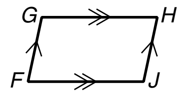
Name ________________________________ Date ____________ Class ____________

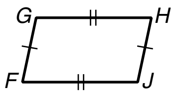
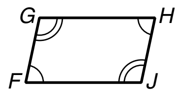
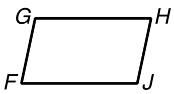
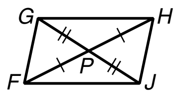
LESSON 6-2

Reteach
Properties of Parallelograms

A parallelogram is a quadrilateral with two pairs of parallel sides. All parallelograms, such as ▱*FGHJ*, have the following properties.

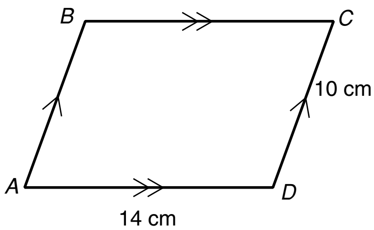


▱*FGHJ*

Properties of Parallelograms	
 $\overline{FG} \cong \overline{HJ}$ $\overline{GH} \cong \overline{JF}$ Opposite sides are congruent.	 $\angle F \cong \angle H$ $\angle G \cong \angle J$ Opposite angles are congruent.
 $m\angle F + m\angle G = 180°$ $m\angle G + m\angle H = 180°$ $m\angle H + m\angle J = 180°$ $m\angle J + m\angle F = 180°$ Consecutive angles are supplementary.	 $\overline{FP} \cong \overline{HP}$ $\overline{GP} \cong \overline{JP}$ The diagonals bisect each other.

Find each measure.

1. *AB*



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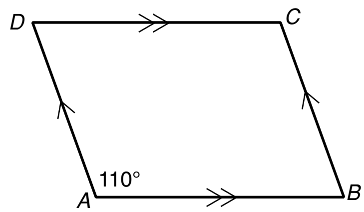
2. m∠*D*



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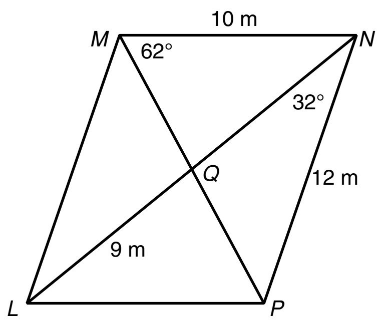
Find each measure in ▱*LMNP*.



3. *ML* ______________________

4. *LP* ______________________

5. m∠*LPM* ______________________

6. *LN* ______________________

7. m∠*MLN* ______________________

8. *QN* ______________________

Name ______________________________ Date ___________ Class ___________

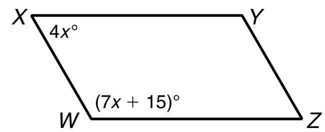
LESSON **6-2**

Reteach

Properties of Parallelograms continued

You can use properties of parallelograms to find measures.

***WXYZ* is a parallelogram. Find m∠*X*.**

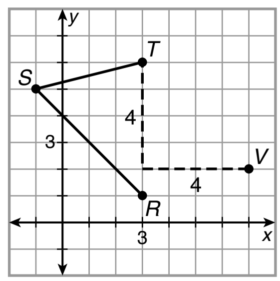


$m\angle W + m\angle X = 180°$	If a quadrilateral is a ▱, then cons. $\angle$s are supp.
$(7x + 15) + 4x = 180°$	Substitute the given values.
$11x + 15 = 180$	Combine like terms.
$11x = 165$	Subtract 15° from both sides.
$x = 15$	Divide both sides by 11.

$m\angle X = (4x)° = [4(15)]° = 60°$

If you know the coordinates of three vertices of a parallelogram, you can use slope to find the coordinates of the fourth vertex.

Three vertices of ▱*RSTV* are *R*(3, 1), *S*(−1, 5), and *T*(3, 6). Find the coordinates of *V*.

