Systems of Equations and Inequalities

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Lab Solve Systems of Linear Inequalities

You can solve a system of equations to decide how many basketball game tickets you can buy at different prices.

Staples Center
Los Angeles, CA
**Vocabulary**

Match each term on the left with a definition on the right.

1. inequality  
   A. a pair of numbers \((x, y)\) that represent the coordinates of a point
2. linear equation  
   B. a statement that two quantities are not equal
3. ordered pair  
   C. the \(y\)-value of the point at which the graph of an equation crosses the \(y\)-axis
4. slope  
   D. a value of the variable that makes the equation true
5. solution of an equation  
   E. the ratio of the vertical change to the horizontal change for a nonvertical line
   F. an equation whose graph is a straight line

**Graph Linear Functions**

Graph each function.

6. \(y = \frac{3}{4}x + 1\)  
7. \(y = -3x + 5\)  
8. \(y = x - 6\)  
9. \(x + y = 4\)  
10. \(y = -\frac{2}{3}x + 4\)  
11. \(y = -5\)

**Solve Multi-Step Equations**

Solve each equation.

12. \(-7x - 18 = 3\)  
13. \(12 = -3n + 6\)  
14. \(\frac{1}{2}d + 30 = 32\)  
15. \(-2p + 9 = -3\)  
16. \(33 = 5y + 8\)  
17. \(-3 + 3x = 27\)

**Solve for a Variable**

Solve each equation for \(y\).

18. \(7x + y = 4\)  
19. \(y + 2 = -4x\)  
20. \(8 = x - y\)  
21. \(x + 2 = y - 5\)  
22. \(2y - 3 = 12x\)  
23. \(y + \frac{3}{4}x = 4\)

**Evaluate Expressions**

Evaluate each expression for the given value of the variable.

24. \(t - 5\) for \(t = 7\)  
25. \(9 - 2a\) for \(a = 4\)  
26. \(\frac{1}{2}x - 2\) for \(x = 14\)  
27. \(n + 15\) for \(n = 37\)  
28. \(9c + 4\) for \(c = \frac{1}{3}\)  
29. \(16 + 3d\) for \(d = 5\)

**Solve and Graph Inequalities**

Solve and graph each inequality.

30. \(b - 9 \geq 1\)  
31. \(-2x < 10\)  
32. \(3y \leq -3\)  
33. \(\frac{1}{3}y \leq 5\)
The information below “unpacks” the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

<table>
<thead>
<tr>
<th>California Standard</th>
<th>Academic Vocabulary</th>
<th>Chapter Concept</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0 Students graph a linear equation and compute the (x)- and (y)-intercepts (e.g., graph (2x + 6y = 4)). They are also able to sketch the region defined by linear inequalities (e.g., they sketch the region defined by (2x + 6y &lt; 4)). (Lessons 6-1, 6-6, 6-7)</td>
<td>define to mark the limits of</td>
<td>You solve a linear inequality that contains two variables and graph the solutions on the coordinate plane.</td>
</tr>
<tr>
<td>8.0 Students understand the concepts of parallel lines and perpendicular lines and how their slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point. (Lesson 6-4)</td>
<td>concept idea or meaning</td>
<td>You understand parallel lines and how they are related in the coordinate plane.</td>
</tr>
<tr>
<td>9.0 Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets. (Lessons 6-1, 6-2, 6-3, 6-4, 6-5, 6-7; Labs 6-2, 6-7)</td>
<td>algebraically having to do with algebra</td>
<td>You use algebra to find solutions that satisfy two linear equations or inequalities, and you understand how the solutions are represented in the coordinate plane.</td>
</tr>
<tr>
<td>15.0 Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems. (Lesson 6-5)</td>
<td>algebraic having to do with algebra</td>
<td>You use algebra to solve real-world problems about rates and mixtures.</td>
</tr>
</tbody>
</table>

Standard 5.0 is also covered in this chapter. To see this standard unpacked, go to Chapter 2, p. 70.
Writing Strategy: Write a Convincing Argument/Explanation

The Write About It icon appears throughout the book. These icons identify questions that require you to write a complete argument or explanation. Writing a convincing argument or explanation shows that you have a solid understanding of a concept.

To be effective, an argument or explanation should include

• reasoning, evidence, work, or facts.
• a complete response that will answer or explain.

From Lesson 3-7

54. Write About It Describe how to use an absolute-value inequality to find all the values on a number line that are within 5 units of −6.

Step 1 Identify what you need to answer or explain.
Explain how an absolute-value inequality can help find values on a number line that are within 5 units of −6.

Step 2 Give evidence, work, or facts that are needed to answer the question.
The distance between two numbers can be found using subtraction. The inequality $|x| < 5$ describes all real numbers whose distance from 0 is less than 5 units. To find all real numbers whose distance from −6 is less than 5 units, you must subtract −6 from $x$.

$$|x - (-6)| < 5$$

Step 3 Write a complete response that answers or explains.
The difference between a number and −6 must be less than 5.

$$|x - (-6)| < 5$$
$$|x + 6| < 5$$

Try This

Write a convincing argument or explanation.

1. What is the least whole number that is a solution of $12x + 15.4 > 118.92$? Explain.

2. Which equation has an error? Explain the error.
   A. $4(6 \cdot 5) = (4)6 \cdot (4)5$
   B. $4(6 \cdot 5) = (4 \cdot 6)5$
Solve Linear Equations by Using a Spreadsheet

You can use a spreadsheet to answer “What if...?” questions. By changing one or more values, you can quickly model different scenarios.

Activity

Company Z makes DVD players. The company's costs are $400 per week plus $20 per DVD player. Each DVD player sells for $45. How many DVD players must company Z sell in one week to make a profit?

Let \( n \) represent the number of DVD players company Z sells in one week.

\[
\begin{align*}
\text{Cost:} & \quad c = 400 + 20n \\
\text{Sales income:} & \quad s = 45n \\
\text{Profit:} & \quad p = s - c
\end{align*}
\]

- \( c = 400 + 20n \) The total cost is $400 plus $20 times the number of DVD players made.
- \( s = 45n \) The total sales income is $45 times the number of DVD players sold.
- \( p = s - c \) The total profit is the sales income minus the total cost.

1. Set up your spreadsheet with columns for number of DVD players, total cost, total income, and profit.
2. Under Number of DVD Players, enter 1 in cell A2.
3. Use the equations above to enter the formulas for total cost, total sales, and total profit in row 2.
   - In cell B2, enter the formula for total cost.
   - In cell C2, enter the formula for total sales income.
   - In cell D2, enter the formula for total profit.
4. Fill columns A, B, C, and D by selecting cells A1 through D1, clicking the small box at the bottom right corner of cell D2, and dragging the box down through several rows.
5. Find the point where the profit is $0. This is known as the breakeven point, where total cost and total income are the same.

Try This

For Exercises 1 and 2, use the spreadsheet from the activity.

1. If company Z sells 10 DVD players, will they make a profit? Explain. What if they sell 16?
2. Company Z makes a profit of $225 dollars. How many DVD players did they sell?

For Exercise 3, make a spreadsheet.

3. Company Y’s costs are $400 per week plus $20 per DVD player. They want the breakeven point to occur with sales of 8 DVD players. What should the sales price be?
6-1 Solving Systems by Graphing

Why learn this?

You can compare costs by graphing a system of linear equations.

Sometimes there are different charges for the same service or product at different places. For example, Bowl-o-Rama charges $2.50 per game plus $2 for shoe rental while Bowling Pinz charges $2 per game plus $4 for shoe rental. A system of linear equations can be used to compare these charges.

A system of linear equations is a set of two or more linear equations containing two or more variables. A solution of a system of linear equations with two variables is an ordered pair that satisfies each equation in the system. So, if an ordered pair is a solution, it will make both equations true.

**Example 1**

Identifying Solutions of Systems

Tell whether the ordered pair is a solution of the given system.

A. \((4, 1)\); \[
\begin{align*}
x + 2y &= 6 \\
x - y &= 3
\end{align*}
\]

\[
\begin{array}{c|c}
4 & 6 \\
4 & 2 \\
6 & \checkmark
\end{array}
\]

Substitute 4 for \(x\) and 1 for \(y\).

The ordered pair \((4, 1)\) makes both equations true.

\((4, 1)\) is a solution of the system.

B. \((-1, 2)\); \[
\begin{align*}
2x + 5y &= 8 \\
3x - 2y &= 5
\end{align*}
\]

\[
\begin{array}{c|c}
2(-1) + 5(2) & 8 \\
-2 + 10 & 8 \\
8 & \checkmark
\end{array}
\] \[
\begin{array}{c|c}
3(-1) - 2(2) & 5 \\
-3 - 4 & 5 \\
-7 & 5\checkmark
\end{array}
\]

Substitute \(-1\) for \(x\) and \(2\) for \(y\).

The ordered pair \((-1, 2)\) makes one equation true, but not the other. \((-1, 2)\) is not a solution of the system.

**Check It Out!**

Tell whether the ordered pair is a solution of the given system.

1a. \((1, 3)\); \[
\begin{align*}
2x + y &= 5 \\
-2x + y &= 1
\end{align*}
\]

1b. \((-2, -1)\); \[
\begin{align*}
x - 2y &= 4 \\
3x + y &= 6
\end{align*}
\]
All solutions of a linear equation are on its graph. To find a solution of a system of linear equations, you need a point that each line has in common. In other words, you need their point of intersection.

\[
\begin{align*}
y &= 2x - 1 \\
y &= -x + 5
\end{align*}
\]

The point \((2, 3)\) is where the two lines intersect and is a solution of both equations, so \((2, 3)\) is the solution of the system.

**Example 2: Solving a System of Linear Equations by Graphing**

Solve each system by graphing. Check your answer.

**A.**
\[
\begin{align*}
y &= x - 3 \\
y &= -x - 1
\end{align*}
\]

**Graph the system.**

The solution appears to be at \((1, -2)\).

**Check**

Substitute \((1, -2)\) into the system.

\[
\begin{array}{c|c}
y &= x - 3 \\
\hline
2 & 1 - 3 \\
-2 & -2 \\
\end{array}
\quad \begin{array}{c|c}
y &= -x - 1 \\
\hline
-2 & -1 - 1 \\
-2 & -2 \\
\end{array}
\]

The solution is \((1, -2)\).

**B.**
\[
\begin{align*}
x + y &= 0 \\
y &= -\frac{1}{2}x + 1
\end{align*}
\]

**Rewrite the first equation in slope-intercept form.**

\[
x + y = 0 \\
-\frac{1}{2}x &= -x \\
y &= \frac{-1}{2}x + 1
\]

**Graph the system.**

The solution appears to be at \((-2, 2)\).

**Check**

Substitute \((-2, 2)\) into the system.

\[
\begin{array}{c|c}
x + y &= 0 \\
-2 + 2 &= 0 \\
0 &= 0 \\
\end{array}
\quad \begin{array}{c|c}
y &= -\frac{1}{2}x + 1 \\
2 &= -\frac{1}{2}(-2) + 1 \\
2 &= 1 + 1 \\
2 &= 2 \\
\end{array}
\]

The solution is \((-2, 2)\).

**Check It Out!**

Solve each system by graphing. Check your answer.

**2a.**
\[
\begin{align*}
y &= -2x - 1 \\
y &= x + 5
\end{align*}
\]

**2b.**
\[
\begin{align*}
y &= \frac{1}{3}x - 3 \\
2x + y &= 4
\end{align*}
\]
**Problem-Solving Application**

Bowl-o-Rama charges $2.50 per game plus $2 for shoe rental, and Bowling Pinz charges $2 per game plus $4 for shoe rental. For how many games will the cost to bowl be the same at both places? What is that cost?

1. **Understand the Problem**
   
The answer will be the number of games played for which the total cost is the same at both bowling alleys. List the important information:
   - Game price: Bowl-o-Rama $2.50 Bowling Pinz: $2
   - Shoe-rental fee: Bowl-o-Rama $2 Bowling Pinz: $4

2. **Make a Plan**
   
   Write a system of equations, one equation to represent the price at each company. Let \( x \) be the number of games played and \( y \) be the total cost.

   \[
   \begin{align*}
   \text{Bowl-o-Rama} & : y = 2.5x + 2 \\
   \text{Bowling Pinz} & : y = 2x + 4
   \end{align*}
   \]

3. **Solve**
   
   Graph \( y = 2.5x + 2 \) and \( y = 2x + 4 \). The lines appear to intersect at \((4, 12)\). So, the cost at both places will be the same for 4 games bowled and that cost will be $12.

4. **Look Back**
   
   Check \((4, 12)\) using both equations.
   - Cost of bowling 4 games at Bowl-o-Rama:
     \[
     2.5(4) + 2 = 10 + 2 = 12
     \]
   - Cost of bowling 4 games at Bowling Pinz:
     \[
     2(4) + 4 = 8 + 4 = 12
     \]

**THINK AND DISCUSS**

1. Explain how to use a graph to solve a system of linear equations.
2. Explain how to check a solution of a system of linear equations.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write a step for solving a linear system by graphing. More boxes may be added.
GUIDED PRACTICE

1. **Vocabulary** Describe a solution of a system of linear equations.

Tell whether the ordered pair is a solution of the given system.

2. (2, –2): \[ \begin{align*} 3x + y &= 4 \\ x - 3y &= -4 \end{align*} \]

3. (3, –1): \[ \begin{align*} x - 2y &= 5 \\ 2x - y &= 7 \end{align*} \]

4. (–1, 5): \[ \begin{align*} -x + y &= 6 \\ 2x + 3y &= 13 \end{align*} \]

Solve each system by graphing. Check your answer.

5. \[ \begin{align*} y &= \frac{1}{2}x \\ y &= -x + 3 \end{align*} \]

6. \[ \begin{align*} y &= x - 2 \\ 2x + y &= 1 \end{align*} \]

7. \[ \begin{align*} -2x - 1 &= y \\ x + y &= 3 \end{align*} \]

8. To deliver mulch, Lawn and Garden charges $30 per cubic yard of mulch plus a $30 delivery fee. Yard Depot charges $25 per cubic yard of mulch plus a $55 delivery fee. For how many cubic yards will the cost be the same? What will that cost be?

PRACTICE AND PROBLEM SOLVING

Tell whether the ordered pair is a solution of the given system.

9. (1, –4): \[ \begin{align*} x - 2y &= 8 \\ 4x - y &= 8 \end{align*} \]

10. (–2, 1): \[ \begin{align*} 2x - 3y &= -7 \\ 3x + y &= -5 \end{align*} \]

11. (5, 2): \[ \begin{align*} 2x + y &= 12 \\ -3y - x &= -11 \end{align*} \]

Solve each system by graphing. Check your answer.

12. \[ \begin{align*} y &= \frac{1}{2}x + 2 \\ y &= -x - 1 \end{align*} \]

13. \[ \begin{align*} y &= x \\ y &= -x + 6 \end{align*} \]

14. \[ \begin{align*} -2x - 1 &= y \\ x &= -y + 3 \end{align*} \]

15. \[ \begin{align*} x + y &= 2 \\ y &= x - 4 \end{align*} \]

16. **Multi-Step** Angelo runs 7 miles per week and increases his distance by 1 mile each week. Marc runs 4 miles per week and increases his distance by 2 miles each week. In how many weeks will Angelo and Marc be running the same distance? What will that distance be?

