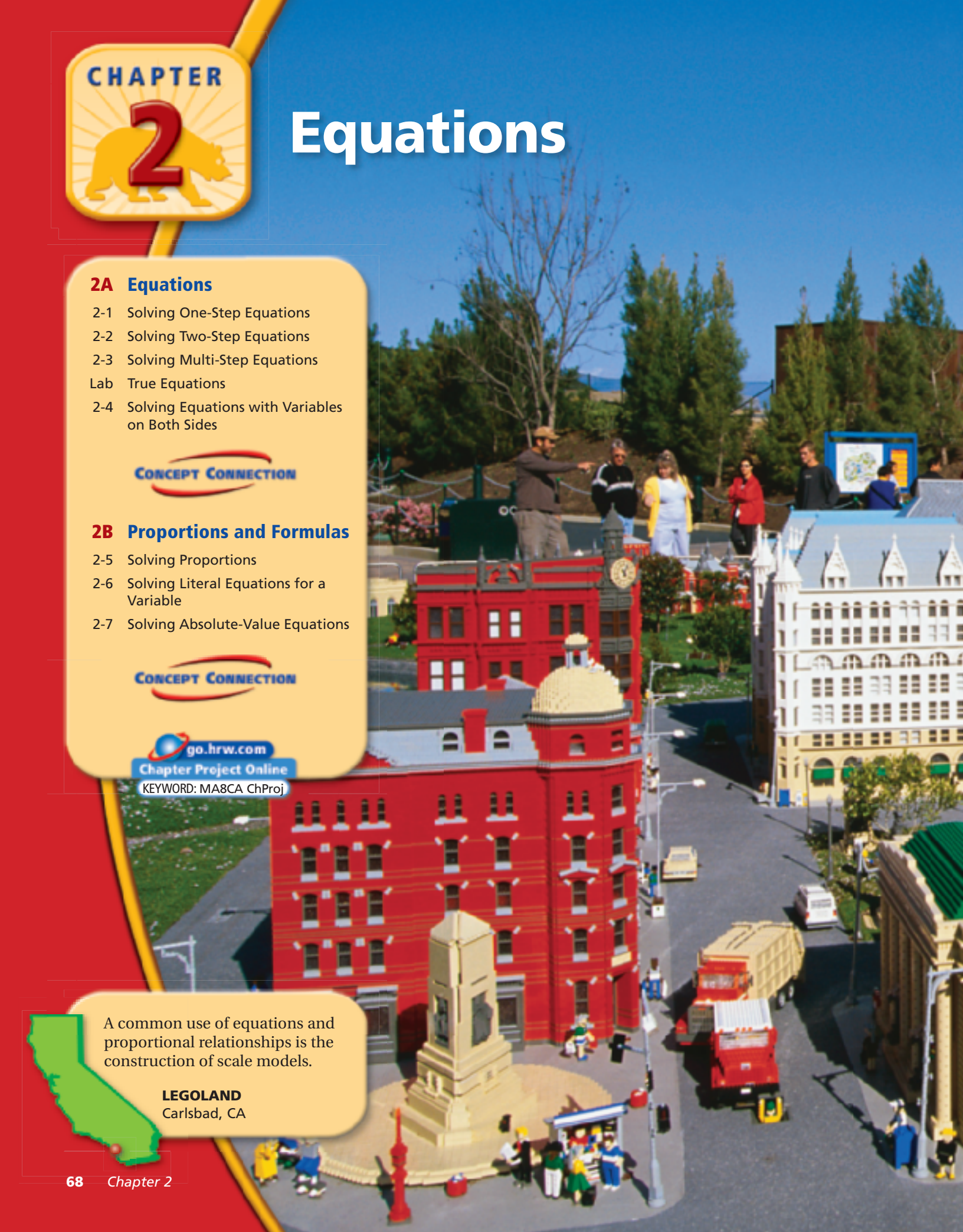
CHAPTER 2

Equations



2A Equations

CONCEPT CONNECTION

2B Proportions and Formulas

CONCEPT CONNECTION

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KEYWORD: MA8CA ChProj

A common use of equations and proportional relationships is the construction of scale models.

LEGOLAND
Carlsbad, CA

ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. constant
2. expression
3. order of operations
4. variable

A. a mathematical phrase that contains operations, numbers, and/or variables

B. a mathematical statement that two expressions are equivalent

C. a process for evaluating expressions

D. a symbol used to represent a quantity that can change

E. a value that does not change

Order of Operations

Simplify each expression.

5. $(7 - 3) \div 2$
6. $4 \cdot 6 \div 3$
7. $12 - 3 + 1$
8. $2 \cdot 10 \div 5$
9. $125 \div 5^2$
10. $7 \cdot 6 + 5 \cdot 4$

Add and Subtract Integers

Add.

11. $-15 + 19$
12. $-6 - (-18)$
13. $6 + (-8)$
14. $-12 + (-3)$

Add and Subtract Fractions

Perform each indicated operation. Give your answer in the simplest form.

15. $\frac{1}{4} + \frac{2}{3}$
16. $1\frac{1}{2} - \frac{3}{4}$
17. $\frac{3}{8} + \frac{2}{3}$
18. $\frac{3}{2} - \frac{2}{3}$

Evaluate Expressions

Evaluate each expression for the given value of the variable.

19. $2x + 3$ for $x = 7$
20. $3n - 5$ for $n = 7$
21. $13 - 4a$ for $a = 2$
22. $3y + 5$ for $y = 5$

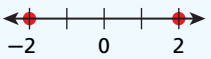
Connect Words and Algebra

23. Janie bought 4 apples and 6 bananas. Each apple cost \$0.75, and each banana cost \$0.60. Write an expression representing the total cost.
24. A rectangle has a width of 13 inches and a length of ℓ inches. Write an expression representing the area of the rectangle.
25. Write a phrase that could be modeled by the expression $n + 2n$.

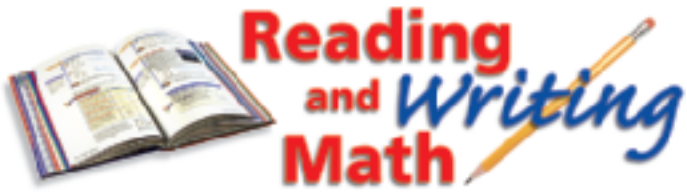
CHAPTER 2

Unpacking the Standards

The information below "unpacks" the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

California Standard	Academic Vocabulary	Chapter Concept
3.0 Students solve equations and inequalities **involving absolute values**. (Lesson 2-7)	**absolute value** a number's distance from 0 on a number line ***Example:***  Both 2 and −2 are 2 units from 0. So $\|2\| = 2$ and $\|-2\| = 2$.	You solve equations that have a variable inside absolute-value symbols.
4.0 Students simplify expressions before solving linear equations and inequalities **in one variable, such as** $3(2x - 5) + (4x - 2) = 12$. (Lessons 2-3, 2-4)	**simplify** (simplification) make things easier **linear equation** an equation whose variable(s) have exponents not greater than 1 **in one variable** containing one variable	You write expressions in their simplest form so that you can find the value of a variable that makes an equation true.
5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities **in one variable** and provide **justification** for each step. (Lessons 2-3, 2-4, 2-7)	**multistep** more than one step **involving** needing the use of **justification** a correct reason	You solve equations when the solution process requires two or more steps.
15.0 Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems. (Lesson 2-5)	**algebraic** having to do with algebra **technique** a way of doing something	You use what you learn in algebra to solve real-world problems about rates.
25.3 Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or **equations** or inequalities, **students determine whether the statement is true sometimes, always, or never.** (Lab 2-4)	**specific** single, exactly one **determine** tell or find out	You decide whether an equation is true for all values of the variable, for some values of the variable, or for no values of the variable.

Standards 2.0, 24.1, 24.2, 25.1, and 25.2 are also covered in this chapter. To see these standards unpacked go to Chapter 5, p. 254; Chapter 3, p. 134; and Chapter 1, p. 4.



CHAPTER 2

California Standards

English-Language Arts Reading
8.1.3

Study Strategy: Use Your Own Words

Explaining a concept using your own words will help you better understand it. For example, learning to solve equations might seem difficult if the textbook doesn't use the same words that you would use.

As you work through each lesson:

- Identify the important ideas from the explanation in the book.
- Use your own words to explain the important ideas you identified.

What Arturo Reads	What Arturo Writes
To evaluate an expression is to find its value.	*Evaluate an expression—find the value.*
To evaluate an algebraic expression, substitute numbers for the variables in the expression and then simplify the expression.	*Substitute a number for each variable (letter), and find the answer.*
A replacement set is a set of numbers that can be substituted for a variable.	*Replacement set—numbers that can be substituted for a letter.*

Try This

Rewrite each paragraph in your own words.

1. Two numbers are opposites if their sum is 0. A number and its opposite are on opposite sides of zero on a number line, but are the same distance from zero.
2. The Commutative and Associative Properties of Addition and Multiplication allow you to rearrange an expression to simplify it.
3. The terms of an expression are the parts to be added or subtracted. Like terms are terms that contain the same variables raised to the same powers. Constants are also like terms.

2-1 Solving One-Step Equations

California Standards

Preparation for 5.0 **Students solve multistep problems, including word problems, involving linear equations** and linear inequalities **in one variable** and provide justification for each step.
Also covered: **2.0**

Vocabulary
equation
solution of an equation
solution set

Why learn this?
You can use an equation to calculate your maximum heart rate. (See Example 4.)

An **equation** is a mathematical statement that two expressions are equal.

A **solution of an equation** is a value of the variable that makes the equation true. A **solution set** is the set of all solutions. Finding the solutions of an equation is also called *solving the equation.*

To find solutions, perform inverse operations until you have *isolated the variable.* A variable is isolated when it appears by itself on one side of an equation, and not at all on the other side.

Inverse Operations
Add x. ⟷ Subtract x.
Multiply by x. ⟷ Divide by x.

An equation is like a balanced scale. To keep the balance, you must perform the same inverse operation on both sides.

Know it! Note

Addition and Subtraction Properties of Equality

WORDS	NUMBERS	ALGEBRA
Addition Property of Equality		
You can add the same number to both sides of an equation, and the statement will still be true.	$3 = 3$ $3 + 2 = 3 + 2$	$a = b$ $a + c = b + c$
Subtraction Property of Equality		
You can subtract the same number from both sides of an equation, and the statement will still be true.	$7 = 7$ $7 - 5 = 7 - 5$	$a = b$ $a - c = b - c$

EXAMPLE 1 **Solving Equations by Using Addition or Subtraction**

Solve each equation.

A $x - 10 = 4$

$x - 10 = 4$
$+10 \quad +10$

Since 10 is subtracted from x, add 10 to both sides to undo the subtraction.

$x = 14$

The solution set is {14}.

Check

$x - 10 = 4$
$14 - 10 \mid 4$
$4 \mid 4$ ✓

To check your solution, substitute 14 for x in the original equation.

Writing Math

Solution sets are written in set notation using braces, { }. Solutions may be given in set notation, or they may be given in the form $x = 14$.

Solve each equation.

B $0.7 = r + 0.4$

$0.7 = r + 0.4$

$-0.4 \quad -0.4$

$0.3 = r$

Since 0.4 is added to r, subtract 0.4 from both sides to undo the addition.

The solution set is {0.3}.

Solve each equation. Check your answer.

1a. $n - 3.2 = 5.6$ **1b.** $-6 = k - 6$ **1c.** $6 + t = 14$

Multiplication and Division Properties of Equality

WORDS	NUMBERS	ALGEBRA
Multiplication Property of Equality		
You can multiply both sides of an equation by the same number, and the statement will still be true.	$6 = 6$ $6(3) = 6(3)$	$a = b$ $ac = bc$
Division Property of Equality		
You can divide both sides of an equation by the same nonzero number, and the statement will still be true.	$8 = 8$ $\frac{8}{4} = \frac{8}{4}$	$a = b$ $(c \neq 0), \frac{a}{c} = \frac{b}{c}$

EXAMPLE 2 Solving Equations by Using Multiplication or Division

Solve each equation.

A $13 = -2w$

$\frac{13}{-2} = \frac{-2w}{-2}$

$w = -\frac{13}{2}$, or -6.5

Since w is multiplied by –2, divide both sides by –2 to undo the multiplication.

The solution set is {–6.5}.

B $-4 = \frac{k}{-5}$

$(-5)(-4) = (-5)\left(\frac{k}{-5}\right)$

$20 = k$

Since k is divided by –5, multiply both sides by –5 to undo the division.

The solution set is {20}.

Check

$-4 = \frac{k}{-5}$

-4	$\frac{20}{-5}$
-4	-4 ✓

To check your solution, substitute 20 for k in the original equation.

Solve each equation. Check your answer.

2a. $\frac{p}{5} = 10$ **2b.** $0.5y = -10$ **2c.** $\frac{c}{8} = 7$

When solving equations, you will sometimes find it easier to add an opposite to both sides instead of subtracting, or to multiply by a reciprocal instead of dividing. This is often true when an equation contains negative numbers or fractions.

EXAMPLE 3 Solving Equations by Using Opposites or Reciprocals

Solve each equation.

A $-8 + b = 2$

$-8 + b = 2$

$+8 \quad +8$

$b = 10$

Since –8 is added to b, add 8 to both sides.

The solution set is {10}.

B $\frac{5}{9}v = 35$

$\left(\frac{9}{5}\right)\frac{5}{9}v = \left(\frac{9}{5}\right)35$

The reciprocal of $\frac{5}{9}$ is $\frac{9}{5}$. Since v is multiplied by $\frac{5}{9}$, multiply both sides by $\frac{9}{5}$.

$v = 63$

The solution set is {63}.

CHECK IT OUT!

Solve each equation. Check your answer.

3a. $-2.3 + m = 7$ **3b.** $-\frac{3}{4} + z = \frac{5}{4}$ **3c.** $\frac{1}{6}w = 102$

EXAMPLE 4 Fitness Application

A person's maximum heart rate is the highest rate, in beats per minute, that the person's heart should reach. A person's age added to his or her maximum heart rate is 220. Write and solve an equation to find the maximum heart rate of a 15-year-old.

Age	added to	maximum heart rate	is	220.
a	$+$	r	$=$	220

$a + r = 220$

Write an equation to represent the relationship.

$15 + r = 220$

$-15 \quad -15$

Substitute 15 for a. Since 15 is added to r, subtract 15 from both sides to undo the addition.

$r = 205$

The maximum heart rate for a 15-year-old is 205 beats per minute.

CHECK IT OUT!

4. The distance in miles from the airport that a plane should begin descending divided by 3 equals the plane's height above the ground in thousands of feet. A plane is 10,000 feet above the ground. Write and solve an equation to find the distance from the airport at which this plane should begin descending.

Student to Student

Zero As a Solution

Ama Walker
Carson High School

I used to get confused when I got a solution of 0. But my teacher reminded me that 0 is a number just like any other number, so it can be a solution of an equation. Just check your answer and see if it works.

$x + 6 = 6$

$-6 \quad -6$

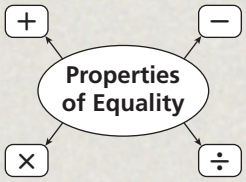
$x = 0$

Check

$x + 6 = 6$	
$0 + 6$	6
6	6 ✓

THINK AND DISCUSS

1. Describe how the Addition and Subtraction Properties of Equality are like a balanced scale.
2. By what number would you multiply both sides of the equation $\frac{b}{4} = 10$ to isolate the variable?
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of an equation that can be solved by using the given property and solve it.



Know it! Note

2-1 Exercises

California Standards Practice

Preparation for 5.0; 2.0, 25.2

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Homework Help Online
KEYWORD: MA8CA 2-1
Parent Resources Online
KEYWORD: MA8CA Parent

GUIDED PRACTICE

1. **Vocabulary** Will the *solution of an equation* such as $x - 3 = 9$ be a variable or a number? Explain.

Solve each equation. Check your answer.

SEE EXAMPLE 1 p. 72

2. $s - 5 = 3$
3. $17 = w - 4$
4. $k - 8 = -7$
5. $t + 5 = -25$
6. $b + \frac{2}{3} = 2$
7. $4.2 = m + 3.6$

SEE EXAMPLE 2 p. 73

8. $\frac{k}{4} = 8$
9. $\frac{g}{1.9} = 10$
10. $-2 = \frac{w}{-7}$
11. $4x = 28$
12. $4m = 10$
13. $-9j = -45$

SEE EXAMPLE 3 p. 74

14. $-10 + d = 7$
15. $20 = -12 + v$
16. $-4.6 + q = 5$
17. $\frac{1}{2}d = 7$
18. $-\frac{2}{3} + c = \frac{2}{3}$
19. $\frac{2}{3}s = -6$

SEE EXAMPLE 4 p. 74

20. **Geology** In 1668, the Hope diamond was reduced from its original weight by about 45 carats, resulting in a diamond weighing about 67 carats. Write and solve an equation to find how many carats the original diamond weighed.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
21–28	1
29–36	2
37–44	3
45	4

Extra Practice

Skills Practice p. EP4
Application Practice p. EP25

Solve each equation. Check your answer.

21. $1 = k - 8$
22. $m + 20 = 3$
23. $x - 7 = 10$
24. $v + 2300 = -800$
25. $b + \frac{1}{2} = \frac{1}{2}$
26. $q - 0.5 = 1.5$
27. $4\frac{2}{3} = r - \frac{1}{3}$
28. $2 = d + \frac{1}{4}$
29. $\frac{x}{2} = 12$
30. $11 = -2z$
31. $5t = -15$
32. $1.6 = \frac{d}{3}$
33. $-\frac{j}{6} = 6$
34. $-12 = -12u$
35. $-8.4 = -4n$
36. $\frac{h}{8.1} = -4$

Solve each equation. Check your answer.

37. $-12 + f = 3$ **38.** $-9 = -4 + g$ **39.** $\frac{4}{7}t = -2$ **40.** $-\frac{4}{5}g = -12$

41. $26 = -4 + y$ **42.** $\frac{5}{2}k = 5$ **43.** $-9 = \frac{3}{4}d$ **44.** $-5.2 + a = -8$

45. Nutrition An orange contains about 80 milligrams of vitamin C, which is 10 times as much as an apple contains. Write and solve an equation to find the amount of vitamin C in an apple.

Write an equation to represent each relationship. Then solve the equation.

46. Ten less than a number is equal to 12.

47. Five times a number is 45.

48. The quotient of a number and 3 is −8.

49. Eight more than a number is 16.

50. ERROR ANALYSIS Below are two possible solutions to $x + 12.5 = 21.6$. Which is incorrect? Explain the error.

A

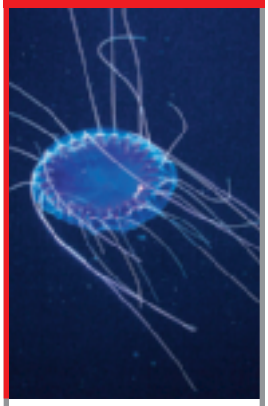
$x + 12.5 = 21.6$
$-12.5 \quad -12.5$
$x = 9.1$

B

$x + 12.5 = 21.6$
$+12.5 \quad +12.5$
$x = 34.1$

California LINK

Oceanography



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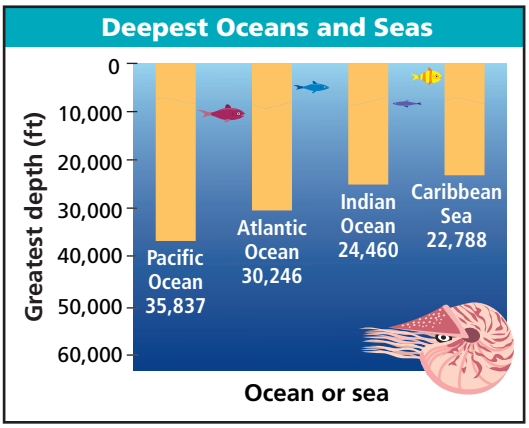
The ocean near Monterey reaches depths of more than 10,000 ft. Below 330 ft, there is little, if any, light. Many deep sea animals, such as jelly fish, make their own light (bioluminescence) to help them survive.

