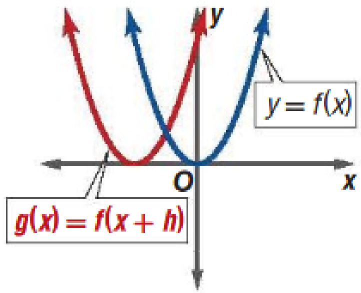


Horizontal Translations

The graph of $g(x) = f(x - h)$ is the graph of $f(x)$ translated

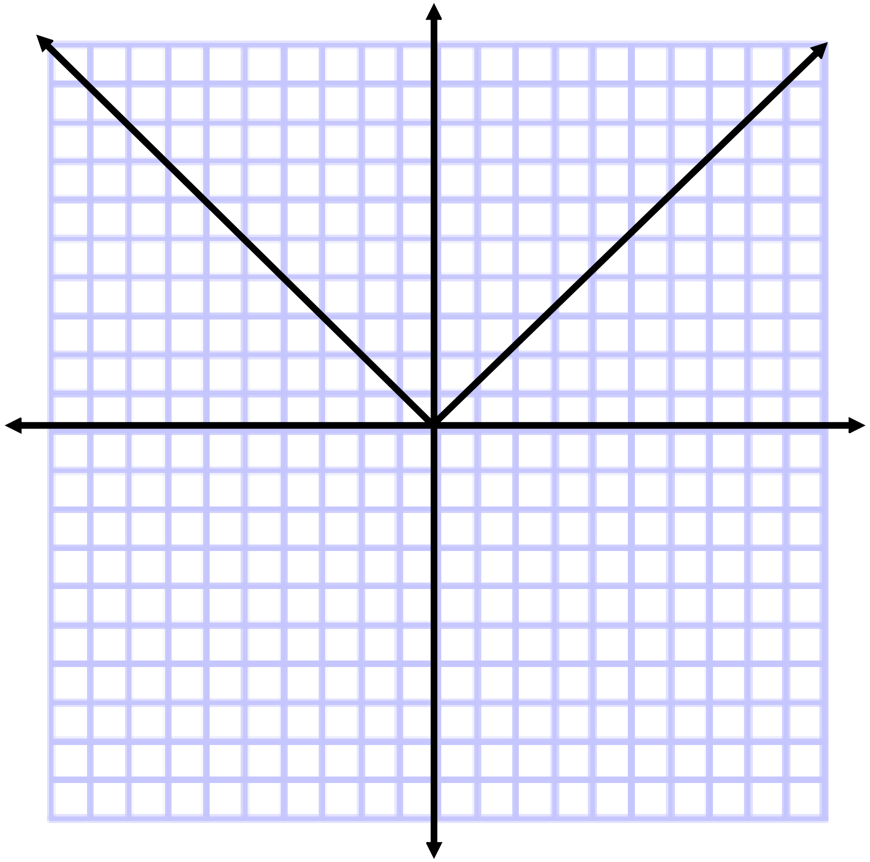
- **h units right** when $h > 0$, and
- **h units left** when $h < 0$.