17. **School** The school band sells carnations on Valentine’s Day for $2 each. They buy the carnations from a florist for $0.50 each, plus a $16 delivery charge.
   a. Write a system of equations to describe the situation.
   b. Graph the system. What does the solution represent?
   c. Explain whether the solution shown on the graph makes sense in this situation. If not, give a reasonable solution.

18. This problem will prepare you for the Concept Connection on page 362.
   a. The Warrior baseball team is selling hats as a fund-raiser. They contacted two companies. Hats Off charges a $50 design fee and $5 per hat. Top Stuff charges a $25 design fee and $6 per hat. Write an equation for each company’s pricing.
   b. Graph the system of equations from part a. For how many hats will the cost be the same? What is that cost?
   c. Explain when it is cheaper for the baseball team to use Top Stuff and when it is cheaper to use Hats Off.
Graphing Calculator  Use a graphing calculator to graph and solve the systems of equations in Exercises 19–22. Round your answer to the nearest tenth.

19. \[
\begin{align*}
  y &= 4.7x + 2.1 \\
  y &= 1.6x - 5.4
\end{align*}
\]

20. \[
\begin{align*}
  4.8x + 0.6y &= 4 \\
  y &= -3.2x + 2.7
\end{align*}
\]

21. \[
\begin{align*}
  y &= \frac{5}{4}x - \frac{2}{3} \\
  \frac{8}{3}x + y &= \frac{5}{9}
\end{align*}
\]

22. \[
\begin{align*}
  y &= 6.9x + 12.4 \\
  y &= -4.1x - 5.3
\end{align*}
\]

23. Landscaping  The gardeners at Middleton Place Gardens want to plant a total of 45 white and pink hydrangeas in one flower bed. In another flower bed, they want to plant 120 hydrangeas. In this bed, they want 2 times the number of white hydrangeas and 3 times the number of pink hydrangeas as in the first bed. Use a system of equations to find how many white and how many pink hydrangeas the gardeners should buy altogether.

24. Fitness  Rusty burns 5 Calories per minute swimming and 11 Calories per minute jogging. In the morning, Rusty burns 200 Calories walking and swims for \( x \) minutes. In the afternoon, Rusty will jog for \( x \) minutes. How many minutes must he jog to burn at least as many Calories \( y \) in the afternoon as he did in the morning? Round your answer up to the next whole number of minutes.

25. A tree that is 2 feet tall is growing at a rate of 1 foot per year. A 6-foot tall tree is growing at a rate of 0.5 foot per year. In how many years will the trees be the same height?

26. Critical Thinking  Write a real-world situation that could be represented by the system \[
\begin{align*}
  y &= 3x + 10 \\
  y &= 5x + 20
\end{align*}
\]

27. Write About It  When you graph a system of linear equations, why does the intersection of the two lines represent the solution of the system?

Multiple Choice  For Exercises 28 and 29, choose the best answer.

28. Taxi company A charges $4 plus $0.50 per mile. Taxi company B charges $5 plus $0.25 per mile. Which system best represents this problem?

   A. \[
   \begin{align*}
   y &= 4x + 0.5 \\
   y &= 5x + 0.25
   \end{align*}
   \]

   B. \[
   \begin{align*}
   y &= 0.5x + 4 \\
   y &= 0.25x + 5
   \end{align*}
   \]

   C. \[
   \begin{align*}
   y &= -4x + 0.5 \\
   y &= -5x + 0.25
   \end{align*}
   \]

   D. \[
   \begin{align*}
   y &= -0.5x + 4 \\
   y &= -0.25x + 5
   \end{align*}
   \]

29. Which system of equations represents the given graph?

   A. \[
   \begin{align*}
   y &= 2x - 1 \\
   y &= \frac{1}{3}x + 3
   \end{align*}
   \]

   B. \[
   \begin{align*}
   y &= -2x + 1 \\
   y &= 2x - 3
   \end{align*}
   \]

   C. \[
   \begin{align*}
   y &= 2x + 1 \\
   y &= \frac{1}{3}x - 3
   \end{align*}
   \]

   D. \[
   \begin{align*}
   y &= -2x - 1 \\
   y &= 3x - 3
   \end{align*}
   \]

30. Gridded Response  Which value of \( b \) will make the system \( y = 2x + 2 \) and \( y = 2.5x + b \) intersect at the point \((2, 6)\)?
31. **Entertainment** If the pattern in the table continues, in what month will the number of sales of VCRs and DVD players be the same? What will that number be?

<table>
<thead>
<tr>
<th>Month</th>
<th>VCRs</th>
<th>DVD Players</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>250</td>
</tr>
<tr>
<td>2</td>
<td>490</td>
<td>265</td>
</tr>
<tr>
<td>3</td>
<td>480</td>
<td>280</td>
</tr>
<tr>
<td>4</td>
<td>470</td>
<td>295</td>
</tr>
</tbody>
</table>

32. Long Distance Inc. charges a $1.45 connection charge and $0.03 per minute. Far Away Calls charges a $1.52 connection charge and $0.02 per minute.

a. For how many minutes will a call cost the same from both companies? What is that cost?

b. When is it better to call using Long Distance Inc.? Far Away Calls? Explain.

c. **What if...?** Long Distance Inc. raised its connection charge to $1.50 and Far Away Calls decreased its connection charge by 2 cents. How will this affect the graphs? Now which company is better to use for calling long distance? Why?

**Spiral Standards Review**  
**Lesson 2-1**

33. \(18 = \frac{3}{7}x\)

34. \(-\frac{x}{5} = 12\)

35. \(-6y = -13.2\)

36. \(\frac{2}{5} = \frac{y}{12}\)

**Lesson 2-3**

37. \(5x + 6x + 5 = 16\)

38. \(6(x + 2) = -2(x + 10)\)

39. \(12 - 6z + 5z = 10\)

**Lesson 3-4**

40. \(4(2x + 1) > 28\)

41. \(3^3 + 9 \leq -4c\)

42. \(\frac{1}{8}x + \frac{3}{5} \leq \frac{3}{8}\)

**Career Path**

**Q:** What math classes did you take in high school?

**A:** Career Math, Algebra, and Geometry

**Q:** What are you studying and what math classes have you taken?

**A:** I am really interested in aviation. I am taking Statistics and Trigonometry. Next year I will take Calculus.

**Q:** How is math used in aviation?

**A:** I use math to interpret aeronautical charts. I also perform calculations involving wind movements, aircraft weight and balance, and fuel consumption. These skills are necessary for planning and executing safe air flights.

**Q:** What are your future plans?

**A:** I could work as a commercial or corporate pilot or even as a flight instructor. I could also work toward a bachelor's degree in aviation management, air traffic control, aviation electronics, aviation maintenance, or aviation computer science.
Model Systems of Linear Equations

You can use algebra tiles to model and solve some systems of linear equations.

**Use with Lesson 6-2**

**Activity**

Use algebra tiles to model and solve

\[
\begin{align*}
y &= 2x - 3 \\
x + y &= 9
\end{align*}
\]

<table>
<thead>
<tr>
<th>MODEL</th>
<th>ALGEBRA</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Algebra Tiles" /></td>
<td><img src="image" alt="Algebra Tiles" /></td>
</tr>
</tbody>
</table>
| The first equation is solved for \( y \). Model the second equation, \( x + y = 9 \), by substituting \( 2x - 3 \) for \( y \). | \[
\begin{align*}
x + y &= 9 \\
x + (2x - 3) &= 9 \\
3x - 3 &= 9
\end{align*}
\] |
| Add 3 yellow tiles on both sides of the mat. This represents adding 3 to both sides of the equation. Remove zero pairs. | \[
\begin{align*}
3x - 3 &= 9 \\
\underline{+3} & \quad \underline{+3} \\
3x &= 12
\end{align*}
\] |
| Divide each group into 3 equal groups. Align one \( x \)-tile with each group on the right side. One \( x \)-tile is equivalent to 4 yellow tiles. \( x = 4 \) | \[
\begin{align*}
\frac{3x}{3} &= \frac{12}{3} \\
x &= 4
\end{align*}
\] |

To solve for \( y \), substitute 4 for \( x \) in one of the equations:

\[
y = 2(4) - 3 = 5
\]

The solution is (4, 5).

**Try This**

Use algebra tiles to model and solve each system of equations.

1. \[
\begin{align*}
y &= x + 3 \\
2x + y &= 6
\end{align*}
\]

2. \[
\begin{align*}
2x + 3 &= y \\
x + y &= 6
\end{align*}
\]

3. \[
\begin{align*}
2x + 3y &= 1 \\
x &= -1 - y
\end{align*}
\]

4. \[
\begin{align*}
y &= x + 1 \\
2x - y &= -5
\end{align*}
\]
Sometimes it is difficult to identify the exact solution to a system by graphing. In this case, you can use a method called substitution.

Substitution is used to reduce the system to one equation that has only one variable. Then you can solve this equation by the methods taught in Chapter 2.

**Why learn this?**

You can solve systems of equations to help select the best value among high-speed Internet providers. (See Example 3.)

**Solving Systems of Equations by Substitution**

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Solve for one variable in at least one equation, if necessary.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2</td>
<td>Substitute the resulting expression into the other equation.</td>
</tr>
<tr>
<td>Step 3</td>
<td>Solve that equation to get the value of the first variable.</td>
</tr>
<tr>
<td>Step 4</td>
<td>Substitute that value into one of the original equations and solve for the other variable.</td>
</tr>
<tr>
<td>Step 5</td>
<td>Write the values from Steps 3 and 4 as an ordered pair, ((x, y)), and check.</td>
</tr>
</tbody>
</table>

**Example 1**

**Solving a System of Linear Equations by Substitution**

Solve each system by substitution.

**A**

\[
\begin{align*}
y &= 2x \\
y &= x + 5
\end{align*}
\]

**Step 1**

Both equations are solved for \(y\).

\[
y = x + 5
\]

**Step 2**

Substitute 2\(x\) for \(y\) in the second equation.

\[
2x = x + 5
\]

**Step 3**

Now solve this equation for \(x\). Subtract \(x\) from both sides to combine like terms.

\[
x = 5
\]

**Step 4**

Write one of the original equations.

\[
y = 2x
\]

Write a number for \(x\).

\[
y = 2(5)
\]

Substitute 5 for \(x\).

\[
y = 10
\]

**Step 5**

Write the solution as an ordered pair.

\[(5, 10)\]

**Check**

Substitute \((5, 10)\) into both equations in the system.

\[
\begin{array}{c|c|c}
| y &= 2x | & y &= x + 5 | \\
| 10 & 2(5) | & 10 & 5 + 5 | \\
| 10 & 10 & | 10 & 10 & \checkmark |
\end{array}
\]
Solve each system by substitution.

B \[
\begin{align*}
2x + y &= 5 \\
y &= x - 4
\end{align*}
\]

Step 1 \[y = x - 4\]  \hspace{1cm} \text{The second equation is solved for } y.

Step 2 \[2x + y = 5\]  \hspace{1cm} \text{Write the first equation.}

\[2x + (x - 4) = 5\]  \hspace{1cm} \text{Substitute } x - 4 \text{ for } y \text{ in the first equation.}

Step 3 \[3x - 4 = 5\]  \hspace{1cm} \text{Simplify. Then solve for } x.

\[3x + 4 = 9\]  \hspace{1cm} \text{Add 4 to both sides.}

\[\frac{3x}{3} = \frac{9}{3}\]  \hspace{1cm} \text{Divide both sides by 3.}

\[x = 3\]

Step 4 \[y = x - 4\]  \hspace{1cm} \text{Write one of the original equations.}

\[y = 3 - 4\]  \hspace{1cm} \text{Substitute 3 for } x.

\[y = -1\]

Step 5 \(3, -1\)  \hspace{1cm} \text{Write the solution as an ordered pair.}

C \[
\begin{align*}
x + 4y &= 6 \\
x + y &= 3
\end{align*}
\]

Step 1 \[x + 4y = 6\]  \hspace{1cm} \text{Solve the first equation for } x \text{ by subtracting 4y from both sides.}

\[x = 6 - 4y\]

Step 2 \[x + y = 3\]  \hspace{1cm} \text{Substitute } 6 - 4y \text{ for } x \text{ in the second equation.}

Step 3 \[6 - 3y = 3\]  \hspace{1cm} \text{Simplify. Then solve for } y.

\[-3y = -3\]  \hspace{1cm} \text{Subtract 6 from both sides.}

\[-3y = -3\]  \hspace{1cm} \text{Divide both sides by } -3.

\[y = 1\]

Step 4 \[x + y = 3\]  \hspace{1cm} \text{Write one of the original equations.}

\[x + 1 = 3\]  \hspace{1cm} \text{Substitute 1 for } y.

\[x = 2\]

Step 5 \(2, 1\)  \hspace{1cm} \text{Write the solution as an ordered pair.}

Helpful Hint

Sometimes neither equation is solved for a variable. You can begin by solving either equation for either \(x\) or \(y\).

Sometimes you substitute an expression for a variable that has a coefficient. When solving for the second variable in this situation, you can use the Distributive Property.
Using the Distributive Property

Solve \[\begin{align*}
4y - 5x &= 9 \\
x - 4y &= 11
\end{align*}\] by substitution.

Step 1 \[x - 4y = 11 \quad + 4y + 4y \quad x = 4y + 11\]

Step 2 \[4y - 5x = 9 \quad \text{Substitute } 4y + 11 \text{ for } x \text{ in the first equation.}\]

Step 3 \[4y - 5(4y + 11) = 9 \quad \text{Distribute } -5 \text{ to the expression in the parentheses. Simplify. Solve for } y.\]

\[4y - 20y - 55 = 9 \quad + 55 + 55 \quad -16y = 64 \quad \frac{-16y}{-16} = \frac{64}{-16} \quad y = -4\]

Step 4 \[x - 4y = 11 \quad \text{Write one of the original equations.}\]

\[x - 4(-4) = 11 \quad \text{Substitute } -4 \text{ for } y.\]

\[x + 16 = 11 \quad \text{Simplify.}\]

\[x = -5 \quad \text{Subtract 16 from both sides.}\]

Step 5 \((-5, -4) \quad \text{Write the solution as an ordered pair.}\)

2. Solve \[\begin{align*}
-2x + y &= 8 \\
3x + 2y &= 9
\end{align*}\] by substitution. Check your answer.

---

**EXAMPLE 2**

When you solve one equation for a variable, you must substitute the value or expression into the other original equation, not the one that has just been solved.

**Caution!**

**Solving Systems by Substitution**

I always look for a variable with a coefficient of 1 or \(-1\) when deciding which equation to solve for \(x\) or \(y\).

