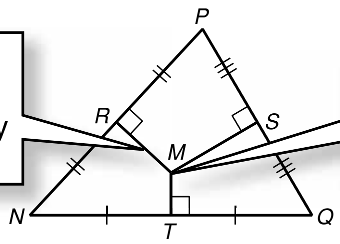
Name ______________________________ Date ___________ Class ___________

LESSON 5-2

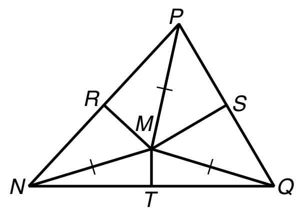
Reteach

Bisectors of Triangles

Perpendicular bisectors $\overline{MR}$, $\overline{MS}$, and $\overline{MT}$ are **concurrent** because they intersect at one point.

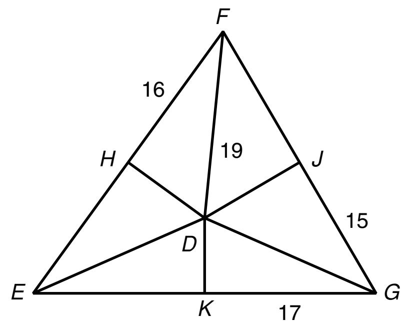


The point of intersection of $\overline{MR}$, $\overline{MS}$, *and* $\overline{MT}$ is called the **circumcenter** of $\triangle NPQ$.

Theorem	Example
Circumcenter Theorem The circumcenter of a triangle is equidistant from the vertices of the triangle.	**Given:** $\overline{MR}$, $\overline{MS}$, and $\overline{MT}$ are the perpendicular bisectors of $\triangle NPQ$. **Conclusion:** $MN = MP = MQ$ 

If a triangle on a coordinate plane has two sides that lie along the axes, you can easily find the circumcenter. Find the equations for the perpendicular bisectors of those two sides. The intersection of their graphs is the circumcenter.

$\overline{HD}$, $\overline{JD}$, and $\overline{KD}$ are the perpendicular bisectors of $\triangle EFG$. Find each length.



1. *DG* ____________________

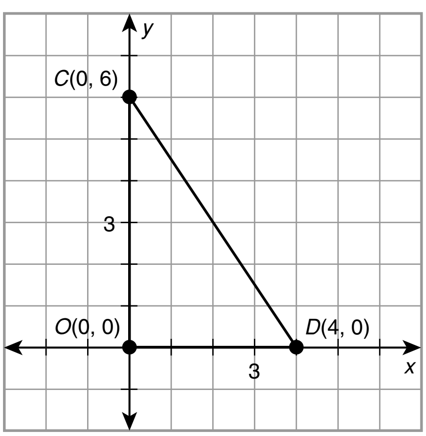
2. *EK* ____________________

3. *FJ* ____________________

4. *DE* ____________________

Find the circumcenter of each triangle.

5.



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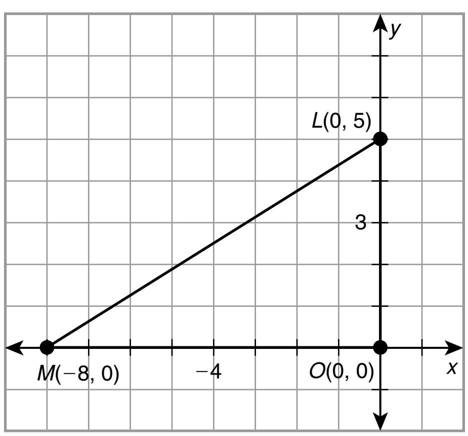
6.