51. Oceanography The Atlantic Ocean's greatest depth (in feet) is 17,366 feet greater than its average depth. Use the information in the graph to write and solve an equation to find the average depth of the Atlantic Ocean.



52. Consumer Economics Dion's long-distance phone bill was $13.80. His long-distance calls cost $0.05 per minute. Write and solve an equation to find the number of minutes he was charged for.

53. Write About It Describe a real-world situation that can be modeled by $x + 5 = 25$. Solve the equation and tell what the solution means in the context of your problem.

54. Critical Thinking Without solving, tell whether the solution of $-2 + z = 10$ is greater than 10 or less than 10. Explain.

CONCEPT CONNECTION

55. This problem will help prepare you for the Concept Connection on page 100.

Rates are often used to describe how quickly something is moving or changing.

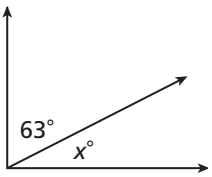
a. A wildfire spreads at a rate of 1000 acres per day. How many acres will the fire cover in 2 days?

b. How many acres will the fire cover in 5 days?

c. Another wildfire spread for 7 days and covered a total of 780 square miles. How can you estimate the number of square miles the fire covered per day?

Geometry The angles in each pair are complementary. Write and solve an equation to find each value of x. (*Hint:* The measures of complementary angles add to 90°.)

56.



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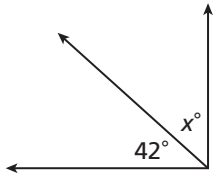
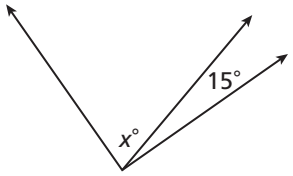


58.



Multiple Choice For Exercises 59 and 60, choose the best answer.

59. Which situation is best represented by $x - 32 = 8$?

Ⓐ Logan withdrew \$32 from her bank account. After her withdrawal, her balance was \$8. How much was originally in her account?

Ⓑ Daniel has 32 baseball cards. Joseph has 8 fewer baseball cards than Daniel has. How many baseball cards does Joseph have?

Ⓒ Room A contains 32 desks. Room B has 8 fewer desks. How many desks are in Room B?

Ⓓ Janelle bought a bag of 32 craft sticks for a project. She used 8 craft sticks. How many craft sticks does she have left?

60. For which equation is $m = 10$ a solution?

Ⓐ $5 = 2m$ Ⓒ $\frac{m}{2} = 5$

Ⓑ $5m = 2$ Ⓓ $\frac{m}{10} = 2$

61. Short Response Luisa bought 6 cans of cat food that each cost the same amount. She spent a total of \$4.80.

a. Write an equation that can be used to determine the cost of one can of cat food.

b. Solve your equation to find the cost of one can of cat food.

CHALLENGE AND EXTEND

Solve each equation. Check your answer.

62. $3\frac{1}{5} + b = \frac{4}{5}$ **63.** $x - \frac{7}{4} = \frac{2}{3}$ **64.** $\left(1\frac{1}{3}\right)x = 2\frac{2}{3}$ **65.** $\left(3\frac{1}{5}\right)b = \frac{4}{5}$

66. If $p - 4 = 2$, find the value of $5p - 20$.

67. If $2p = 4$, find the value of $6p + 10$.

68. If $3x = 15$, find the value of $12 - 4x$.

69. If $2 + n = -11$, find the value of $6n$.

SPIRAL STANDARDS REVIEW

7NS1.2, 1.0, 2.0

Multiply or divide. *(Lesson 1-3)*

70. $-63 \div (-7)$ **71.** $\frac{3}{7} \div \left(-\frac{4}{7}\right)$ **72.** $(-12)(-6)$

Give the side length of a square with the given area. *(Lesson 1-5)*

73. 225 m^2 **74.** 36 ft^2 **75.** 100 cm^2

Write each product using the Distributive Property. Then simplify. *(Lesson 1-6)*

76. $11(104)$ **77.** $12(43)$ **78.** $3(46)$

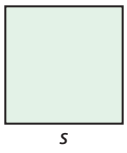
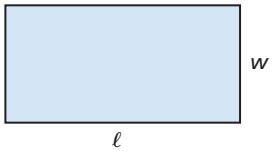
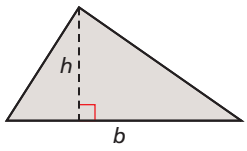
Connecting Algebra to Geometry

Area of Composite Figures

Review the area formulas for squares, rectangles, and triangles in the table below.

California Standards

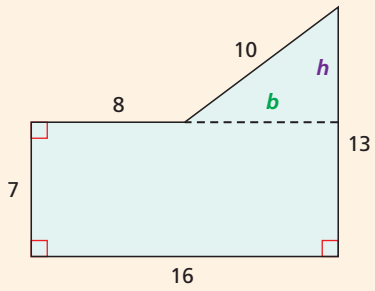
Reinforcement of 7MG2.2 Estimate and **compute the area of more complex or irregular two-** and three-**dimensional figures by breaking the figure down into more basic geometric objects.**

Squares	Rectangles	Triangles
		
$A = s^2$	$A = \ell w$	$A = \frac{1}{2}bh$

A *composite figure* is a figure that is composed of basic shapes. You can divide composite figures into combinations of squares, rectangles, and triangles to find their areas.

Example

Find the area of the figure shown.



Divide the figure into a rectangle and a right triangle. Notice that you do not know the base or the height of the triangle. Use b and h to represent these lengths.

The bottom of the rectangle is 16 units long; the top of the rectangle is 8 units long plus the base of the triangle. Use this information to write and solve an equation.

$$\begin{array}{r} b + 8 = 16 \\ \underline{-8 \quad -8} \\ b \quad = 8 \end{array}$$

The right side of the figure is 13 units long: 7 units from the rectangle plus the height of the triangle. Use this information to write and solve an equation.

$$\begin{array}{r} h + 7 = 13 \\ \underline{-7 \quad -7} \\ h \quad = 6 \end{array}$$

The area of the figure is the sum of the areas of the rectangle and the triangle.

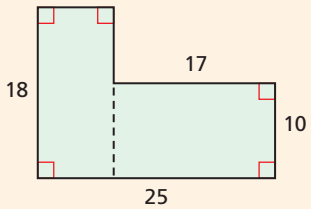
Area of rectangle ↓ Area of triangle ↓

$$A = \ell w + \frac{1}{2}bh$$
$$A = 16(7) + \frac{1}{2}(8)(6)$$
$$A = 112 + 24$$
$$A = 136 \text{ square units}$$

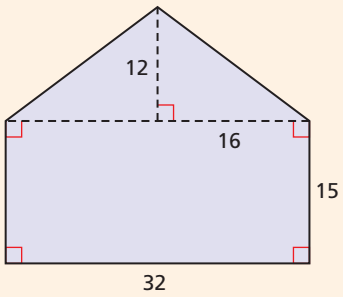
Try This

Find the area of each composite figure.

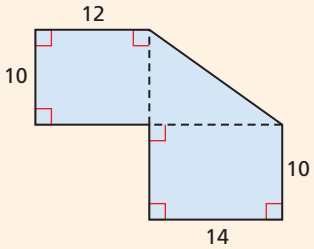
1.



2.



3.



2-2 Solving Two-Step Equations

California Standards

Preparation for 5.0 **Students solve multistep problems, including word problems, involving linear equations** and linear inequalities **in one variable** and provide justification for each step.

Vocabulary
equivalent equations

Why learn this?
Equations containing two operations can model the cost of a music club membership. (See Example 3.)

Many equations contain more than one operation, such as $2x + 5 = 11$.

This equation contains multiplication and addition. Equations that contain two operations require two steps to solve. Identify the operations in the equation and the order in which they are applied to the variable. Then use inverse operations to undo them in reverse order one at a time.

$$2x + 5 = 11$$

Operations in the Equation	To Solve
❶ First x is **multiplied** by 2.	❶ **Subtract** 5 from both sides of the equation.
❷ Then 5 is **added**.	❷ Then **divide** both sides by 2.

$2x + 5 = 11$

$-5 \quad -5$ **Subtract 5 from both sides of the equation.**

$2x = 6$

$\frac{2x}{2} = \frac{6}{2}$ **Divide both sides of the equation by 2.**

$x = 3$ The solution set is {3}.

Each time you perform an inverse operation, you create an equation that is *equivalent* to the original equation. **Equivalent equations** have the same solutions, or the same solution set. In the example above, $2x + 5 = 11$, $2x = 6$, and $x = 3$ are all equivalent equations.

EXAMPLE 1 Solving Two-Step Equations

Solve $10 = 6 - 2x$.

$10 = 6 - 2x$ *First x is multiplied by –2. Then 6 is added.*

$-6 \quad -6$ *Subtract 6 from both sides.*

$4 = -2x$ *4 = –2x is equivalent to 10 = 6 – 2x.*

$\frac{4}{-2} = \frac{-2x}{-2}$ *Since x is multiplied by –2, divide both sides by –2 to undo the multiplication.*

$-2 = x$ *The solution set is {–2}.*

Helpful Hint

Check your answer.

$10 = 6 - 2x$

10	$6 - 2(-2)$
10	$6 - (-4)$
10	$6 + 4$
10	10 ✓

CHECK IT OUT! **Solve each equation. Check your answer.**

1a. $-4 + 7x = 3$ **1b.** $1.5 = 1.2y - 5.7$ **1c.** $\frac{n}{7} + 2 = 2$

EXAMPLE 2 Solving Two-Step Equations That Contain Fractions

Solve each equation.

A $\frac{q}{15} - \frac{1}{5} = \frac{3}{5}$

Method 1 Use fraction operations.

$\frac{q}{15} - \frac{1}{5} = \frac{3}{5}$

$+\frac{1}{5} \quad +\frac{1}{5}$ — *Since $\frac{1}{5}$ is subtracted from $\frac{q}{15}$, add $\frac{1}{5}$ to both sides to undo the subtraction.*

$\frac{q}{15} = \frac{4}{5}$

$15\left(\frac{q}{15}\right) = 15\left(\frac{4}{5}\right)$ — *Since q is divided by 15, multiply both sides by 15 to undo the division.*

$q = \frac{15 \cdot 4}{5}$ — *Simplify.*

$q = \frac{60}{5}$

$q = 12$ — *The solution set is {12}.*

Helpful Hint

You can multiply both sides of the equation by any common denominator of the fractions. Using the LCD is the most efficient.
To review fraction operations, including LCD, see Skills Bank pages SB8–SB9.

Method 2 Multiply by the least common denominator (LCD) to clear the fractions.

$\frac{q}{15} - \frac{1}{5} = \frac{3}{5}$

$15\left(\frac{q}{15} - \frac{1}{5}\right) = 15\left(\frac{3}{5}\right)$ — *Multiply both sides by 15, the LCD of the fractions.*

$15\left(\frac{q}{15}\right) - 15\left(\frac{1}{5}\right) = 15\left(\frac{3}{5}\right)$ — *Distribute 15 on the left side.*

$q - 3 = 9$

$+3 \quad +3$ — *Simplify. Since 3 is subtracted from q, add 3 to both sides to undo the subtraction.*

$q = 12$ — *The solution set is {12}.*

Check *To check your solution, substitute 12 for q in the original equation.*

$\frac{q}{15} - \frac{1}{5} = \frac{3}{5}$

$\frac{12}{15} - \frac{1}{5}$	$\frac{3}{5}$
$\frac{3}{5}$	$\frac{3}{5}$ ✓

B $\frac{j}{4} + 2 = 9$

$\frac{j}{4} + 2 = 9$

$-2 \quad -2$ — *Since 2 is added to $\frac{j}{4}$, subtract 2 from both sides to undo the addition.*

$\frac{j}{4} = 7$

$4\left(\frac{j}{4}\right) = 4(7)$ — *Since j is divided by 4, multiply both sides by 4 to undo the division.*

$j = 28$ — *The solution set is {28}.*

CHECK IT OUT! **Solve each equation. Check your answer.**

2a. $\frac{2x}{5} - \frac{1}{2} = 5$ **2b.** $\frac{3}{4}u + \frac{1}{2} = \frac{7}{8}$ **2c.** $\frac{1}{5}n - \frac{1}{3} = \frac{8}{3}$

EXAMPLE 3 *Problem-Solving Application*

PROBLEM SOLVING

Alex belongs to the Student Music Club and bought a discount card for $19.95. After one year, Alex has spent $63.40. Write and solve an equation to find how many CDs Alex bought during the year.

1 Understand the Problem

The **answer** will be the number of CDs that Alex bought during the year.

List the important information:

- Alex paid $19.95 for a student discount card.
- Alex paid $3.95 for each CD he purchased.
- After one year, Alex has spent $63.40.

2 Make a Plan

Let c represent the number of CDs that Alex purchased. That means Alex has spent $3.95c. However, Alex must also add the amount he spent on the card. Write an equation to represent this situation.

total cost	=	cost of CDs	+	cost of discount card
63.40	=	$3.95c$	+	19.95

3 Solve

$$63.40 = 3.95c + 19.95$$
$$-19.95 \qquad -19.95$$
$$43.45 = 3.95c$$
$$\frac{43.45}{3.95} = \frac{3.95c}{3.95}$$
$$11 = c$$

Since 19.95 is added to 3.95c, subtract 19.95 from both sides to undo the addition.

Since c is multiplied by 3.95, divide both sides by 3.95 to undo the multiplication.

Alex bought 11 CDs during the year.

4 Look Back

Check that the answer is reasonable. The cost per CD is about $4, so if Alex bought 11 CDs, this amount is about $11(4) = \$44$.

Add the cost of the discount card, which is about $20: $44 + 20 = 64$. So the total cost was about $64, which is close to the amount given in the problem, $63.40.

CHECK IT OUT!

3a. Sara paid $15.95 to become a member at a gym. She then paid a monthly membership fee. Her total cost for 12 months was $735.95. How much was the monthly fee?

3b. Lynda has 12 records in her collection. She adds the same number of new records to her collection each month. After 7 months Lynda has 26 records. How many records does Lynda add each month?

THINK AND DISCUSS

1. Explain the steps you would follow to solve $2x + 1 = 7$. How is this procedure different from the one you would follow to solve $2x - 1 = 7$?

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write and solve a two-step equation. Use addition, subtraction, multiplication, and division.

Solving Two-Step Equations	

2-2 Exercises

California Standards Practice
Preparation for 5.0

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GUIDED PRACTICE

SEE EXAMPLE 1 p. 79

Solve each equation. Check your answer.

1. $4a + 3 = 11$
2. $8 = 3r - 1$
3. $\frac{x}{6} + 4 = 15$
4. $x + 0.3 = 3.3$
5. $15y + 31 = 61$
6. $9 - c = -13$

SEE EXAMPLE 2 p. 80

7. $\frac{1}{3}y + \frac{1}{4} = \frac{5}{12}$
8. $\frac{2}{7}j - \frac{1}{7} = \frac{3}{14}$
9. $\frac{x}{8} - \frac{1}{2} = 6$
10. $\frac{1}{2} + 12x = \frac{9}{2}$
11. $\frac{5}{6}x - \frac{1}{3} = \frac{5}{2}$
12. $3 - \frac{1}{2}r = 12$

SEE EXAMPLE 3 p. 81

13. Transportation Paul bought a student discount card for the bus. The card cost $7 and allows him to buy daily bus passes for $1.50. After one month, Paul spent $29.50. How many daily bus passes did Paul buy?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–19	1
20–25	2
26	3

Extra Practice
Skills Practice p. EP4
Application Practice p. EP25

Solve each equation. Check your answer.

14. $5 = 2g + 1$
15. $6h - 7 = 17$
16. $15 = \frac{a}{3} - 2$
17. $3x + 3 = 18$
18. $0.6g + 11 = 5$
19. $32 = 5 - 3t$
20. $2d + \frac{1}{5} = \frac{3}{5}$
21. $1 = 2x + \frac{1}{2}$
22. $\frac{z}{2} + 1 = \frac{3}{2}$
23. $\frac{2}{3} = \frac{4j}{6}$
24. $\frac{3}{4} = \frac{3}{8}x - \frac{3}{2}$
25. $\frac{1}{5} - \frac{x}{5} = -\frac{2}{5}$

26. Consumer Economics Jennifer is saving money to buy a bike. The bike costs $245. She has $125 saved, and each week she adds $15 to her savings. How many weeks will it take her to save enough money to buy the bike?

Write an equation to represent each relationship. Then solve.

27. Seven less than twice a number equals 19.

28. Eight decreased by 3 times a number equals 2.

29. 30 increased by 5 times a number equals 80.

30. 30 less than 4 times a number is equal to 14.

31. The sum of 64 and 3 times a number is −2.

32. 6 added to twice a number is equal to −8.

LINK

History

Martin Luther King Jr. entered college at age 15. During his life he earned 3 degrees and was awarded 20 honorary degrees.

Source: lib.lsu.edu

33. History In 1963, Dr. Martin Luther King Jr. began his famous "I have a dream" speech with the words "Five score years ago, a great American, in whose symbolic shadow we stand, signed the Emancipation Proclamation." The Proclamation was signed by President Abraham Lincoln in 1863.

a. Using the dates given, write and solve an equation that can be used to find the number of years in a score.

b. How many score would represent 60?

Solve each equation. Check your answer.

34. $3t + 44 = 50$

35. $3x - 6 = 18$

36. $15 = \frac{c}{3} - 2$

37. $2x + 6.5 = 15.5$

38. $3.9w - 17.9 = -2.3$

39. $20 = x - 3x$

40. $5x + 9 = 39$

41. $15 + 5.5m = 70$

42. $7j + 3 = 24$

43. $\frac{3}{4} + \frac{x}{2} = 3$

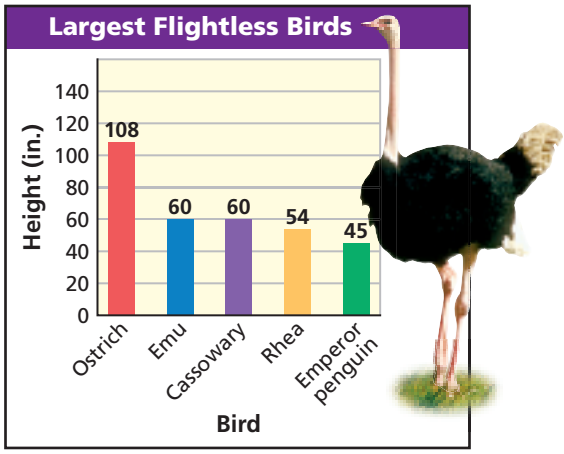
44. $50 = 3t - 4$

45. $14.5 = 5.5n - 2$

Biology **Use the graph for Exercises 46 and 47.**

46. The maximum height of an ostrich is 20 inches more than 4 times the maximum height of a kiwi. Write and solve an equation to find the maximum height of a kiwi.

47. Five times the maximum height of a kakapo minus 70 equals the maximum height of an emu. Write and solve an equation to find the maximum height of a kakapo.



Source: The Top Ten of Everything

48. Transportation A taxi company charges $1.10 plus $0.95 per mile. Karen's total fare was $12.50. How many miles did Karen travel?