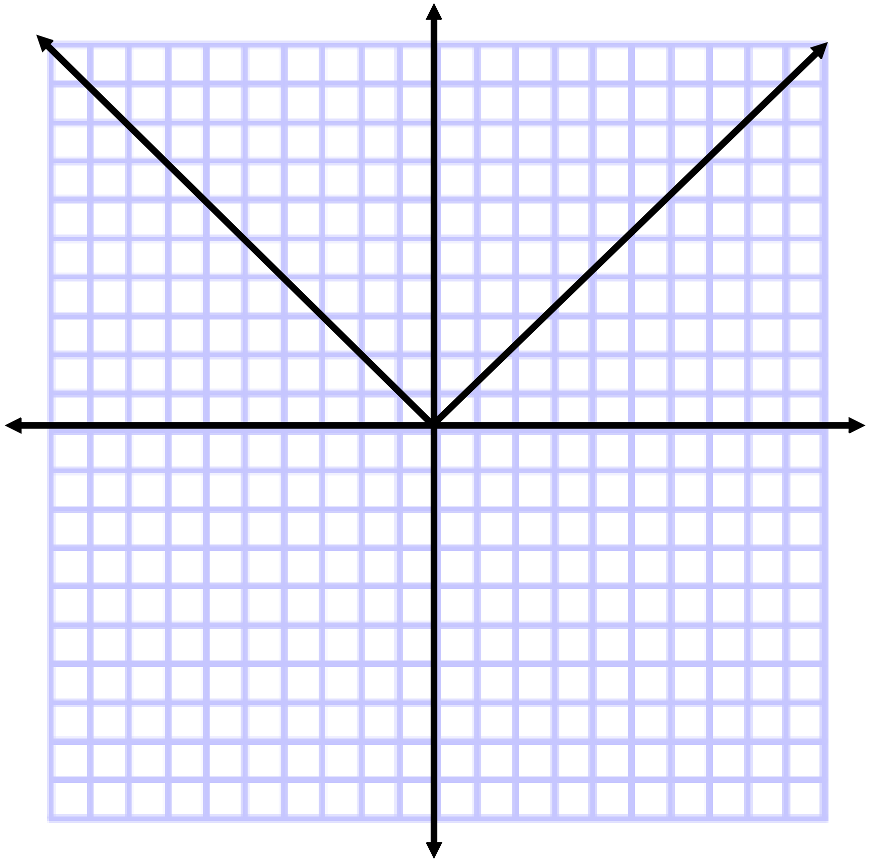
Use the graph of $f(x) = |x|$ to graph each function.

a. $g(x) = |x| + 4$



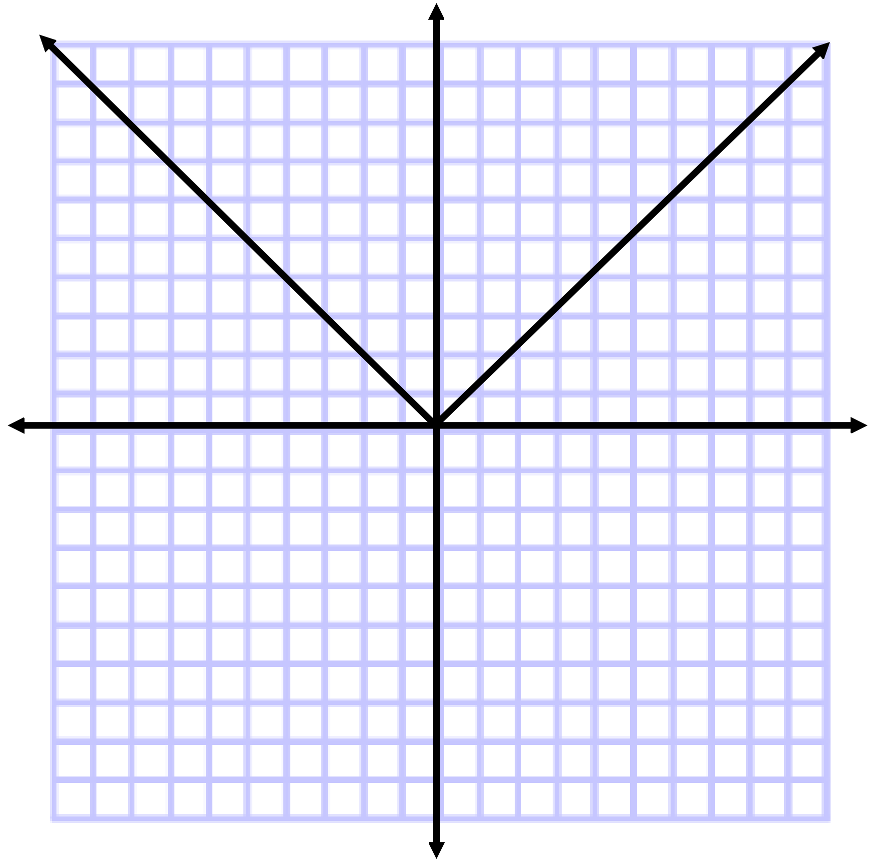
Use the graph of $f(x) = |x|$ to graph each function.

b. $g(x) = |x + 3|$



Use the graph of $f(x) = |x|$ to graph each function.