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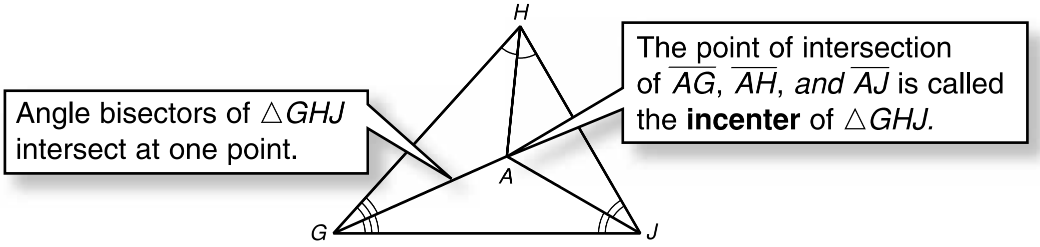
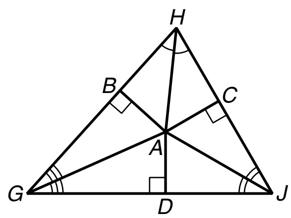
Name ________________________________ Date ____________ Class ____________

LESSON 5-2

Reteach

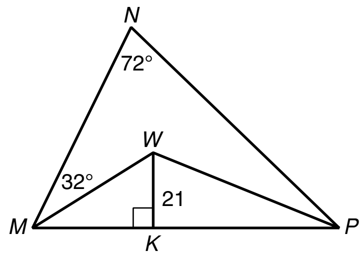
Bisectors of Triangles continued



Theorem	Example
Incenter Theorem The incenter of a triangle is equidistant from the sides of the triangle.	**Given:** $\overline{AG}$, $\overline{AH}$, and $\overline{AJ}$ are the angle bisectors of △*GHJ*. **Conclusion:** $AB = AC = AD$ 

$\overline{WM}$ and $\overline{WP}$ are angle bisectors of △*MNP*, and *WK* = 21.

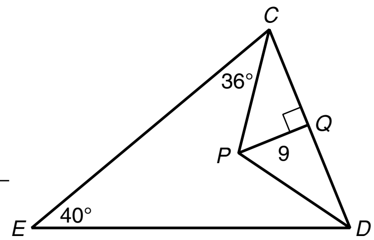
Find m∠*WPN* and the distance from *W* to $\overline{MN}$ and $\overline{NP}$.



$m\angle NMP = 2m\angle NMW$	Def. of ∠ bisector
$m\angle NMP = 2(32°) = 64°$	Substitute.
$m\angle NMP + m\angle N + m\angle NPM = 180°$	△ Sum Thm.
$64° + 72° + m\angle NPM = 180°$	Substitute.
$m\angle NPM = 44°$	Subtract 136° from each side.
$m\angle WPN = \frac{1}{2}m\angle NPM$	Def. of ∠ bisector
$m\angle WPN = \frac{1}{2}(44°) = 22°$	Substitute.

The distance from *W* to $\overline{MN}$ and $\overline{NP}$ is 21 by the Incenter Theorem.

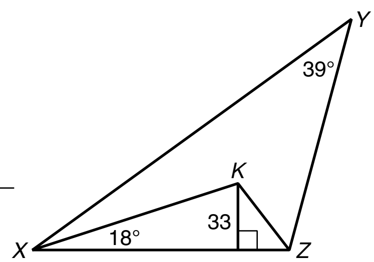
$\overline{PC}$ and $\overline{PD}$ are angle bisectors of △*CDE*. Find each measure.



7. the distance from *P* to $\overline{CE}$ ____________________

8. m∠*PDE* ____________________

$\overline{KX}$ and $\overline{KZ}$ are angle bisectors of △*XYZ*. Find each measure.



9. the distance from *K* to $\overline{YZ}$ ____________________

10. m∠*KZY* ____________________

LESSON 5-2

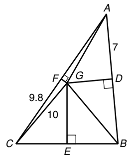
Practice A

Bisectors of Triangles

Fill in the blanks to complete each definition or theorem.

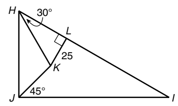
1. The circumcenter of a triangle is equidistant from the **vertices** of the triangle.
2. When three or more lines **intersect** at one point, the lines are said to be concurrent.
3. The incenter of a triangle is the point where the three **angle** bisectors of a triangle are concurrent.
4. The **incenter** of a triangle is equidistant from the sides of the triangle.
5. The **circumcenter** of a triangle is the point where the three perpendicular bisectors of a triangle are concurrent.

Use the figure for Exercises 6–8. $\overline{DG}$, $\overline{EG}$, and $\overline{FG}$ are perpendicular bisectors of $\triangle ABC$. Find each length.