CONCEPT CONNECTION

49. This problem will help prepare you for the Concept Connection on page 100.

a. The cost of fighting a particular forest fire is $225 per acre. Complete the table.

b. Write an equation for the relationship between the cost c of fighting the fire and the number of acres n.

Cost of Fighting Fire	
Acres	Cost ($)
100	22,500
200	
500	
1000	
1500	
n	

50. Critical Thinking The equation $2 - m = 17$ has more than one solution method. Give at least two different "first steps" to solve this equation.

51. Write About It Write a series of steps that you can use to solve any two-step equation.

Multiple Choice **For Exercises 52 and 53, choose the best answer.**

52. The equation $c = 48 + 0.06m$ represents the cost c of renting a car and driving m miles. Which statement best describes this cost?

Ⓐ The cost is a flat rate of \$0.06 per mile.

Ⓑ The cost is \$0.48 for the first mile and \$0.06 for each additional mile.

Ⓒ The cost is a \$48 fee plus \$0.06 per mile.

Ⓓ The cost is a \$6 fee plus \$0.48 per mile.

53. Which equation is equivalent to $4m - 3 = 21$?

Ⓐ $4m + 3 = 24$ Ⓒ $4m = 18$

Ⓑ $4m - 3 = 18$ Ⓓ $4m = 24$

54. Gridded Response A telemarketer earns \$150 a week plus \$2 for each call that results in a sale. Her pay last week was \$204. How many of her calls last week resulted in sales?

CHALLENGE AND EXTEND

Solve each equation. Check your answer.

55. $\frac{11}{2} + 3x = \frac{-5^2}{2}$

56. $\frac{15}{2^2}x - 15 = \frac{33}{2^2}$

57. $-5.2x + 1.69 = -8.71$

58. $\frac{1}{2}x - 12.75 = 21.25$

59. $169 = 37x - 4^2$

60. $8.49 = 4.6x - 5.31$

61. Business The formula $p = nc - e$ gives the profit p when a number of items n are each sold at a cost c and expenses e are subtracted.

a. If $p = 2500$, $n = 2000$, and $e = 800$, what is the value of c?

b. If $p = 2500$, $n = 1000$, and $e = 800$, what is the value of c?

c. What if...? If n is divided in half while p and e remain the same, what is the effect on c?

SPIRAL STANDARDS REVIEW

6AF1.1, 7NS1.4, 7NS1.5, 1.0

Write all classifications that apply to each real number. *(Lesson 1-5)*

62. $\sqrt{3}$ **63.** -58 **64.** $2\frac{1}{3}$ **65.** 0.17

Write each product using the Distributive Property. Then simplify. *(Lesson 1-6)*

66. $8(61)$ **67.** $9(28)$ **68.** $11(28)$ **69.** $13(21)$

70. $3(45)$ **71.** $7(19)$ **72.** $9(72)$ **73.** $8(33)$

Solve each equation. *(Lesson 2-1)*

74. $17 = k + 4$ **75.** $x - 18 = 3$ **76.** $a + 6 = -12$ **77.** $-7 = q - 7$

78. $12b = 60$ **79.** $7 = \frac{z}{4}$ **80.** $3a = 24$ **81.** $\frac{t}{6} = -7$

2-3 Solving Multi-Step Equations

California Standards

4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x - 5) + 4(x - 2) = 12$.

5.0 Students solve multi-step problems, including word problems, involving linear equations and linear inequalities **in one variable** and provide justification for each step.

Why learn this?

Martial arts instructors can model enrollment costs with multi-step equations.

A martial arts school is offering a special where students can enroll for half price, after a $12.50 application fee.

Ten students enrolled and paid a total of $325. To find the regular price of enrollment, you can solve an equation.

Regular price of enrollment

Number of students → $10\left(\frac{p}{2} + 12.50\right) = 325$ ← Total cost

Application fee

Notice that this equation contains multiplication, division, and addition. An equation that contains multiple operations will require multiple steps to solve. You will create an equivalent equation at each step.

EXAMPLE 1 Solving Multi-Step Equations

Solve $\frac{4x + 1}{5} = 5$. Check your answer.

$5\left(\frac{4x + 1}{5}\right) = 5(5)$ — *Since 4x + 1 is divided by 5, multiply both sides by 5 to undo the division.*

$4x + 1 = 25$

$-1 \quad -1$ — *Since 1 is added to 4x, subtract 1 from both sides to undo the addition.*

$4x = 24$

$\frac{4x}{4} = \frac{24}{4}$ — *Since x is multiplied by 4, divide both sides by 4 to undo the multiplication.*

$x = 6$ — *The solution set is {6}.*

Check $\frac{4x + 1}{5} = 5$ — *To check your solution, substitute 6 for x in the original equation.*

$\frac{4(6) + 1}{5}$	5
$\frac{24 + 1}{5}$	5
$\frac{25}{5}$	5
5	5 ✓

CHECK IT OUT!

Solve each equation. Check your answer.

1a. $\frac{5m + 13}{2} = 1$

1b. $\frac{4 - 2z}{4} = -2$

You may have to combine like terms or use the Distributive Property before you begin solving.

EXAMPLE 2 Simplifying Before Solving Equations

Solve $6x + 3 - 8x = 13$.

$6x + 3 - 8x = 13$

$6x - 8x + 3 = 13$ *Use the Commutative Property of Addition.*

$-2x + 3 = 13$ *Combine like terms.*

$-3 \quad -3$ *Since 3 is added to $-2x$, subtract 3 from both sides to undo the addition.*

$-2x = 10$

$\frac{-2x}{-2} = \frac{10}{-2}$ *Since x is multiplied by -2, divide both sides by -2 to undo the multiplication.*

$x = -5$ *The solution set is $\{-5\}$.*

Solve each equation. Check your answer.

2a. $2a + 3 - 8a = 8$ **2b.** $-8 - 2d + 2 = 4$ **2c.** $4x - 8 + 2x = 40$

EXAMPLE 3 Simplifying Using the Distributive Property

Solve each equation.

A $9 = 6 - (x + 2)$

$9 = 6 + (-1)(x + 2)$ *Write subtraction as addition of the opposite.*

$9 = 6 + (-1)(x) + (-1)(2)$ *Distribute -1.*

$9 = 6 - x - 2$ *Simplify.*

$9 = 6 - 2 - x$ *Commutative Property of Addition*

$9 = 4 - x$ *Combine like terms.*

$-4 \quad -4$ *Since 4 is added to $-x$, subtract 4 from both sides.*

$5 = -x$

$\frac{5}{-1} = \frac{-x}{-1}$ *Since x is multiplied by -1, divide both sides by -1.*

$-5 = x$

Helpful Hint

You can think of a negative sign as a coefficient of -1.
$-(x + 2) = -1(x + 2)$
$-x = -1x$

B $4(x + 1) + 2(x - 7) = 50$

$4(x + 1) + 2(x - 7) = 50$

$4(x) + 4(1) + 2(x) + 2(-7) = 50$ *Distribute 4 and 2.*

$4x + 4 + 2x - 14 = 50$ *Simplify.*

$4x + 2x + 4 - 14 = 50$ *Commutative Property of Addition*

$6x - 10 = 50$ *Combine like terms.*

$+10 \quad +10$ *Since 10 is subtracted from 6x, add 10 to both sides.*

$6x = 60$

$\frac{6x}{6} = \frac{60}{6}$ *Since x is multiplied by 6, divide both sides by 6.*

$x = 10$

Solve each equation. Check your answer.

3a. $3(a+1)-4=5$ **3b.** $-4(2-y)=8$ **3c.** $d+3(d-4)=20$

EXAMPLE 4 *Fitness Application*

A martial arts school is offering a special where a new student can enroll for half price, after paying a $12.50 application fee. Ten students enrolled, and the instructor collected a total of $325. Write and solve an equation to find the regular price of enrollment.

Let p represent the regular price of enrollment.

10 students each paid	(half the regular price	plus	$12.50)	for a total of	$325
10	$\left(\frac{p}{2}\right.$	+	$12.50\Big)$	=	325

$$10\left(\frac{p}{2}+12.50\right)=325$$

$$10\left(\frac{p}{2}\right)+10(12.50)=325 \quad \textit{Distribute 10.}$$

$$5p+125=325 \quad \textit{Simplify.}$$

$$5p+125=325$$

$$-125 \quad -125$$

$$5p=200 \quad \textit{Since 125 is added to 5p, subtract 125 from both sides to undo the addition.}$$

$$\frac{5p}{5}=\frac{200}{5} \quad \textit{Since p is multiplied by 5, divide both sides by 5 to undo the multiplication.}$$

$$p=40$$

The regular price of enrollment is $40.

4a. At a local gym, there is a joining fee of $59.95 and a monthly membership fee. Sara and Martin both joined this gym. Their combined cost for 12 months was $1319.90. How much is the monthly fee?

4b. Lily and 4 of her friends want to enroll in a yoga class. After enrollment, the studio requires a $7 processing fee. The 5 girls pay a total of $125.75. How much does the class cost?

THINK AND DISCUSS

1. What would be your first step in solving the equation $3(z+12)=16$?

2. When an equation contains several operations, how do you know which operation to undo first?

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write and solve a multi-step equation. Use addition, subtraction, multiplication, and division at least one time each.

Solving Multi-Step Equations	

2-3 Exercises

California Standards Practice
4.0, 5.0, 25.2

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GUIDED PRACTICE

SEE EXAMPLE 1 p. 85

Solve each equation. Check your answer.

1. $\frac{d+3}{4} = 2$
2. $\frac{3x-2}{8} = 2$
3. $\frac{6a-1}{7} = 1$
4. $\frac{n+3}{4} = 12$
5. $\frac{2h-5}{2} = 0.5$
6. $\frac{10k+9}{4} = 5$

SEE EXAMPLE 2 p. 86

7. $9 - 2c + c = -13$
8. $15y + 21 + 10 = 61$
9. $8 = 3r - 5 + 4$
10. $42 = 4d - 6d + 6$
11. $2x + 0.3 - x = 3.3$
12. $2a + 3 + 2a = 11$

SEE EXAMPLE 3 p. 86

13. $3(x-4) = 36$
14. $t(4-1) + 9 = 27$
15. $5(1-2w) + 8w = 15$
16. $17 = 4(a-2) + 2a$
17. $\frac{1}{2}(m-6) = 12$
18. $2\left(\frac{x}{4} - 1\right) = 8$

SEE EXAMPLE 4 p. 87

19. Kathryn organized her books onto 4 shelves. The top shelf holds 5 books, the second shelf holds 7, and the 2 bottom shelves hold the same number of books. Kathryn has a total of 24 books. How many books does each bottom shelf hold?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
20–22	1
23–28	2
29–34	3
35–36	4

Extra Practice
Skills Practice p. EP4
Application Practice p. EP25

Solve each equation. Check your answer.

20. $\frac{x-1}{2} = 5$
21. $6 = \frac{2w+3}{5}$
22. $\frac{-3+y}{4} = 25$
23. $5x - 2x + 3 = 24$
24. $11 = 2g + 6 - 5$
25. $2 + 0.5g + 9 = 61$
26. $4h - 7 + 2h = 7$
27. $34 = -5 - 6t + 3t$
28. $1.5v - 0.9v + 2.1 = 4.5$
29. $3(d+5) = 23$
30. $8(x+2) = 32$
31. $-12 = 5(k-2)$
32. $15 = 3(k-6)$
33. $5 = \frac{1}{2}(x-6)$
34. $6(y-4) = 0$

35. **Consumer Economics** Amanda and Casey's total restaurant bill, including tip, was $34. Amanda's portion of the bill was twice as much as Casey's, and they each left a tip of $2. How much did each person pay?

36. Marissa is buying a shirt for each of her brothers and each of her 2 sisters. Each shirt costs $7.50, and she spends a total of $30. How many brothers does Marissa have?

Solve each equation. Check your answer.

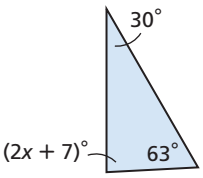
37. $5(w+10) = 45$
38. $9(p-2) = 54$
39. $\frac{1}{2} = \frac{1}{4}(y-8)$
40. $2x + \frac{3}{4} + \frac{6}{4} = 16.75$
41. $2.3w - 3.2 + 4.1 = -6$
42. $19 = -4(h+5) + h$
43. $3(2x-4) + \frac{x}{2} = 1$
44. $2.5(4-2m) = 50$
45. $-4(d+1) + 2(d-2) = \frac{10}{3}$

Write an equation to represent each relationship. Then solve.

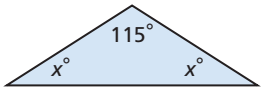
46. Three increased by a number, all multiplied by 6, equals 36.
47. Sixteen plus seven decreased by 4 times some number equals 3.
48. Three times the sum of a number and 4, minus the number, is equal to 18.
49. One-half of a number added to twice the difference of the number and 5 equals 0.

Geometry Write and solve an equation to find the value of x for each triangle. (*Hint:* The sum of the angle measures in any triangle is 180°.)

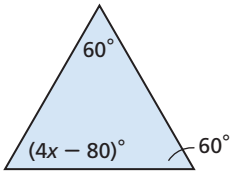
50.



51.



52.



Solve each equation.

53. $x + 3 - 2x + 5 = 10$

54. $9x - 5 - 6x - 1.3 = 0$

55. $-5 = 7g + 5 - 6g$

56. $3(x - 2) - 5(2x + 1) = 3$

57. $5(2y - 7) + 8(y + 6) = 31$

58. $4x - 3(6 + x) - 1 = 2$

59. $17 - (4 - 6z) + 4 = 42$

60. $42r - 2(13 - 5r) + 7 = 85$

61. The sum of two consecutive whole numbers is 5. What are the two numbers? (*Hint:* Let n represent the first number. Then $n + 1$ is the next consecutive whole number.)

62. Stan's, Mark's, and Wayne's ages are consecutive whole numbers. Stan is the youngest, and Wayne is the oldest. The sum of their ages is 111. Find their ages.

63. The sum of two consecutive even whole numbers is 206. What are the two numbers? (*Hint:* Let n represent the first number. What expression can you use to represent the second number?)

64. **Multi-Step** Alexis and Martin helped the school set chairs in rows for an assembly. They put the same number of chairs in each row. Using the 92 chairs available, Alexis made 4 rows with 2 chairs left over while Martin made 5 rows with no chairs left over.

a. Let c represent the number of chairs in each row. Write an equation that can be used to find c.

b. Solve your equation from part **a**.

c. Alexis and Martin are asked to remove 2 chairs from each row. Then how many of the 92 chairs were not used for the assembly?

65. **ERROR ANALYSIS** Below are two possible solutions to $3x + 5 - 4x = 19$. Which is incorrect? Explain the error.

A

$3x + 5 - 4x = 19$
$3x + 4x - 5 = 19$
$7x + 5 = 19$
$7x = 14$
$x = 2$

B

$3x + 5 - 4x = 19$
$3x - 4x + 5 = 19$
$-x + 5 = 19$
$-x = 14$
$x = -14$

CONCEPT CONNECTION

66. This problem will help prepare you for the Concept Connection on page 100.

a. Suppose firefighters can extinguish a wildfire at a rate of 60 acres per day. Use this information to complete the table.

b. Use the last row in the table to write an equation for acres A extinguished in terms of the number of days d.

Days	Acres
1	60
2	
3	180
4	
5	
d	

67. Critical Thinking The equation $3(2x - 5) + 4(x - 2) = 12$ requires several properties of multiplication and addition to solve. Name the properties.

68. Write About It Write a series of steps that you can use to solve any multi-step equation.

Multiple Choice **For Exercises 69 and 70, choose the best answer.**

69. Josh and Howard collect comic books. Josh has 12 more than Howard has, and together they want to triple their collection for a total of 66 comic books. Which equation can be used to find the number of comic books Howard owns?

Ⓐ $3(2h - 12) = 66$ Ⓒ $3h + 12 = 66$

Ⓑ $2h + 6 = 66$ Ⓓ $3(2h + 12) = 66$

70. What is the first incorrect step in the solution shown?

$$\frac{4g - 3}{7} = 3$$
Step 1: $4g - 3 = 21$
Step 2: $4g = 18$
Step 3: $g = 4.5$

Ⓐ Step 1 Ⓒ Step 3

Ⓑ Step 2 Ⓓ All steps are correct.

71. Gridded Response A band earns \$50 a show plus a bonus for every show they play. Last month they played 5 times for a total of \$300. How much is the bonus?

CHALLENGE AND EXTEND

Solve each equation. Check your answer.

72. $\frac{9}{2}x + 18 + 3x = \frac{11}{2}$ **73.** $12\left(\frac{1}{4}x - 1\right) = 12$

74. $(x + 6) - (2x + 7) - 3x = -9$ **75.** $(4x + 2) - (12x + 8) + 2(5x - 3) = 6 + 11$

76. $2(5 - y) - 5(y + 3) = -26$ **77.** $t(3 + 2) - 6(t - 5) - 22 = 6$

78. Given the equation $\frac{3x + 2(x + 6)}{2(5 + 4)} = 14$, answer the following questions.

a. Find two equivalent equations.

b. Reasoning Without solving the equation, predict what would happen to the solution if the numerator changed to $3x + 3(x + 6)$.

SPIRAL STANDARDS REVIEW

7AF1.3, 7AF4.1, 1.0

Name the property that is illustrated in each equation. *(Lesson 1-6)*

79. $-19 + n = n - 19$ **80.** $6(k + b) = (k + b)6$

Simplify each expression by combining like terms. *(Lesson 1-7)*

81. $5m + 3m$ **82.** $22c^2 - 14c$ **83.** $102v + 16v$

84. $51b - b$ **85.** $12c + x$ **86.** $\frac{1}{2}p + \frac{1}{2}$

Solve each equation. Check your answer. *(Lesson 2-2)*

87. $10w + 4 = 34$ **88.** $11 = 3d - 4$ **89.** $38 = -6d + 2$

2-4 Technology LAB

True Equations

Use with Lesson 2-4

An equation such as $2x + 2 = 6$ is neither true nor false until a value is substituted for x. In Lessons 2-1 through 2-3, you have been solving equations like this one to find the value or values of x that make the equation true. These equations are considered "sometimes true"— they are true when x equals a solution and false when x equals any other value.

Equations may also be "always true" or "never true".

California Standards

25.3 Given a specific algebraic statement involving **linear,** quadratic, or absolute value expressions or **equations** or inequalities, **students determine whether the statement is true sometimes, always, or never.**

Activity

Determine whether each equation is sometimes, always, or never true.

$3 + x = x + 3$ $\qquad 2x = x + 3$ $\qquad 3x + 2 = x + 2x$

Use a spreadsheet to test several values of x in each equation.

1. Set up a column for x and a column for each equation. Under x, enter several values.

	A	B	C	D
1	x	3 + x = x + 3	2x = x + 3	3x+2=x+2x
2	-4	=3+A2=A2+3	=2*A2=A2+3	=3*A2+2=A2+2*A2
3	-3			

2. Enter the formulas as shown into row 2. These formulas will return TRUE if the equation is true for the value of x and FALSE if it is not.

3. Use the mouse to click on the lower right corner of cell B2. Hold down the mouse button and drag the cursor down to the last row in which you have entered an x-value. The equation $3 + x = x + 3$ appears to always be true. In fact, you know it is always true because it is an example of the Commutative Property of Addition.