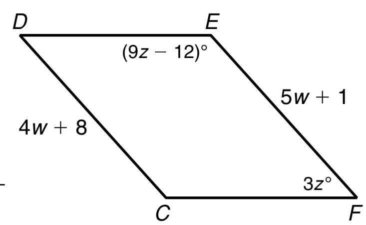


Since opposite sides must be parallel, the rise and the run from *S* to *R* must be the same as the rise and the run from *T* to *V*.

From *S* to *R*, you go down 4 units and right 4 units. So, from *T* to *V*, go down 4 units and right 4 units. Vertex *V* is at *V*(7, 2).

You can use the slope formula to verify that $\overline{ST} \parallel \overline{RV}$.

***CDEF* is a parallelogram. Find each measure.**



9. *CD* ______________________

10. *EF* ______________________

11. m∠*F* ______________________

12. m∠*E* ______________________

The coordinates of three vertices of a parallelogram are given. Find the coordinates of the fourth vertex.

13. ▱*ABCD* with *A*(0, 6), *B*(5, 8), *C*(5, 5)

____

14. ▱*KLMN* with *K*(−4, 7), *L*(3, 6), *M*(5, 3)

____

LESSON **6-2**

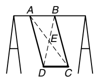
Practice A

Properties of Parallelograms

Fill in the blanks to complete each definition or theorem.

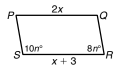
1. If a quadrilateral is a parallelogram, then its consecutive angles are **supplementary**.
2. If a quadrilateral is a parallelogram, then its opposite sides are **congruent or parallel**.
3. A parallelogram is a quadrilateral with two pairs of **parallel** sides.
4. If a quadrilateral is a parallelogram, then its diagonals **bisect** each other.
5. If a quadrilateral is a parallelogram, then its opposite angles are **congruent**.

The figure shows a swing blown to one side by a breeze. As long as the seat of the swing is parallel to the top bar, the swing makes a parallelogram. In ▱*ABCD*, *DC* = 2 ft, *BE* = $4\frac{1}{2}$ ft, and m∠*BAD* = 75°. Find each measure.



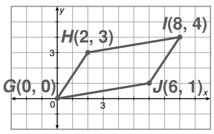
6. *AB* **2 ft**
7. *ED* **$4\frac{1}{2}$ ft**
8. *BD* **9 ft**
9. m∠*ABC* **105°**
10. m∠*BCD* **75°**
11. m∠*ADC* **105°**

***PQRS* is a parallelogram. Find each measure.**



12. *RS* **6**
13. m∠*S* **100°**
14. m∠*R* **80°**

Three vertices of ▱*GHIJ* are *G*(0, 0), *H*(2, 3), and *J*(6, 1). Complete Exercises 15–21 to find the coordinates of vertex *I*.



15. Plot vertices *G*, *H*, and *J* on the coordinate plane.
16. Find the rise (difference in the *y*-coordinates) from *G* to *H*. **3**
17. Find the run (difference in the *x*-coordinates) from *G* to *H*. **2**
18. Using your answers from Exercises 16 and 17, add the rise to the *y*-coordinate of vertex *J* and add the run to the *x*-coordinate of vertex *J*. These are the coordinates of vertex *I*. (**8** , **4**)
19. Plot vertex *I*. Connect the points to draw ▱*GHIJ*.
20. Check your answer by finding the slopes of $\overline{IH}$ and $\overline{JG}$.

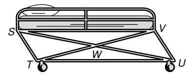
    slope of $\overline{IH}$ = **$\frac{1}{6}$**   slope of $\overline{JG}$ = **$\frac{1}{6}$**
21. Parallel lines have equal slopes. Are the slopes of $\overline{IH}$ and $\overline{JG}$ equal? **yes**

LESSON **6-2**

Practice B

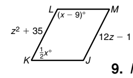
Properties of Parallelograms

A gurney is a wheeled cot or stretcher used in hospitals. Many gurneys are made so that the base will fold up for easy storage in an ambulance. When partially folded, the base forms a parallelogram. In ▱*STUV*, *VU* = 91 centimeters, *UW* = 108.8 centimeters, and m∠*TSV* = 57°. Find each measure.