6. AG **10**
7. DB **7**
8. AF **9.8**
9. GB **10**

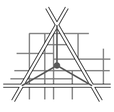
Use the figure for Exercises 10–13. $\overline{HK}$ and $\overline{JK}$ are angle bisectors of $\triangle HIJ$. Find each measure.



10. the distance from K to $\overline{JI}$ **25**
11. $m\angle HJK$ **45°**
12. $m\angle JHK$ **30°**
13. $m\angle HJI$ **90°**

Millsville is a town with three large streets that form a triangle. The town council wants to place a fire station so that it is the same distance from each of the three streets.

14. Why would the town council want the fire station equidistant from the large streets?
    Possible answer: If the fire station is the same distance from all the main streets, the fire trucks can quickly get to a fire near any of the three main streets.
15. Tell whether the circumcenter or the incenter of the triangle should be the location of the fire station. **incenter**
16. Bisect each angle of the triangle to find the location of the firehouse. You may use a compass and straightedge or a protractor.

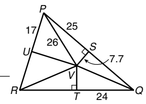


LESSON 5-2

Practice B

Bisectors of Triangles

Use the figure for Exercises 1 and 2. $\overline{SV}$, $\overline{TV}$, and $\overline{UV}$ are perpendicular bisectors of the sides of $\triangle PQR$. Find each length.

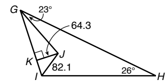


1. RV **26**
2. TR **24**

Find the circumcenter of the triangle with the given vertices.

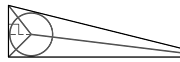
3. $A(0, 0)$, $B(0, 5)$, $C(5, 0)$
   (**2.5**, **2.5**)
4. $D(0, 7)$, $E(-3, 1)$, $F(3, 1)$
   (**0**, **3.25**)

Use the figure for Exercises 7 and 8. $\overline{GJ}$ and $\overline{IJ}$ are angle bisectors of $\triangle GHI$. Find each measure.

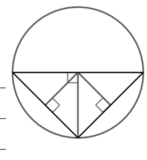


5. the distance from J to $\overline{GH}$ **64.3**
6. $m\angle JGK$ **23°**

Raleigh designs the interiors of cars. He is given two tasks to complete on a new production model.



7. A triangular surface as shown in the figure is molded into the driver's side door as an armrest. Raleigh thinks he can fit a cup holder into the triangle, but he'll have to put the largest possible circle into the triangle. Explain how Raleigh can do this. Sketch his design on the figure.
   Possible answer: Raleigh needs to find the incircle of the triangle. The incircle just touches all three sides of the triangle, so it is the largest circle that will fit. The incenter can be found by drawing the angle bisector from each vertex of the triangle. The incircle is drawn with the incenter as the center and a radius equal to the distance to one of the sides.
8. The car's logo is the triangle shown in the figure. Raleigh has to use this logo as the center of the steering wheel. Explain how Raleigh can do this. Sketch his design on the figure.
   Possible answer: Raleigh needs to find the circumcircle of the triangle. The circumcircle just touches all three vertices of the triangle, so it fits just around it. The circumcenter can be found by drawing the perpendicular bisectors of the sides of the triangle. The circumcircle is drawn with the circumcenter as center and a radius equal to the distance from the center to one of the vertices.

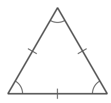


LESSON 5-2

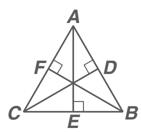
Practice C

Bisectors of Triangles

1. Sketch a triangle whose incenter and circumcenter are the same point. What kind of triangle is it?
   equilateral triangle

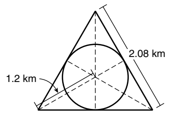


2. Draw a diagram and write a paragraph proof showing that the incenter and the circumcenter are the same point for the kind of triangle you drew in Exercise 1. (*Hint:* Show that the angle bisector is the same line as the perpendicular bisector, or vice versa.)



