# AP Calculus ---Notecards 1 – 20

**NC 1**

- **For a limit to exist, the left-handed limit must equal the right sided limit**

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$$

- **A function can have a limit at x = c even if there is a hole in the graph at that point. (Limit means "what y-value are you approaching?"**

- **Graphically, $\lim_{x \to \infty} f(x) = k$ is a horizontal asymptote of the graph.**

- **Graphically, $\lim_{x \to c} f(x) = \infty$ is a vertical asymptote of the graph.**

**NC 2**

- **Algebraically, to find a limit, $\lim_{x \to c} f(x)$,**
  - **PLUG IN *c*, and that is your limit**

    **Ex: $\lim_{x \to 2} \frac{x^2-1}{x-1} = \frac{3}{1} = 3$**

  - **If you plug in and get $\frac{0}{0}$, there is a hole at x = c.**

    **Factor, cancel, and plug in---OR use L'Hospital's Rule and plug in**

    **Ex: $\lim_{x \to 1} \frac{x^2-1}{x-1} = \frac{0}{0}$ $\quad L'HRule: \frac{2x}{1} = 2x = 2(1) = 2$**

  - **If you plug in and get $\frac{\#}{0}$, there is a vertical asymptote at x = c.**

    **To find the direction, plug in values very close to x on either side to find out which infinity the graph is going.**

    **Ex: $\lim_{x \to 1} \frac{x^2-2}{x-1} = \frac{-1}{0}$ $\quad DNE\ (you\ get +\infty\ on\ left\ and -\infty\ on\ right)$**

**NC 3**

- **Limits to Infinity for $\frac{f(x)}{g(x)}$**

  - **BETC (Bottom Equals Top Coefficients)**

    $\lim_{x\to\infty} \frac{x^2-1}{2x^2-1} = \frac{1}{2}$

  - **BOBO (Bigger On Bottom Zero)**

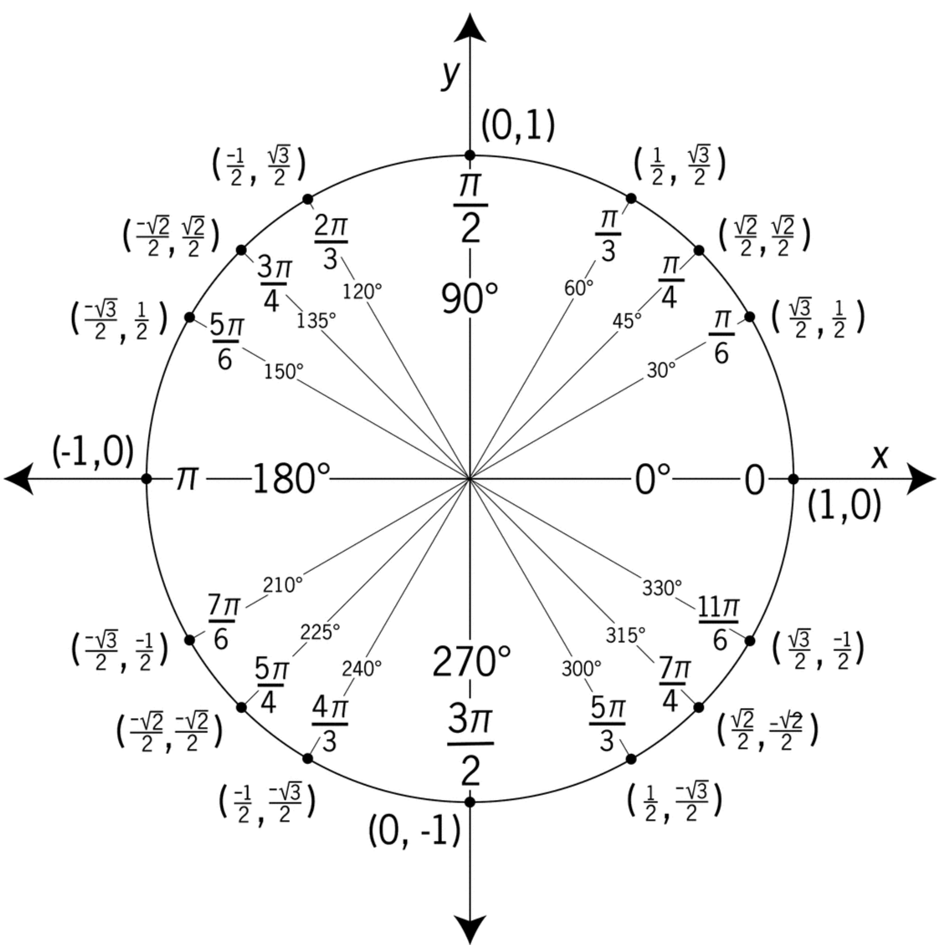
    $$\lim_{x\to\infty} \frac{x-1}{2x^2-1} = 0$$