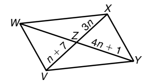
1. *SW* **108.8 cm**
2. *TS* **91 cm**
3. *US* **217.6 cm**
4. m∠*SVU* **123°**
5. m∠*STU* **123°**
6. m∠*TUV* **57°**

***JKLM* is a parallelogram. Find each measure.**



7. m∠*L* **117°**
8. m∠*K* **63°**
9. *MJ* **71**

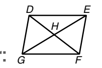
***VWXY* is a parallelogram. Find each measure.**



10. *VX* **21**
11. *XZ* **10.5**
12. *ZW* **15**
13. *WY* **30**
14. Three vertices of ▱*ABCD* are *B*(−3, 3), *C*(2, 7), and *D*(5, 1). Find the coordinates of vertex *A*. **(0, −3)**

Write a two-column proof.

15. **Given:** *DEFG* is a parallelogram.
    Prove: m∠*DHG* = m∠*EDH* + m∠*FGH*



Possible answer:

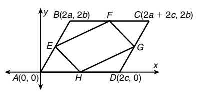
Statements	Reasons
1. *DEFG* is a parallelogram.	1. Given
2. m∠*EDG* = m∠*EDH* + m∠*GDH*, m∠*FGD* = m∠*FGH* + m∠*DGH*	2. Angle Add. Post.
3. m∠*EDG* + m∠*FGD* = 180°	3. ▱ → cons. ∠s supp.
4. m∠*EDH* + m∠*GDH* + m∠*FGH* + m∠*DGH* = 180°	4. Subst. (Steps 2, 3)
5. m∠*GDH* + m∠*DGH* + m∠*DHG* = 180°	5. Triangle Sum Thm.
6. m∠*GDH* + m∠*DGH* + m∠*DHG* = m∠*EDH* + m∠*GDH* + m∠*FGH* + m∠*DGH*	6. Trans. Prop. of =
7. m∠*DHG* = m∠*EDH* + m∠*FGH*	7. Subtr. Prop. of =

LESSON **6-2**

Practice C

Properties of Parallelograms

The area of a parallelogram is given by the formula $A = bh$, where A is the area, b is the length of a base, and h is the height perpendicular to the base. *ABCD* is a parallelogram. *E*, *F*, *G*, and *H* are the midpoints of the sides.

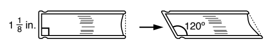


1. Show that the area of *EFGH* is half the area of *ABCD*. **Possible answer: The height of *ABCD* is 2*b* and the length of the base is 2*c*, so the area of *ABCD* is 4*bc*. Because *ABCD* is a parallelogram, *AB* = *DC* and *BC* = *AD* and ∠*A* is congruent to ∠*C* and ∠*B* is congruent to ∠*D*. Furthermore, because *E*, *F*, *G*, and *H* are midpoints, *AE* = *BE* = *CG* = *DG* and *BF* = *CF* = *AH* = *DH*. So by SAS, △*AEH* is congruent to △*CGF* and △*BEF* is congruent to △*DGH*. Now find the coordinates of the midpoints: *E*(*a*, *b*), *F*(*c* + 2*a*, 2*b*), *G*(2*c* + *a*, *b*), *H*(*c*, 0). The height of △*AEH* is *b* and the length of the base is *c*, so its area is $\frac{1}{2}bc$. The areas of congruent triangles are equal, so the area of △*CGF* is also $\frac{1}{2}bc$. The height of △*DGH* is *b* and the length of the base is *c*, so its area is $\frac{1}{2}bc$. The area of △*BEF* is also $\frac{1}{2}bc$. The area of all four triangles is thus 2*bc*. The area of *EFGH* is the area of *ABCD* minus the area of the triangles, or 4*bc* − 2*bc* = 2*bc*. And the area of *EFGH* is $2bc = \frac{1}{2}(4bc) = \frac{1}{2}$(area of *ABCD*).**
2. Show that *EFGH* is a parallelogram.