Possible answer: Triangle ABC is equilateral, so $\overline{AC}$ is congruent to $\overline{AB}$. If $\overline{AE}$ is the angle bisector of $\angle A$, then $\triangle CAE$ and $\triangle BAE$ are congruent by SAS and $\overline{AE}$ is the perpendicular bisector of $\overline{BC}$ by CPCTC; and if two congruent angles are supplementary, then they are right angles. Or if $\overline{AE}$ is the perpendicular bisector of $\overline{BC}$, then $\triangle CAE$ and $\triangle BAE$ are congruent by HL and $\overline{AE}$ is the angle bisector of $\angle A$ by CPCTC. Therefore $\overline{AE}$ is both the perpendicular bisector of $\overline{BC}$ and the angle bisector of $\angle A$. Similar reasoning will show that this is true for the other angle bisectors and perpendicular bisectors. The circumcenter is the point of concurrency of the perpendicular bisectors; the incenter is the point of concurrency of the angle bisectors. Because here the angle bisectors are also the perpendicular bisectors, the incenter and the circumcenter must be the same point.

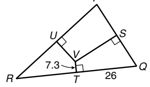
Meteor Crater in northern Arizona was created by the impact of a relatively small meteor—about 80 feet in diameter. The roughly circular crater is not only the best preserved impact crater on Earth but was also the first crater proven to have been caused by a meteor.



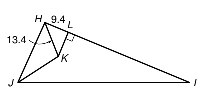
3. If the landowners at Meteor Crater built an equilateral triangle-shaped roadway around the crater as shown in the figure, find the diameter of Meteor Crater. **1.2 km**
4. A right triangle has a hypotenuse with length 17. What is the radius of the circle that can be circumscribed about this triangle? **8.5**

Round answers to the nearest tenth in Exercises 5 and 6.

5. $\overline{VS}$, $\overline{VT}$, and $\overline{VU}$ are perpendicular bisectors of the sides of $\triangle PQR$. Find the circumference of the circle that can be circumscribed about this triangle.
   about 169.7



6. $\overline{KH}$ and $\overline{KJ}$ are angle bisectors of $\triangle HIJ$. Find the area of the circle that can be inscribed in this triangle.
   91.2π unit^2 or 286.5 unit^2

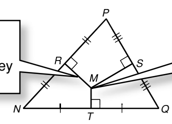


LESSON 5-2

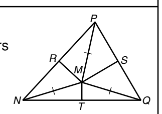
Reteach

Bisectors of Triangles

Perpendicular bisectors $\overline{MR}$, $\overline{MS}$, and $\overline{MT}$ are **concurrent** because they intersect at one point.

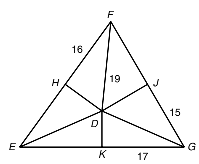


The point of intersection of $\overline{MR}$, $\overline{MS}$, *and* $\overline{MT}$ is called the **circumcenter** of $\triangle NPQ$.

Theorem	Example
Circumcenter Theorem The circumcenter of a triangle is equidistant from the vertices of the triangle.	**Given:** $\overline{MR}$, $\overline{MS}$, and $\overline{MT}$ are the perpendicular bisectors of $\triangle NPQ$. **Conclusion:** $MN = MP = MQ$ 

If a triangle on a coordinate plane has two sides that lie along the axes, you can easily find the circumcenter. Find the equations for the perpendicular bisectors of those two sides. The intersection of their graphs is the circumcenter.

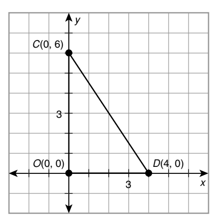
$\overline{HD}$, $\overline{JD}$, and $\overline{KD}$ are the perpendicular bisectors of $\triangle EFG$. Find each length.



1. DG **19**
2. EK **17**
3. FJ **15**
4. DE **19**

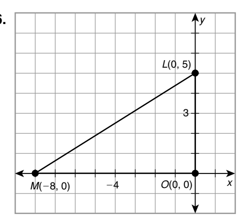
Find the circumcenter of each triangle.

5.



(2, 3)

6.

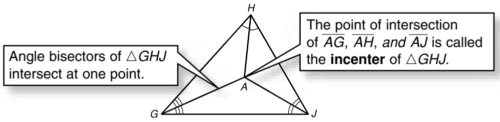


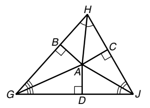
(−4, 2.5)

LESSON 5-2

Reteach

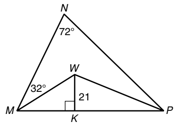
Bisectors of Triangles continued



Theorem	Example
Incenter Theorem The incenter of a triangle is equidistant from the sides of the triangle.	**Given:** $\overline{AG}$, $\overline{AH}$, and $\overline{AJ}$ are the angle bisectors of △GHJ. **Conclusion:** $AB = AC = AD$ 

$\overline{WM}$ and $\overline{WP}$ are angle bisectors of △MNP, and WK = 21.