For the system

\[\begin{align*}
2x + y &= 14 \\
-3x + 4y &= -10
\end{align*}\]

I would solve the first equation for \(y\) because it has a coefficient of 1.

\[2x + y = 14 \quad y = -2x + 14\]

Then I use substitution to find the values of \(x\) and \(y\).

\[-3x + 4y = -10 \quad -3x + 4(-2x + 14) = -10 \quad -3x + (-8x) + 56 = -10 \quad -11x + 56 = -10 \quad -11x = -66 \quad x = 6\]

\[y = -2x + 14 \quad y = -2(6) + 14 = 2\]

The solution is \((6, 2)\).
**Consumer Economics Application**

One high-speed Internet provider has a $50 setup fee and costs $30 per month. Another provider has no setup fee and costs $40 per month.

a. In how many months will both providers cost the same? What will that cost be?

Write an equation for each option. Let \( t \) represent the total amount paid and \( m \) represent the number of months.

<table>
<thead>
<tr>
<th>Total paid</th>
<th>is setup fee plus cost per month times months.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Option 1</td>
<td>( t = 50 + 30m )</td>
</tr>
<tr>
<td>Option 2</td>
<td>( t = 0 + 40m )</td>
</tr>
</tbody>
</table>

Step 1 \( t = 50 + 30m \)  
Both equations are solved for \( t \).

Step 2 \( 50 + 30m = 40m \)  
Substitute 50 + 30m for \( t \) in the second equation.

Step 3 \( -30m = 10m \)  
Solve for \( m \). Subtract 30m from both sides to combine like terms.

Step 4 \( 50 = 10m \)  
Divide both sides by 10.

Step 5 \( 5, 200 \)  
Write one of the original equations. Substitute 5 for \( m \).

In 5 months, the total cost for each option will be the same—$200.

b. If you plan to cancel in 1 year, which is the cheaper provider? Explain.

Option 1: \( t = 50 + 30(12) = 410 \)  
Option 2: \( t = 40(12) = 480 \)

Option 1 is cheaper.

---

**Check it Out!**

3. One cable television provider has a $60 setup fee and $80 per month, and the second has a $160 equipment fee and $70 per month.

a. In how many months will the cost be the same? What will that cost be?

b. If you plan to move in 6 months, which is the cheaper option? Explain.

---

**THINK AND DISCUSS**

1. If you graphed the equations in Example 1A, where would the lines intersect?

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, solve the system by substitution using the first step given. Show that each method gives the same solution.
Solve each system by substitution. Check your answer.

1. \[ \begin{align*}
    y &= 5x - 10 \\
    y &= 3x + 8
\end{align*} \]

2. \[ \begin{align*}
    3x + y &= 2 \\
    4x + y &= 20
\end{align*} \]

3. \[ \begin{align*}
    y &= x + 5 \\
    4x + y &= 20
\end{align*} \]

4. \[ \begin{align*}
    x - 2y &= 10 \\
    \frac{1}{2}x - 2y &= 4
\end{align*} \]

5. \[ \begin{align*}
    y - 2x &= 3 \\
    2x - 3y &= 21
\end{align*} \]

6. \[ \begin{align*}
    x &= y - 8 \\
    -x - y &= 0
\end{align*} \]

7. **Consumer Economics** The Strauss family is deciding between two lawn-care services. Green Lawn charges a $49 startup fee, plus $29 per month. Grass Team charges a $25 startup fee, plus $37 per month.
   a. In how many months will both lawn-care services cost the same? What will that cost be?
   b. If the family will use the service for only 6 months, which is the better option? Explain.

8. \[ \begin{align*}
    y &= x + 3 \\
    y &= 2x + 4
\end{align*} \]

9. \[ \begin{align*}
    y &= 2x + 10 \\
    y &= -2x - 6
\end{align*} \]

10. \[ \begin{align*}
    x + 2y &= 8 \\
        x + 3y &= 12
\end{align*} \]

11. \[ \begin{align*}
    2x + 2y &= 2 \\
    -4x + 4y &= 12
\end{align*} \]

12. \[ \begin{align*}
    y &= 0.5x + 2 \\
    -y &= -2x + 4
\end{align*} \]

13. \[ \begin{align*}
    -x + y &= 4 \\
    3x - 2y &= -7
\end{align*} \]

14. \[ \begin{align*}
    3x + y &= -8 \\
    -2x - y &= 6
\end{align*} \]

15. \[ \begin{align*}
    x + 2y &= -1 \\
    4x - 4y &= 20
\end{align*} \]

16. \[ \begin{align*}
    4x &= y - 1 \\
    6x - 2y &= -3
\end{align*} \]

17. **Recreation** Casey wants to buy a gym membership. One gym has a $150 joining fee and costs $35 per month. Another gym has no joining fee and costs $60 per month.
   a. In how many months will both gym memberships cost the same? What will that cost be?
   b. If Casey plans to cancel in 5 months, which is the better option for him? Explain.

18. \[ \begin{align*}
    x &= 5 \\
    x + y &= 8
\end{align*} \]

19. \[ \begin{align*}
    y &= -3x + 4 \\
    x &= 2y + 6
\end{align*} \]

20. \[ \begin{align*}
    3x - y &= 11 \\
    5y - 7x &= 1
\end{align*} \]

21. \[ \begin{align*}
    \frac{1}{2}x + \frac{1}{3}y &= 6 \\
    x - y &= 2
\end{align*} \]

22. \[ \begin{align*}
    x &= 7 - 2y \\
    2x + y &= 5
\end{align*} \]

23. \[ \begin{align*}
    y &= 1.2x - 4 \\
    2.2x + 5 &= y
\end{align*} \]

24. Justin and Lacee are taking a walk. Justin walks at a rate of 6 ft/s, while Lacee walks at 4 ft/s. Lacee starts 10 ft ahead of Justin.
   a. After how many seconds will Lacee and Justin be next to each other? What distance will they have walked?
   b. How many seconds will it take for Justin to catch up to Lacee if she starts 32 ft ahead of Justin?
25. Ian and Jessica each save their quarters. Ian starts out with 34 quarters and saves 8 quarters a month. Jessica starts out with 2 quarters and saves 16 quarters a month. In how many months will Jessica have the same number of quarters as Ian? How many quarters will each of them have?

26. **Multi-Step** Use the receipts below to write and solve a system of equations to find the cost of a large popcorn and the cost of a small drink.

27. **Finance** Helene invested a total of $1000 in two simple-interest bank accounts. One account paid 5% annual interest; the other paid 6% annual interest. The total amount of interest she earned after one year was $58. Write and solve a system of equations to find the amount invested in each account. *(Hint: Change the interest rates into decimals first.)*

28. \[
\begin{align*}
  x + y &= 90 \\
  y &= 4x - 10
\end{align*}
\]

29. \[
\begin{align*}
  x &= 2y \\
  x + y &= 90
\end{align*}
\]

30. \[
\begin{align*}
  y &= 2(x - 15) \\
  x + y &= 90
\end{align*}
\]

31. \[
\begin{align*}
  x + y &= 90 \\
  y &= 2x + 3
\end{align*}
\]

32. Tricia and Michael share a cell phone plan. Together, they made a total of 52 calls last month for a total of 620 min. Tricia averaged 15 min for each of her calls, while Michael averaged 10 min.
   a. How many calls did Tricia make last month? Michael?
   b. How many calls did Tricia make if the total number of calls was 60?

33. **Write About It** Explain how to solve a system of equations by substitution.

34. **Critical Thinking** Explain the connection between the solution of a system solved by graphing and the solution to the same system solved by substitution.

35. This problem will prepare you for the Concept Connection on page 362.
   At the school store, Juanita bought 2 books and a backpack for a total of $26 before tax. Each book cost $8 less than the backpack.
   a. Write a system of equations that can be used to find the price of each book and the price of the backpack.
   b. Solve this system by substitution.
   c. Solve this system by graphing. Discuss advantages and disadvantages of solving by substitution and solving by graphing.
36. **Estimation** Use the graph to estimate the solution to
\[
\begin{align*}
2x - y &= 6 \\
x + y &= -0.6
\end{align*}
\]
Round your answer to the nearest tenth.
Then solve the system by substitution.

**Multiple Choice** For Exercises 37 and 38, choose the best answer.

37. Elizabeth met 24 of her cousins at a family reunion. The number of male cousins \(m\) was 6 less than twice the number of female cousins \(f\). Which system can be used to find the number of male cousins and female cousins?

A) \[
\begin{align*}
m + f &= 24 \\
f &= 2m - 6
\end{align*}
\]

B) \[
\begin{align*}
m + f &= 24 \\
f &= m + 6
\end{align*}
\]

C) \[
\begin{align*}
m = 24 + f \\
m = f - 6
\end{align*}
\]

D) \[
\begin{align*}
f &= 24 - m \\
m &= 2f - 6
\end{align*}
\]

38. Which problem is best represented by the system
\[
\begin{align*}
d &= n + 5 \\
d + n &= 12
\end{align*}
\]

A) Roger has 12 coins in dimes and nickels. There are 5 more dimes than nickels.

B) Roger has 5 coins in dimes and nickels. There are 12 more dimes than nickels.

C) Roger has 12 coins in dimes and nickels. There are 5 more nickels than dimes.

D) Roger has 5 coins in dimes and nickels. There are 12 more nickels than dimes.

**CHALLENGE AND EXTEND**

39. A car dealership has 378 cars on its lot. The ratio of new cars to used cars is 5:4. Write and solve a system of equations to find the number of new and used cars on the lot.

Solve each system by substitution. Check your answer.

40. \[
\begin{align*}
2r - 3s - t &= 12 \\
s + 3t &= 10 \\
t &= 4
\end{align*}
\]

41. \[
\begin{align*}
x + y + z &= 7 \\
y + z &= 5 \\
2y - 4z &= -14
\end{align*}
\]

42. \[
\begin{align*}
a + 2b + c &= 19 \\
b + c &= -5 \\
3b + 2c &= 15
\end{align*}
\]

**SPIRAL STANDARDS REVIEW**

Without graphing, tell whether each point is on the graph of \(y = 3x - 6x - 9\). (Lesson 5-1)

43. \((0, -9)\)  

44. \((3, 0)\)  

45. \(-\frac{1}{3}, -8\)  

Find the \(x\)- and \(y\)-intercepts. (Lesson 5-2)

46. \(6x - 2y = 12\)  

47. \(-3y + x = 15\)  

48. \(4y - 40 = -5x\)

Tell whether each ordered pair is a solution of the given system. (Lesson 6-1)

49. \(3, 0); \begin{align*}
2x - y &= -6 \\
x + y &= 3
\end{align*}\)

50. \((-1, 4); \begin{align*}
y - 2x &= 6 \\
x + 4y &= 15
\end{align*}\)

51. \((5, 6); \begin{align*}
\frac{1}{3}y + x &= 7 \\
2x &= 12
\end{align*}\)
Solving Systems by Elimination

Why learn this?
You can solve a system of linear equations to determine how many flowers of each type you can buy to make a bouquet. (See Example 4.)

Another method for solving systems of equations is elimination. Like substitution, the goal of elimination is to get one equation that has only one variable.

Remember that an equation stays balanced if you add equal amounts to both sides. Consider the system

\[
\begin{align*}
  x - 2y &= -19 \\
  5x + 2y &= 1
\end{align*}
\]

Since \(5x + 2y = 1\), you can add \(5x + 2y\) to one side of the first equation and 1 to the other side and the balance is maintained.

\[
\begin{align*}
  x - 2y + 5x + 2y &= -19 + 1 \\
  6x &= -18
\end{align*}
\]

Since \(-2y\) and \(2y\) have opposite coefficients, you can eliminate the \(y\)-term by adding the two equations. The result is one equation that has only one variable: \(6x = -18\).

Solving Systems of Equations by Elimination

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Write the system so that like terms are aligned.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2</td>
<td>Eliminate one of the variables.</td>
</tr>
<tr>
<td>Step 3</td>
<td>Solve for the variable not eliminated in Step 2.</td>
</tr>
<tr>
<td>Step 4</td>
<td>Substitute the value of the variable into one of the original equations and solve for the other variable.</td>
</tr>
<tr>
<td>Step 5</td>
<td>Write the answers from Steps 3 and 4 as an ordered pair, ((x, y)), and check your answer.</td>
</tr>
</tbody>
</table>

Later in this lesson you will learn how to multiply one or more equations by a number in order to produce opposites that can be eliminated.
**Example 1**

Elimination Using Addition

Solve \( \begin{cases} x - 2y = -19 \\ 5x + 2y = 1 \end{cases} \) by elimination.

**Step 1** \( x - 2y = -19 \) \( \text{Write the system so that like terms are aligned.} \)

**Step 2** \( + \) \( 5x + 2y = 1 \)

\[
\begin{array}{c|c|c|c}
& x & - & 2y & = & -19 \\
\hline
-3 & - & 2 & (8) & = & -19 \\
-3 & - & 16 & = & -19 \\
\hline
5x & + & 2y & = & 1 \\
\hline
5(-3) & + & 2(8) & = & 1 \\
-15 & + & 16 & = & 1 \checkmark
\end{array}
\]

**Step 3** \( 6x = -18 \) \( \text{Add the equations to eliminate the y-terms.} \)

\[
\begin{array}{c|c|c|c|c|c}
& 6x & + & 0 & = & -18 \\
\hline
6 & = & 6 \\
\hline
x & = & -3
\end{array}
\]

**Step 4** \( x - 2y = -19 \) \( \text{Write one of the original equations.} \)

\[
\begin{array}{c|c|c|c|c|c}
-3 & - & 2y & = & -19 \\
- & 2y & = & -16 \\
\hline
-2 & = & -16 \\
y & = & 8
\end{array}
\]

**Step 5** \( (-3, 8) \) \( \text{Write the solution as an ordered pair.} \)

**Check It Out!**

1. Solve \( \begin{cases} y + 3x = -2 \\ 2y - 3x = 14 \end{cases} \) by elimination. Check your answer.

When two equations each contain the same term, you can subtract one equation from the other to solve the system. To subtract an equation, add the opposite of each term.

**Example 2**

Elimination Using Subtraction

Solve \( \begin{cases} 3x + 4y = 18 \\ -2x + 4y = 8 \end{cases} \) by elimination.

**Step 1** \( \frac{3x + 4y = 18}{-2x + 4y = 8} \) \( \text{Add the opposite of each term in the second equation.} \)

**Step 2** \( \frac{3x + 4y = 18}{+ 2x - 4y = -8} \) \( \text{Eliminate the y-term.} \)

\[
\begin{array}{c|c|c|c|c|c}
& 3x & + & 4y & = & 18 \\
\hline
-2 & + & 4 & y & = & 8 \\
\hline
5x & + & 0 & = & 10 \\
\hline
x & = & 2
\end{array}
\]

**Step 3** \( 5x = 10 \) \( \text{Simplify and solve for x.} \)

\[
\begin{array}{c|c|c|c|c|c}
& -2x & + & 4y & = & 8 \\
\hline
-2(2) & + & 4y & = & 8 \\
\hline
-4 & + & 4y & = & 8 \\
\hline
4y & = & 12 \\
y & = & 3
\end{array}
\]

**Step 4** \( -2x + 4y = 8 \) \( \text{Write one of the original equations.} \)

\[
\begin{array}{c|c|c|c|c|c}
-2 & + & 4y & = & 8 \\
\hline
\text{Add 4 to both sides.} \\
\text{Simplify and solve for y.}
\end{array}
\]

**Step 5** \( (2, 3) \) \( \text{Write the solution as an ordered pair.} \)
2. Solve \[
\begin{align*}
3x + 3y &= 15 \\
-2x + 3y &= -5
\end{align*}
\] by elimination. Check your answer.