	A	B	C	D
1	x	3 + x = x + 3	2x = x + 3	3x+2=x+2x
2	-4	TRUE	FALSE	FALSE
3	-3	TRUE	FALSE	FALSE
4	-2	TRUE	FALSE	FALSE
5	-1	TRUE	FALSE	FALSE
6	0	TRUE	FALSE	FALSE
7	1	TRUE	FALSE	FALSE
8	2	TRUE	FALSE	FALSE
9	3	TRUE	TRUE	FALSE
10	4	TRUE	FALSE	FALSE
11	5	TRUE	FALSE	FALSE

4. Repeat Step 3 for columns C and D. The equation $2x = x + 3$ is true only when $x = 3$. In other words, it is sometimes true. The equation $3x + 2 = x + 2x$ appears to never be true. If you simplify the right side of this equation, you get $3x + 2 = 3x$. You can see that this equation is never true because $3x + 2$ will always be greater than $3x$ for any value of x.

To test more values of x, enter more values of x into column A and copy the formulas as described above.

Try This

Determine whether each equation is sometimes, always, or never true.

1. $4a + 2 = 3a$
2. $-3z = 4z - 7z$
3. $5c + 8 = 5c + 8$
4. $6x = 2x - 8x$
5. $4x + 10 = 7x$
6. $6 + 2a = a + 5 + a$
7. $-4 + 3c + 6 = 2c + 2 + c$
8. $6g + 8 = -9 + 6g$
9. $3y + 4 = -2 + 5y$

2-4 Solving Equations with Variables on Both Sides

California Standards

4.0 Students simplify expressions before solving linear equations and inequalities **in one variable,** such as $3(2x - 5) + 4(x - 2) = 12$.

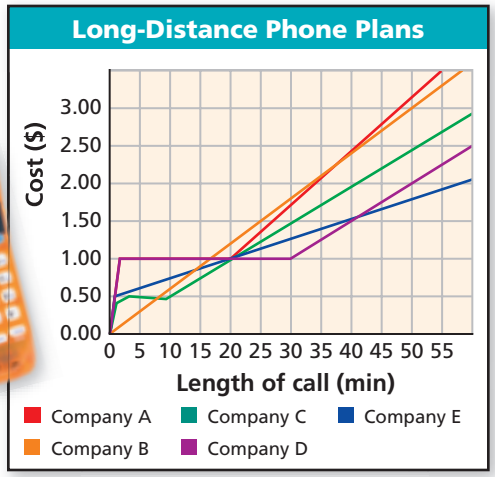
5.0 Students solve multi-step problems, including word problems, involving linear equations and linear inequalities **in one variable** and provide justification for each step.

Vocabulary
identity

Why learn this?
You can compare prices and find the best value.

Many phone companies offer low rates for long-distance calls without requiring customers to sign up for their services. To compare rates, solve an equation with variables on both sides.

To solve an equation like this, use inverse operations to "collect" variable terms on one side of the equation.



EXAMPLE 1 Solving Equations with Variables on Both Sides

Solve each equation.

A $7k = 4k + 15$

$7k = 4k + 15$
$-4k \quad -4k$
$3k = 15$
$\frac{3k}{3} = \frac{15}{3}$
$k = 5$

To collect the variable terms on one side, subtract 4k from both sides.

Since k is multiplied by 3, divide both sides by 3 to undo the multiplication.

B $5x - 2 = 3x + 4$

$5x - 2 = 3x + 4$
$-3x \quad -3x$
$2x - 2 = 4$
$+2 \quad +2$
$2x = 6$
$\frac{2x}{2} = \frac{6}{2}$
$x = 3$

To collect the variable terms on one side, subtract 3x from both sides.

Since 2 is subtracted from 2x, add 2 to both sides to undo the subtraction.

Since x is multiplied by 2, divide both sides by 2 to undo the multiplication.

Check

$5x - 2 = 3x + 4$	
$5(3) - 2$	$3(3) + 4$
$15 - 2$	$9 + 4$
13	13 ✓

To check your solution, substitute 3 for x in the original equation.

Helpful Hint

Equations are often easier to solve when the variable has a positive coefficient. Keep this in mind when deciding on which side to "collect" variable terms.

CHECK IT OUT! **Solve each equation. Check your answer.**

1a. $4b + 2 = 3b$

1b. $0.5 + 0.3y = 0.7y - 0.3$

To solve more complicated equations, you may need to first simplify by using the Distributive Property or combining like terms.

EXAMPLE 2 Simplifying Each Side Before Solving Equations

Solve each equation.

A $2(y+6)=3y$

$2(y+6) = 3y$ — *Distribute 2 to the expression in parentheses.*

$2(y)+2(6) = 3y$

$2y+12 = 3y$

$-2y \quad -2y$ — *To collect the variable terms on one side, subtract 2y from both sides.*

$12 = y$

Check $2(y+6)=3y$ — *To check your solution, substitute 12 for y in the original equation.*

$2(12+6)$	$3(12)$
$2(18)$	36
36	36 ✓

B $2k-5=3(1-2k)$

$2k-5 = 3(1-2k)$ — *Distribute 3 to the expression in parentheses.*

$2k-5 = 3(1)-3(2k)$

$2k-5 = 3-6k$

$+6k \quad +6k$ — *To collect the variable terms on one side, add 6k to both sides.*

$8k-5 = 3$

$+5 \quad +5$ — *Since 5 is subtracted from 8k, add 5 to both sides.*

$8k = 8$

$\frac{8k}{8} = \frac{8}{8}$ — *Since k is multiplied by 8, divide both sides by 8.*

$k = 1$

C $3-5b+2b=-2-2(1-b)$

$3-5b+2b = -2-2(1-b)$

$3-5b+2b = -2-2(1)-2(-b)$ — *Distribute –2 to the expression in parentheses.*

$3-5b+2b = -2-2+2b$

$3-3b = -4+2b$ — *Combine like terms.*

$+3b \quad +3b$ — *Add 3b to both sides.*

$3 = -4+5b$

$+4 \quad +4$ — *Since –4 is added to 5b, add 4 to both sides.*

$7 = 5b$

$\frac{7}{5} = \frac{5b}{5}$ — *Since b is multiplied by 5, divide both sides by 5.*

$1.4 = b$

Solve each equation. Check your answer.

2a. $\frac{1}{2}(b+6)=\frac{3}{2}b-1$ **2b.** $3x+15-9=2(x+2)$

An **identity** is an equation that is always true, no matter what value is substituted for the variable. The solution set of an identity is all real numbers. Some equations are always false. Their solution sets are empty. In other words, their solution sets contain no elements.

EXAMPLE 3 Infinitely Many Solutions or No Solutions

Solve each equation.

A $x + 4 - 6x = 6 - 5x - 2$

$x + 4 - 6x = 6 - 5x - 2$ *Identify like terms.*

$4 - 5x = 4 - 5x$ *Combine like terms on the left and the right.*

The statement $4 - 5x = 4 - 5x$ is true for all values of x. The equation $x + 4 - 6x = 6 - 5x - 2$ is an identity. All values of x will make the equation true. In other words, all real numbers are solutions.

B $-8x + 6 + 9x = -17 + x$

$-8x + 6 + 9x = -17 + x$ *Identify like terms.*

$x + 6 = -17 + x$ *Combine like terms.*

$-x \quad -x$ *Subtract x from both sides.*

$6 = -17$ ✗ *False statement; the solution set is ∅.*

The equation $-8x + 6 + 9x = -17 + x$ is always false. There is no value of x that will make the equation true. There are no solutions.

Writing Math

The empty set can be written as ∅ or {}.

CHECK IT OUT!

Solve each equation.

3a. $4y + 7 - y = 10 + 3y$

3b. $2c + 7 + c = -14 + 3c + 21$

EXAMPLE 4 *Consumer Application*

The long-distance rates of two phone companies are shown in the table. How long is a call that costs the same amount no matter which company is used? What is the cost of that call?

Phone Company	Charges
Company A	36¢ plus 3¢ per minute
Company B	6¢ per minute

Let m represent minutes, and write expressions for each company's cost.

When is	36¢	plus	3¢ per minute	times number of minutes	the same as	6¢ per minute	times number of minutes	?
	36	+	3	(m)	=	6	(m)	

$36 + 3m = 6m$

$-3m \quad -3m$ *To collect the variable terms on one side, subtract 3m from both sides.*

$36 = 3m$

$\frac{36}{3} = \frac{3m}{3}$ *Since m is multiplied by 3, divide both sides by 3 to undo the multiplication.*

$12 = m$

The charges will be the same for a 12-minute call using either phone service. To find the cost of this call, evaluate either expression for $m = 12$:

$36 + 3m = 36 + 3(12) = 36 + 36 = 72 \qquad 6m = 6(12) = 72$

The cost of a 12-minute call through either company is 72¢.

CHECK IT OUT!

4. Four times Greg's age, decreased by 3 is equal to 3 times Greg's age, increased by 7. How old is Greg?

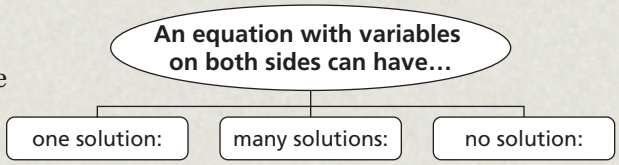
THINK AND DISCUSS

1. Tell which of the following is an identity. Explain your answer.

a. $4(a + 3) - 6 = 3(a + 3) - 6$ **b.** $8.3x - 9 + 0.7x = 2 + 9x - 11$

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of an equation that has the indicated number of solutions.



2-4 Exercises

California Standards Practice

4.0, 5.0, 25.3

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GUIDED PRACTICE

1. Vocabulary How can you recognize an *identity*?

SEE EXAMPLE 1 p. 92

Solve each equation. Check your answer.

2. $2c - 5 = c + 4$ **3.** $8r + 4 = 10 + 2r$

4. $2x - 1 = x + 11$ **5.** $28 - 0.3y = 0.7y - 12$

SEE EXAMPLE 2 p. 93

6. $-2(x + 3) = 4x - 3$ **7.** $3c - 4c + 1 = 5c + 2 + 3$

8. $1 + \frac{3}{5}(q - 4) = \frac{2}{5}(q + 1)$ **9.** $5 - (t + 3) = -1 + 2(t - 3)$

SEE EXAMPLE 3 p. 94

10. $7x - 4 = -2x + 1 + 9x - 5$ **11.** $8x + 6 - 9x = 2 - x - 15$

12. $6y = 8 - 9 + 6y$ **13.** $6 - 2x - 1 = 4x + 8 - 6x - 3$

SEE EXAMPLE 4 p. 94

14. Consumer Economics A house-painting company charges $376 plus $12 per hour. Another painting company charges $280 plus $15 per hour.

a. How long is a job for which both companies will charge the same amount?

b. What will that cost be?

PRACTICE AND PROBLEM SOLVING

Solve each equation. Check your answer.

15. $7a - 17 = 4a + 1$ **16.** $2b - 5 = 8b + 1$ **17.** $4x - 2 = 3x + 4$

18. $2x - 5 = 4x - 1$ **19.** $8x - 2 = 3x + 12.25$ **20.** $5x + 2 = 3x$

21. $3c - 5 = 2c + 5$ **22.** $-17 - 2x = 6 - x$ **23.** $3(t - 1) = 9 + t$

24. $5 - x - 2 = 3 + 4x + 5$ **25.** $2(x + 4) = 3(x - 2)$ **26.** $3m - 10 = 2(4m - 5)$

27. $5 - (n - 4) = 3(n + 2)$ **28.** $6(x + 7) - 20 = 6x$ **29.** $8(x + 1) = 4x - 8$

30. $x - 4 - 3x = -2x - 3 - 1$ **31.** $-2(x + 2) = -2x + 1$ **32.** $2(x + 4) - 5 = 2x + 3$

Independent Practice

For Exercises	See Example
15–22	1
23–29	2
30–32	3
33	4

Extra Practice

Skills Practice p. EP5

Application Practice p. EP25

33. Multi-Step Justin and Tyson are beginning an exercise program to train for football season. Justin weighs 150 lb and hopes to gain 2 lb per week. Tyson weighs 195 lb and hopes to lose 1 lb per week.

a. If the plan works, in how many weeks will the boys weigh the same amount?

b. What will that weight be?

Reasoning **Tell whether each equation is sometimes, always, or never true.**

34. $5(x + 3) - 2 = 5x + 13$

35. $2n + 1 = 2(n + 1) - 1$

36. $3(k - 1) - 3(2k + 2) = 24$

Solve each equation. Check your answer.

37. $2x - 2 = 4x + 6$

38. $3x + 5 = 2x + 2$

39. $4x + 3 = 5x - 4$

40. $-\frac{2}{5}p + 2 = \frac{1}{5}p + 11$

41. $5x + 24 = 2x + 15$

42. $5x - 10 = 14 - 3x$

43. $12 - 6x = 10 - 5x$

44. $5x - 7 = -6x - 29$

45. $1.8x + 2.8 = 2.5x + 2.1$

46. $2.6x + 18 = 2.4x + 22$

47. $1 - 3x = 2x + 8$

48. $\frac{1}{2}(8 - 6h) = h$

49. $\frac{1}{3}(x + 1) = \frac{2}{9}x + \frac{7}{9}$

50. $9x - 8 + 4x = 7x + 16$

51. $3(2x - 1) + 5 = 6(x + 1)$

52. Travel Rapid Rental Car company charges a $40 rental fee, $15 for gas, and $0.25 per mile driven. For the same car, Capital Cars charges $45 for rental and gas and $0.35 per mile.

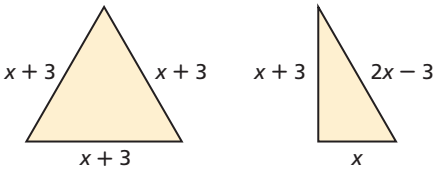
a. Find the number of miles for which the companies' charges will be the same. Then find that charge.

b. The Barre family estimates that they will drive about 95 miles during their vacation to Hershey, Pennsylvania. Which company should they rent their car from? Explain.

c. **What if...?** The Barres have extended their vacation and now estimate that they will drive about 120 miles. Should they still rent from the same company as in part **b**? Why or why not?

d. Give a general rule for deciding which company to rent from.

53. Geometry The triangles shown have the same perimeter. What is the value of x?



CONCEPT CONNECTION

54. This problem will prepare you for the Concept Connection on page 100.

a. A fire currently covers 420 acres and continues to spread at a rate of 60 acres per day. How many total acres will be covered in the next 2 days?

b. Write an expression for the total area covered by the fire in d days.

c. The firefighters estimate that they can put out the fire at a rate of 80 acres per day. Write an expression for the total area that the firefighters can put out in d days.

d. Set the expressions in parts **b** and **c** equal. Solve for d. What does d represent?

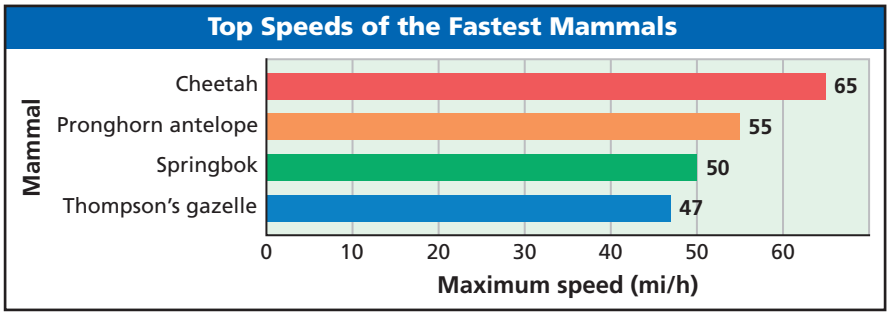
Biology

A cheetah's body is well designed for fast running. Its tail acts like a boat's rudder to help it make sharp turns. Its spine acts like a spring to propel it forward.

Source: www.cheetahspot.com

55. Critical Thinking Write an equation with variables on both sides that has no solution.

56. Biology The graph shows the maximum recorded speeds of the four fastest mammals.



Source: The Top 10 of Everything

a. Write an expression for the distance in miles that a Thompson's gazelle can run at top speed in x hours.

b. Write an expression for the distance in miles that a cheetah can run at top speed in x hours.

c. A cheetah and a Thompson's gazelle are running at their top speeds. The cheetah is one mile behind the gazelle. Write an expression for the distance the cheetah must run to catch up with the gazelle.

d. Write and solve an equation that represents how long the cheetah will have to run at top speed to catch up with the gazelle.

e. A cheetah can maintain its top speed for only 300 yards. Will the cheetah be able to catch the gazelle? Explain. (*Hint:* 1 mile = 1760 yards)

57. Write About It Write a series of steps that you can use to solve any equation with variables on both sides.

Multiple Choice **For Exercises 58–61, choose the best answer.**

58. Lindsey's monthly magazine subscription costs \$1.25 per issue. Kenzie's monthly subscription costs \$1.50 per issue, but she received her first 2 issues free. Which equation can be used to find the number of months after which the girls will have paid the same amount?

Ⓐ $1.25m = 1.50m - 2$ Ⓒ $1.25m = 1.50(m - 2)$

Ⓑ $1.25m = 1.50m - 2m$ Ⓓ $1.25m = 3m - 1.50$

59. What is the numerical solution of the equation *7 times a number equals 3 less than 5 times that number*?

Ⓐ -1.5 Ⓑ 0.25 Ⓒ $\frac{2}{3}$ Ⓓ 4

60. Three packs of markers cost \$9.00 less than 5 packs of markers. Which equation best represents this situation?

Ⓐ $5x + 9 = 3x$ Ⓒ $3x - 9 = 5x$

Ⓑ $3x + 9 = 5x$ Ⓓ $9 - 3x = 5x$

61. Nicole has \$120. If she saves \$20 per week, in how many days will she have \$500?

Ⓐ 19 Ⓑ 25 Ⓒ 133 Ⓓ 175

62. Gridded Response Solve $-2(x - 1) + 5x = 2(2x - 1)$.

CHALLENGE AND EXTEND

Solve each equation.

63. $4x + 2[4 - 2(x + 2)] = 2x - 4$

64. $\frac{x+5}{2} + \frac{x-1}{2} = \frac{x-1}{3}$

65. $\frac{2}{3}w - \frac{1}{4} = \frac{2}{3}\left(w - \frac{1}{4}\right)$

66. $-5 - 7 - 3f = -f - 2(f + 6)$

67. $\frac{2}{3}x + \frac{1}{2} = \frac{3}{5}x - \frac{5}{6}$

68. $x - \frac{1}{4} = \frac{x}{3} + 7\frac{3}{4}$

69. Find three consecutive integers such that twice the greatest integer is 2 less than 3 times the least integer.

70. Find three consecutive integers such that twice the least integer is 12 more than the greatest integer.

71. Rob had twice as much money as Sam. Then Sam gave Rob 1 quarter, 2 nickels, and 3 pennies. Rob then gave Sam 8 dimes. If they now have the same amount of money, how much money did Rob originally have? Check your answer.

SPIRAL STANDARDS REVIEW

6NS2.1, 6AF1.2, 7NS1.2, 7AF4.1

Write an expression for the perimeter of each figure. *(Lesson 1-1)*

72. square with side x cm

73. equilateral triangle with side y cm

Multiply or divide. *(Lesson 1-3)*

74. $6.1 \div 0$

75. $3(-21)$

76. $0 \div \frac{7}{8}$

77. $\frac{2}{5} \div \frac{1}{10}$

78. $5 \div (-5)$

79. $\frac{-16}{-8}$

80. $-1000 \div (-0.001)$

81. $500(-0.25)$

Solve each equation. *(Lesson 2-2)*

82. $4x - 44 = 8$

83. $2x - 6 = 24$

84. $-1 = \frac{x}{4} - 3$

85. $2x + 6 = 12$

Career Path

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KEYWORD: MA8CA Career

Beth Simmons
Biology major

Q: **What math classes did you take in high school?**

A: Algebra 1 and 2, Geometry, and Precalculus

Q: **What math classes have you taken in college?**

A: Two calculus classes and a calculus-based physics class

Q: **How do you use math?**

A: I use math a lot in physics. Sometimes I would think a calculus topic was totally useless, and then we would use it in physics class! In biology, I use math to understand populations.