Possible answer: Use the slope formula to find the slope of each side: slope of $\overline{EF} = \frac{b}{a + c}$, slope of $\overline{GH} = \frac{b}{a + c}$, slope of $\overline{FG} = \frac{b}{a - c}$, slope of $\overline{EH} = \frac{b}{a - c}$. Segments with equal slopes are parallel, so $\overline{EF}$ is parallel to $\overline{GH}$ and $\overline{FG}$ is parallel to $\overline{EH}$. Therefore *EFGH* is a parallelogram.

Mr. Nguyen is blessed (or cursed) with an abundance of books. They litter his apartment. Mr. Nguyen is trying to clean up his living room, so he bought a new three-shelf bookcase. Each shelf is $2\frac{1}{2}$ feet long.

3. Mr. Nguyen measures a few books and finds they average $1\frac{1}{8}$ inch thick. Calculate the maximum number of books Mr. Nguyen can fit on the bookcase. **80 books**
4. Mr. Nguyen finds that if he stresses a book by pushing back on the front cover, the book gets thinner. Mr. Nguyen discovers that he can stress a book by about 30° without harming the book's binding. Calculate the maximum number of stressed books Mr. Nguyen can fit on the bookcase. **92 books**



Find the range of possible diagonal lengths in a parallelogram with the given side lengths.

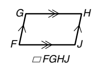
5. 3 and 12 **$9 < \ell < 15$**
6. x and $2x$ **$x < \ell < 3x$**
7. x and x **$0 < \ell < 2x$**

LESSON **6-2**

Reteach

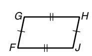
Properties of Parallelograms

A parallelogram is a quadrilateral with two pairs of parallel sides. All parallelograms, such as ▱*FGHJ*, have the following properties.



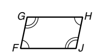
▱*FGHJ*

Properties of Parallelograms



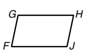
$\overline{FG} \cong \overline{HJ}$
$\overline{GH} \cong \overline{JF}$

Opposite sides are congruent.



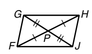
∠*F* ≅ ∠*H*
∠*G* ≅ ∠*J*

Opposite angles are congruent.



m∠*F* + m∠*G* = 180°
m∠*G* + m∠*H* = 180°
m∠*H* + m∠*J* = 180°
m∠*J* + m∠*F* = 180°

Consecutive angles are supplementary.

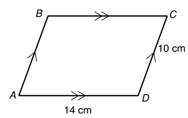


$\overline{FP} \cong \overline{HP}$
$\overline{GP} \cong \overline{JP}$

The diagonals bisect each other.

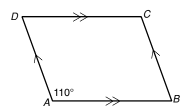
Find each measure.

1. *AB*



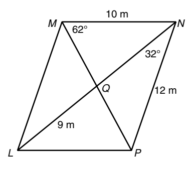
10 cm

2. m∠*D*



70°

Find each measure in ▱*LMNP*.



3. *ML* **12 m**
4. *LP* **10 m**
5. m∠*LPM* **62°**
6. *LN* **18 m**
7. m∠*MLN* **32°**
8. *QN* **9 m**

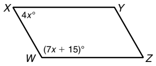
LESSON 6-2

Reteach

Properties of Parallelograms continued

You can use properties of parallelograms to find measures.

***WXYZ* is a parallelogram. Find m∠*X*.**

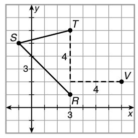


m∠*W* + m∠*X* = 180°	If a quadrilateral is a ▱, then cons. ∠s are supp.
(7*x* + 15) + 4*x* = 180°	Substitute the given values.
11*x* + 15 = 180	Combine like terms.
11*x* = 165	Subtract 15° from both sides.
x = 15	Divide both sides by 11.

m∠*X* = (4*x*)° = [4(15)]° = 60°

If you know the coordinates of three vertices of a parallelogram, you can use slope to find the coordinates of the fourth vertex.