In some cases, you will first need to multiply one or both of the equations by a number so that one variable has opposite coefficients.

**Example 3**

**Elimination Using Multiplication First**

Solve each system by elimination.

**A** \[
\begin{align*}
2x + y &= 3 \\
-x + 3y &= -12
\end{align*}
\]

Step 1 \[
\begin{align*}
2x + y &= 3
\end{align*}
\]

Step 2 \[
\begin{align*}
2(-x + 3y &= -12) \\
2x + y &= 3 \\
+(-2x + 6y &= -24) \\
\hline
7y &= -21
\end{align*}
\]

Step 3 \[
\begin{align*}
y &= -3
\end{align*}
\]

Step 4 \[
\begin{align*}
2x + y &= 3 \\
2x - 3 &= 3 \\
+3+3 \\
2x &= 6 \\
x &= 3
\end{align*}
\]

Step 5 \((3, -3)\) Write the solution as an ordered pair.

**B** \[
\begin{align*}
7x - 12y &= -22 \\
5x - 8y &= -14
\end{align*}
\]

Step 1 \[
\begin{align*}
2(7x - 12y &= -22)
\end{align*}
\]

Step 2 \[
\begin{align*}
(-3)(5x - 8y &= -14) \\
14x - 24y &= -44 \\
+(-15x + 24y &= 42) \\
\hline
-x + 0 &= -2
\end{align*}
\]

Step 3 \[
\begin{align*}
x &= 2
\end{align*}
\]

Step 4 \[
\begin{align*}
7x - 12y &= -22 \\
7(2) - 12y &= -22 \\
14 - 12y &= -22 \\
-14-14 \\
-12y &= -36 \\
y &= 3
\end{align*}
\]

Step 5 \((2, 3)\) Write the solution as an ordered pair.

Solve each system by elimination. Check your answer.

3a. \[
\begin{align*}
3x + 2y &= 6 \\
-x + y &= -2
\end{align*}
\]

3b. \[
\begin{align*}
2x + 5y &= 26 \\
-3x - 4y &= -25
\end{align*}
\]
Consumer Economics Application

Sam spent $24.75 to buy 12 flowers for his mother. The bouquet contained roses and daisies. How many of each type of flower did Sam buy?

Write a system. Use \( r \) for the number of roses and \( d \) for the number of daisies.

**Step 1**

2.50\( r \) + 1.75\( d \) = 24.75  \( \text{The cost of roses and daisies totals $24.75.} \)

\( r + d = 12 \)  \( \text{The total number of roses and daisies is 12.} \)

**Step 2**

\[
egin{align*}
2.50r + 1.75d &= 24.75 \\
(-2.50)(r + d &= 12) \\
2.50r + 1.75d &= 24.75 \\
+ (-2.50r - 2.50d &= -30.00) \\
0 &= -5.25 \\
\end{align*}
\]

Multiply the second equation by \(-2.50\) to get opposite \( r \)-coefficients.

Add this equation to the first equation to eliminate the \( r \)-term.

**Step 3**

\[
\begin{align*}
d &= 7 \\
\end{align*}
\]

Simplify and solve for \( d \).

**Step 4**

\[
\begin{align*}
r + d &= 12 \quad \text{Write one of the original equations.} \\
r + 7 &= 12 \quad \text{Substitute 7 for \( d \).} \\
7 &= 5 \quad \text{Subtract 7 from both sides.} \\
r &= 5 \\
\end{align*}
\]

**Step 5**

\((5, 7)\)  \( \text{Write the solution as an ordered pair.} \)

Sam can buy 5 roses and 7 daisies.

4. **What if...?** Sally spent $14.85 to buy 13 flowers. She bought lilies, which cost $1.25 each, and tulips, which cost $0.90 each. How many of each flower did Sally buy?

All systems can be solved in more than one way. For some systems, some methods may be more appropriate than others.
THINK AND DISCUSS

1. Explain how multiplying the second equation in a system by \(-1\) and eliminating by adding is the same as elimination by subtraction. Give an example of a system for which this applies.

2. Explain why it does not matter which variable you solve for first when solving a system by elimination.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of a system of equations that you could solve using the given method.

GUIDED PRACTICE

Solve each system by elimination. Check your answer.

1. \[
\begin{align*}
-x + y &= 5 \\
x - 5y &= -9
\end{align*}
\]

2. \[
\begin{align*}
x + y &= 12 \\
x - y &= 2
\end{align*}
\]

3. \[
\begin{align*}
2x + 5y &= -24 \\
x + 4y &= -8
\end{align*}
\]

4. \[
\begin{align*}
x - 10y &= 60 \\
x + 14y &= 12
\end{align*}
\]

5. \[
\begin{align*}
5x + y &= 0 \\
x + 2y &= 30
\end{align*}
\]

6. \[
\begin{align*}
-5x + 7y &= 11 \\
-5x + 3y &= 19
\end{align*}
\]

7. \[
\begin{align*}
2x + 3y &= 12 \\
5x - y &= 13
\end{align*}
\]

8. \[
\begin{align*}
-3x + 4y &= 12 \\
2x + y &= -8
\end{align*}
\]

9. \[
\begin{align*}
2x + 4y &= -4 \\
3x + 5y &= -3
\end{align*}
\]

10. **Consumer Economics** Each family in a neighborhood is contributing $20 worth of food to the neighborhood picnic. The Harlin family is bringing 12 packages of buns. The hamburger buns cost $2.00 per package. The hot-dog buns cost $1.50 per package. How many packages of each type of bun did they buy?

PRACTICE AND PROBLEM SOLVING

Solve each system by elimination. Check your answer.

11. \[
\begin{align*}
-x + y &= -1 \\
2x - y &= 0
\end{align*}
\]

12. \[
\begin{align*}
-2x + y &= -20 \\
2x + y &= 48
\end{align*}
\]

13. \[
\begin{align*}
3x - y &= -2 \\
-2x + y &= 3
\end{align*}
\]

14. \[
\begin{align*}
x - y &= 4 \\
x - 2y &= 10
\end{align*}
\]

15. \[
\begin{align*}
x + 2y &= 5 \\
3x + 2y &= 17
\end{align*}
\]

16. \[
\begin{align*}
3x - 2y &= -1 \\
3x - 4y &= 9
\end{align*}
\]

17. \[
\begin{align*}
x - y &= -3 \\
5x + 3y &= 1
\end{align*}
\]

18. \[
\begin{align*}
9x - 3y &= 3 \\
3x + 8y &= -17
\end{align*}
\]

19. \[
\begin{align*}
5x + 2y &= -1 \\
3x + 7y &= 11
\end{align*}
\]

20. **Multi-Step** Mrs. Gonzalez bought centerpieces to put on each table at a graduation party. She spent $31.50. There are 8 tables each requiring either a candle or vase. Candles cost $3 and vases cost $4.25. How many of each type did she buy?
21. **Geometry** The difference between the length and width of a rectangle is 2 units. The perimeter is 40 units. Write and solve a system of equations to determine the length and width of the rectangle. *(Hint: The perimeter of a rectangle is \(2\ell + 2w\).)*

22. **ERROR ANALYSIS** Which is incorrect? Explain the error.

23. A music school Terry is interested in is offering a special for new students. If Terry enrolls in 2 classes, he has to pay a fee in addition to the price of classes for a total of $18. If he decides to take 6 classes, the fee is subtracted from his total for a total of $38. Follow the steps below to find the cost of each class and the price of the fee.

<table>
<thead>
<tr>
<th>Classes</th>
<th>Fee</th>
<th>Total Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price for 2 classes</td>
<td>(2x + y)</td>
<td>$18</td>
</tr>
<tr>
<td>Price for 6 classes</td>
<td>(6x + y)</td>
<td>$38</td>
</tr>
</tbody>
</table>

a. Copy and complete the table.

b. Use the information in the table to write a system of equations.

c. Solve the system of equations to find the price of each class and the price of the fee that Terry has to pay.

**Critical Thinking** Solve each system. Which method did you use to solve each system? Explain.

24. \[\begin{align*}
\frac{1}{2}x - 5y &= 30 \\
\frac{1}{2}x + 7y &= 6
\end{align*}\]

25. \[\begin{align*}
-x + 2y &= 3 \\
4x - 5y &= -3
\end{align*}\]

26. \[\begin{align*}
3x - y &= 10 \\
2x - y &= 7
\end{align*}\]

27. \[\begin{align*}
3y + x &= 10 \\
x &= 4y + 2
\end{align*}\]

28. \[\begin{align*}
y &= -4x \\
y &= 2x + 3
\end{align*}\]

29. \[\begin{align*}
2x + 6y &= 12 \\
4x + 5y &= 15
\end{align*}\]

30. **Business** A local boys club sold 176 bags of mulch and made a total of $520. They did not sell any of the expensive cocoa mulch. Use the table to determine how many bags of each type of mulch they sold.

<table>
<thead>
<tr>
<th>Mulch Prices ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cocoa</td>
</tr>
<tr>
<td>Hardwood</td>
</tr>
<tr>
<td>Pine Bark</td>
</tr>
</tbody>
</table>

31. This problem will prepare you for the Concept Connection on page 362.

a. The school store is running a promotion on school supplies. Different supplies are placed on two shelves. You can purchase 3 items from shelf A and 2 from shelf B for $16. Or you can purchase 2 items from shelf A and 3 from shelf B for $14. Write a system of equations that can be used to find the individual prices for the supplies on shelf A and on shelf B.

b. Solve the system of equations by elimination. Check your answer.

c. If the supplies on shelf A are normally $6 each and the supplies on shelf B are normally $3 each, how much will you save on each package plan from part a?
32. **Write About It** Solve the system \[
\begin{align*}
3x + y &= 1 \\
2x + 4y &= -6
\end{align*}
\] Explain how you can check your solution algebraically and graphically.

**Multiple Choice** For Exercises 33 and 34, choose the best answer.

33. A math test has 25 problems. Some are worth 2 points, and some are worth 3 points. The test is worth 60 points total. Which system can be used to determine the number of 2-point problems and the number of 3-point problems on the test?

- **A** \[
\begin{align*}
x + y &= 25 \\
2x + 3y &= 60
\end{align*}
\]
- **B** \[
\begin{align*}
x + y &= 60 \\
2x + 3y &= 25
\end{align*}
\]
- **C** \[
\begin{align*}
x - y &= 25 \\
2x + 3y &= 60
\end{align*}
\]
- **D** \[
\begin{align*}
x - y &= 60 \\
2x - 3y &= 25
\end{align*}
\]

34. An electrician charges $15 plus $11 per hour. Another electrician charges $10 plus $15 per hour. For what amount of time will the cost be the same? What is that cost?

- **A** 1 hour; $25
- **B** \(1\frac{1}{2}\) hours; $30
- **C** \(1\frac{3}{4}\) hours; $32.50
- **D** \(1\frac{3}{4}\) hours; $32.50

35. **Short Response** Three hundred and fifty-eight tickets to the school basketball game on Friday were sold. Student tickets were $1.50, and nonstudent tickets were $3.25. The school made $752.25.

a. Write a system of linear equations that could be used to determine how many student and how many nonstudent tickets were sold. Define the variables you use.

b. Solve the system you wrote in part a. How many student and how many nonstudent tickets were sold?

**CHALLENGE AND EXTEND**

Solve each system by any method. Check your answer.

36. \[
\begin{align*}
x + 16 \frac{1}{2} &= -\frac{3}{4}y \\
y &= \frac{1}{2}x
\end{align*}
\]

37. \[
\begin{align*}
2x + y + z &= 17 \\
\frac{1}{2}z &= 5 \\
x - y &= 5
\end{align*}
\]

38. \[
\begin{align*}
x - 2y - z &= -1 \\
x + 2y + 4z &= -11 \\
2x + y + z &= 1
\end{align*}
\]

39. Three students participated in a fund-raiser for school. Each sold a combination of pens, notebooks, and bags. The first student sold 2 pens, 7 notebooks, and 3 bags for a total of $73. The second student sold 10 pens, 2 notebooks, and 2 bags for a total of $50. The third student sold 1 pen, 4 notebooks, and 5 bags for a total of $71. Find the price of each item.

**SPIRAL STANDARDS REVIEW**

Determine whether each relation defines a function. Write an equation if possible. *(Lesson 4-3)*

40. \[
\begin{array}{cccc}
x & 1 & 2 & 3 \\ y & 6 & 7 & 8 & 9
\end{array}
\]

41. \[
\begin{array}{cccc}
x & 1 & 2 & 3 & 4 \\ y & 3 & 6 & 9 & 12
\end{array}
\]

Write an equation in slope-intercept form for the line with the given slope that contains the given point. *(Lesson 5-6)*

43. slope = 2; (5, 1) 

44. slope = -4; (3, -1) 

45. slope = \(\frac{1}{2}\); (-2, 9)

Solve each system by substitution. Check your answer. *(Lesson 6-2)*

46. \[
\begin{align*}
y &= x - 1 \\
x + y &= 10
\end{align*}
\]

47. \[
\begin{align*}
x &= y - 5 \\
2x + 1 &= y
\end{align*}
\]

48. \[
\begin{align*}
y &= 2x - 1 \\
x - y &= 3
\end{align*}
\]
Chapter 6 Systems of Equations and Inequalities

6-4 Solving Special Systems

Why learn this?
Linear systems can be used to analyze business growth, such as comic book sales. (See Example 4.)

In Lesson 6-1, you saw that when two lines intersect at a point, there is exactly one solution to the system. Systems with at least one solution are consistent systems.

When the two lines in a system do not intersect, they are parallel lines. There are no ordered pairs that satisfy both equations, so there is no solution. A system that has no solution is an inconsistent system.

**EXAMPLE 1**

Systems with No Solution

Solve \( \begin{cases} y = x - 1 \\ -x + y = 2 \end{cases} \).

**Method 1** Compare slopes and \( y \)-intercepts.

\[
\begin{align*}
y &= x - 1 \\
-x + y &= 2
\end{align*}
\]

Write both equations in slope-intercept form. \( y = 1x - 1 \) The lines are parallel because they have the same slope and different \( y \)-intercepts.

These lines do not intersect so the system is an inconsistent system.

**Method 2** Solve the system algebraically. Use the substitution method because the first equation is solved for \( y \).

\[
-\frac{1}{2} + (x - 1) = 2 \quad \text{Substitute } x - 1 \text{ for } y \text{ in the second equation, and solve.}
\]

\[
-1 = 2x \quad \text{False statement. The equation has no solutions.}
\]

This system has no solution so it is an inconsistent system.

**Check** Graph the system to confirm that the lines are parallel.

1. Solve \( \begin{cases} y = -2x + 5 \\ 2x + y = 1 \end{cases} \).
If two linear equations in a system have the same graph, the graphs are coincident lines, or the same line. There are infinitely many solutions of the system because every point on the line represents a solution of both equations.