Q: **What career options are you considering?**

A: When I graduate, I could teach, or I could go to graduate school and do more research. I have a lot of options.

Use with Lessons 2-1 through 2-4

Deductive Reasoning and Equations

Deductive reasoning is the process of using logic along with known facts, definitions, and properties to reach a conclusion. In mathematics, deductive reasoning can be used to prove whether given statements are true.

You may not realize it, but you use deductive reasoning every time you solve an equation. In fact, solving an equation can be thought of as a proof.

Example

California Standards

24.1 Students explain the difference between inductive and deductive reasoning and identify and **provide examples of each.**
Also covered: **5.0, 24.2, 25.1, 25.2**

Solve $3(x + 5) + 2(x + 3) = 26$. Give a reason for each step in your solution process. Identify the conditional statement that is proved and its hypothesis and conclusion.

One way to write down deductive reasoning is to use two columns—one for each step or statement and one for the facts, definitions, and/or properties that support each step.

Statements	Reasons
1. $3(x + 5) + 2(x + 3) = 26$	Given
2. $3x + 15 + 2x + 6 = 26$	Distributive Property
3. $(3x + 2x) + (15 + 6) = 26$	Commutative and Associative Properties of Addition
4. $5x + 21 = 26$	Combine like terms.
5. $5x = 5$	Subtraction Property of Equality (Subtract 21 from both sides.)
6. $x = 1$	Division Property of Equality (Divide both sides by 5.)

The above proves the conditional statement "If $3(x + 5) + 2(x + 3) = 26$, then $x = 1$."

Hypothesis: $3(x + 5) + 2(x + 3) = 26$ Conclusion: $x = 1$

Try This

Solve each equation. Give a reason for each step in your solution process. Identify the conditional statement that is proved and its hypothesis and conclusion.

1. $x - 2 = 4$ **2.** $x + 6 = 16$ **3.** $-5x = 25$ **4.** $\frac{x}{4} = 13$

5. $-2x + 5 = 9$ **6.** $6x - 5 = 2x - 21$ **7.** $6(x - 5) = 10$ **8.** $6x + 1 + x = 10 - 12$

9. What is the error in the solution below? Write a correct solution.

Statements	Reasons
1. $5x + 1 = 7$	Given
2. $5x = 8$	Addition Property of Equality (Add 1 to both sides.)
3. $x = \frac{8}{5}$	Division Property of Equality (Divide both sides by 5.)

CHAPTER 2

CONCEPT CONNECTION

SECTION 2A

Equations and Formulas

All Fired Up A large forest fire in the western United States has been burning for 14 days, spreading to cover approximately 3850 acres. Firefighters have been doing their best to contain the fire, but hot temperatures and high winds have prompted them to request additional help.

1. Find the average number of acres the fire covered each day for the first 14 days.
2. When the fire began, officials estimated that, with no additional firefighting help, the fire would spread to cover 9075 acres before being contained. Write and solve an equation to find the total number of days it would take for the fire to cover 9075 acres.
3. Additional help arrives on day 15, and when the firefighters contain the fire, it has spread to cover a total of only 5775 acres. How many days did it take to contain the fire after the help arrived?
4. The total cost of fighting the fire was approximately $1,440,000. Write and solve an equation to find the approximate cost per day of fighting the fire.

Quiz for Lessons 2-1 Through 2-4

2-1 Solving One-Step Equations

Solve each equation.

1. $x - 32 = -18$ **2.** $1.1 = m - 0.9$ **3.** $j + 4 = -17$ **4.** $\frac{9}{8} = g + \frac{1}{2}$

5. $\frac{h}{3} = -12$ **6.** $-2.8 = \frac{w}{-3}$ **7.** $42 = 3c$ **8.** $-0.1b = 3.7$

9. When she first purchased it, Soledad's computer had 400 GB of hard drive space. After six months, there were only 313 GB available. Write and solve an equation to find the amount of hard drive space that Soledad used in the first six months.

2-2 Solving Two-Step Equations

Solve each equation.

10. $2k + 15 = 29$ **11.** $-6a - 12 = 24$ **12.** $7 + 3b = -11$ **13.** $1.6 = 0.4n - 2$

14. $2r + 20 = 200$ **15.** $21 = 3g - 6$ **16.** $\frac{3}{5}k + 5 = 7$ **17.** $2.5x + 4.5 = -8$

18. Christine's video store membership cost \$5.00, and it costs \$3.50 to rent a DVD. Christine spent \$29.50 her first month. How many DVDs did she rent?

19. A fund-raiser raised \$2400, which was $\frac{3}{5}$ of the goal. Write and solve an equation to find the amount of the goal.

2-3 Solving Multi-Step Equations

Solve each equation.

20. $4(x - 7) = 2$ **21.** $\frac{2}{3} - \frac{y}{4} = \frac{5}{12}$

22. $5n + 6 - 3n = -12$ **23.** $\frac{3}{4}(8 + 2y) = 3$

24. A taxicab company charges \$2.10 plus \$0.80 per mile. Carmen paid a fare of \$11.70. Write and solve an equation to find the number of miles she traveled.

25. If $2(x + 3) = 24$, find the value of $x - 6$.

2-4 Solving Equations with Variables on Both Sides

Solve each equation.

26. $4x - 3 = 2x + 5$ **27.** $3(2x - 5) = 2(3x - 2)$

28. $2(2t - 3) = 6(t + 2)$ **29.** $7(x + 5) = -7(x + 5)$

30. On the first day of the year, Diego had \$700 in his savings account and started spending \$35 a week. His brother Juan had \$450 and started saving \$15 a week. After how many weeks will the brothers have the same amount? What will that amount be?

2-5 Solving Proportions

California Standards

15.0 Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

Why learn this?

Proportions are used to draw accurate maps. (See Example 5.)

A **ratio** is a comparison of two quantities. The ratio of a to b can be written $a:b$ or $\frac{a}{b}$, where $b \neq 0$.

A statement that two ratios are equal, such as $\frac{1}{12} = \frac{2}{24}$, is called a **proportion**.

Vocabulary

ratio	proportion
rate	unit rate
cross products	percent
scale	scale
scale drawing	scale model

EXAMPLE 1 Using Ratios

The ratio of faculty members to students at a college is 1:15. There are 675 students. How many faculty members are there?

$\frac{\text{faculty}}{\text{students}} \rightarrow \frac{1}{15}$ — *Write a ratio comparing faculty to students.*

$\frac{1}{15} = \frac{x}{675}$ — *Write a proportion. Let x be the number of faculty members.*

$675\left(\frac{x}{675}\right) = 675\left(\frac{1}{15}\right)$ — *Since x is divided by 675, multiply both sides by 675.*

$x = 45$ — There are 45 faculty members.

CHECK IT OUT!

1. The ratio of red marbles to green marbles is 6:5. There are 18 red marbles. How many green marbles are there?

A common application of proportions is *rates*. A **rate** is a ratio of two quantities with different units, such as $\frac{34 \text{ mi}}{2 \text{ gal}}$. Rates are usually written as *unit rates*. A **unit rate** is a rate with a second quantity of 1 unit, such as $\frac{17 \text{ mi}}{1 \text{ gal}}$ or 17 mi/gal. You can convert any rate to a unit rate.

EXAMPLE 2 Finding Unit Rates

Takeru Kobayashi of Japan ate 53.5 hot dogs in 12 minutes to win a contest. Find the unit rate. Round your answer to the nearest hundredth.

$\frac{53.5}{12} = \frac{x}{1}$ — *Write a proportion to find an equivalent ratio with a second quantity of 1.*

$4.46 \approx x$ — *Divide on the left side to find x.*

The unit rate is approximately 4.46 hot dogs per minute.

CHECK IT OUT!

Find each unit rate. Round to the nearest hundredth if necessary.

2a. Cory earns $52.50 in 7 hours.

2b. A machine seals 138 envelopes in 23 minutes.

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ and $b \cdot c$ are called **cross products**. You can solve a proportion for a missing value by using the Cross Products Property.

Cross Products Property

WORDS	NUMBERS	ALGEBRA
In a proportion, cross products are equal.	$\frac{2}{3} \times \frac{4}{6}$ $2 \cdot 6 = 3 \cdot 4$	If $\frac{a}{b} \times \frac{c}{d}$ and $b \neq 0$ and $d \neq 0$, then $ad = bc$.

EXAMPLE 3 Solving Proportions

Solve each proportion.

A $\frac{5}{9} = \frac{3}{w}$

$\frac{5}{9} \times \frac{3}{w}$

$5(w) = 9(3)$ *Use cross products.*

$5w = 27$

$\frac{5w}{5} = \frac{27}{5}$ *Divide both sides by 5.*

$w = \frac{27}{5}$

B $\frac{8}{x+10} = \frac{1}{12}$

$\frac{8}{x+10} \times \frac{1}{12}$

$8(12) = 1(x+10)$ *Use cross products.*

$96 = x + 10$

$-10 \quad -10$ *Subtract 10 from both sides.*

$86 = x$

Solve each proportion. Check your answer.

3a. $\frac{-5}{2} = \frac{y}{8}$

3b. $\frac{g+3}{5} = \frac{7}{4}$

Another common application of proportions is *percents*. A **percent** is a ratio that compares a number to 100. For example, $25\% = \frac{25}{100}$.

You can use the proportion $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$ to find unknown values.

EXAMPLE 4 Percent Problems

A **Find 50% of 20.**

Method 1 Use a proportion.

$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$ *Use the percent proportion.*

$\frac{x}{20} = \frac{50}{100}$ *Let x represent the part.*

$100x = 1000$ *Find the cross products. Since x is multiplied by 100, divide both sides by 100 to undo the multiplication.*

$\frac{100x}{100} = \frac{1000}{100}$

$x = 10$

50% of 20 is 10.

B **440 is what percent of 400?**

Method 2 Use an equation.

$440 = x \cdot 400$ *Write an equation. Let x represent the percent.*

$440 = 400x$

$\frac{440}{400} = \frac{400x}{400}$ *Since x is multiplied by 400, divide both sides by 400 to undo the multiplication.*

$1.1 = x$ *The answer is a decimal.*

$110\% = x$ *Write the decimal as a percent. This answer is reasonable; 440 is more than 100% of 400.*

440 is 110% of 400.

CHECK IT OUT!

4a. Find 20% of 60.

4b. 48 is 15% of what number?

Proportions are used to create *scale drawings* and *scale models*. A **scale** is a ratio between two sets of measurements, such as 1 in:5 mi. A **scale drawing** or **scale model** uses a scale to represent an object as smaller or larger than the actual object. A map is an example of a scale drawing.

EXAMPLE 5 Scale Drawings and Scale Models

On the map, the distance from Chicago to Evanston measures 0.625 in. What is the actual distance?

$\frac{\text{map}}{\text{actual}} \rightarrow \frac{1 \text{ in.}}{18 \text{ mi}}$ *Write the scale as a fraction.*

$\frac{1}{18} \times \frac{0.625}{x}$ *Let x be the actual distance.*

$x \cdot 1 = 18(0.625)$ *Use cross products to solve.*

$x = 11.25$

The actual distance is 11.25 mi.

Reading Math

A scale written without units, such as 32:1, means that 32 units of any measure correspond to 1 unit of that same measure.

CHECK IT OUT!

5a. The actual distance between North Chicago and Waukegan is 4 mi. What is this distance on the map above?

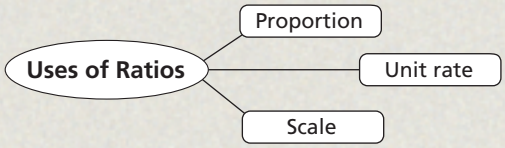
5b. A scale model of a human heart is 16 ft long. The scale is 32:1. How many inches long is the actual heart that the model represents?

THINK AND DISCUSS

1. Explain two ways to solve the proportion $\frac{t}{4} = \frac{3}{5}$.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of each use of ratios.



2-5 Exercises

California Standards Practice
5.0, 15.0, 25.1, 25.2

go.hrw.com
Homework Help Online
KEYWORD: MA8CA 2-5
Parent Resources Online
KEYWORD: MA8CA Parent

GUIDED PRACTICE

Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. What does it mean when two ratios form a *proportion*?
2. In your own words, write a definition of *percent*.

SEE EXAMPLE 1 p. 102

3. The ratio of the sale price of a jacket to the original price is 3:4. The original price is $64. What is the sale price?
4. **Chemistry** The ratio of hydrogen atoms to oxygen atoms in water is 2:1. If an amount of water contains 341 trillion atoms of oxygen, how many hydrogen atoms are there?

SEE EXAMPLE 2 p. 102

Find each unit rate.

5. A computer's fan rotates 2000 times in 40 seconds.
6. Twelve cows produce 224,988 pounds of milk.

SEE EXAMPLE 3 p. 103

Solve each proportion. Check your answer.

7. $\frac{3}{z} = \frac{1}{8}$
8. $\frac{x}{3} = \frac{1}{5}$
9. $\frac{b}{4} = \frac{3}{2}$
10. $\frac{f+3}{12} = \frac{7}{2}$
11. $\frac{-1}{5} = \frac{3}{2d}$
12. $\frac{3}{14} = \frac{s-2}{21}$

SEE EXAMPLE 4 p. 103

13. Find 75% of 40.
14. Find $12\frac{1}{2}$% of 168.
15. Find 115% of 57.
16. Find 70% of 8.
17. What percent of 40 is 25?
18. What percent of 225 is 180?

SEE EXAMPLE 5 p. 104

19. **Archaeology** Stonehenge II in Hunt, Texas, is a scale model of the ancient construction in Wiltshire, England. The scale of the model to the original is 3:5. The Altar Stone of the original construction is 4.9 meters tall. Write and solve a proportion to find the height of the Texas model of the Altar Stone.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
20–21	1
22–23	2
24–27	3
28–33	4
34	5

Extra Practice

Skills Practice p. EP5
Application Practice p. EP25

20. **Gardening** The ratio of the height of a bonsai ficus tree to the height of a full-size ficus tree is 1:9. The bonsai ficus is 6 inches tall. What is the height of a full-size ficus?
21. **Manufacturing** At one factory, the ratio of defective light bulbs produced to total light bulbs produced is about 3:500. How many light bulbs are expected to be defective when 12,000 are produced?

Find each unit rate.

22. Four gallons of gasoline weigh 25 pounds.
23. Fifteen ounces of gold cost $6,058.50.

Solve each proportion. Check your answer.

24. $\frac{v}{6} = \frac{1}{2}$
25. $\frac{2}{5} = \frac{4}{y}$
26. $\frac{2}{h} = \frac{-5}{6}$
27. $\frac{3}{10} = \frac{b+7}{20}$

Find each value. Round to the nearest tenth if necessary.

28. 60% of 80 **29.** 35% of 90 **30.** $\frac{1}{2}$% of 500 **31.** 210% of 30

32. What percent of 52 is 13? **33.** What percent of 9 is 27?

34. Science The image shows a dust mite as seen under a microscope. The actual length of this dust mite is 0.3 millimeter. Use a ruler to measure the length of the dust mite in the image in millimeters. What is the scale of the drawing?

Solve each proportion.

35. $\frac{x-1}{3} = \frac{x+1}{5}$ **36.** $\frac{m}{3} = \frac{m+4}{7}$

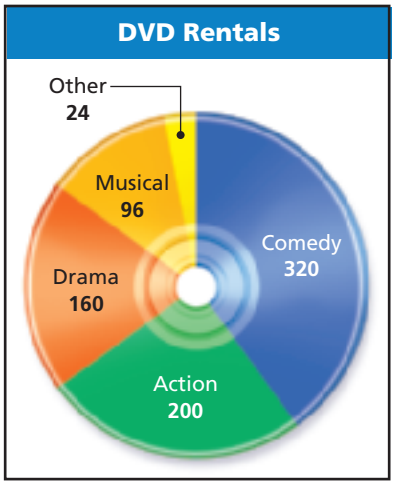
37. $\frac{1}{x-3} = \frac{3}{x-5}$ **38.** $\frac{a}{2} = \frac{a-4}{30}$

39. $\frac{3}{2y} = \frac{16}{y+2}$ **40.** $\frac{n+3}{5} = \frac{n-1}{2}$

41. Multi-Step According to the 2000 U.S. Census, 138,053,563 Americans are male and 143,368,343 Americans are female. About what percent of the population is male? female? Round your answers to the nearest percent.

42. Write About It Give three examples of proportions. How do you know they are proportions? Then give three nonexamples of proportions. How do you know they are not proportions?

43. Entertainment The numbers of various types of movies rented over a period of time are indicated in the graph.



a. What percent of the movies rented were comedies?

b. What type of movie made up 25% of the rentals?

c. What percent of the movies rented were in the "other" category?

d. What if...? If 25 of the comedy rentals had instead been action rentals, what percent of the movies rented would have been comedies? Round your answer to the nearest tenth.

CONCEPT CONNECTION

44. This problem will help prepare you for the Concept Connection on page 120.

Two notes are separated by an interval of a fourth if the ratio of the notes' frequencies is 4:3. The note D_4 has a frequency of 297 hertz (Hz).

a. Write and solve a proportion that you can use to find the frequency of a note that has higher pitch (that is, a greater frequency) than the note D_4 and is separated from the note D_4 by an interval of a fourth.

b. Find the frequency of a note with a higher pitch that is separated by an interval of a fourth from the note you found in part **a.**

45. **ERROR ANALYSIS** Below is a bonus question that appeared on an algebra test and a student's response.

> The ratio of junior varsity members to varsity members on the track team is 3:5. There are 24 members on the team. Write a proportion to find the number of junior varsity members. $\frac{3}{5} = \frac{x}{24}$

The student did not receive the bonus points. Why is this proportion incorrect?

Multiple Choice **For Exercises 46 and 47, choose the best answer.**

46. One day the U.S. dollar was worth approximately 100 Japanese yen. An exchange of 2500 yen was made that day. What was the value of the exchange in dollars?

Ⓐ $25 Ⓑ $2,400 Ⓒ $2,500 Ⓓ $270,000

47. Which proportion can be used to find 14% of 60?

Ⓐ $\frac{x}{100} = \frac{60}{14}$ Ⓑ $\frac{14}{100} = \frac{60}{x}$ Ⓒ $\frac{x}{100} = \frac{14}{60}$ Ⓓ $\frac{14}{100} = \frac{x}{60}$

48. **Gridded Response** Raul surveyed 35 students about their preferred lunch. Fourteen preferred chicken. Half of those students preferred chicken with barbecue sauce. What percent should Raul report as preferring chicken with barbecue sauce?

CHALLENGE AND EXTEND

49. **Geometry** Complementary angles are two angles whose measures add to 90°. The ratio of the measures of two complementary angles is 4 : 5. What are the measures of the angles?

50. **Population** The population density of Jackson, Mississippi, is 672.2 people per square kilometer. What is the population density in people per square meter? Show that your answer is reasonable. (*Hint:* There are 1000 meters in 1 kilometer. How many square meters are in 1 square kilometer?)

Find each value. Round to the nearest tenth if necessary.

51. What percent of 16 is 2.75?

52. 22 is 73.5% of what number?

53. **Reasoning** Without using cross products, show that $\frac{3}{x} = \frac{5}{x-1}$ is equivalent to $3(x-1) = 5x$.