  - **BOTI (Bigger On Top Infinity (check for +/-))**

    $$\lim_{x\to\infty} \frac{x^5-1}{2x^2-1} = \frac{+}{+} = +\infty$$

**NC 4**

**Unit Circle**



**NC 5**

Definition of Continuity at a Point:

A function f is continuous at a point $c$ if:

(1) $f(c)$ is defined

(2) $\lim_{x \to c} f(x)$ exists

(3) $\lim_{x \to c} f(x) = f(c)$

**NC 6**

- **Average Rate of Change (AROC) = slope of secant line = slope between two points =** $\frac{f(b)-f(a)}{b-a}$
- **Instantaneous Rate of Change (IROC) = slope of tangent line = slope at one point =** $f'(x)$.
- **AROC can be used to "estimate IROC at x = c"**

**Ex:**

**1) For $f(x) = 1 - 3x^2$, find AROC on [-1,1]. Then find IROC at x = 3.**

**AROC = 0** **IROC = -18**

**2) For the chart below, find AROC between t = 22 and t = 26.**

**AROC =** $\frac{29-11}{26-22} = \frac{18}{4} = \frac{9}{2}$

**3) For the chart below, find IROC at x = 27. Must use AROC to estimate:** $\frac{29-18}{28-26} = \frac{11}{2}$

| $t$ | 20 | 22 | 24 | 26 | 28 | 30 |
|---|---|---|---|---|---|---|
| $f(t)$ | 5 | 7 | 11 | 18 | 29 | 45 |

**NC 7**

- **Notations and expressions for finding the derivative:**
  - $y'$
  - $\frac{dy}{dx}$ **or** $\frac{d}{dx}$
  - $f'(x)$
  - **IROC**
  - **Slope of tangent line**
  - **Differentiate**

**NC 8**

- **Writing an equation for a tangent line:** $y - y_1 = m(x - x_1)$
- **m = slope = derivative=f ’(x)**
- $(x_1, y_1)$ **is a point on f(x)**
- **To find** $y_1$**, plug x into f(x)**

**NC 9**

- **Techniques of Differentiation**

  - If $f(x) = k$, then $f'(x) = 0$.
  - If $f(x) = kx$, then $f'(x) = k$.
  - If $f(x) = x^n$, then $f'(x) = nx^{n-1}$
  - If $h(x) = f(x)g(x)$, then $h'(x) = f'(x)g(x) + g'(x)f(x)$
  - If $h(x) = \frac{f(x)}{g(x)}$, then $h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$
  - If $h(x) = f\big(g(x)\big)$, then $h'(x) = f'(g(x)) \cdot g'(x)$
  - If $f(x) = lnx$, then $f'(x) = \frac{1}{x}$
  - If $f(x) = \ln u$, then $f'(x) = \frac{1}{u} \cdot \frac{du}{dx}$
  - If $f(x) = e^x$, then $f'(x) = e^x$
  - If $f(x) = e^u$, then $f'(x) = e^u \cdot \frac{du}{dx}$

**NC 10** **Trig Derivatives**

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

**NC 11**

- If $s(t)$ is the position of an object, then $s'(t) = v(t) =$ velocity.
- The velocity, $v(t)$, is the rate of change (slope) of the position, $s(t)$.
  - If $v(t) > 0$, the object is moving right or up
  - If $v(t) < 0$, the object is moving left or down
  - If $v(t) = 0$, the object is stationary/has stopped
- Velocity tells the speed and direction of the object.
- Speed = $|v(t)|$

**NC 12**

- If $s(t)$ is the position of an object, then $s''(t) = v'(t) =$ acceleration.
- The acceleration, $a(t)$, is the rate of change (slope) of the velocity, $v(t)$.
  - If $a(t) > 0$, the object is being pushed forward
  - If $a(t) < 0$, the object is being pulled backwards
- An objects speed increases when $a(t)$ and $v(t)$ have the SAME signs.
- An objects speed decreases when $a(t)$ and $v(t)$ have DIFFERENT signs.