c. $g(x) = |x - 2| - 1$

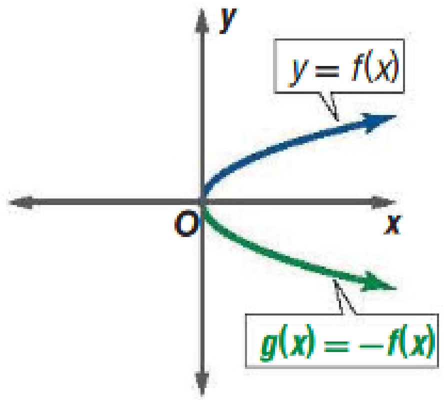


Another type of rigid transformation is a **reflection**, which produces a mirror image of the graph of a function with respect to a specific line.

KeyConcept Reflections in the Coordinate Axes

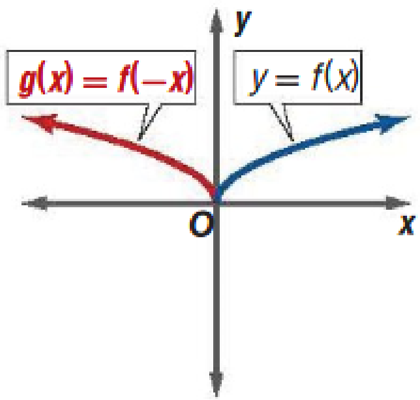
Reflection in x-axis

$g(x) = -f(x)$ is the graph of $f(x)$ **reflected in the x-axis.**

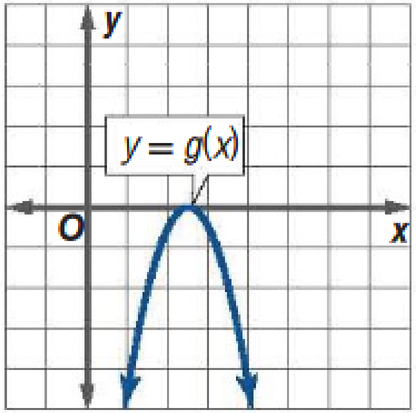


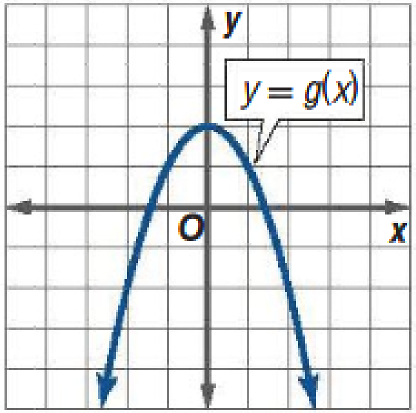
Reflection in y-axis

$g(x) = f(-x)$ is the graph of $f(x)$ **reflected in the y-axis.**



Describe how the graphs of $f(x) = x^2$ and $g(x)$ are related. Then write an equation for $g(x)$.

a. 

b. 

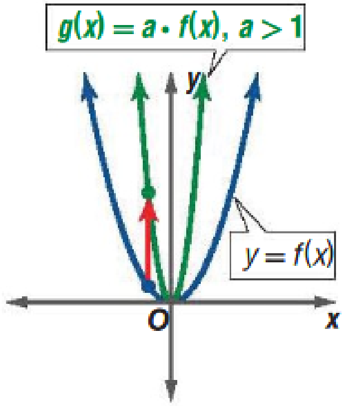
A **dilation** is a nonrigid transformation that has the effect of compressing (shrinking) or expanding (enlarging) the graph of a function vertically or horizontally.

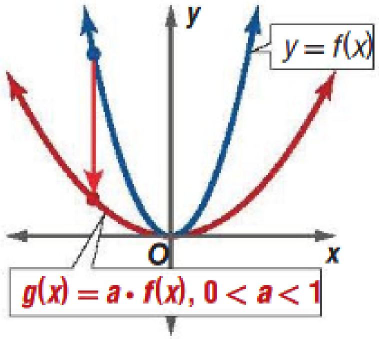
KeyConcept Vertical and Horizontal Translations

Vertical Dilations

If a is a positive real number, then $g(x) = a \cdot f(x)$, is

- the graph of $f(x)$ **expanded vertically**, if $a > 1$.
- the graph of $f(x)$ **compressed vertically**, if $0 < a < 1$.