SPIRAL STANDARDS REVIEW

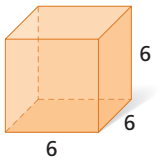
7AF2.1, 2.0, 4.0

Evaluate each expression. *(Lesson 1-4)*

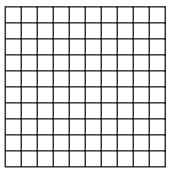
54. 8^2 **55.** $(-3)^3$ **56.** $(-3)^2$ **57.** $-\left(\frac{1}{2}\right)^5$

Write the power represented by each geometric model. *(Lesson 1-4)*

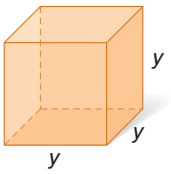
58.



59.



60.



Solve each equation. Check your answer. *(Lesson 2-4)*

61. $2x - 12 = 5x + 3$ **62.** $3a - 4 = 6 - 7a$ **63.** $3x - 4 = 2x + 4$

Use with Lesson 2-5

Proving Conditional Statements

In Lesson 2-5, you used the Cross Products Property:

If $\frac{a}{b} = \frac{c}{d}$ $(b \neq 0$ and $d \neq 0)$, then $ad = bc$.

Notice that the Cross Products Property is a conditional statement. You can use deductive reasoning to prove that it is true.

California Standards

25.1 **Students use properties of numbers to construct simple, valid arguments (direct** and indirect) **for,** or formulate counterexamples to, **claimed assertions.**

Also covered: **Ext. of 5.0, 24.1, 24.2**

Example

Prove the Cross Products Property: If $\frac{a}{b} = \frac{c}{d}$ $(b \neq 0$ and $d \neq 0)$, then $ad = bc$.

Statements	Reasons
1. $\frac{a}{b} = \frac{c}{d}$ $(b \neq 0$ and $d \neq 0)$	Given
2. $\left(\frac{a}{b}\right)bd = \left(\frac{c}{d}\right)bd$	Multiplication Property of Equality (Multiply both sides by bd.)
3. $\frac{abd}{b} = \frac{cbd}{d}$	Multiply.
4. $ad = cb$	Simplify.
5. $ad = bc$	Commutative Property of Multiplication

Try This

1. Below is an incomplete proof of the following statement: If a and b are even, then $a + b$ is even. Fill in the blanks to complete the proof.

Statements	Reasons
1. *a and b are* ____?____.	Given
2. a and b are each divisible by ▢. In other words, they can each be written as the product of ▢ and some other number: $a = 2m$ and $b = 2n$.	Definition of ____?____
3. $a + b = 2m + 2n$	Substitute ▢ for a and ▢ for b.
4. $a + b = 2($▢$)$	____?____
5. Because $a + b$ is divisible by 2, ____?____ is even.	Definition of ____?____

2. Prove the following statement: If a and b are even, then ab is even.

2-6 Solving Literal Equations for a Variable

California Standards

Extension of 5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

Vocabulary
formula
literal equation

Who uses this?

Athletes can "rearrange" the distance formula to calculate their average speed.

Many wheelchair athletes compete in marathons, which cover about 26.2 miles. Using the time t it took to complete the race, the distance d, and the *formula* $d = rt$, racers can find their average speed r.

A **formula** is an equation that states a rule for a relationship among quantities.

In the formula $d = rt$, d is isolated. You can "rearrange" a formula to isolate any variable by using inverse operations. This is called *solving for a variable*.

Know it! Note

Solving for a Variable	
Step 1	Locate the variable you are asked to solve for in the equation.
Step 2	Identify the operations on this variable and the order in which they are applied.
Step 3	Use inverse operations to undo operations and isolate the variable.

EXAMPLE 1 *Sports Application*

In 2004, Ernst Van Dyk won the wheelchair race of the Boston Marathon with a time of about 1.3 hours. The race was about 26.2 miles. What was his average speed? Use the formula $d = rt$ and round your answer to the nearest tenth.

The question asks for speed, so first solve the formula $d = rt$ for r.

$d = rt$ *Locate r in the equation.*

$\frac{d}{t} = \frac{rt}{t}$ *Since r is multiplied by t, divide both sides by t to undo the multiplication.*

$\frac{d}{t} = r$, or $r = \frac{d}{t}$

Now use this formula and the information given in the problem.

$r = \frac{d}{t} \approx \frac{26.2}{1.3}$

≈ 20.2

Van Dyk's average speed was about 20.2 miles per hour.

Helpful Hint

A nonzero number divided by itself equals 1. For $t \neq 0$, $\frac{t}{t} = 1$.

CHECK IT OUT!

1. Solve the formula $d = rt$ for t. Find the time in hours that it would take Van Dyk to travel 26.2 miles if his average speed was 18 miles per hour. Round to the nearest hundredth.

EXAMPLE 2 Solving Formulas for a Variable

A **The formula for a Fahrenheit temperature in terms of degrees Celsius is $F = \frac{9}{5}C + 32$. Solve for C.**

$F = \frac{9}{5}C + 32$ — *Locate C in the equation.*

$-32 \quad -32$ — *Since 32 is added to $\frac{9}{5}C$, subtract 32 from both sides to undo the addition.*

$F - 32 = \frac{9}{5}C$

$\left(\frac{5}{9}\right)(F - 32) = \left(\frac{5}{9}\right)\frac{9}{5}C$ — *Since C is multiplied by $\frac{9}{5}$, divide both sides by $\frac{9}{5}$ (multiply by $\frac{5}{9}$) to undo the multiplication.*

$\frac{5}{9}(F - 32) = C$

Remember!

Dividing by a fraction is the same as multiplying by the reciprocal.

B **The formula for a person's typing speed is $s = \frac{w - 10e}{m}$, where s is speed in words per minute, w is number of words typed, e is number of errors, and m is number of minutes typing. Solve for w.**

$s = \frac{w - 10e}{m}$ — *Locate w in the equation.*

$m(s) = m\left(\frac{w - 10e}{m}\right)$ — *Since w − 10e is divided by m, multiply both sides by m to undo the division.*

$ms = w - 10e$

$+10e \quad +10e$ — *Since 10e is subtracted from w, add 10e to both sides to undo the subtraction.*

$ms + 10e = w$

CHECK IT OUT! **2.** The formula for an object's final velocity f is $f = i - gt$, where i is the object's initial velocity, g is acceleration due to gravity, and t is time. Solve for i.

A formula is a type of *literal equation*. A **literal equation** is an equation with two or more variables. To solve for one of the variables, use inverse operations.

EXAMPLE 3 Solving Literal Equations for a Variable

A **Solve $m - n = 5$ for m.**

$m - n = 5$ — *Locate m in the equation.*

$+n \quad +n$ — *Since n is subtracted from m, add n to both sides to undo the subtraction.*

$m = 5 + n$

B **Solve $\frac{m}{k} = x$ for k.**

$\frac{m}{k} = x$ — *Locate k in the equation.*

$k\left(\frac{m}{k}\right) = kx$ — *Since k appears in the denominator, multiply both sides by k.*

$m = kx$

$\frac{m}{x} = \frac{kx}{x}$ — *Since k is multiplied by x, divide both sides by x to undo the multiplication.*

$\frac{m}{x} = k$

CHECK IT OUT! **3a.** Solve $5 - b = 2t$ for t. **3b.** Solve $D = \frac{m}{V}$ for V.

THINK AND DISCUSS

1. Describe a situation in which a formula could be used more easily if it were "rearranged." Include the formula in your description.
2. Explain how to solve $P = 2\ell + 2w$ for w.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Write a formula that is used in each subject. Then solve the formula for each of its variables.

Common Formulas	
Subject	**Formula**
Geometry	
Physical science	
Earth science	

2-6 Exercises

California Standards Practice
Extension of 5.0

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GUIDED PRACTICE

SEE EXAMPLE 1 p. 109

1. **Vocabulary** Explain why a *formula* is a type of *literal equation*.
2. **Construction** The formula $a = 46c$ gives the floor area a in square meters that can be wired using c circuits.
   a. Solve $a = 46c$ for c.
   b. If a room is 322 square meters, how many circuits are required to wire this room?

SEE EXAMPLE 2 p. 110

3. The formula for the volume of a rectangular prism with length ℓ, width w, and height h is $V = \ell wh$. Solve this formula for w.

SEE EXAMPLE 3 p. 110

4. Solve $st + 3t = 6$ for s.
5. Solve $m - 4n = 8$ for m.
6. Solve $\frac{f+4}{g} = 6$ for f.
7. Solve $b + c = \frac{10}{a}$ for a.

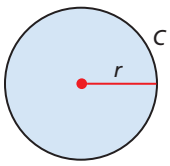
PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
8	1
9	2
10–13	3

Extra Practice
Skills Practice p. EP5
Application Practice p. EP25

8. **Geometry** The formula $C = 2\pi r$ relates the circumference C of a circle to its radius r. (Recall that π is the constant ratio of circumference to diameter.)
   a. Solve $C = 2\pi r$ for r.
   b. If a circle's circumference is 15 inches, what is its radius? Leave the symbol π in your answer.



C is the distance around the circle.
r is the distance from the center of the circle to any point on the circle.

9. **Finance** The formula $A = P + I$ shows that the total amount of money A received from an investment equals the principal P (the original amount of money invested) plus the interest I. Solve this formula for I.
10. Solve $-2 = 4r + s$ for s.
11. Solve $xy - 5 = k$ for x.
12. Solve $\frac{m}{n} = p - 6$ for n.
13. Solve $\frac{x-2}{y} = z$ for y.

Solve for the indicated variable.

14. $S = 180n - 360$ for n

15. $\frac{x}{5} - g = a$ for x

16. $A = \frac{1}{2}bh$ for b

17. $y = mx + b$ for x

18. $a = 3n + 1$ for n

19. $PV = nRT$ for T

20. $T + M = R$ for T

21. $M = T - R$ for T

22. $PV = nRT$ for R

23. $2a + 2b = c$ for b

24. $5p + 9c = p$ for c

25. $ax + r = 7$ for r

26. $3x + 7y = 2$ for y

27. $4y + 3x = 5$ for x

28. $y = 3x + 3b$ for b

29. **Estimation** The table shows the flying time and distance traveled for five flights on a certain airplane.

a. Use the data in the table to write a rule that *estimates* the relationship between flying time t and distance traveled d.

b. Use your rule from part **a** to estimate the time that it takes the airplane to fly 1300 miles.

c. Solve your rule for d.

d. Use your rule from part **c** to estimate the distance the airplane can fly in 8 hours.

Flying Times

Flight	Time (h)	Distance (mi)
A	2	1018
B	3	1485
C	4	2103
D	5	2516
E	6	2886

30. **Sports** To find a baseball pitcher's earned run average (ERA), you can use the formula $Ei = 9r$, where E represents ERA, i represents number of innings pitched, and r represents number of earned runs allowed. Solve the equation for E. What is a pitcher's ERA if he allows 5 earned runs in 18 innings pitched?

31. **Meteorology** For altitudes up to 36,000 feet, the relationship between temperature and altitude can be described by the formula $t = -0.0035a + g$, where t is the temperature in degrees Fahrenheit, a is the altitude in feet, and g is the ground temperature in degrees Fahrenheit. Solve this formula for a.

32. **Write About It** In your own words, explain how to solve a literal equation for one of the variables.

33. **Critical Thinking** How is solving $a - ab = c$ for a different from the problems in this lesson? How might you solve this equation for a?

CONCEPT CONNECTION

34. This problem will help prepare you for the Concept Connection on page 120.

The formula $s = fw$ relates the speed of sound s in meters per second, the frequency of a note f in Hertz (Hz), and the wavelength of the note's sound wave w in meters.

a. Solve the formula for f.

b. The speed of sound is approximately 340 m/s. The note A_4 has a wavelength of approximately 0.773 m. Find the frequency of the note A_4 to the nearest whole number.

c. As the wavelength of a note increases, what happens to the frequency? Explain.

Multiple Choice **For Exercises 35–37, choose the best answer.**

35. Which equation is the result of solving $9 + 3x = 2y$ for x?

Ⓐ $\frac{9 + 3y}{2} = x$ Ⓑ $\frac{2}{3}y - 9 = x$ Ⓒ $x = \frac{2}{3}y - 3$ Ⓓ $x = 2y - 3$

36. Which of the following is a correct method for solving $2a - 5b = 10$ for b?

Ⓐ Add $5b$ to both sides, then divide both sides by 2.

Ⓑ Subtract $5b$ from both sides, then divide both sides by 2.

Ⓒ Divide both sides by 5, then add $2a$ to both sides.

Ⓓ Subtract $2a$ from both sides, then divide both sides by -5.

37. Anna wants to make a cardboard box with a length of 7 inches, a width of 5 inches, and a volume of 210 cubic inches. In the formula for the volume of a rectangular prism, which variable does Anna need to solve for in order to build the box?

Ⓐ V Ⓑ ℓ Ⓒ w Ⓓ h

CHALLENGE AND EXTEND

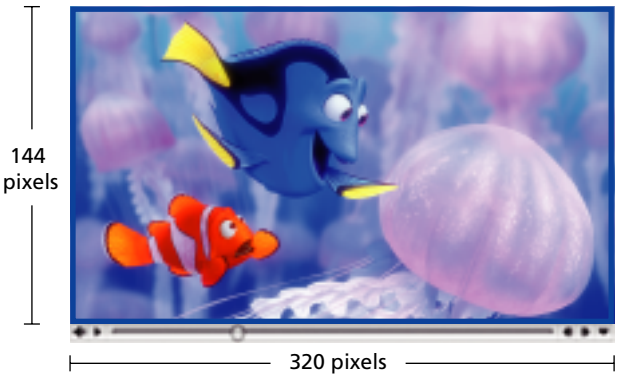
Solve for the indicated variable.

38. $3.3x + r = 23.1$ for x **39.** $\frac{2}{5}a - \frac{3}{4}b = c$ for a **40.** $\frac{3}{5}x + 1.4y = \frac{2}{5}$ for y

41. $t = \frac{d}{500} + \frac{1}{2}$ for d **42.** $s = \frac{1}{2}gt^2$ for g **43.** $v^2 = u^2 + 2as$ for s

44. Solve $y = mx + 6$ for m. What can you say about y if $m = 0$?

45. **Entertainment** The formula $S = \frac{h \cdot w \cdot f \cdot t}{35{,}000}$ gives the approximate size in kilobytes (Kb) of a compressed video. The variables h and w represent the height and width of the frame measured in pixels, f is the number of frames per second (fps) the video plays, and t is the time the video plays in seconds. Estimate the time the movie trailer shown will play if it plays at 15 fps and has a size of 2370 Kb.



SPIRAL STANDARDS REVIEW

6NS2.1, 6AF1.1, 7AF1.2

46. Jill spent $\frac{1}{4}$ of the money she made baby-sitting. She made $40 baby-sitting. How much did she spend? *(Previous course)*

47. In one class, $\frac{3}{5}$ of the students are boys. There are 30 students in the class. How many are girls? *(Previous course)*

Evaluate each expression for the given value of x. *(Lesson 1-7)*

48. $3 + 2 \cdot x + 4$ for $x = 3$ **49.** $24 \div 4 - x$ for $x = 12$ **50.** $43 - 62 + x$ for $x = 15$

Solve each equation. *(Lesson 2-1)*

51. $18 = -2 + w$ **52.** $2 = -3 + c$ **53.** $-8 + k = 4$ **54.** $-15 + a = -27$

2-7 Solving Absolute-Value Equations

California Standards

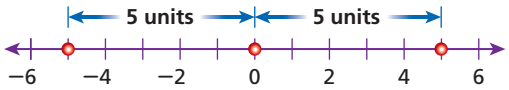
3.0 Students solve equations and inequalities **involving absolute values.**

5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities **in one variable** and provide justification for each step.

Why learn this?

Engineers can solve absolute-value equations to calculate the length of the deck of a bridge. (See Example 3.)

Recall that the absolute value of a number is that number's distance from zero on a number line. For example, $|-5| = 5$ and $|5| = 5$.

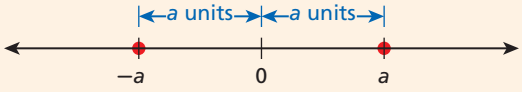


For any nonzero absolute value, there are exactly two numbers with that absolute value. For example, both 5 and −5 have an absolute value of 5.

To write this statement using algebra, you would write $|x| = 5$. This equation asks, "What values of x have an absolute value of 5?" The solutions are 5 and −5. Notice that this equation has two solutions.

Know it! Note

Absolute-Value Equations

WORDS	NUMBERS
The equation $\|x\| = a$ asks, "What values of x have an absolute value of a?" The solutions are a and the opposite of a.	$\|x\| = 5$ $x = 5$ or $x = -5$
GRAPH	**ALGEBRA**
	$\|x\| = a$ $x = a$ or $x = -a$ $(a \geq 0)$

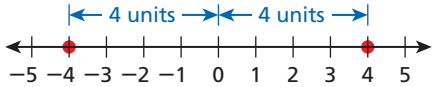
To solve absolute-value equations, perform inverse operations to isolate the absolute-value expression on one side of the equation. Then you must consider two cases.

EXAMPLE 1 Solving Absolute-Value Equations

Solve each equation.

A $|x| = 4$

$|x| = 4$ *Think: What numbers are 4 units from 0?*



Case 1	Case 2	*Rewrite the equation as two cases.*
$x = -4$	$x = 4$	

The solutions are −4 and 4. You can write the solution set as $\{-4, 4\}$.

Helpful Hint

Be sure to check both solutions when you solve an absolute-value equation.

$\|x\| = 4$		$\|x\| = 4$	
$\|-4\|$	4	$\|4\|$	4
4	4 ✓	4	4 ✓

Solve each equation.

B $4|x+2| = 24$

$\frac{4|x+2|}{4} = \frac{24}{4}$ — *Since $|x+2|$ is multiplied by 4, divide both sides by 4 to undo the multiplication.*

$|x+2| = 6$ — *Think: What numbers are 6 units from 0?*

Case 1	Case 2
$x+2=-6$	$x+2=6$
$-2 \quad -2$	$-2 \quad -2$
$x=-8$	$x=4$

Rewrite the equation as two cases. Since 2 is added to x, subtract 2 from both sides of the equation.

The solution set is $\{-8, 4\}$.

Solve each equation. Check your answer.

1a. $|x| - 3 = 4$ **1b.** $8 = |x - 2.5|$

The table summarizes the steps for solving absolute-value equations.

Solving an Absolute-Value Equation
1. Use inverse operations to isolate the absolute-value expression.
2. Rewrite the resulting equation as two cases that do not involve absolute values.
3. Solve the equation in each of the two cases.

Not all absolute-value equations have two solutions. If the absolute-value expression equals 0, there is one solution. If an equation states that an absolute value is negative, there are no solutions.

EXAMPLE 2 Special Cases of Absolute-Value Equations

Solve each equation.

A $|x+3|+4=4$

$|x+3|+4=4$ — *Since 4 is added to $|x+3|$, subtract 4 from both sides to undo the addition.*

$-4 \quad -4$

$|x+3| = 0$

$x+3=0$ — *There is only one case. Since 3 is added to x, subtract 3 from both sides to undo the addition.*

$-3 \quad -3$

$x=-3$

The solution set is $\{-3\}$.

B $5=|x+2|+8$

$5=|x+2|+8$ — *Since 8 is added to $|x+2|$, subtract 8 from both sides to undo the addition.*

$-8 \quad -8$

$-3=|x+2|$ ✗ — *Absolute value cannot be negative.*

This equation has no solution. The solution set is the empty set, $\varnothing$.

Remember!

Absolute value must be nonnegative because it represents a distance.

Solve each equation.

2a. $2 - |2x - 5| = 7$ **2b.** $-6 + |x - 4| = -6$

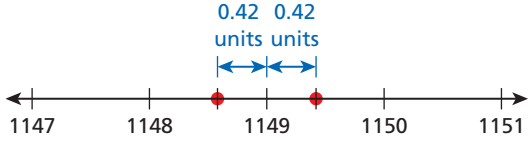
EXAMPLE 3 *Engineering Application*

Sydney Harbour Bridge in Australia is 1149 meters long. Because of changes in temperature, the bridge can expand or contract by as much as 420 millimeters. Write and solve an absolute-value equation to find the minimum and maximum lengths of the bridge.