**Example 2**

**Systems with Infinitely Many Solutions**

Solve \[\begin{cases} y = 2x + 1 \\ 2x - y + 1 = 0 \end{cases}\]

Compare slopes and \( y \)-intercepts.

\[y = 2x + 1 \rightarrow y = 2x + 1\] \(2x - y + 1 = 0 \rightarrow y = 2x + 1\]

Write both equations in slope-intercept form. The lines have the same slope and the same \( y \)-intercept.

If this system were graphed, the graphs would be the same line. There are infinitely many solutions.

Every point on this line is a solution of the system.

2. Solve \[\begin{cases} y = x - 3 \\ x - y - 3 = 0 \end{cases}\]

Consistent systems can either be independent or dependent.

- An **independent system** has exactly one solution. The graph of an independent system consists of two intersecting lines.
- A **dependent system** has infinitely many solutions. The graph of a dependent system consists of two coincident lines.

**Classification of Systems of Linear Equations**

<table>
<thead>
<tr>
<th>CLASSIFICATION</th>
<th>CONSISTENT AND INDEPENDENT</th>
<th>CONSISTENT AND DEPENDENT</th>
<th>INCONSISTENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Solutions</td>
<td>Exactly one</td>
<td>Infinitely many</td>
<td>None</td>
</tr>
<tr>
<td>Description</td>
<td>Different slopes</td>
<td>Same slope, same ( y )-intercept</td>
<td>Same slope, different ( y )-intercepts</td>
</tr>
<tr>
<td>Graph</td>
<td>Intersecting lines</td>
<td>Same line</td>
<td>Parallel lines</td>
</tr>
</tbody>
</table>
**Example 3**

Classifying Systems of Linear Equations

Classify each system. Give the number of solutions.

**A**

\[
\begin{align*}
2y &= x + 2 \\
\frac{1}{2}x + y &= 1
\end{align*}
\]

Write both equations in slope-intercept form.

\[2y = x + 2 \rightarrow y = \frac{1}{2}x + 1\]

\[-\frac{1}{2}x + y = 1 \rightarrow y = \frac{1}{2}x + 1\]

The system is consistent and dependent. It has infinitely many solutions.

**B**

\[
\begin{align*}
y &= 2(x - 1) \\
y &= x + 1
\end{align*}
\]

Write both equations in slope-intercept form.

\[y = 2(x - 1) \rightarrow y = 2x - 2\]

\[y = x + 1 \rightarrow y = 1x + 1\]

The system is consistent and independent. It has one solution.

**Example 4**

Business Application

The sales manager at Comics Now is comparing its sales with the sales of its competitor, Dynamo Comics. If the sales patterns continue, will the sales for Comics Now ever equal the sales for Dynamo Comics? Explain.

Use the table to write a system of linear equations. Let \(y\) represent the sales total and \(x\) represent the increase in sales.

<table>
<thead>
<tr>
<th></th>
<th>Sales total</th>
<th>equals</th>
<th>increase in sales per year</th>
<th>times</th>
<th>years</th>
<th>plus</th>
<th>beginning sales</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Comics Now</strong></td>
<td>(y)</td>
<td>40</td>
<td>(x)</td>
<td></td>
<td></td>
<td></td>
<td>130</td>
</tr>
<tr>
<td><strong>Dynamo Comics</strong></td>
<td>(y)</td>
<td>40</td>
<td>(x)</td>
<td></td>
<td></td>
<td></td>
<td>180</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
y &= 40x + 130 \\
y &= 40x + 180
\end{align*}
\]

Both equations are in slope-intercept form.

\[y = 40x + 130\]

\[y = 40x + 180\]

The graphs of the two equations are parallel lines, so there is no solution. If the patterns continue, sales for the two companies will never be equal.

4. Matt has $100 in a checking account and deposits $20 per month. Ben has $80 in a checking account and deposits $30 per month. Will the accounts ever have the same balance? Explain.
GUIDED PRACTICE

1. **Vocabulary** A ________ system can be independent or dependent. (consistent or inconsistent)

Solve each system of linear equations.

2. \[
\begin{align*}
y &= x + 1 \\
-x + y &= 3
\end{align*}
\]

3. \[
\begin{align*}
y &= 3x + y = 6 \\
y &= -3x + 2
\end{align*}
\]

4. \[
\begin{align*}
y &= -4x + 1 \\
4x + y &= 2
\end{align*}
\]

5. \[
\begin{align*}
y &= -x + 3 \\
x + y &= 3 = 0
\end{align*}
\]

6. \[
\begin{align*}
y &= 2x - 4 \\
2x - y &= -4
\end{align*}
\]

7. \[
\begin{align*}
y &= -7x + y = -2 \\
7x - y &= 2
\end{align*}
\]

Classify each system. Give the number of solutions.

8. \[
\begin{align*}
y &= 2(x + 3) \\
-2y &= 2x + 6
\end{align*}
\]

9. \[
\begin{align*}
y &= -3x - 1 \\
3x + y &= 1
\end{align*}
\]

10. \[
\begin{align*}
y &= 9y = 3x + 18 \\
1/3x - y &= -2
\end{align*}
\]

11. **Athletics** Micah walks on a treadmill at 4 miles per hour. He has walked 2 miles when Luke starts running at 6 miles per hour on the treadmill next to him. If their rates continue, will Luke’s distance ever equal Micah’s distance? Explain.

PRACTICE AND PROBLEM SOLVING

Solve each system of linear equations.

12. \[
\begin{align*}
y &= 2x - 2 \\
-2x + y &= 1
\end{align*}
\]

13. \[
\begin{align*}
x + y &= 3 \\
y &= -x - 1
\end{align*}
\]

14. \[
\begin{align*}
x + 2y &= -4 \\
y &= -1/2x - 4
\end{align*}
\]

15. \[
\begin{align*}
-6 + y &= 2x \\
y &= 2x - 36
\end{align*}
\]

16. \[
\begin{align*}
y &= -2x + 3 \\
2x + y &= -3 = 0
\end{align*}
\]

17. \[
\begin{align*}
y &= x - 2 \\
x - y &= 2 = 0
\end{align*}
\]

18. \[
\begin{align*}
x + y &= -4 \\
y &= -x - 4
\end{align*}
\]

19. \[
\begin{align*}
-9x - 3y &= -18 \\
3x + y &= 6
\end{align*}
\]
Classify each system. Give the number of solutions.

20. \[\begin{align*}
  y &= -x + 5 \\
  x + y &= 5
\end{align*}\]

21. \[\begin{align*}
  y &= -3x + 2 \\
  y &= 3x
\end{align*}\]

22. \[\begin{align*}
  y - 1 &= 2x \\
  y &= 2x - 1
\end{align*}\]

23. **Sports** Mandy is skating at 5 miles per hour. Nikki is skating at 6 miles per hour and started 1 mile behind Mandy. If their rates stay the same, will Mandy catch up with Nikki? Explain.

24. **Multi-Step** Photocopier A can print 35 copies per minute. Photocopier B can print 35 copies per minute. Copier B is started and makes 10 copies. Copier A is then started. If the copiers continue, will the number of copies from machine A ever equal the number of copies from machine B? Explain.

25. **Entertainment** One week Trey rented 4 DVDs and 2 video games for $18. The next week he rented 2 DVDs and 1 video game for $9. Find the rental costs for each video game and DVD. Explain your answer.

26. Rosa bought 1 pound of cashews and 2 pounds of peanuts for $10. At the same store, Sabrina bought 2 pounds of cashews and 1 pound of peanuts for $11. Find the cost per pound for cashews and peanuts.

27. **Geology** Pam and Tommy collect geodes. Pam’s parents gave her 2 geodes to start her collection, and she buys 4 every year. Tommy has 2 geodes that were given to him for his birthday the same year Pam started her collection. He buys 4 every year. If Pam and Tommy continue to buy the same amount of geodes per year, when will Tommy have as many geodes as Pam? Explain your answer.

28. Use the data given in the tables.

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>24</td>
<td>26</td>
<td>28</td>
<td>30</td>
</tr>
</tbody>
</table>

**a.** Write an equation to describe the data in each table.
**b.** Graph the system of equations from part **a.** Describe the graph.
**c.** How could you have predicted the graph by looking at the equations?
**d.** **What if…?** Each y-value in the second table increases by 1. How does this affect the graphs of the two equations? How can you tell how the graphs would be affected without actually graphing?

29. **Critical Thinking** Describe the graphs of two equations if the result of solving the system by substitution or elimination is the statement \(1 = 3\).

30. **Concept Connection** This problem will prepare you for the Concept Connection on page 362.
   The Crusader pep club is selling team buttons that support the sports teams. They contacted Buttons, Etc. which charges $50 plus $1.10 per button, and Logos, which charges $40 plus $1.10 per button.
   **a.** Write an equation for each company’s cost.
   **b.** Use the system from part **a** to find when the price for both companies is the same. Explain.
   **c.** What part of the equation should the pep club negotiate to change so that the cost of Buttons, Etc. is the same as Logos? What part of the equation should change in order to get a better price?
31. \(\text{ERROR ANALYSIS}\) Student A says there is no solution to the graphed system of equations. Student B says there is one solution. Which student is incorrect? Explain the error.

32. Write About It Compare the graph of a system that is consistent and independent with the graph of a system that is consistent and dependent.

**Multiple Choice** For Exercises 33 and 34, choose the best answer.

33. Which of the following classifications fit the following system?

\[
\begin{align*}
2x - y &= 3 \\
6x - 3y &= 9
\end{align*}
\]

- (A) Inconsistent and independent
- (B) Consistent and independent
- (C) Inconsistent and dependent
- (D) Consistent and dependent

34. Which of the following would be enough information to classify a system of two linear equations?

- (A) The graphs have the same slope.
- (B) The \(y\)-intercepts are the same.
- (C) The graphs have different slopes.
- (D) The \(y\)-intercepts are different.

**CHALLENGE AND EXTEND**

35. What conditions are necessary for the system \(\begin{align*} y &= 2x + p \\ y &= 2x + q \end{align*}\) to have infinitely many solutions? no solution?

36. Reasoning Solve the systems in parts a and b. Use this information to make a conjecture about all solutions that exist for the system in part c.

\[
\begin{align*}
a. \quad & \begin{cases} 3x + 4y = 0 \\ 4x + 3y = 0 \end{cases} \\
b. \quad & \begin{cases} 2x + 5y = 0 \\ 5x + 2y = 0 \end{cases} \\
c. \quad & \begin{cases} ax + by = 0 \\ bx + ay = 0 \end{cases}, \text{ for } a > 0, b > 0, a \neq b
\end{align*}
\]

**Spiral Standards Review**

Use the map to find the actual distances between each pair of cities. (Lesson 2-5)

37. from Hon to Averly
38. from Averly to Lewers

Solve each equation. (Lesson 2-7)

39. \(|x - 2.5| = 6\)
40. \(|4x + 6| = -7\)
41. \(|3z + 5| = 8\)

Solve each system by graphing. (Lesson 6-1)

42. \(\begin{align*} y &= x - 2 \\ y &= -x + 4 \end{align*}\)
43. \(\begin{align*} y &= 2x \\ x + y &= -6 \end{align*}\)
44. \(\begin{align*} y &= -\frac{1}{2}x \\ y - x &= 9 \end{align*}\)
Applying Systems

Who uses this?
Kayakers can calculate their rate of speed by solving a system of equations.

When a kayaker paddles downstream, the river’s current helps the kayaker move faster, so the speed of the current is added to the kayaker’s speed in still water to find the total speed. When a kayaker is going upstream, the speed of the current is subtracted from the kayaker’s speed in still water.

You can use these ideas and a system of equations to solve problems about rates of speed.

EXAMPLE
Solving Rate Problems
Ben paddles his kayak 8 miles upstream in 4 hours. He turns around and paddles downstream to his starting point in 2 hours. What is the rate at which Ben paddles in still water? What is the rate of the river’s current?

Let \( b \) be the rate at which Ben paddles in still water, and let \( c \) be the rate of the current.

Use a table to set up two equations—one for the upstream trip and one for the downstream trip.

<table>
<thead>
<tr>
<th>Rate</th>
<th>( \cdot )</th>
<th>Time</th>
<th>=</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upstream</td>
<td>( b - c )</td>
<td>4</td>
<td>=</td>
<td>8</td>
</tr>
<tr>
<td>Downstream</td>
<td>( b + c )</td>
<td>2</td>
<td>=</td>
<td>8</td>
</tr>
</tbody>
</table>

Solve the system \( \begin{cases} 4(b - c) = 8 \\ 2(b + c) = 8 \end{cases} \). First write the system as \( \begin{cases} 4b - 4c = 8 \\ 2b + 2c = 8 \end{cases} \), and then use elimination.

**Step 1**
\( 4b - 4c = 8 \)

**Step 2**
\[
\begin{align*}
2(2b + 2c &= 8) \\
4b - 4c &= 8 \\
+ (4b + 4c &= 16) \\
8b &= 24 \\
b &= 3
\end{align*}
\]
Multiply each term in the second equation by 2 to get opposite coefficients of \( c \).

Add the new equation to the first equation.

**Step 3**
\[
\begin{align*}
8b &= 24 \\
b &= 3
\end{align*}
\]
Simplify and solve for \( b \).

**Step 4**
\[
\begin{align*}
4b - 4c &= 8 \\
4(3) - 4c &= 8 \\
12 - 4c &= 8 \\
- 12 - 12 \\
- 4c &= -4 \\
c &= 1
\end{align*}
\]
Subtract 12 from both sides.

Simplify and solve for \( c \).

**Step 5**
\( (3, 1) \)
Write the solution as an ordered pair.

Ben paddles at 3 mi/h in still water. The rate of the current is 1 mi/h.
1. Ben paddles his kayak along a course on a different river. Going upstream, it takes him 6 hours to complete the course. Going downstream, it takes him 2 hours to complete the same course. What is the rate of the current, and how long is the course?

**Example 2: Solving Mixture Problems**

A pharmacist wants to mix an ointment that is 6% zinc oxide with an ointment that is 12% zinc oxide to make 30 grams of an ointment that is 10% zinc oxide. How many grams of each ointment should the pharmacist mix together?

Let $s$ be the number of grams of the 6% ointment, and let $t$ be the number of grams of the 12% ointment.

Use a table to set up two equations—one for the amount of ointment and one for the amount of zinc oxide in the ointment.

<table>
<thead>
<tr>
<th>Amount of Ointment (g)</th>
<th>6% Ointment</th>
<th>+</th>
<th>12% Ointment</th>
<th>=</th>
<th>10% Ointment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount of Ointment (g)</td>
<td>$s$</td>
<td>+</td>
<td>$t$</td>
<td>=</td>
<td>30</td>
</tr>
<tr>
<td>Amount of Zinc Oxide (g)</td>
<td>0.06$s$</td>
<td>+</td>
<td>0.12$t$</td>
<td>=</td>
<td>0.1(30) = 3</td>
</tr>
</tbody>
</table>

Solve the system $\begin{cases} s + t = 30 \\ 0.06s + 0.12t = 3 \end{cases}$. Use substitution.

