

6-3

Binomial Radical Expressions

Vocabulary Builder

binomial (adjective) by NOH mee ul

Definition: A binomial expression is an expression made up of two terms.

Related Words: monomial, binomial expression, trinomial

Examples: monomial: $a, x^2, -3, 17c^3, \sqrt{5}$

binomial: $a - 7, x^2 + 0.9, -3 - ab, 17c^3 + 1, b - \sqrt{5}$

trinomial: $a - 7 + x, x^2 + x + 0.9, -3 - ab + a, 17c^3 = c^2 + 1, b^3 + b - \sqrt{5}$

Take note

Property Combining Radical Expressions: Sums and Differences

Use the Distributive Property to add or subtract like radicals.

* Index
AND radicand must match to add or subtract

$$a\sqrt[n]{x} + b\sqrt[n]{x} = (a + b)\sqrt[n]{x} \quad a\sqrt[n]{x} - b\sqrt[n]{x} = (a - b)\sqrt[n]{x}$$


Problem 1 Adding and Subtracting Radical Expressions

Got It? What is the simplified form of each expression?

$7\sqrt[3]{5} - 4\sqrt[3]{5}$
Simplest form

$3x\sqrt{xy} + 4x\sqrt{xy}$
 $7x\sqrt{xy}$

6-3

Binomial Radical Expressions

**Problem 3** Simplifying Before Adding or Subtracting

Got It? What is the simplified form of the expression $\sqrt[3]{250} + \sqrt[3]{54} - \sqrt[3]{16}$?

$$\begin{array}{l} 1 \\ 8 \\ 27 \\ 64 \\ 125 \\ 216 \\ \vdots \end{array} \quad \begin{array}{l} \sqrt[3]{250} + \sqrt[3]{54} - \sqrt[3]{16} \\ \sqrt[3]{125} \sqrt[3]{2} + \sqrt[3]{27} \sqrt[3]{2} - \sqrt[3]{8} \sqrt[3]{2} \\ 5\sqrt[3]{2} + 3\sqrt[3]{2} - 2\sqrt[3]{2} \\ 6\sqrt[3]{2} \end{array}$$

**Problem 4** Multiplying Binomial Radical Expressions

Got It? What is the product $(3 + 2\sqrt{5})(2 + 4\sqrt{5})$?

$$\begin{array}{l} 6 + 12\sqrt{5} + 4\sqrt{5} + 8\sqrt{5} \\ 6 + 16\sqrt{5} + 8(5) \\ 6 + 16\sqrt{5} + 40 \\ 46 + 16\sqrt{5} \end{array}$$

6-3

Binomial Radical Expressions

**Problem 5** Multiplying Conjugates**Got It?** What is the product of the expression $(6 - \sqrt{12})(6 + \sqrt{12})$?

$$36 + \cancel{6\sqrt{12}} - \cancel{6\sqrt{12}} - 12$$

$$\boxed{24}$$

**Problem 6** Rationalizing the Denominator**Got It?** How can you write the expression $\frac{2\sqrt{7}}{\sqrt{3} - \sqrt{5}}$ with a rationalized denominator?

If you add/subtract radicals in the denominator, multiply by the conjugate.

$$\frac{2\sqrt{7}}{\sqrt{3} - \sqrt{5}} \cdot \frac{(\sqrt{3} + \sqrt{5})}{(\sqrt{3} + \sqrt{5})} = \frac{2\sqrt{21} + 2\sqrt{35}}{3 + \sqrt{15} - \sqrt{15} - 5} = \frac{2\sqrt{21} + 2\sqrt{35}}{-2}$$

$$= \frac{2\sqrt{21}}{-2} + \frac{2\sqrt{35}}{-2}$$

$$= -\sqrt{21} - \sqrt{35}$$

6-3

Binomial Radical Expressions

6.3 problems



Practice and Problem-Solving Exercises



Practice

Simplify if possible.

See Problem 1.

10. $5\sqrt{6} + \sqrt{6}$

11. $6\sqrt[3]{3} - 2\sqrt[3]{3}$

12. $4\sqrt{3} + 4\sqrt[3]{3}$

13. $3\sqrt{x} - 5\sqrt{x}$

14. $14\sqrt{x} + 3\sqrt{y}$

15. $7\sqrt[3]{x^2} - 2\sqrt[3]{x^2}$

16. The design of a garden path uses stone pieces shaped as squares with a side length of 15 in. Find the length of the path.



See Problem 2.

Simplify.

See Problem 3.

17. $6\sqrt{18} + 3\sqrt{50}$

18. $14\sqrt{20} - 3\sqrt{125}$

19. $\sqrt{18} + \sqrt{32}$

20. $\sqrt[3]{54} + \sqrt[3]{16}$

21. $3\sqrt[3]{81} - 2\sqrt[3]{54}$

22. $\sqrt[4]{32} + \sqrt[4]{48}$

Multiply.

See Problem 4.

23. $(3 + \sqrt{5})(1 + \sqrt{5})$

24. $(2 + \sqrt{7})(1 + 3\sqrt{7})$

25. $(3 - 4\sqrt{2})(5 - 6\sqrt{2})$

26. $(\sqrt{3} + \sqrt{5})^2$

27. $(\sqrt{13} + 6)^2$

28. $(2\sqrt{5} + 3\sqrt{2})^2$

Multiply each pair of conjugates.

See Problem 5.

29. $(5 - \sqrt{11})(5 + \sqrt{11})$

30. $(4 - 2\sqrt{3})(4 + 2\sqrt{3})$

31. $(2\sqrt{6} + 8)(2\sqrt{6} - 8)$

32. $(\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5})$

Rationalize each denominator. Simplify your answer.

See Problem 6.

33. $\frac{4}{1 + \sqrt{3}}$

34. $\frac{4}{3\sqrt{3} - 2}$

35. $\frac{5 + \sqrt{3}}{2 - \sqrt{3}}$

36. $\frac{3 + \sqrt{8}}{2 - 2\sqrt{8}}$

$$35.) \frac{(5 + \sqrt{3})(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})} = \frac{10 + 5\sqrt{3} + 2\sqrt{3} + 3}{4 + 2\sqrt{3} - 2\sqrt{3} - 3} = \frac{13 + 7\sqrt{3}}{1} = 13 + 7\sqrt{3}$$