First convert millimeters to meters.

420 mm = 0.4 2 0 m *Move the decimal point three places to the left.*

The length of the bridge can vary by 0.42 m, so find two numbers that are 0.42 units away from 1149 on a number line.



You can find these numbers by using the absolute-value equation $|x - 1149| = 0.42$. Solve the equation by rewriting it as two cases.

Case 1	**Case 2**
$x - 1149 = -0.42$	$x - 1149 = 0.42$
$+ 1149 \quad + 1149$	$+ 1149 \quad + 1149$
$x = 1148.58$	$x = 1149.42$

Since 1149 is subtracted from x, add 1149 to both sides of each equation.

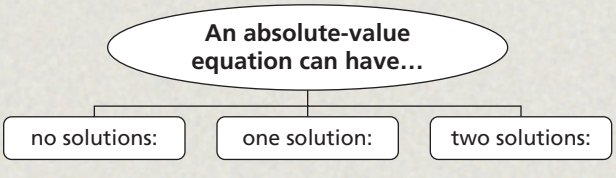
The minimum length of the bridge is 1148.58 m, and the maximum length is 1149.42 m.

3. Sydney Harbour Bridge is 134 meters tall. The height of the bridge can rise or fall by 180 millimeters because of changes in temperature. Write and solve an absolute-value equation to find the minimum and maximum heights of the bridge.

THINK AND DISCUSS

1. **Explain** the steps you would use to solve the equation $\frac{1}{5}|x - 3| = 2$.

2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of an absolute-value equation that has the indicated number of solutions, and then solve.



2-7 Exercises

California Standards Practice
3.0, 5.0, 25.1, 25.3

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GUIDED PRACTICE

SEE EXAMPLE 1 p. 114

Solve each equation.

1. $|x| = 6$
2. $9 = |x + 5|$
3. $|3x| + 2 = 8$
4. $2|x| = 18$
5. $\left|x + \frac{1}{2}\right| = 1$
6. $|x - 3| - 6 = 2$

SEE EXAMPLE 2 p. 115

7. $-8 = |x|$
8. $|x| = 0$
9. $|x + 4| = -7$
10. $7 = |3x + 9| + 7$
11. $|2.8 - x| + 1.5 = 1.5$
12. $5|x + 7| + 14 = 8$

SEE EXAMPLE 3 p. 116

13. **Communication** Barry's walkie-talkie has a range of 2 mi. Barry is traveling on a straight highway and is at mile marker 207. Write and solve an absolute-value equation to find the minimum and maximum mile marker from 207 that Barry's walkie-talkie will reach.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–22	1
23–28	2
29	3

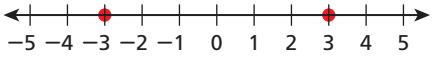
Extra Practice
Skills Practice p. EP5
Application Practice p. EP25

Solve each equation.

14. $|x| = \frac{1}{5}$
15. $|2x - 4| = 22$
16. $18 = 3|x - 1|$
17. $-2|x| = -4$
18. $3|x| - 12 = 18$
19. $|x - 42.04| = 23.24$
20. $\left|\frac{2}{3}x - \frac{2}{3}\right| = \frac{2}{3}$
21. $|3x + 1| = 13$
22. $|-2x + 3| = 5.8$
23. $|4x| + 9 = 9$
24. $8 = 7 - |x|$
25. $|x| + 6 = 12 - 6$
26. $|x - 3| + 14 = 5$
27. $0 = \left|\frac{2}{3} - x\right|$
28. $3 + |x - 1| = 3$

29. **Space Shuttle** The diameter of a valve for the space shuttle must be within 0.001 mm of 5 mm. Write and solve an absolute-value equation to find the boundary values for the acceptable diameters of the valve.
30. The two numbers that are 5 units from 3 on the number line are represented by the equation $|n - 3| = 5$. What are these two numbers? Graph the solutions.
31. Write and solve an absolute-value equation that represents two numbers x that are 2 units from 7 on a number line. Graph the solutions.
32. **Manufacturing** A quality control inspector at a bolt factory examines random bolts that come off the assembly line. Any bolt whose diameter differs by more than 0.04 mm from 6.5 mm is sent back. Write and solve an absolute-value equation to find the maximum and minimum diameters of an acceptable bolt.
33. **Construction** A brick company guarantees to fill a contractor's order to within 5% accuracy. A contractor orders 1500 bricks. Write and solve an absolute-value equation to find the maximum and minimum number of bricks guaranteed.
34. **Multi-Step** A machine prints posters and then trims them to the correct size. The equation $|\ell - 65.1| = 0.2$ gives the maximum and minimum acceptable lengths for the posters in inches. Does a poster with a length of 64.8 inches fall within the acceptable range? Why or why not?

Write an absolute-value equation whose solutions are graphed on the number line.

35.


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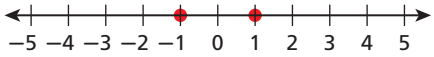

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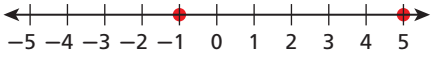

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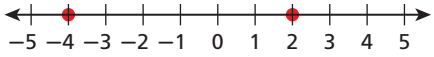


Reasoning **Tell whether each statement is sometimes, always, or never true. Explain.**

39. An absolute-value equation has two solutions.

40. The value of $|x + 4|$ is equal to the value of $|x| + 4$.

41. The absolute value of a number is nonnegative.

42. Temperature A thermostat is set so that the temperature in a laboratory freezer stays within 2.5°F of 2°F. Write and solve an absolute-value equation to find the maximum and minimum temperatures in the freezer.

43. Recreation To ensure safety, boaters must be aware of wind conditions while they are on the water. A particular anemometer gives a measurement of wind speed within a certain amount of the true wind speed, as shown in the table.

Measured Wind Speed (mi/h)	True Wind Speed (mi/h)
20	15–25
22	17–27
24	19–29
26	21–31
28	23–33
30	25–35

a. Use the table to write an absolute-value equation for the minimum and maximum possible true wind speeds t for the measured wind speed shown on the anemometer.

b. Solve your equation from part **a.** Check that the solution is correct by comparing it to the values given in the table when the measured wind speed is 24 mi/h.

c. Will your equation work for all of the values in the table? Explain.

d. Explain what your equation says about the instrument's measurements.

CONCEPT CONNECTION

44. This problem will help prepare you for the Concept Connection on page 120.

A violin can produce a range of notes. The center of the violin's frequency range is 1666 Hz.

a. Write an absolute-value expression that gives the distance on the number line of a violin note's frequency f from the value 1666.

b. The lowest and highest notes that a violin can produce have frequencies that differ by 1470 Hz from the frequency at the center of the range. Write an absolute-value equation for the frequencies of these notes.

c. Find the least and greatest frequencies that a violin can produce.

45. Write About It Do you agree with the following statement: "To solve an absolute-value equation, you need to solve two equations." Why or why not?

46. Critical Thinking Is there a value of a for which the equation $|x - a| = 1$ has exactly one solution? Explain.

Multiple Choice **For Exercises 47–49, choose the best answer.**

47. Which situation could be modeled by the equation $|x - 65| = 3$?

Ⓐ Two numbers on the number line are 65 units away from 3.

Ⓑ The length of a carpet is 3 inches less than 65 inches.

Ⓒ The maximum and minimum weights of wrestlers on the team are within 3 kg of 65 kg.

Ⓓ The members of an exercise club for seniors are all between 63 and 67 years old.

48. For which of the following is $n = -3$ a solution?

Ⓐ $|n - 1| = 2$ Ⓑ $|n + 2| = -1$ Ⓒ $|n - 2| = 1$ Ⓓ $|n + 1| = 2$

49. The minimum and maximum sound levels at a rock concert are 90 decibels and 95 decibels. Which equation models this situation?

Ⓐ $|x - 90| = 5$ Ⓑ $|x - 92.5| = 2.5$ Ⓒ $|x - 92.5| = 5$ Ⓓ $|x - 95| = 2.5$

CHALLENGE AND EXTEND

50. The perimeter of a rectangle is 100 inches. The length of the rectangle is $|2x - 4|$ inches, and the width is x inches. What are the possible values of x? Explain.

51. Reasoning Fill in the missing reasons to justify each step in solving the equation $3|2x + 1| = 21$.

Statements	Reasons
1. $3\|2x + 1\| = 21$	**1.** Given
2. $\|2x + 1\| = 7$	**2.** _____?_____
3. $2x + 1 = -7$ or $2x + 1 = 7$	**3.** Definition of absolute value
4. $2x = -8$ or $2x = 6$	**4.** _____?_____
5. $x = -4$ or $x = 3$	**5.** _____?_____

52. Solve $|x| = |x + 1|$. (*Hint:* Consider two cases: $x \geq 0$ and $x < 0$.)

SPIRAL STANDARDS REVIEW 6AF1.1, 6NS1.3

Solve each equation. Check your answer. *(Lesson 2-1)*

53. $5 = p - 4.5$ **54.** $-2 = y + 6\frac{1}{2}$ **55.** $-12 + q = 3$ **56.** $y - 4.3 = -5.7$

Solve each proportion. Check your answer. *(Lesson 2-5)*

57. $\frac{m}{8} = \frac{3}{4}$ **58.** $\frac{16}{y} = \frac{12}{18}$ **59.** $\frac{4}{5} = \frac{12}{x + 6}$ **60.** $\frac{-2}{3} = \frac{5}{2x}$

Solve for the indicated variable. *(Lesson 2-6)*

61. $m + 5n = 7$ for m **62.** $S = T + R$ for T **63.** $2y + 3x = 1$ for y

64. $\frac{3 + w}{z} = x$ for w **65.** $c + d = \frac{5}{e}$ for e **66.** $6M - N = S$ for N

CONCEPT CONNECTION

Proportions and Formulas

Make a Note of It Sounds are produced by vibrating objects, such as guitar strings. The number of vibrations per second is called the frequency, and the frequency determines the note that you hear. The table shows the approximate frequency for the notes in a scale, where 1 Hertz (Hz) = 1 vibration/second.

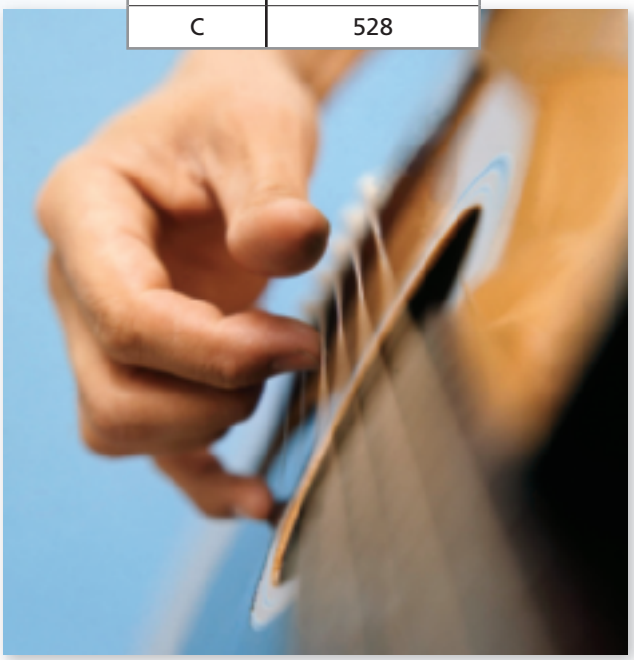
Musical Notes	
Note	**Frequency (Hz)**
C	264
D	297
E	330
F	352
G	396
A	440
B	495
C	528

1. Notes that sound pleasing when played at the same time have a special relationship. For example, two notes whose frequencies have a ratio of 5 : 4 are said to be separated by an interval of a third. Which note is separated by an interval of a third from the note F?

2. The speed of sound is approximately 340 m/s. The following formula relates the speed of sound s, the frequency f, and the wavelength w.

$$s = fw$$

Solve the formula for w to find the wavelength for the note G.



3. A piano can produce a wide range of notes. The center of the piano's frequency range is of 2106.75 Hz, while the frequencies at the extreme ends of the range differ from this value by 2079.25 Hz. Write and solve an absolute-value equation to find the minimum and maximum frequencies for notes on a piano.

Quiz for Lessons 2-5 Through 2-7

2-5 Solving Proportions

1. Last week, the ratio of laptops to desktops sold at a computer store was 2:3. Eighteen desktop models were sold. How many laptop models were sold?
2. Anita read 150 pages in 5 hours. What is her reading rate in pages per minute?

Find the unit rate.

3. Twenty-six crackers contain 156 Calories.
4. A store developed 1024 photographs in 8 hours.

Solve each proportion.

5. $\frac{-18}{n} = \frac{9}{2}$
6. $\frac{d}{5} = \frac{2}{4}$
7. $\frac{4}{12} = \frac{r+2}{16}$
8. $\frac{-3}{7} = \frac{6}{x+6}$

Find each value. Round to the nearest tenth if necessary.

9. Find 40% of 25.
10. Find 130% of 9.
11. 35 is what percent of 70?
12. What percent of 400 is 640?
13. 16 is 80% of what number?
14. 200% of what number is 28?
15. A volunteer at the zoo is responsible for feeding the animals in 15 exhibits in the reptile house. This represents 20% of the total exhibits in the reptile house. How many exhibits are in the reptile house?

2-6 Solving Literal Equations for a Variable

16. Solve $5j + s = t - 2$ for t.
17. Solve $h + p = 3(k - 8)$ for k.
18. Solve $2x + 3y = 12$ for x.
19. Solve $\frac{x}{r} = v$ for x
20. The formula for the area of a triangle is $A = \frac{1}{2}bh$. Solve the formula for h. If the area of a triangle is 48 cm^2, and its base measures 12 cm, what is the height of a triangle?

2-7 Solving Absolute-Value Equations

Solve each equation.

21. $|r| = 7$
22. $|h + 4| = 11$
23. $|2x + 4| = 0$
24. $16 = 7|p + 3| + 30$
25. Collette is a contestant on a game show. She can win a car if she can guess the price of the car within $725. The price of the car is $16,785. Write and solve an absolute-value equation to find the values for the maximum and minimum price that Collette can guess to win the car.

CHAPTER 2

Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. A formula is a type of a(n) ____?____.

2. A(n) ____?____ is used to compare two quantities.

2-1 Solving One-Step Equations *(pp. 72–77)*

 2.0, Prep for 5.0

EXAMPLES

Solve each equation. Check your answer.

■ $x - 12 = -8$

$+12 \quad +12$ *Add 12 to both sides.*

$x = 4$

Check

$x - 12 = -8$	
$4 - 12$	-8
-8	-8 ✓

■ $-8x = 148$

$\frac{-8x}{-8} = \frac{148}{-8}$ *Divide both sides by –8.*

$x = -18.5$

Check

$-8x = 148$	
$-8\,(-18.5)$	148
148	148 ✓

EXERCISES

Solve each equation. Check your answer.

3. $b - 16 = 20$

4. $4 + x = 2$

5. $9 + a = -12$

6. $-7 + y = 11$

7. $z - \frac{1}{4} = \frac{7}{8}$

8. $w + \frac{2}{3} = 3$

9. $7.2 + t = 12.3$

10. $22.5 = x - 3$

11. $35 = 5x$

12. $-6s = 42$

13. $\frac{f}{15} = 8$

14. $3m = 45$

15. $4j = 28$

16. $\frac{n}{3} = 12$

17. $4k = 3.5$

18. $\frac{c}{5} = 2.5$

19. Robin needs 108 signatures for her petition. So far, she has 27. Write and solve an equation to determine how many more signatures she needs.

20. Mrs. Wilson gave some money to each of her 6 children. Each child received $35.50. How much money did Mrs. Wilson give to her 6 children in total?

2-2 Solving Two-Step Equations *(pp. 79–84)*

Prep for 5.0

EXAMPLES

Solve each equation.

■ $\frac{z}{2.4} + 2 = 14$

$\frac{z}{2.4} + 2 = 14$

$-2 \quad -2$ *Subtract 2 from both sides.*

$\frac{z}{2.4} = 12$

$(2.4)\frac{z}{2.4} = (2.4)12$ *Multiply both sides by 2.4.*

$z = 28.8$

■ $5x - 6 = 79$

$5x - 6 = 79$

$+6 \quad +6$ *Add 6 to both sides.*

$5x = 85$

$\frac{5x}{5} = \frac{85}{5}$ *Divide both sides by 5.*

$x = 17$

EXERCISES

Solve each equation. Check your answer.

21. $4t - 13 = 57$

22. $5 - 2y = 15$

23. $3z + 1 = 19$

24. $18 = 30 - 3h$

25. $\frac{k}{5} - 6 = 2$

26. $\frac{1}{6}f + \frac{3}{4} = \frac{1}{2}$

27. $6h - 7 = 25$

28. $17 = 29 - 4k$

29. $\frac{1}{4} + \frac{3}{4}t = 10$

30. $18 - 4a = 34$

31. $0.2x + 0.5 = 0.9$

32. $-0.6x + 1 = 0.4$

33. Thomas had the same score for 3 different quizzes. On one quiz he received 5 extra bonus points. His total score for the 3 quizzes was 281 points. What was his score on each quiz?

2-3 Solving Multi-Step Equations *(pp. 85–90)*

4.0, 5.0

EXAMPLE

Solve each equation.

■ $\frac{3x}{5} - \frac{x}{4} + \frac{1}{2} = \frac{6}{5}$

$20\left(\frac{3x}{5} - \frac{x}{4} + \frac{1}{2}\right) = 20\left(\frac{6}{5}\right)$ *Multiply by the LCD.*

$12x - 5x + 10 = 24$ *Combine like terms.*

$7x + 10 = 24$

$-10 \quad -10$ *Subtract 10 from both sides.*

$7x = 14$

$\frac{7x}{7} = \frac{14}{7}$ *Divide both sides by 7.*

$x = 2$

■ $3(b + 1) = -9$

$3(b) + 3(1) = -9$ *Distribute 3.*

$3b + 3 = -9$

$-3 \quad -3$ *Subtract 3 from both sides.*

$3b = -12$

$\frac{3b}{3} = \frac{-12}{3}$ *Divide both sides by 3.*

$b = -4$

EXERCISES

Solve each equation. Check your answer.

34. $a - 12 + 2a = 27$

35. $5y + 3 - 7y = 15$

36. $4 + 3a - 6 = 43$

37. $3(x + 2) = 24$

38. $h(5 - 2) + 8 = 17$

39. $8(z + 10) + z = 98$

40. $-6 = 2.5(w - 2)$

41. $0.3x - 1.1 - x = 0.3$

42. If $8n + 22 = 70$, find the value of $3n$.

43. If $0 = 6n - 36$, find the value of $n - 5$.

44. The sum of the measures of two angles is 180°. One angle measures $3a$, and the other angle measures $2a - 25$ Find a. Then find the measure of each angle.

45. A drama club sold 120 child tickets and 80 adult tickets to a play. A child ticket cost \$3 less than an adult ticket. The club collected a total of \$640 from ticket sales. What is the cost of an adult ticket? a child ticket?

2-4 Solving Equations with Variables on Both Sides *(pp. 92–98)*

 4.0, 5.0

EXAMPLE

Solve each equation.