- **Step 1** $s + t = 30$
  
  $$s = 30 - t$$

- **Step 2** $0.06s + 0.12t = 3$
  
  Substitute $30 - t$ for $s$ in the second equation.

  $$0.06(30 - t) + 0.12t = 3$$
  
  Distribute $0.06$ to the expression in parentheses.

  $$1.8 - 0.06t + 0.12t = 3$$

  Simplify. Solve for $t$.

  $$1.8 + 0.06t = 3$$

  Subtract $1.8$ from both sides.

  $$0.06t = 1.2$$

  Divide both sides by $0.06$.

  $$t = 20$$

- **Step 4** $s + t = 30$
  
  Write one of the original equations.

  $$s + 20 = 30$$

  Subtract $20$ from both sides.

  $$s = 10$$

- **Step 5** $(10, 20)$
  
  Write the solution as an ordered pair.

The pharmacist should use 10 grams of the 6% ointment and 20 grams of the 12% ointment.

2. Suppose the pharmacist wants to get the same result by mixing an ointment that is 9% zinc oxide with an ointment that is 15% zinc oxide. How many grams of each ointment should the pharmacist mix together?
**Example 3**

**Solving Number-Digit Problems**

The sum of the digits of a two-digit number is 7. When the digits are reversed, the new number is 45 less than the original number. What is the original number? Check your answer.

Let \( t \) represent the tens digit of the original number, and let \( u \) represent the units digit. Write the original number and the new number in expanded form.

**Original number:** \( 10t + u \)  
**New number:** \( 10u + t \)

Now set up two equations.

The sum of the digits in the original number is 7.

First equation: \( t + u = 7 \)

The new number is 45 less than the original number.

Second equation: \( 10u + t = (10t + u) - 45 \)

Simplify the second equation, so that the variables are only on the left side.

\[
10u + t = 10t + u - 45
\]

Subtract 10t from both sides.

\[
10u - 9t = u - 45
\]

Subtract \( u \) from both sides.

\[
9u - 9t = -45
\]

Divide both sides by 9.

\[
9u - 9t = \frac{-45}{9}
\]

\[
u - t = -5
\]

\[-t + u = -5
\]

Write the left side with the variable \( t \) first.

Now solve the system \[
\begin{cases}
t + u = 7 \\
-t + u = -5
\end{cases}
\]

Use elimination.

**Step 1** \( t + u = 7 \)

**Step 2** \( -t + u = -5 \)

**Step 3** \( 2u = 2 \)

Add the equations to eliminate the \( t \)-terms.

\[
\frac{2u}{2} = \frac{2}{2}
\]

\( u = 1 \)

**Step 4** \( t + u = 7 \)

Write one of the original equations.

\( t + 1 = 7 \)

Substitute 1 for \( u \).

\(-1 - 1 = 6 \)

Subtract 1 from both sides.

\( t = 6 \)

**Step 5** \((6, 1)\)

Write the solution as an ordered pair.

The original number is 61.

**Check**

Check the solution using the original problem.

The sum of the digits is \( 6 + 1 = 7 \). \( \checkmark \)

When the digits are reversed, the new number is 16, and \( 61 - 16 = 45 \). \( \checkmark \)

---

3. The sum of the digits of a two-digit number is 17. When the digits are reversed, the new number is 9 more than the original number. What is the original number? Check your answer.
GUIDED PRACTICE

1. Recreation It takes Cathy 1.5 hours to paddle her canoe 6 miles upstream. Then she turns her canoe around and paddles 6 miles downstream in 1 hour. What is the rate of the current? What is Cathy's paddling rate in still water?

2. Chemistry A chemist mixed a 15% glucose solution with a 35% glucose solution. This mixture produced 35 liters of a 19% glucose solution. How many liters of each solution did the chemist use in the mixture?

3. The sum of the digits of a two-digit number is 14. When the digits are reversed, the new number is 36 more than the original number. What is the original number? Check your answer.

PRACTICE AND PROBLEM SOLVING

4. Aviation With a tailwind, a jet flew 2000 miles in 4 hours. The jet's return trip against the same wind required 5 hours. Find the jet's speed and the wind speed.

5. Chemistry A 4% salt solution is mixed with a 16% salt solution. How many milliliters of each solution are needed to obtain 600 milliliters of a 10% salt solution?

6. The sum of the digits of a two-digit number is 10. If 18 is added to the number, the digits will be reversed. Find the number. Check your answer.

7. A coin bank contains 250 dimes and quarters worth a total of $39.25.
   a. Let \( q \) be the number of quarters, and let \( d \) be the number of dimes. Copy and complete the table.

<table>
<thead>
<tr>
<th>Number of Coins</th>
<th>Quarters</th>
<th>+</th>
<th>Dimes</th>
<th>=</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( q )</td>
<td></td>
<td></td>
<td></td>
<td>250</td>
</tr>
<tr>
<td>Value in Dollars</td>
<td>0.25( q )</td>
<td>+</td>
<td></td>
<td>=</td>
<td></td>
</tr>
</tbody>
</table>

   b. Use the information in the table to write a system of equations.
   c. Find the number of quarters and the number of dimes in the bank.
8. **Business** A grocery store sells a mixture of peanuts and raisins for $1.75 per pound. Peanuts cost $1.25 per pound, and raisins cost $2.75 per pound. Follow the steps below to find the amount of raisins and peanuts that go into one pound of the mixture.
   a. Let \( p \) be the amount of peanuts, and let \( r \) be the amount of raisins in one pound of the mixture. Copy and complete the table.

<table>
<thead>
<tr>
<th>Weight (lb)</th>
<th>Peanuts +</th>
<th>Raisins =</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>+</td>
<td>( r ) =</td>
<td></td>
</tr>
<tr>
<td>Cost ($)</td>
<td>1.25</td>
<td>+</td>
<td>1.75</td>
</tr>
</tbody>
</table>

   b. Use the information in the table to write a system of equations.
   c. Solve to find the amount of peanuts and raisins in the one-pound mixture.

9. A father is 32 years older than his daughter. In 4 years, the father will be 5 times as old as his daughter. Follow these steps to find their present ages.
   a. Let \( f \) be the father's present age, and let \( d \) be the daughter's present age.
      Write expressions that give the father's age and the daughter's age in 4 years.
   b. Write a system of equations based on the information in the problem.
   c. Solve the system to find the present age of the father and daughter.

10. **Multi-Step** The manager of a food store wants to create a blend of herbs that she can sell for $1 per ounce. She decides to make 8 ounces of a blend of oregano and sage. What will be the ratio of oregano to sage in the mixture?

11. The sum of the digits of a two-digit number is 13. Twice the first digit is 1 less than the second digit. What is the two-digit number?

12. **ERROR ANALYSIS** The sum of Anna's age and Mario's age is 30. Mario is 6 years older than Anna. Two students found Anna's age \( a \) as shown. Which solution is incorrect? Explain.

   \[
   \begin{align*}
   A: & \quad \begin{cases} a + m = 30 \\ a = m + 6 \end{cases} \\
   & \quad 2a = 36 \\
   & \quad a = 18 \\
   B: & \quad \begin{cases} a + m = 30 \\ a = m - 6 \end{cases} \\
   & \quad +(a - m = -6) \\
   & \quad 2a = 24 \\
   & \quad a = 12
   \end{align*}
   \]

13. This problem will prepare you for the Concept Connection on page 362.
A pep club is planning to sell adults’ T-shirts and children’s T-shirts as a fund-raiser. The club will order a total of 100 shirts. The club's president wants to raise $1100 from the sale of the shirts and proposes selling adults’ shirts for $13 each and children's shirts for $8 each.
   a. Write a system of equations that the president could use to determine the number of each type of shirt to order.
   b. How many adults’ shirts and children’s shirts should the club order?
   c. How many of each type of shirt should the club order to be able to raise $1200?
14. **Critical Thinking** A chemist wants to mix a 10% saline solution with a 15% saline solution to make 20 milliliters of an 18% saline solution. What happens when you try to solve a system of equations to determine the amount of each saline solution that the chemist should use? Why does this happen?

15. **Write About It** Write your own number-digit problem. Include a complete solution to the problem.

**Multiple Choice** For Exercises 16–18, choose the best answer.

16. With a tailwind, a plane makes a 3000-mile trip in 5 hours. On the return trip, the plane flies against the same wind and covers the 3000 miles in 6 hours. What is the speed of the wind?

(A) 40 mi/h  
(B) 50 mi/h  
(C) 100 mi/h  
(D) 550 mi/h

17. A jar contains quarters and dimes. There are 15 more quarters than dimes. The total value of the coins is $23. Which system of equations can be used to find the number of quarters $q$ and the number of dimes $d$?

$$\begin{align*}
\text{A} & : \begin{cases} q = d - 15 \\ 0.25q + 0.1d = 0.23 \end{cases} \\
\text{B} & : \begin{cases} q = d + 15 \\ 0.25q + 0.1d = 0.23 \end{cases}
\end{align*}$$

(C) $d = q + 15$  
(D) $q = d + 15$  
(D) $0.25q + 0.1d = 23$

18. Donnell wants to make a 2-pound mixture of cashews and pecans that costs $2.60 per pound. How many pounds of cashews should he use?

(A) 0.4 pound  
(B) 0.8 pound  
(C) 1.2 pounds  
(D) 1.6 pounds

<table>
<thead>
<tr>
<th>Item</th>
<th>Price per Pound ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cashews</td>
<td>2.50</td>
</tr>
<tr>
<td>Pecans</td>
<td>3.00</td>
</tr>
</tbody>
</table>

**CHALLENGE AND EXTEND**

19. In 15 years, Maya will be twice as old as David is now. In 15 years, David will be as old as Maya will be 10 years from now. How old are Maya and David now?

20. To train for a marathon, Mei runs an 18-mile course at a constant speed. If she doubles her usual speed, she can complete the course in an hour and a half less than her usual time. What is Mei's usual speed and her usual time to complete the course?

21. Write a word problem that can be solved by solving this system of equations.

$$\begin{align*}
a + b &= 20 \\
0.25a + 0.5b &= 6
\end{align*}$$

**SPIRAL STANDARDS REVIEW**

Use properties and operations to show that the first expression simplifies to the second expression. (Lesson 1-7)

22. $4(x - 1) + x, 5x - 4$  
23. $7a - 2(a + 1), 5a - 2$  
24. $4x + 5x + x^2 - 3x, 6x + x^2$

Solve each equation. Check your answer. (Lesson 2-7)

25. $2|x| + 5 = 11$  
26. $3 + 4|x| = 3$  
27. $|2x + 1| = 7$  
28. $12 = 3|x + 2|$

Solve each system of linear equations. Check your answer. (Lesson 6-4)

29. $\begin{cases} 2x - y = 1 \\ x = \frac{1}{2}y + 1 \end{cases}$  
30. $\begin{cases} -2y = x - 1 \\ x + 2y = 1 \end{cases}$  
31. $\begin{cases} x - y = 2 \\ 2x = 2y + 4 \end{cases}$  
32. $\begin{cases} x + y = 3 \\ x + 1 = -y \end{cases}$
Systems of Equations

We’ve Got Spirit  Some cheerleaders are going to sell spirit bracelets and foam fingers to raise money for traveling to away games.

1. Two companies, Spirit for You and Go Team, are interested in providing the foam fingers. The cheerleaders plan to sell 100 foam fingers. Based on this information, which company should they choose? Explain your reasoning.

<table>
<thead>
<tr>
<th>Company</th>
<th>Design fee</th>
<th>Cost per item</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spirit for You</td>
<td>$35</td>
<td>$2.50</td>
</tr>
<tr>
<td>Go Team</td>
<td>$20</td>
<td>$3.00</td>
</tr>
</tbody>
</table>

2. The cheerleaders sold foam fingers for $5 and spirit bracelets for $4. They sold 40 more foam fingers than bracelets, and they earned $965. Write a system of equations to describe this situation.

3. Solve this system using at least two different methods. Explain each method.

4. Using the company you chose in Problem 1, how much profit did the cheerleaders make from the foam fingers alone? (Hint: profit = amount earned – expenses)

5. What is the maximum price the cheerleaders could pay for each spirit bracelet in order to make a total profit of $500?
Quiz for Lessons 6-1 Through 6-5

**6-1 Solving Systems by Graphing**

Tell whether the ordered pair is a solution of the given system.

1. \((-2, 1); \begin{cases} y = -2x - 3 \\ y = x + 3 \end{cases}\)

2. \((9, 2); \begin{cases} x - 4y = 1 \\ 2x - 3y = 3 \end{cases}\)

3. \((3, -1); \begin{cases} y = -\frac{1}{3}x \\ y + 2x = 5 \end{cases}\)

Solve each system by graphing. Check your answer.

4. \(\begin{cases} y = x + 5 \\ y = \frac{1}{2}x + 4 \end{cases}\)

5. \(\begin{cases} y = -x - 2 \\ 2x - y = 2 \end{cases}\)

6. \(\begin{cases} \frac{2}{3}x + y = -3 \\ 4x + y = 7 \end{cases}\)

7. **Banking** Christiana and Marlena opened their first savings accounts on the same day. Christiana opened her account with $50 and plans to deposit $10 every month. Marlena opened her account with $30 and plans to deposit $15 every month. After how many months will their two accounts have the same amount of money? What will that amount be?

**6-2 Solving Systems by Substitution**

Solve each system by substitution. Check your answer.

8. \(\begin{cases} y = -x + 5 \\ 2x + y = 11 \end{cases}\)

9. \(\begin{cases} 4x - 3y = -1 \\ 3x - y = -2 \end{cases}\)

10. \(\begin{cases} y = -x \\ y = -2x - 5 \end{cases}\)

**6-3 Solving Systems by Elimination**

Solve each system by elimination. Check your answer.

11. \(\begin{cases} x + 3y = 15 \\ 2x - 3y = -6 \end{cases}\)

12. \(\begin{cases} x + y = 2 \\ 2x + y = -1 \end{cases}\)

13. \(\begin{cases} -2x + 5y = -1 \\ 3x + 2y = 11 \end{cases}\)

14. It takes Akira 10 minutes to make a black and white drawing and 25 minutes for a color drawing. On Saturday he made a total of 9 drawings in 2 hours. Write and solve a system of equations to determine how many drawings of each type Akira made.

**6-4 Solving Special Systems**

Classify each system. Give the number of solutions.

15. \(\begin{cases} 3x = -6y + 3 \\ 2y = -x + 1 \end{cases}\)

16. \(\begin{cases} y = -4x + 2 \\ 4x + y = -2 \end{cases}\)

17. \(\begin{cases} 4x - 3y = 8 \\ y = 4(x + 2) \end{cases}\)

**6-5 Applying Systems**

18. The sum of the digits of a two-digit number is 6. When the digits are reversed, the new number is 18 more than the original number. What is the original number? Check your answer.
Who uses this?

Consumers can use linear inequalities to determine how much food they can buy for an event. (See Example 3.)