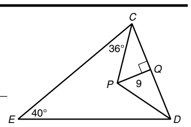
Find m∠WPN and the distance from W to $\overline{MN}$ and $\overline{NP}$.



m∠NMP = 2m∠NMW	Def. of ∠ bisector
m∠NMP = 2(32°) = 64°	Substitute.
m∠NMP + m∠N + m∠NPM = 180°	△ Sum Thm.
64° + 72° + m∠NPM = 180°	Substitute.
m∠NPM = 44°	Subtract 136° from each side.
m∠WPN = $\frac{1}{2}$m∠NPM	Def. of ∠ bisector
m∠WPN = $\frac{1}{2}$(44°) = 22°	Substitute.

The distance from W to $\overline{MN}$ and $\overline{NP}$ is 21 by the Incenter Theorem.

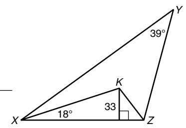
$\overline{PC}$ and $\overline{PD}$ are angle bisectors of △CDE. Find each measure.



7. the distance from P to $\overline{CE}$ **9**

8. m∠PDE **34°**

$\overline{KX}$ and $\overline{KZ}$ are angle bisectors of △XYZ. Find each measure.



9. the distance from K to $\overline{YZ}$ **33**

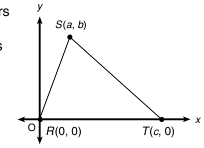
10. m∠KZY **52.5°**

LESSON 5-2

Challenge

The Perpendicular Bisectors of the Sides of a Triangle

In Lesson 5-2, you investigated the three perpendicular bisectors of the sides of a triangle and discovered some surprising facts about them. On this page, you will see how coordinate methods can help you prove those facts.



1. Refer to the figure at right. Follow these steps to prove that the perpendicular bisectors of the sides of △RST are *concurrent*. That is, you will prove that all three perpendicular bisectors intersect at a single point.

   a. Write equations for the perpendicular bisectors of $\overline{RS}$, $\overline{ST}$, and $\overline{RT}$. (*Hint:* If a line having slope m passes through a point $P(x_1, y_1)$, then an equation of the line is $y - y_1 = m(x - x_1)$.)

   $\overline{RS}$: $y - \frac{b}{2} = -\left(\frac{a}{b}\right)\left(x - \frac{a}{2}\right)$; $\overline{ST}$: $y - \frac{b}{2} = -\left(\frac{a-c}{b}\right)\left(x - \frac{a+c}{2}\right)$; $\overline{RT}$: $x = \frac{c}{2}$

   b. Use a system of equations to find the coordinates of the point where the perpendicular bisectors of $\overline{RS}$ and $\overline{RT}$ intersect.

   $\left(\frac{c}{2}, \frac{a^2 + b^2 - ac}{2b}\right)$

   c. Use a system of equations to find the coordinates of the point where the perpendicular bisectors of $\overline{ST}$ and $\overline{RT}$ intersect.

   $\left(\frac{c}{2}, \frac{a^2 + b^2 - ac}{2b}\right)$

   d. Use the results of parts b and c to complete the proof.

   Since the perpendicular bisectors of $\overline{RS}$ and $\overline{RT}$ intersect in the same point as the perpendicular bisectors of $\overline{ST}$ and $\overline{RT}$, all three lines intersect the same point. Thus the perpendicular bisectors of the sides of △RST are concurrent.

2. Let point Z be the point of concurrency of the three perpendicular bisectors of the sides of △RST above. Follow these steps to prove that point Z is equidistant from the vertices of △RST. In other words, prove that RZ = SZ = TZ. Use the Distance Formula to write expressions for $(RZ)^2$, $(SZ)^2$, and $(TZ)^2$.