■ $3y - 5 = 2y + 5$

$3y - 5 = 2y + 5$

$-2y \quad -2y$ *Subtract 2y from both sides.*

$y - 5 = 5$

$+5 \quad +5$ *Add 5 to both sides.*

$y = 10$

■ $x + 7 = 12 + 3x - 7x$

$x + 7 = 12 - 4x$ *Combine like terms.*

$+4x \quad +4x$ *Add 4x to both sides.*

$5x + 7 = 12$

$-7 \quad -7$ *Subtract 7 from both sides.*

$5x = 5$

$\frac{5x}{5} = \frac{5}{5}$ *Divide both sides by 5.*

$x = 1$

EXERCISES

Solve each equation. Check your answer.

46. $4x + 2 = 3x$

47. $-3r - 8 = -5r - 12$

48. $-a - 3 + 7 = 3a$

49. $-(x - 4) = 2x + 6$

50. $\frac{2}{3}n = 4n - \frac{10}{3}n - \frac{1}{2}$

51. $0.2(7 + 2t) = 0.4t + 1.4$

52. $0.5x - 1.7 = 0.3x - 1$

53. $-2c + 8 = 5c - 10$

54. $7x - 28 = 3x$

55. $9x - 3 = 6x + 15$

56. $\frac{3}{4}n + 1 = \frac{5}{4}n + 2$

57. $\frac{1}{3}x + \frac{2}{3} = \frac{5}{3}x$

58. One photo shop charges \$0.36 per print. Another photo shop charges \$2.52 plus \$0.08 per print. Juan finds that the cost of developing his photos is the same at either shop. How many photos does Juan have to develop?

2-5 Solving Proportions *(pp. 102–107)*

 15.0

EXAMPLES

■ **Solve** $\frac{3w - 7}{21} = \frac{3}{7}$.

$\frac{3w - 7}{21} \times \frac{3}{7}$

$7(3w - 7) = 21(3)$ *Use cross products.*

$21w - 49 = 63$

$+49 \quad +49$ *Add 49 to both sides.*

$21w = 112$

$\frac{21w}{21} = \frac{112}{21}$ *Divide both sides by 21.*

$w = \frac{16}{3}$

EXERCISES

59. In the ninth grade there are 320 students and 20 teachers. What is the student-to-teacher ratio?

Solve each proportion. Check your answer.

60. $\frac{n}{8} = \frac{2}{10}$

61. $\frac{2}{9} = \frac{12}{x}$

62. Amelia Earhart made her 1932 solo flight across the Atlantic Ocean in a Lockheed Vega. One model of the plane is $\frac{1}{30}$ the size of the real airplane. The wingspan of the Lockheed was 41 feet. What is the wingspan of the model? Round your answer to the nearest hundredth.

■ **Earth has a surface area of approximately 197 million square miles. About 58 million square miles is land. Find the percent of Earth's surface area that is water.**

$197 \text{ million} - 58 \text{ million} = 139 \text{ million}$

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

$$\frac{139 \text{ million}}{197 \text{ million}} \times \frac{n}{100}$$

$197n = 13{,}900$ *Use cross products.*

$n = 70.56$

About 71% of Earth's surface area is water.

63. Find 2.3% of 230.

64. Find 115% of 2700.

65. What percent of 18 is 12? Round your answer to the nearest tenth of a percent.

66. What percent of 14 is 56?

67. 90% of what number is 120? Round your answer to the nearest tenth.

68. 90 is 37.5% of what number?

69. A student answered 32 questions correctly and 8 incorrectly. What percent of the questions were answered correctly?

2-6 Solving Literal Equations for a Variable *(pp. 109–113)*

Ext. of 5.0

EXAMPLE

■ **Solve $A = P + Prt$ for r.**

$A = P + Prt$

$-P \quad -P$ *Subtract P from both sides.*

$A - P = Prt$

$\frac{A - P}{Pt} = \frac{Prt}{Pt}$ *Divide both sides by Pt.*

$\frac{A - P}{Pt} = r$

EXERCISES

Solve for the indicated variable.

70. $C = \frac{360}{n}$ for n

71. $S = \frac{n}{2}(a + \ell)$ for a

72. $0.25x + y = 225$ for x

73. The formula $a = \frac{d}{g}$ gives the average gas mileage a of a vehicle that uses g gallons of gas to travel d miles. Use the formula to find how many gallons of gas a vehicle with an average gas mileage of 20.2 miles per gallon will use to travel 75 miles. Round your answer to the nearest tenth.

2-7 Solving Absolute-Value Equations *(pp. 114–119)*

3.0, 5.0

EXAMPLE

■ **Solve $3|y + 4| = 30$.**

$\frac{3|y + 4|}{3} = \frac{30}{3}$ *Divide both sides by 3.*

$|y + 4| = 10$

Case 1

$y + 4 = 10$

$-4 \quad -4$

$y = 6$

Case 2

$y + 4 = -10$

$-4 \quad -4$

$y = -14$

EXERCISES

Solve each equation. Check your answer.

74. $|x + 6| = 21$

75. $7|y - 5| = 14$

76. $3|y| + 4 = 31$

77. $12 = |x - 5.4|$

78. $|g + 6| + 12 = 14$

79. $|x| = \frac{5}{7}$

80. Jason is driving his car at 55 mi/h. He needs to keep his car within 5 mi/h of his current speed. Write and solve an absolute-value equation to find Jason's maximum and minimum speeds.

CHAPTER TEST

Solve each equation.

1. $y - 7 = 2$
2. $x + 12 = 19$
3. $-5 + z = 8$
4. $9x = 72$
5. $\frac{m}{-8} = -2.5$
6. $\frac{7}{8}a = 42$
7. $15 = 3 - 4x$
8. $\frac{2a}{3} + \frac{1}{5} = \frac{7}{6}$
9. $8 - (b - 2) = 11$
10. $-2x + 4 = 5 - 3x$
11. $3(q - 2) + 2 = 5q - 7 - 2q$
12. $5z = -3(z + 7)$
13. $m - 2.7 = -1.5m + 1$
14. $x - 3.6 = 10.2 - 3.2$
15. $c + 13.5 = 20$

Solve for the indicated variable.

16. $r - 2s = 14$ for s
17. $V = \frac{1}{3}bh$ for b
18. $P = 2(\ell + w)$ for ℓ
19. $2x + a = 4$ for x
20. $4x + 6y = 12$ for x
21. $\frac{x}{r} = n$ for r
22. The ratio of red marbles to blue marbles in a bag is 4 : 7. There are 16 red marbles. How many blue marbles are there?

Find each unit rate. Round to the nearest hundredth if necessary.

23. A store sells 3 videotapes for $4.99.
24. Twenty-five students use 120 sheets of paper.

Solve each proportion.

25. $\frac{5}{4} = \frac{x}{12}$
26. $\frac{8}{2z} = \frac{15}{60}$
27. $\frac{x + 10}{10} = \frac{18}{12}$
28. $\frac{x}{8} = \frac{1}{4}$
29. $\frac{5}{12} = \frac{-4}{f}$
30. $\frac{3}{10} = \frac{x + 1}{15}$
31. $\frac{c - 4}{5} = \frac{-c}{2}$
32. $\frac{3n}{2} = \frac{2}{3}$
33. $\frac{w}{6} = \frac{5}{2}$
34. The scale on a map is 1 inch : 500 miles. If two cities are 875 miles apart, how far apart are they on the map?
35. Order the following from least to greatest: 0.625, $\frac{1}{8}$, $\frac{1}{2}$, 1, 20%, 30%.
36. What is 23% of 46?
37. 37.5 is 60% of what number?
38. What percent of 175 is 35?
39. What is 29% of 32?
40. 84.41 is 23% of what number?

Solve each equation.

41. $|x - 14| = 21$
42. $13 = |y + 2| - 3$
43. $4|z| = 20$
44. $3|x| + 5 = 8$
45. $3|g + 1| + 5 = 7$
46. $|2v| = 6$

College Entrance Exam Practice

CHAPTER 2

FOCUS ON ACT

The ACT Mathematics Test is one of four tests in the ACT. You have 60 minutes to answer 60 multiple-choice questions. The questions cover material typically taught through the end of eleventh grade. You will need to know some basic formulas.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

HOT TIP!

There is no penalty for incorrect answers on the ACT. If you are unsure of the correct answer, eliminate as many answer choices as possible. Then make your best guess. Be sure you have marked an answer for every question before time runs out.

---

1. At a certain high school, the ratio of left-handed to right-handed basketball players is 1 : 4. If there are a total of 20 players on the team, how many players are right-handed?

(A) 1

(B) 4

(C) 5

(D) 12

(E) 16

---

2. If $y - 3 = \frac{2}{5}(x + 1)$, then $x =$?

(F) $\frac{5(y-3)-2}{2}$

(G) $y - \frac{22}{5}$

(H) $\frac{2(y-3)}{5} - 1$

(J) $\frac{2(y+1)+15}{5}$

(K) $\frac{5}{2}y - 4$

---

3. What is $\frac{1}{5}\%$ of 20?

(A) 0.004

(B) 0.04

(C) 0.4

(D) 4

(E) 100

---

4. If $x - 3 = 4 - 2(x + 5)$, then $x =$?

(F) -3

(G) -1

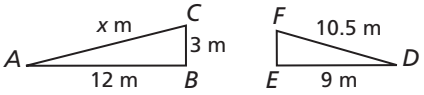
(H) 1

(J) $\frac{3}{2}$

(K) $\frac{11}{3}$

---

5. If $\triangle ABC \sim \triangle DEF$, what is the length of $\overline{AC}$?



(A) 2.6 meters

(B) 3.5 meters

(C) 7 meters

(D) 14 meters

(E) 15 meters

---

6. A movie theater makes 30% of its revenue from concession sales. If concession sales were $174,000, what was the total revenue?

(F) $52,200

(G) $121,800

(H) $248,570

(J) $580,000

(K) $746,000

CHAPTER 2

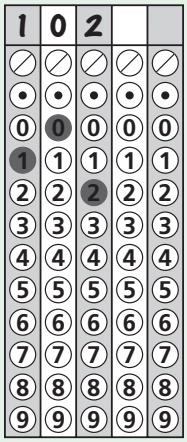
STRATEGIES FOR SUCCESS

Gridded Response: Fill in Answer Grids Correctly

When responding to a test item that requires you to place your answer in a grid, you must fill out the grid on your answer sheet correctly, or the item will be marked as incorrect.

EXAMPLE 1

Gridded Response: Simplify the expression $12^2 - 3(10 + 4)$.



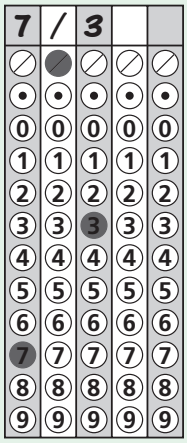
$$12^2 - 3(10 + 4)$$
$$12^2 - 3(14)$$
$$144 - 3(14)$$
$$144 - 42$$
$$102$$

The expression simplifies to 102.

- Write your answer in the answer boxes at the top of the grid.
- Put only one digit in each box. Do not leave a blank box in the middle of an answer.
- Shade the bubble for each digit in the same column as the digit in the answer box.

EXAMPLE 2

Gridded Response: Evaluate the expression $ba \div c$ for $a = -7$, $b = 2$, and $c = -6$.



$$ba \div c$$
$$(2)(-7) \div (-6)$$
$$-14 \div (-6)$$
$$\frac{7}{3} = 2\frac{1}{3} = 2.\overline{3}$$

The expression simplifies to $\frac{7}{3}$, $2\frac{1}{3}$, or $2.\overline{3}$.

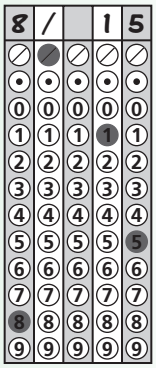
- Mixed numbers and repeating decimals cannot be gridded, so you must grid the answer as $\frac{7}{3}$.
- Write your answer in the answer boxes at the top of the grid.
- Put only one digit or symbol in each box. On some grids, the fraction bar and the decimal point have a designated box. Do not leave a blank box in the middle of an answer.
- Shade the bubble for each digit or symbol in the same column as the digit in the answer box.

Grid formats may vary from test to test. The grid in this book is used often, but it is not used on every test that has gridded response questions. Always examine the grid when taking a standardized test to be sure you know how to fill it in correctly.

Read each sample and then answer the questions that follow.

Sample A

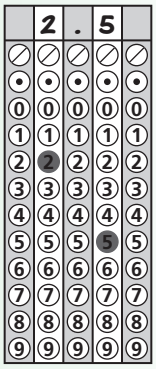
A student correctly evaluated an expression and got $\frac{8}{15}$ as a result. Then the student filled in the grid as shown.



1. What error did the student make when filling out the grid?
2. Explain how to fill in the answer correctly.

Sample B

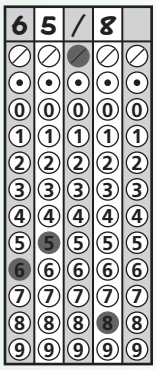
The square root of 6.25 is 2.5. This answer is displayed in the grid.



3. What error did the student make when filling in the grid?
4. Explain how to fill in the answer correctly.

Sample C

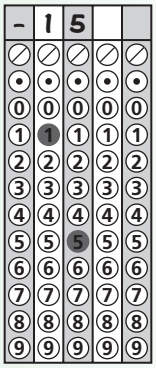
A student correctly simplified the expression $2\frac{1}{8} + 3\frac{5}{8} + \frac{7}{8}$. Then the student filled in the grid as shown.



5. What answer does the grid show?
6. Explain why you cannot fill in a mixed number.
7. Write the answer in two forms that could be entered in the grid correctly.

Sample D

A student added −10 and 25 and got an answer of 15. Then the student filled in the grid as shown.



8. What error does the grid show?
9. Another student got an answer of −15. Explain why the student knew this answer was wrong.

CHAPTER 2

MASTERING THE STANDARDS

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KEYWORD: MA8CA Practice

CUMULATIVE ASSESSMENT, CHAPTERS 1–2

Multiple Choice

1. What operation does ◊ represent if $x \lozenge 2.2 = 4.5$ when $x = 9.9$?

Ⓐ Addition
Ⓑ Subtraction
Ⓒ Multiplication
Ⓓ Division

2. A couple earns $4819.25 a month. They pay 9.5% of their monthly income as the monthly payment on their car. To the nearest dollar, how much does the couple pay for their monthly car payment?

Ⓐ $458
Ⓑ $507
Ⓒ $4578
Ⓓ $4810

3. Naomi runs 8 miles each day. If she slows her pace by half, she runs this distance in 2 hours and 40 minutes. What is her normal pace?

Ⓐ 6 miles per hour
Ⓑ 8 miles per hour
Ⓒ 4.75 miles per hour
Ⓓ 6.75 miles per hour

4. A clock loses 5 minutes every day. How much time will it lose in 2 hours?

Ⓐ 0.417 second
Ⓑ 25 seconds
Ⓒ 240 seconds
Ⓓ 600 seconds

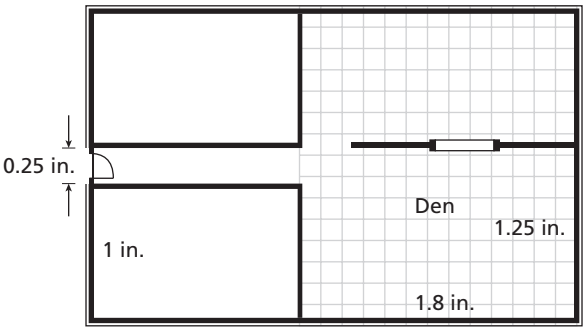
5. A statue is 8 feet tall. The display case for a model of the statue is 18 inches tall. Which scale allows for the tallest model of the statue that will fit in the display case?

Ⓐ 1 inch : 2 inches
Ⓑ 1 inch : 7 inches
Ⓒ 1 inch : 5 inches
Ⓓ 1 inch : 10 inches

6. What is the value of $-|6^2|$?

Ⓐ -36
Ⓑ -12
Ⓒ -8
Ⓓ -3

7. Mr. Phillips wants to install hardwood flooring in his den. The flooring costs $25.86 per square yard. The blueprint below shows his house. What other information do you need in order to find the total cost of the flooring?



Ⓐ The lengths and widths of the adjoining rooms in the blueprint
Ⓑ The total area of the blueprint
Ⓒ The scale of inches in the blueprint to yards in the house
Ⓓ The width of the den

8. What value of n makes the equation below have no solution?

$$2x + 2 = nx - 3$$

Ⓐ -2
Ⓑ 0
Ⓒ 2
Ⓓ 3

9. Which of the equations below represents the second step of the solution process?

Step 1: $3(5x - 2) + 27 = -24$
Step 2: ☐
Step 3: $15x + 21 = -24$
Step 4: $15x = -45$
Step 5: $x = -3$

Ⓐ $3(5x + 27) - 2 = -24$
Ⓑ $3(5x + 25) = -24$
Ⓒ $15x - 2 + 27 = -24$
Ⓓ $15x - 6 + 27 = -24$

If you are stuck on a problem, skip it and come back later. Another problem might remind you of something that will help. If you feel yourself become tense, take a few deep breaths to relax.

10. Cass drove 3 miles to school, and then she drove m miles to a friend's house. The total mileage for these two trips was 8 miles. Which equation CANNOT be used to determine the number of miles Cass drove?

Ⓐ $3 + m = 8$

Ⓑ $3 - m = 8$

Ⓒ $8 - 3 = m$

Ⓓ $8 - m = 3$

11. If $\frac{20}{x} = \frac{4}{x-5}$, which of the following is a true statement?

Ⓐ $x(x-5) = 80$

Ⓑ $20x = 4(x-5)$

Ⓒ $20(x-5) = 4x$

Ⓓ $24 = 2x - 5$

Gridded Response

12. Four times a number is two less than six times the same number minus ten. What is the number?

13. On August 1st, Melissa invested $6000 in a retirement account. A portion of her account record is shown below. Her balance can be found using the equation $B = 192m + 6000$, where B is the balance and m is the number of months after August. Find the missing balance, in dollars, in the table.

Date	Balance ($)
8/1	6000
9/1	6192
10/1	6384
1/1	

14. At 2:45 P.M. you are 112 miles from Dallas. You want to be in Dallas at 4:30 P.M. What is the average number of miles per hour you must travel to be on time?

15. A cyclist travels 45 miles in 4 hours. How many feet does she travel in one second?

16. A bike rental shop charges a one-time charge of $8 plus an hourly fee to rent a bike. Dan paid $24.50 to rent a bike for $5\frac{1}{2}$ hours. Find the bike shop's hourly fee in dollars.

Short Response

17. Alex buys 5 calendars to give as gifts. Each calendar has the same price. When the cashier rings up Alex's calendars, the total cost before tax is $58.75.

a. Write and solve an equation to find the cost of each calendar.

b. The total cost of Alex's calendars after tax is $63.45. Find the percent sales tax. Show your work and explain in words how you found your answer.

c. Alex's friend Keisha buys some calendars for the same price. She uses her 15% discount card. The total cost before tax is $39.95. How many calendars did Keisha buy? Show your work and explain in words how you found your answer.

18. A student's solution for the absolute-value equation $6|x + 4| = 36$ was {2}. Explain why this answer is incorrect.

Extended Response

19. Korena is putting a decorative border around her rectangular flower garden. The total perimeter of the garden is 200 feet.

a. Draw three different rectangles that could represent Korena's flower garden. Label the dimensions of your rectangles.

b. Use the table to show the lengths and widths of five different rectangles that could represent Korena's flower garden. Do not use any of your rectangles from part **a.**

Possible Dimensions of Korena's Garden		
Length (ℓ)	Width (w)	Perimeter (P)

c. The length of Korena's garden is 4 times its width. Explain how to use the perimeter formula $P = 2\ell + 2w$ to find the dimensions of Korena's garden.

d. Find the dimensions of Korena's garden.