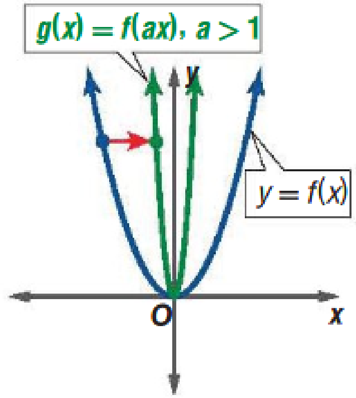


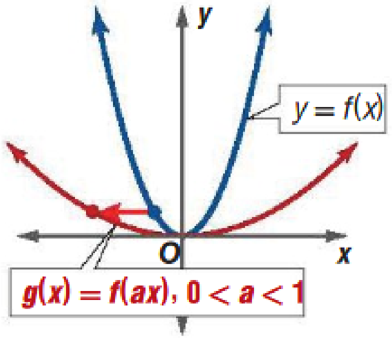


Horizontal Dilations

If a is a positive real number, then $g(x) = f(ax)$, is

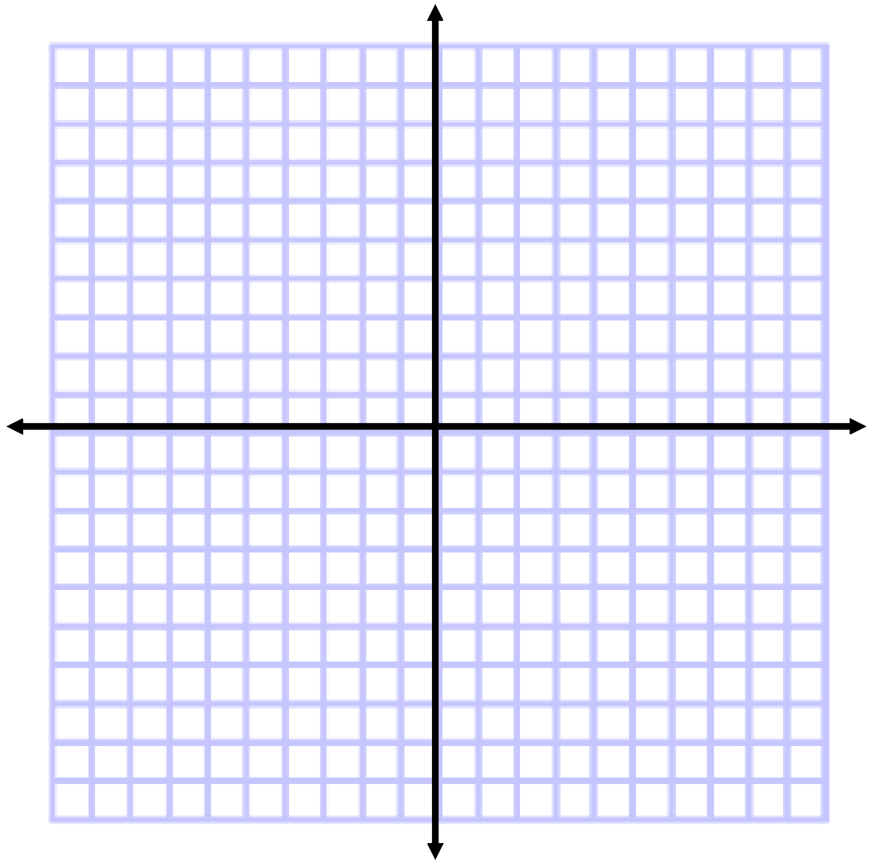
- the graph of $f(x)$ **compressed horizontally**, if $a > 1$.
- the graph of $f(x)$ **expanded horizontally**, if $0 < a < 1$.