A **linear inequality** is similar to a linear equation, but the equal sign is replaced with an inequality symbol. A **solution of a linear inequality** is any ordered pair that makes the inequality true.

### Example 1

**Identifying Solutions of Inequalities**

Tell whether the ordered pair is a solution of the inequality.

**A** \((7, 3)\); \(y < x - 1\)

\[
\begin{align*}
3 &< 7 - 1 \\
3 &< 6 \checkmark
\end{align*}
\]

\((7, 3)\) is a solution.

**B** \((4, 5)\); \(y > 3x + 2\)

\[
\begin{align*}
5 &> 3(4) + 2 \\
5 &> 12 + 2 \\
5 &> 14 \times
\end{align*}
\]

\((4, 5)\) is not a solution.

### Check It Out

Tell whether the ordered pair is a solution of the inequality.

1a. \((4, 5)\); \(y < x + 1\)  
1b. \((1, 1)\); \(y > x - 7\)

A linear inequality describes a region of a coordinate plane called a **half-plane**. All points in the region are solutions of the linear inequality. The boundary line of the region is the graph of the related equation.

When the inequality is written as \(y \leq \) or \(y \geq\), the points on the boundary line are solutions, and the line is **solid**.

When the inequality is written as \(y < \) or \(y >\), the points on the boundary line are not solutions, and the line is **dashed**.

When the inequality is written as \(y > \) or \(y \geq\), the points **above** the boundary line are also solutions.

When the inequality is written as \(y < \) or \(y \leq\), the points **below** the boundary line are also solutions.
Graphing Linear Inequalities

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Solve the inequality for ( y ) (slope-intercept form).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2</td>
<td>Graph the boundary line. Use a solid line for ( \leq ) or ( \geq ). Use a dashed line for ( &lt; ) or ( &gt; ).</td>
</tr>
<tr>
<td>Step 3</td>
<td>Shade the half-plane above the line for ( y &gt; ) or ( y \geq ). Shade the half-plane below the line for ( y &lt; ) or ( y \leq ). Check your answer.</td>
</tr>
</tbody>
</table>

**Example 2**

Graphing Linear Inequalities in Two Variables

Graph the solutions of each linear inequality. Check your answer.

**A** \( y < 3x + 4 \)

- **Step 1** The inequality is already solved for \( y \).
- **Step 2** Graph the boundary line \( y = 3x + 4 \).
  Use a dashed line for \( < \).
- **Step 3** The inequality is \( < \), so shade below the line.

**Check**

<table>
<thead>
<tr>
<th>( y )</th>
<th>( 3(0) + 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>0 + 4</td>
</tr>
<tr>
<td>0</td>
<td>&lt; 4 ✓</td>
</tr>
</tbody>
</table>

Substitute \((0, 0)\) for \((x, y)\) because it is not on the boundary line.
The point \((0, 0)\) satisfies the inequality, so the graph is shaded correctly.

**B** \( 3x + 2y \geq 6 \)

- **Step 1** Solve the inequality for \( y \).
  \[
  3x + 2y \geq 6 \\
  -3x \quad -3x \\
  2y \geq -3x + 6 \\
  y \geq -\frac{3}{2}x + 3
  \]
- **Step 2** Graph the boundary line \( y = -\frac{3}{2}x + 3 \).
  Use a solid line for \( \geq \).
- **Step 3** The inequality is \( \geq \), so shade above the line.

**Check**

<table>
<thead>
<tr>
<th>( y )</th>
<th>( -\frac{3}{2}(0) + 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>0 + 3</td>
</tr>
<tr>
<td>0</td>
<td>( \geq 3 ) x</td>
</tr>
</tbody>
</table>

A false statement means that the half-plane containing \((0, 0)\) should NOT be shaded. \((0, 0)\) is not one of the solutions, so the graph is shaded correctly.

Graph the solutions of each linear inequality. Check your answer.

2a. \( 4x - 3y > 12 \)  
2b. \( 2x - y - 4 > 0 \)  
2c. \( y \geq -\frac{2}{3}x + 1 \)
Consumer Economics Application

Sarah can spend at most $7.50 on vegetables. Broccoli costs $1.25 per bunch and carrots cost $0.75 per package.

a. Write a linear inequality to describe the situation.

Let \( x \) represent the number of bunches of broccoli and let \( y \) represent the number of packages of carrots.

Write an inequality. Use \( \leq \) for “at most.”

\[
\text{Cost of broccoli} + \text{cost of carrots} \leq 7.50
\]

Solve the inequality for \( y \).

\[
1.25x + 0.75y \leq 7.50
\]

\[
100(1.25x + 0.75y) \leq 100(7.50)
\]

\[
125x + 75y \leq 750
\]

\[
-125x -125x
\]

\[
75y \leq 750 - 125x
\]

\[
y \leq \frac{750 - 125x}{75}
\]

\[
y \leq 10 - \frac{5}{3}x
\]

b. Graph the solutions.

Step 1 Since Sarah cannot buy a negative amount of vegetables, the system is graphed only in Quadrant I. Graph the boundary line \( y = -\frac{5}{3}x + 10 \). Use a solid line for \( \leq \).

Step 2 Shade below the line. Sarah must buy whole numbers of bunches or packages. All the points on or below the line with whole number coordinates are the different combinations of broccoli and carrots that Sarah can buy.

Vegetable Combinations

3. What if...? Dirk is going to bring two types of olives to the Honor Society induction and can spend no more than $6. Green olives cost $2 per pound and black olives cost $2.50 per pound.

a. Write a linear inequality to describe the situation.

b. Graph the solutions.

c. Give two combinations of olives that Dirk could buy.
EXAMPLE 4 Writing an Inequality from a Graph

Write an inequality to represent each graph.

A. \( y \)-intercept: 2; slope: \(-\frac{1}{3}\)
Write an equation in slope-intercept form.
\[ y = mx + b \rightarrow y = -\frac{1}{3}x + 2 \]
The graph is shaded below a dashed boundary line.
Replace = with < to write the inequality \( y < -\frac{1}{3}x + 2 \).

B. \( y \)-intercept: -2; slope: 5
Write an equation in slope-intercept form.
\[ y = mx + b \rightarrow y = 5x + (-2) \]
The graph is shaded above a solid boundary line.
Replace = with \( \geq \) to write the inequality \( y \geq 5x - 2 \).

C. \( y \)-intercept: none; slope: undefined
The graph is a vertical line at \( x = -2 \).
The graph is shaded on the right side of a solid boundary line.
Replace = with \( \geq \) to write the inequality \( x \geq -2 \).

THINK AND DISCUSS
1. Tell how graphing a linear inequality is the same as graphing a linear equation. Tell how it is different.
2. Explain how you would write a linear inequality from a graph.
3. GET ORGANIZED Copy and complete the graphic organizer.

<table>
<thead>
<tr>
<th>Inequality</th>
<th>( y &lt; 5x + 2 )</th>
<th>( y &gt; 7x - 3 )</th>
<th>( y \leq 9x + 1 )</th>
<th>( y \geq -3x - 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>&lt;</td>
<td>&gt;</td>
<td>( \leq )</td>
<td>( \geq )</td>
</tr>
<tr>
<td>Boundary Line</td>
<td>Dashed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shading</td>
<td>Below</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
GUIDED PRACTICE

1. **Vocabulary** Can a solution of a linear inequality lie on a dashed boundary line? Explain.

SEE EXAMPLE 1

2. $(0, 3); y \leq -x + 3$
3. $(2, 0); y > -2x - 2$
4. $(-2, 1); y < 2x + 4$

SEE EXAMPLE 2

5. $y \leq -x$
6. $y > 3x + 1$
7. $-y < -x + 4$
8. $-y \geq x + 1$

SEE EXAMPLE 3

9. **Multi-Step** Jack is making punch with orange juice and pineapple juice. He can make at most 16 cups of punch.
   a. Write an inequality to describe the situation.
   b. Graph the solutions.
   c. Give two possible combinations of cups of orange juice and pineapple juice that Jack can use in his punch.

SEE EXAMPLE 4

10. Write an inequality to represent each graph.

11. Write an inequality to represent each graph.

PRACTICE AND PROBLEM SOLVING

Tell whether the ordered pair is a solution of the given inequality.

12. $(2, 3); y \geq 2x + 3$
13. $(1, -1); y < 3x - 3$
14. $(0, 7); y > 4x + 7$

Graph the solutions of each linear inequality. Check your answer.

15. $y > -2x + 6$
16. $-y \geq 2x$
17. $x + y \leq 2$
18. $x - y \geq 0$

19. **Multi-Step** Beverly is serving hamburgers and hot dogs at her cookout. Hamburger meat costs $3 per pound, and hot dogs cost $2 per pound. She wants to spend no more than $30.
   a. Write an inequality to describe the situation.
   b. Graph the solutions.
   c. Give two possible combinations of pounds of hamburger and hot dogs that Beverly can buy.

Write an inequality to represent each graph.
22. **Business** An electronics store makes $125 profit on every DVD player it sells and $100 on every CD player it sells. The store owner wants to make a profit of at least $500 a day selling DVD players and CD players.
   a. Write a linear inequality to determine the number of DVD players \( x \) and the number of CD players \( y \) that the owner needs to sell to meet his goal.
   b. Graph the linear inequality.
   c. Describe the possible values of \( x \). Describe the possible values of \( y \).
   d. List three possible combinations of DVD players and CD players that the owner could sell to meet his goal.

**Graph the solutions of each linear inequality. Check your answer.**

23. \( y \leq 2 - 3x \)  
24. \(-y < 7 + x \)  
25. \( 2x - y \leq 4 \)  
26. \( 3x - 2y > 6 \)

27. **Geometry** Marvin has 18 yards of fencing that he can use to put around a rectangular garden.
   a. Write a linear inequality that describes the possible lengths and widths of the garden.
   b. Graph the inequality and list three possible solutions to the problem.
   c. What are the dimensions of the largest square garden that can be fenced in with whole-number dimensions?

28. **Hobbies** Stephen wants to buy yellow tangs and clown fish for his saltwater aquarium. He wants to spend no more than $77 on fish. At the store, yellow tangs cost $15 each and clown fish cost $11 each. Write and graph a linear inequality to find the number of yellow tangs \( x \) and the number of clown fish \( y \) that Stephen could purchase. Name a solution of your inequality that is not reasonable for the situation. Explain.

**Graph each inequality on a coordinate plane.**

29. \( y > 1 \)  
30. \(-2 < x \)  
31. \( x \geq -3 \)  
32. \( y \leq 0 \)  
33. \( 0 \geq x \)  
34. \(-12 + y > 0 \)  
35. \( x + 7 < 7 \)  
36. \(-4 \geq x - y \)

37. **School** At a high school football game, tickets at the gate cost $7 per adult and $4 per student. Write a linear inequality to determine the number of adult and student tickets that need to be sold so that the amount of money taken in at the gate is at least $280. Graph the inequality and list three possible solutions.

38. **Critical Thinking** Why must a region of a coordinate plane be shaded to show all solutions of a linear inequality?

39. **Write About It** Give a real-world situation that can be described by a linear inequality. Then graph the inequality and give two solutions.

40. This problem will prepare you for the Concept Connection on page 378.
   Gloria is making teddy bears. She is making boy and girl bears. She has enough stuffing to create 50 bears. Let \( x \) represent the number of girl bears and \( y \) represent the number of boy bears.
   a. Write an inequality that shows the possible number of boy and girl bears Jenna can make.
   b. Graph the inequality.
   c. Give three possible solutions for the numbers of boy and girl bears that can be made.
41. **ERROR ANALYSIS** Student A wrote \( y < 2x - 1 \) as the inequality represented by the graph. Student B wrote \( y \leq 2x - 1 \) as the inequality represented by the graph. Which student is incorrect? Explain the error.

42. **Write About It** How do you decide to shade above or below an inequality? What does this shading represent?

**Multiple Choice** For Exercises 43–45, choose the best answer.

43. Which point is a solution of the inequality \( y > -x + 3 \)?
   - A \((0, 3)\)
   - B \((1, 4)\)
   - C \((-1, 4)\)
   - D \((0, -3)\)

44. Which inequality is represented by the graph at right?
   - A \(2x + y \geq 3\)
   - B \(2x + y > 3\)
   - C \(2x + y \leq 3\)
   - D \(2x + y < 3\)

45. Which of the following describes the graph of \(3 \leq x\)?
   - A The boundary line is dashed, and the shading is to the right.
   - B The boundary line is dashed, and the shading is to the left.
   - C The boundary line is solid, and the shading is to the right.
   - D The boundary line is solid, and the shading is to the left.

**CHALLENGE AND EXTEND**
Graph each inequality. Check your answer.

46. \(0 \geq -6 - 2x - 5y\)
47. \(y > |x|\)
48. \(y \geq |x - 3|\)

49. A linear inequality has the points \((0, 3)\) and \((-3, 1.5)\) as solutions on the boundary line. Also, the point \((1, 1)\) is not a solution. Write the linear inequality.

50. Two linear inequalities are graphed on the same coordinate plane. The point \((0, 0)\) is a solution of both inequalities. The entire coordinate plane is shaded except for Quadrant I. What are the two inequalities?

**Spiral Standards Review**

Graph each equation. Then tell whether the equation represents a function. (Lesson 4-3)

51. \(y = 2x - 4\)
52. \(y = x^2 + 2\)
53. \(y = 3\)

Write an equation in slope-intercept form for the line through the two points. (Lesson 5-6)

54. \((0, 9)\) and \((5, 2)\)
55. \((-5, -2)\) and \((7, 7)\)
56. \((0, 0)\) and \((-8, -10)\)
57. \((-1, -2)\) and \((1, 4)\)
58. \((2, 2)\) and \((6, 5)\)
59. \((-3, 2)\) and \((3, -1)\)

Solve each system by elimination. Check your answer. (Lesson 6-3)

60. \[
\begin{align*}
    x + 6y &= 14 \\
    x - 6y &= -10
\end{align*}
\]
61. \[
\begin{align*}
    x + y &= 13 \\
    3x + y &= 9
\end{align*}
\]
62. \[
\begin{align*}
    2x - 4y &= 18 \\
    5x - y &= 36
\end{align*}
\]
63. \[
\begin{align*}
    2y + x &= 12 \\
    y - 2x &= 1
\end{align*}
\]
64. \[
\begin{align*}
    2y - 6x &= -8 \\
    y &= -5x + 12
\end{align*}
\]
65. \[
\begin{align*}
    2x + 3y &= 33 \\
    y &= \frac{1}{3}x
\end{align*}
\]
**Who uses this?**

The owner of a surf shop can use systems of linear inequalities to determine how many surfboards and wakeboards need to be sold to make a certain profit. (See Example 4.)

A system of linear inequalities is a set of two or more linear inequalities containing two or more variables. The solutions of a system of linear inequalities consists of all the ordered pairs that satisfy all the linear inequalities in the system.

### Example 1

**Identifying Solutions of Systems of Linear Inequalities**

Tell whether the ordered pair is a solution of the given system.