   $(RZ)^2$: $\left(\frac{c}{2}\right)^2 + \left(\frac{a^2 + b^2 - ac}{2b}\right)^2$;

   $(SZ)^2$: $\left(a - \frac{c}{2}\right)^2 + \left(b - \frac{a^2 + b^2 - ac}{2b}\right)^2$;

   $(TZ)^2$: $\left(c - \frac{c}{2}\right)^2 + -\left(\frac{a^2 + b^2 - ac}{2b}\right)^2$

LESSON 5-2

Problem Solving

Bisectors of Triangles

1. A new dog park is being planned. Describe how to find a location for the park so that it is the same distance from three suburbs.

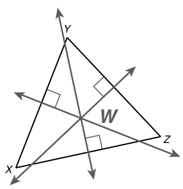
   Draw a triangle that has the suburbs as its vertices. Find the circumcenter of the triangle by drawing the perpendicular bisector of each side.

2. A fountain is in a triangular sitting area of a mall, △ABC. A diagram shows that the fountain is at the point where the angle bisectors of △ABC are concurrent. If the distance from the fountain to one wall is 15 feet, what is the distance from the fountain to another wall? Explain.

   15 ft; By the Incenter Thm., the incenter of a triangle is equidistant from the sides of the triangle.

3. A water tower is to be built so that it is the same distance from the cities at X, Y, and Z. Draw a sketch on △XYZ to show the location W where the water tower should be built. Justify your sketch.

   Draw the perpendicular bisectors of $\overline{XY}$, $\overline{YZ}$, and $\overline{ZX}$. By the Circumcenter Thm., W is equidistant from X, Y, and Z.



Choose the best answer.

4. The circumcenter of △FGH is at (4, −5). If G is at (0, 0), which of the following are possible coordinates of F and H?

   A F(0, −8), H(10, 0)

   B F(0, 8), H(−10, 0)

   (C) F(0, −10), H(8, 0)

   D F(0, 10), H(−8, 0)

5. A triangle has vertices Q(–9, 10), R(0, 1), and S(8, 4). Which is a correct statement about the incenter and circumcenter of △QRS?

   F Both points are on △QRS.

   G Both points are inside △QRS.

   H Both points are outside △QRS.

   (J) One point is inside △QRS, and one point is outside △QRS.

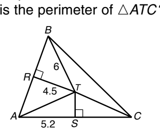
6. $\overline{RT}$ and $\overline{TS}$ are perpendicular bisectors of △ABC. What is the perimeter of △ATC?

   A 17.2 units

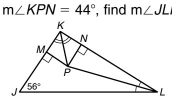
   B 19.4 units

   C 20.9 units

   (D) 22.4 units



7. If m∠KPN = 44°, find m∠JLP.



   (F) 16° H 23°

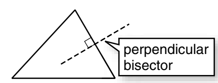
   G 18° J 32°

LESSON 5-2

Reading Strategies

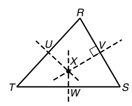
Vocabulary Development

A **perpendicular bisector** is a line that intersects the side of a triangle at 90° and passes through its midpoint.



If three or more lines intersect at the same point, the lines are **concurrent.** This may be easy to remember if you think about three of your favorite television programs being broadcast concurrently. This means they are on at the same time.

The perpendicular bisectors of the sides of a triangle are concurrent as shown here.



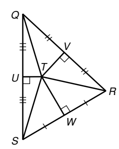
1. What is a bisector of a triangle?

   It is a line that divides a side of a triangle into two congruent parts.

2. What is the perpendicular bisector of RS in △RST above?

   $\overline{VX}$

The point where the perpendicular bisectors meet is called the **circumcenter of the triangle** and is equidistant from the vertices of the triangle. It is a **point of concurrency** because the three lines intersect at one point.



3. What is the point of concurrency for △QRS?

   point T

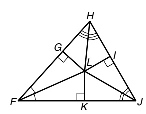
4. What does *equidistant* mean?

   the same distance from two or more objects or of equal distance

5. In the triangle shown above, ST = **QT** = **RT**.

The **angle bisectors of a triangle** are also concurrent. Consider the diagram at right:

In this diagram, point L is the point of concurrency of the angle bisectors of △FHJ. It is called the **incenter of a triangle.**



6. What is the angle bisector of ∠H?

   $\overline{HL}$

7. What is the angle bisector of ∠F?

   $\overline{FL}$