Three vertices of ▱*RSTV* are *R*(3, 1), *S*(−1, 5), and *T*(3, 6). Find the coordinates of *V*.

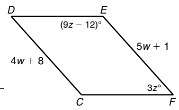


Since opposite sides must be parallel, the rise and the run from *S* to *R* must be the same as the rise and the run from *T* to *V*.

From *S* to *R*, you go down 4 units and right 4 units. So, from *T* to *V*, go down 4 units and right 4 units. Vertex *V* is at *V*(7, 2).

You can use the slope formula to verify that $\overline{ST} \parallel \overline{RV}$.

***CDEF* is a parallelogram. Find each measure.**



9. *CD* — 36

10. *EF* — 36

11. m∠*F* — 48°

12. m∠*E* — 132°

The coordinates of three vertices of a parallelogram are given. Find the coordinates of the fourth vertex.

13. ▱*ABCD* with *A*(0, 6), *B*(5, 8), *C*(5, 5) — *D*(0, 3)

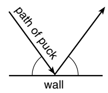
14. ▱*KLMN* with *K*(−4, 7), *L*(3, 6), *M*(5, 3) — *N*(−2, 4)

LESSON 6-2

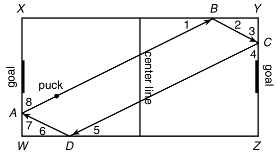
Challenge

Finding Special Quadrilaterals on an Air Hockey Table

In the game of air hockey, a puck glides on a thin layer of air above a rectangular table that is 4 feet wide and 8 feet long. The object of the game is to hit the puck into your opponent's goal. The movement of the puck is confined to the table by walls at the edges of the table. Because of a physical principle, the *law of reflection,* the angle at which the puck bounces off a wall is congruent to the angle at which it strikes the wall.



In the figure at right, a puck has been hit in such a way that it bounces off each wall of the table exactly once. Complete the following proof that the path shown is a parallelogram.



Given: *WXYZ* is a rectangle; m∠1 = m∠2; m∠3 = m∠4; m∠5 = m∠6; m∠7 = m∠8

Prove: *ABCD* is a parallelogram.

By the 1. **Triangle Sum** Theorem, m∠1 + m∠8 + m∠*X* = 180° and m∠6 + m∠7 + m∠*W* = 180°. So m∠1 = 180° − m∠8 − m∠*X* and m∠6 = 180° − m∠7 − m∠*W*. It is given that m∠8 = 2. **m∠7**. Because *WXYZ* is a rectangle, m∠*X* = m∠*W* = 3. **90°**. So, using properties of equality, m∠1 = m∠6. By similar reasoning, m∠2 = 4. **m∠5**. You know that m∠1 + m∠*ABC* + m∠2 = 5. **180°** and that m∠5 + m∠*CDA* + m∠6 = 6. **180°**. So m∠*ABC* = 180° − m∠1 − m∠2 and m∠*CDA* = 180° − m∠6 − m∠5. Using properties of equality, it follows that m∠*ABC* = 7. **m∠*CDA***. By similar reasoning, m∠*BCD* = 8. **m∠*DAB***. Because *ABCD* is a quadrilateral, m∠*ABC* + m∠*BCD* + m∠*CDA* + m∠*DAB* = 9. **360°**. So, using properties of equality and substitution, m∠*ABC* + m∠*BCD* + m∠*ABC* + m∠*BCD* = 360°, or 2(m∠*ABC*) + 2(m∠*BCD*) = 360°. It follows that 2(m∠*ABC* + m∠*BCD*) = 360°, or m∠*ABC* + m∠*BCD* = 180°. This means that ∠*ABC* and ∠*BCD* are a pair of 10. **supplementary** angles. Similarly, ∠*BCD* and ∠*CDA* are a pair of 11. **supplementary** angles. So $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$ by the 12. **Converse of the Same-Side Interior Angles** Theorem.