**NC 13**

- Differentiability implies continuity, but continuity doesn't imply differentiability.
- Types of functions that are continuous, but not differentiable: CUSP, CORNER, VERTICAL TANGENT
- To check differentiability, first use the definition of continuity. If the function is continuous, then check definition of differentiability (same rules as continuity except use derivatives)

## NC 14

**Log/Exponential Rules**

- $e^0 = 1$
- $ln1 = 0$
- $e^{lnx} = x$ **(ex: $e^{ln6} = 6$)----remember e and ln cancel out!**
- $lne^x = x$ **(ex: $lne^9 = 9$)---remember e and ln cancel out**

## NC 15

*****Limit Definition of the Derivative**

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = f\,'(x) \qquad \lim_{h\to 0}\frac{f(c+h)-f(c)}{h} = f\,'(c)$$

**Step 1: f ’(x) or f ‘ (c) ?**

**Step 2: Replace “x + h” or “c + h” with “x”**

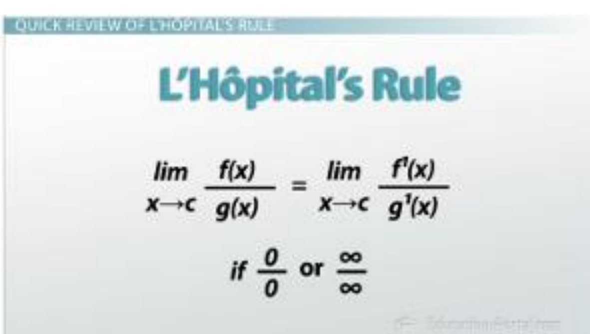
**Step 3: Find f ’(x) or f ‘ (c)**

**Ex:** $\lim_{h\to 0}\frac{(x+h)^2-2(x+h)-x^2+2x}{h} =$ **2x-2**

**Ex:** $\lim_{h\to 0}\frac{2(3+h)^2-2(3+h)-12}{h}$ **= 10**

## NC 16

****L’Hospital’s Rule:**



**NC 17**

- **Increasing/Decreasing Behavior of f(x) & Relative Extrema**
    - **Find the critical values (where f ' (x) = 0 or f ' (x) is undefined**
    - **Use a Number Line Analysis to evaluate f ' (x) on both sides of the critical values**
    - **If f ' (x) > 0, then f(x) is increasing**
    - **If f ' (x) < 0, then f(x) is decreasing**
    - **If f ' (x) = 0, then f(x) has a horizontal tangent (min, max, or layout)**
    - **If f ' (x) is undefined, then f(x) has a cusp, corner or vertical tangent**
    - **A relative maximum happens where the graph goes from increasing to decreasing on either side of a critical value**
    - **A relative minimum happens where the graph goes from decreasing to increasing on either side of a critical value**

**NC 18**

- **Concavity of f(x)**
    - **Find where f '' (x) = 0 or f '' (x) is undefined**
    - **Use a Number Line Analysis to evaluate f ''(x) on both sides**
    - **If f '' (x) > 0, then f(x) is concave up**
    - **If f '' (x) < 0, then f(x) is concave down**
    - **If f '' (x) = 0 or is undefined, there is a possible point of inflection**
    - **A point of inflection happens only when there is a change from ccu to ccd, or vice versa**

**NC 19**

**2nd Derivative Test**

- **If f ' (a) = 0 and f '' (a) > 0, then x = a is a relative minimum.**
- **If f ' (a) = 0 and f '' (a) < 0, then x = a is a relative maximum.**
- **If f ' (a) = 0 and f '' (a) = 0, then the test is inconclusive and you would need to use the first derivative number line to determine if the point was a max or min.**

**NC 20**

**To find a derivative using the GC (graphing calculator)**

- **Math 8**
- **Plug in "x", f(x) and x = c**

**Example: For f(x) = $\sqrt{x+9x^3}$, find f ' (5) using the GC.**

$\frac{d}{dx}\left(\sqrt{x+9x^3}\right)|x=5$ **(Answer: 10.055)**