Identify the parent function $f(x)$ of $g(x)$, and describe how the graphs of $g(x)$ and $f(x)$ are related. Then graph $f(x)$ and $g(x)$ on the same axes.

a. $g(x) = \frac{1}{4}x^3$



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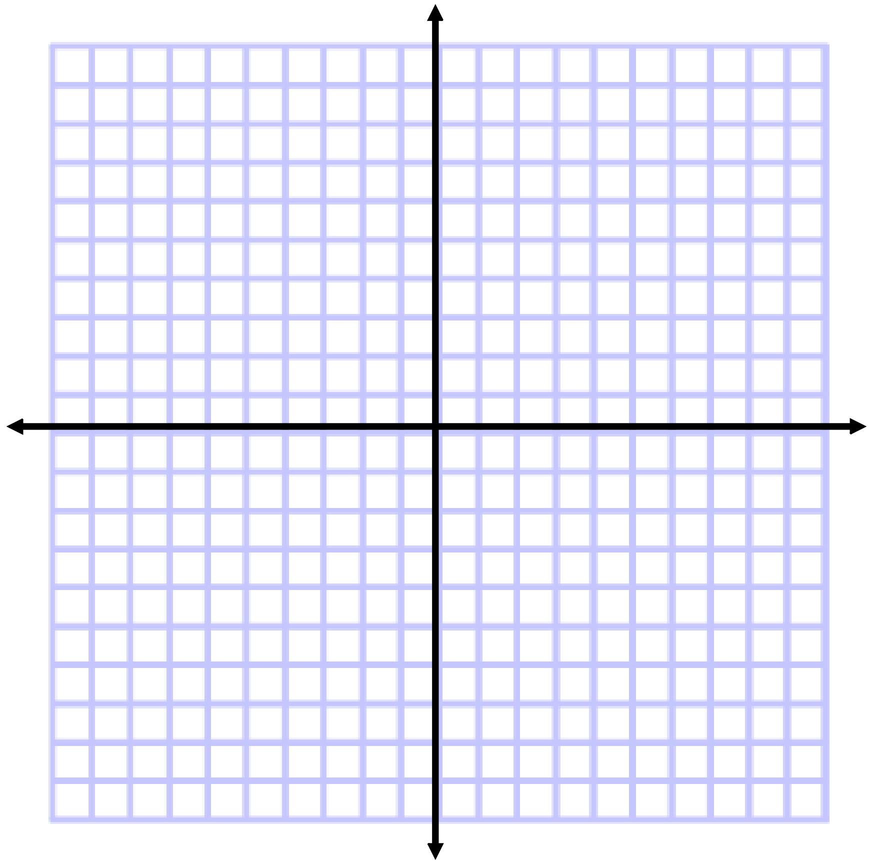
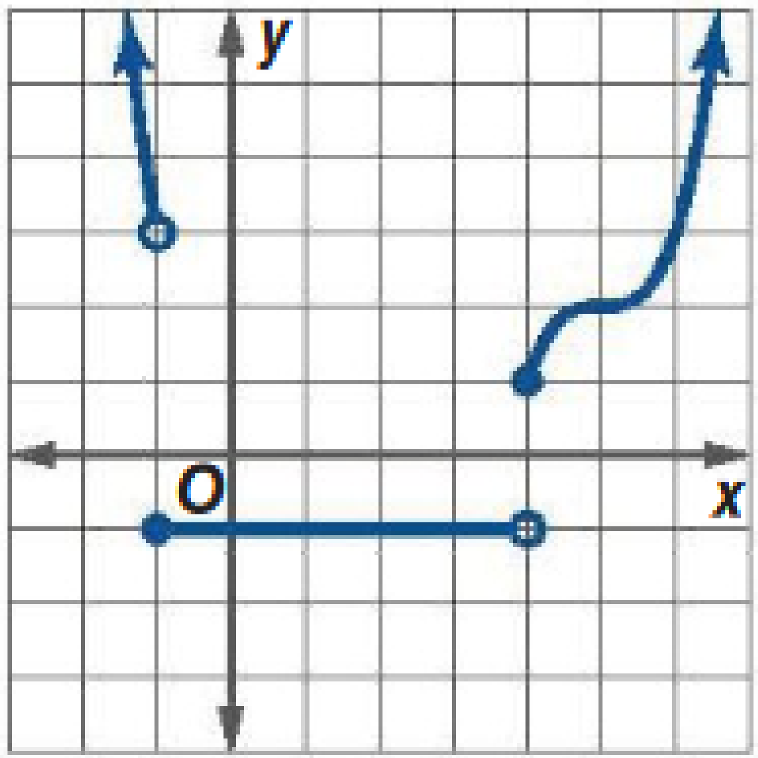
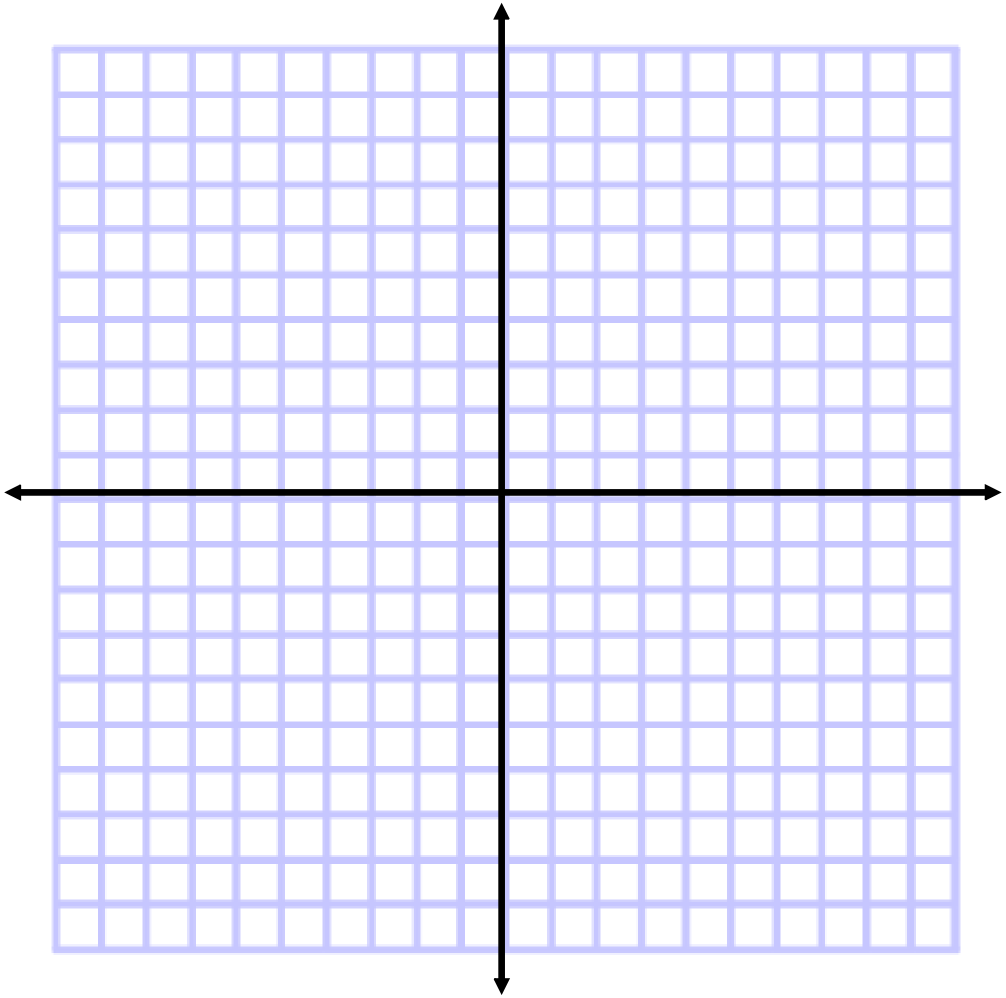


b. $g(x) = -(0.2x)^2$

Graph $f(x) = \begin{cases} 3x^2 & \text{if } x < -1 \\ -1 & \text{if } -1 \leq x < 4. \\ (x-5)^3 + 2 & \text{if } x \geq 4 \end{cases}$



5B. $h(x) = \begin{cases} (x+6)^2 & \text{if} \quad x < -5 \\ 7 & \text{if} \quad -5 \le x \le 2 \\ |4 - x| & \text{if} \quad x > 2 \end{cases}$



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