Therefore, *ABCD* is a parallelogram by the 13. **definition** of a parallelogram.

Can the path of a puck have the given shape? Write an explanation on a separate sheet of paper.

14. rhombus — Yes; explanations will vary.

15. rectangle — No; the puck will have to land in the goal.

16. kite — No; explanations will vary.

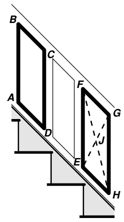
LESSON 6-2

Problem Solving

Properties of Parallelograms

Use the diagram for Exercises 1 and 2.

The wall frames on the staircase wall form parallelograms *ABCD* and *EFGH*.



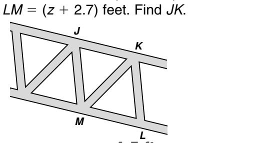
1. In ▱*ABCD*, the measure of ∠*A* is three times the measure of ∠*B*. What are the measures of ∠*C* and ∠*D*?

   m∠*C* = 135°; m∠*D* = 45°

2. In ▱*EFGH*, *FH* = 5*x* inches, *EG* = (2*x* + 4) inches, and *JG* = 8 inches. What is the length of *JH*?

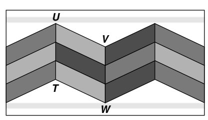
   15 in.

3. The diagram shows a section of the support structure of a roller coaster. In ▱*JKLM*, *JK* = (3*z* − 0.9) feet, and *LM* = (*z* + 2.7) feet. Find *JK*.



   4.5 ft

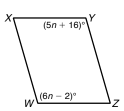
4. In ▱*TUVW*, part of a ceramic tile pattern, m∠*TUV* = (8*x* + 1)° and m∠*UVW* = (12*x* + 19)°. Find m∠*TUV*.



   65°

Choose the best answer.

5. What is the measure of ∠*Z* in parallelogram *WXYZ*?

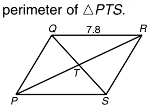


   A 18°
   B 74° (circled)
   C 106°
   D 108°

6. The perimeter of ▱*CDEF* is 54 centimeters. Find the length of $\overline{FC}$ if $\overline{DE}$ is 5 centimeters longer than $\overline{EF}$.

   F 11 cm
   G 14 cm
   H 16 cm (circled)
   J 44 cm

7. In ▱*PQRS*, *QT* = 7*x*, *TS* = 2*x* + 2.5, *RT* = 2*y*, and *TP* = *y* + 3. Find the perimeter of △*PTS*.

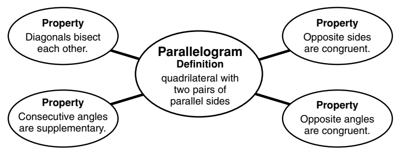


   A 6
   B 9.5
   C 12
   D 17.3 (circled)

LESSON 6-2

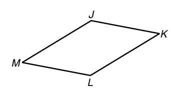
Reading Strategies

Use a Graphic Aid



Use the graphic aid above to help answer Exercises 1–10.

In ▱*JKLM*, *LM* = 86 millimeters, *LK* = 100 millimeters, and m∠*JML* = 42°. Find each measure.



1. *JM* — 100 mm

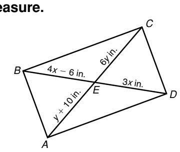
2. m∠*KJM* — 138°

3. *KJ* — 86 mm

4. m∠*LKJ* — 42°

5. m∠*MLK* — 138°

Given: ▱*ABCD*. Find each measure.



6. *AE* — 12 in.

7. *BE* — 18 in.

8. *CE* — 12 in.

9. *AC* — 24 in.

10. *BD* — 36 in.