Chapter 12 Limits and an Introduction to Calculus

Course Number

Instructor

Date

Section 12.1 Introduction to Limits

Objective: In this lesson you learned how to estimate limits and use properties and operations of limits.

I. The Limit Concept and Definition of Limit (Pages 806–808)

What you should learn
How to use the definition of a limit to estimate limits

Define **limit.**

If $f(x)$ becomes arbitrarily close to a unique number L as x approaches c from either side, the limit of $f(x)$ as x approaches c is L. This written as $\lim_{x \to c} f(x) = L$.

Describe how to estimate the limit $\lim_{x \to -2} \dfrac{x^2 + 4x + 4}{x + 2}$ numerically.

Let $f(x) = (x^2 + 4x + 4)/(x + 2)$. Then construct a table that shows values of $f(x)$ when x is close to -2. Use the table to look for a numerical trend in the value of $f(x)$ as x approaches -2. This is an estimate of the limit.

The existence or nonexistence of $f(x)$ when $x = c$ has no bearing on the existence of . . . the limit of $f(x)$ as x approaches c.

II. Limits That Fail to Exist (Pages 809–810)

What you should learn
How to decide whether limits of functions exist

The limit of $f(x)$ as $x \to c$ does not exist if any of the following conditions is true:

1. $f(x)$ approaches a different number from the right side of c than from the left side of c.
2. $f(x)$ increases or decreases without bound as x approaches c.
3. $f(x)$ oscillates between two fixed values as x approaches c.

Give an example of a limit that does not exist.

Answers will vary.

III. Properties of Limits (Pages 811–812)

What you should learn
How to use properties and operations of limits to find limits

Let b and c be real numbers and let n be a positive integer. Complete each of the following properties of limits.

1. $\lim_{x \to c} b =$ ___*b*___

2. $\lim_{x \to c} x =$ ___*c*___

3. $\lim_{x \to c} x^n =$ ___c^n___

4. $\lim_{x \to c} \sqrt[n]{x} =$ ___$\sqrt[n]{c}$, for n even and $c > 0$___

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = K$$

Complete each of the following statements about operations with limits.

1. Scalar multiple: $\lim_{x \to c}[b\, f(x)] =$ ___*bL*___

2. Sum or difference: $\lim_{x \to c}[f(x) \pm g(x)] =$ ___$L \pm K$___

3. Product: $\lim_{x \to c}[f(x) \cdot g(x)] =$ ___*LK*___

4. Quotient: $\lim_{x \to c} \dfrac{f(x)}{g(x)} =$ ___L/K, provided $K \neq 0$___

5. Power: $\lim_{x \to c}[f(x)]^n =$ ___L^n___

Example 1: Find the limit: $\lim_{x \to 2} \dfrac{4 - x^2}{x}$.

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Homework Assignment

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Exercises