**A** 
$(2, 1)$; \[
\begin{align*}
y &< -x + 4 \\
y &\leq x + 1
\end{align*}
\]

$(2, 1)$

\[
\begin{array}{c|c|c}
y & -x + 4 & 2 \\
1 & 2 & 4
\end{array}
\]

$1 < |2 + 4|$

$(2, 1)$ is a solution to the system because it satisfies both inequalities.

**B** 
$(2, 0)$; \[
\begin{align*}
y &\geq 2x \\
y &< x + 1
\end{align*}
\]

$(2, 0)$

\[
\begin{array}{c|c|c}
y & 2x & 2 \\
0 & 2 & (2)
\end{array}
\]

$0 < |4 - 0|$

$(2, 0)$ is not a solution to the system because it does not satisfy both inequalities.

**CHECK IT OUT!**

Tell whether the ordered pair is a solution of the given system.

**1a.** $(0, 1)$; \[
\begin{align*}
y &< -3x + 2 \\
y &\geq x - 1
\end{align*}
\]

**1b.** $(0, 0)$; \[
\begin{align*}
y &> -x + 1 \\
y &> x - 1
\end{align*}
\]

To show all the solutions of a system of linear inequalities, graph the solutions of each inequality. The solutions of the system are represented by the overlapping shaded regions. Below are graphs of Examples 1A and 1B.
EXAMPLE 2  
Solving a System of Linear Inequalities by Graphing

Graph the system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

\[
\begin{aligned}
8x + 4y &\leq 12 \\
y &> \frac{1}{2}x - 2
\end{aligned}
\]

Write the first inequality in slope-intercept form.

\[
\begin{aligned}
8x + 4y &\leq 12 \\
4y &\leq -8x + 12 \\
y &\leq -2x + 3
\end{aligned}
\]

\[
\begin{aligned}
\text{Graph the system.} \\
\begin{aligned}
y &\leq -2x + 3 \\
y &> \frac{1}{2}x - 2
\end{aligned}
\end{aligned}
\]

\[
\begin{aligned}
(-3, 4) \text{ satisfies both inequalities.} \\
(-1, 1) \text{ satisfies both inequalities.} \\
(2, -1) \text{ satisfies only } y \leq -2x + 3. \\
(2, -4) \text{ satisfies only } y \leq -2x + 3.
\end{aligned}
\]

\[
\begin{aligned}
(-1, 1) \text{ and } (-3, 4) \text{ are solutions.} \\
(2, -1) \text{ and } (2, -4) \text{ are not solutions.}
\end{aligned}
\]

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

2a. \[
\begin{aligned}
y &\leq x + 1 \\
y &> 2
\end{aligned}
\]

2b. \[
\begin{aligned}
y &> x - 7 \\
3x + 6y &\leq 12
\end{aligned}
\]

In Lesson 6-4, you saw that in systems of linear equations, if the lines are parallel, there are no solutions. With systems of linear inequalities, that is not always true.

EXAMPLE 3  
Graphing Systems with Parallel Boundary Lines

Graph each system of linear inequalities.

A \[
\begin{aligned}
y &< 2x - 3 \\
y &> 2x + 2
\end{aligned}
\]

This system has no solution.

B \[
\begin{aligned}
y &> x - 3 \\
y &\leq x + 1
\end{aligned}
\]

The solutions are all points between the parallel lines and on the solid line.

C \[
\begin{aligned}
y &\leq -3x - 2 \\
y &\leq -3x + 4
\end{aligned}
\]

The solutions are the same as the solutions of \( y \leq -3x - 2 \).
Graph each system of linear inequalities.

3a. \[
\begin{align*}
  y &> x + 1 \\
  y &\leq x - 3
\end{align*}
\]

3b. \[
\begin{align*}
  y &\geq 4x - 2 \\
  y &\leq 4x + 2
\end{align*}
\]

3c. \[
\begin{align*}
  y &> -2x + 3 \\
  y &> -2x
\end{align*}
\]

**Business Application**

A surf shop makes the profits given in the table. The shop owner sells at least 10 surfboards and at least 20 wakeboards per month. He wants to earn at least $2000 a month. Show and describe all possible combinations of surfboards and wakeboards that the store owner needs to sell to meet his goals. List two possible combinations.

**Step 1** Write a system of inequalities.

Let \( x \) represent the number of surfboards and \( y \) represent the number of wakeboards.

\[
\begin{align*}
  x &\geq 10 & \text{He sells at least 10 surfboards.} \\
  y &\geq 20 & \text{He sells at least 20 wakeboards.} \\
  150x + 100y &\geq 2000 & \text{He wants to earn a total of at least $2000.}
\end{align*}
\]

**Step 2** Graph the system.

The graph should be in only the first quadrant because sales are not negative.

**Step 3** Describe all possible combinations.

To meet the sales goals, the shop could sell any combination represented by an ordered pair of whole numbers in the solution region. Answers must be whole numbers because the shop cannot sell part of a surfboard or wakeboard.

**Step 4** List two possible combinations.

Two possible combinations are:

- 15 surfboards and 25 wakeboards
- 25 surfboards and 20 wakeboards

4. At her party, Alice is serving pepper jack cheese and cheddar cheese. She wants to have at least 2 pounds of each. Alice wants to spend at most $20 on cheese. Show and describe all possible combinations of the two cheeses Alice could buy. List two possible combinations.

**Price per Pound (\$)**

<table>
<thead>
<tr>
<th>Cheese</th>
<th>Price per Pound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pepper Jack</td>
<td>4</td>
</tr>
<tr>
<td>Cheddar</td>
<td>2</td>
</tr>
</tbody>
</table>

**Think and Discuss**

1. How would you write a system of linear inequalities from a graph?

2. **Get Organized** Copy and complete each part of the graphic organizer. In each box, draw a graph and list one solution.
### GUIDED PRACTICE

1. **Vocabulary** A solution of a system of inequalities is a solution of _____ ? _____ of the inequalities in the system. (at least one or all)

   Tell whether the ordered pair is a solution of the given system.

2. \((0, 0)\); \(\begin{cases} y < -x + 3 \\ y < x + 2 \end{cases}\)

3. \((0, 0)\); \(\begin{cases} y < 3 \\ y > x - 2 \end{cases}\)

4. \((1, 0)\); \(\begin{cases} y > 3x \\ y \leq x + 1 \end{cases}\)

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

5. \(\begin{cases} y < 2x - 1 \\ y > 2 \end{cases}\)

6. \(\begin{cases} x < 3 \\ y > x - 2 \end{cases}\)

7. \(\begin{cases} y \geq 3x \\ 3x + y \geq 3 \end{cases}\)

8. \(\begin{cases} 2x - 4y \leq 8 \\ y > x - 2 \end{cases}\)

Graph each system of linear inequalities.

9. \(\begin{cases} y > 2x + 3 \\ y < 2x \end{cases}\)

10. \(\begin{cases} y \leq -3x - 1 \\ y \geq -3x + 1 \end{cases}\)

11. \(\begin{cases} y > 4x - 1 \\ y \leq 4x + 1 \end{cases}\)

12. \(\begin{cases} y < -x + 3 \\ y > -x + 2 \end{cases}\)

13. \(\begin{cases} y > 2x - 1 \\ y > 2x - 4 \end{cases}\)

14. \(\begin{cases} y \leq -3x + 4 \\ y \leq -3x - 3 \end{cases}\)

15. **Business** Sandy makes $2 profit on every cup of lemonade that she sells and $1 on every cupcake that she sells. Sandy wants to sell at least 5 cups of lemonade and at least 5 cupcakes per day. She wants to earn at least $25 per day. Show and describe all the possible combinations of lemonade and cupcakes that Sandy needs to sell to meet her goals. List two possible combinations.

### PRACTICE AND PROBLEM SOLVING

Tell whether the ordered pair is a solution of the given system.

16. \((0, 0)\); \(\begin{cases} y > -x - 1 \\ y < 2x + 4 \end{cases}\)

17. \((0, 0)\); \(\begin{cases} x + y < 3 \\ x + y < 3 \end{cases}\)

18. \((1, 0)\); \(\begin{cases} y > 3x \\ y > 3x + 1 \end{cases}\)

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

19. \(\begin{cases} y < -3x - 3 \\ y \geq 0 \end{cases}\)

20. \(\begin{cases} y < -1 \\ y > 2x - 1 \end{cases}\)

21. \(\begin{cases} y > 2x + 4 \\ 6x + 2y \geq -2 \end{cases}\)

22. \(\begin{cases} 9x + 3y \leq 6 \\ y > x \end{cases}\)

Graph each system of linear inequalities.

23. \(\begin{cases} y < 3 \\ y > 5 \end{cases}\)

24. \(\begin{cases} y < x - 1 \\ y > x - 2 \end{cases}\)

25. \(\begin{cases} x \geq 2 \\ x \leq 2 \end{cases}\)

26. \(\begin{cases} y > -4x - 3 \\ y < -4x + 2 \end{cases}\)

27. \(\begin{cases} y > -1 \\ y > 2 \end{cases}\)

28. \(\begin{cases} y \leq 2x + 1 \\ y \leq 2x - 4 \end{cases}\)
29. **Multi-Step** Linda works at a pharmacy for $15 an hour. She also baby-sits for $10 an hour. Linda needs to earn at least $90 per week, but she does not want to work more than 20 hours per week. Show and describe the number of hours Linda could work at each job to meet her goals. List two possible solutions.

30. **Farming** Tony wants to plant at least 40 acres of corn and at least 50 acres of soybeans. He wants no more than 200 acres of corn and soybeans. Show and describe all the possible combinations of the number of acres of corn and of soybeans Tony could plant. List two possible combinations.

Graph each system of linear inequalities.

31. \[
\begin{align*}
  y \geq -3 \\
  y \geq 2
\end{align*}
\]

32. \[
\begin{align*}
  y > -2x - 1 \\
  y > -2x - 3
\end{align*}
\]

33. \[
\begin{align*}
  x \leq -3 \\
  x \geq 1
\end{align*}
\]

34. \[
\begin{align*}
  y < 4 \\
  y > 0
\end{align*}
\]

Write a system of linear inequalities to represent each graph.

35. \[
\begin{align*}
  y \geq -3 \\
  y \geq 2
\end{align*}
\]

36. \[
\begin{align*}
  y > -2x - 1 \\
  y > -2x - 3
\end{align*}
\]

37. \[
\begin{align*}
  x \leq -3 \\
  x \geq 1
\end{align*}
\]

38. **Military** For males to enter the United States Air Force Academy, located in Colorado Springs, CO, they must be at least 17 but less than 23 years of age. Their standing height must be not less than 60 inches and not greater than 80 inches. Graph all possible heights and ages for eligible male candidates. Give three possible combinations.

39. **ERROR ANALYSIS** Two students wrote a system of linear inequalities to describe the graph. Which student is incorrect? Explain the error.

40. **Recreation** Vance wants to fence in a rectangular area for his dog. He wants the length of the rectangle to be at least 30 feet and the perimeter to be no more than 150 feet. Graph all possible dimensions of the rectangle.

41. **Reasoning** Can the solutions of a system of linear inequalities be the points on a line? Explain.

42. This problem will prepare you for the Concept Connection on page 378. Gloria is starting her own company making teddy bears. She has enough bear bodies to create 40 bears. She will make girl bears and boy bears.

   a. Write an inequality to show this situation.

   b. Gloria will charge $15 for girl bears and $12 for boy bears. She wants to earn at least $540 a week. Write an inequality to describe this situation.

   c. Graph this situation and locate the solution region.
Chapter 6 Systems of Equations and Inequalities

43. Write About It  What must be true of the boundary lines in a system of two linear inequalities if there is no solution of the system? Explain.

Multiple Choice  For Exercises 44 and 45, choose the best answer.

44. Which point is a solution of \[ \begin{cases} 2x + y \geq 3 \\ y \geq -2x + 1 \end{cases} \]?

A (0, 0)  B (0, 1)  C (1, 0)  D (1, 1)

45. Which system of inequalities best describes the graph?

A \[ \begin{cases} y < 2x - 3 \\ y > 2x + 1 \end{cases} \]  C \[ \begin{cases} y < 2x - 3 \\ y < 2x + 1 \end{cases} \]

B \[ \begin{cases} y > 2x - 3 \\ y < 2x + 1 \end{cases} \]  D \[ \begin{cases} y > 2x - 3 \\ y > 2x + 1 \end{cases} \]

46. Short Response  Graph and describe \[ \begin{cases} y + x > 2 \\ y \leq -3x + 4 \end{cases} \]. Give two possible solutions of the system.

CHALLENGE AND EXTEND

47. Estimation  Graph the given system of inequalities. Estimate the area of the overlapping solution regions.

\[ \begin{cases} y \geq 0 \\ y \leq x + 3.5 \\ y \leq -x + 3.5 \end{cases} \]

48. Write a system of linear inequalities for which \((-1, 1)\) and \((1, 4)\) are solutions and \((0, 0)\) and \((2, -1)\) are not solutions.

49. Graph \(|y| < 1\).

50. Write a system of linear inequalities for which the solutions are all the points in the third quadrant.

Spiral Standards Review  \(\rightarrow 2.0, \rightarrow 6.0\)

Use the diagram to find each of the following. (Lesson 1-4)

51. area of the square
52. area of the yellow triangle
53. combined area of the blue triangles

Tell whether the given ordered pairs satisfy a linear function. (Lesson 5-1)

54. \(\{(3, 8), (4, 6), (5, 4), (6, 2), (7, 0)\}\)  55. \(\{(6, 1), (7, 2), (8, 4), (9, 7), (10, 11)\}\)
56. \(\{(2, 10), (7, 9), (12, 8), (17, 7), (22, 6)\}\)  57. \(\{(1, -9), (3, -7), (5, -5), (7, -3), (9, -1)\}\)

Graph the solutions of each linear inequality. Check your answer. (Lesson 6-6)

58. \(y \leq 2x - 1\)  59. \(-\frac{1}{4}x + y > 6\)  60. \(5 - x \geq 0\)
**6-7 Technology Lab**

**Solve Systems of Linear Inequalities**

A graphing calculator gives a visual solution to a system of linear inequalities.

**Use with Lesson 6-7**

**Activity**

Graph the system \( \begin{cases} y > 2x - 4 \\ 2.75y - x < 6 \end{cases} \). Give two ordered pairs that are solutions.

1. Write the first boundary line in slope-intercept form.
   \[ y > 2x - 4 \quad \Rightarrow \quad y = 2x - 4 \]

2. Press \( \text{Y=} \) and enter \( 2x - 4 \) for \( Y_1 \).
   
   The inequality contains the symbol \( > \). The solution region is above the boundary line. Press \( \text{ } \downarrow \text{ } \) to move the cursor to the left of \( Y_1 \). Press \( \text{ENTER} \) until the icon that looks like a region above a line appears. Press \( \text{GRAPH} \).

3. Solve the second inequality for \( y \).
   
   \[ 2.75y - x < 6 \]
   
   \[ 2.75y < x + 6 \]
   
   \[ y < \frac{x + 6}{2.75} \quad \Rightarrow \quad y = \frac{x + 6}{2.75} \]

4. Press \( \text{Y=} \) and enter \( (x + 6)/2.75 \) for \( Y_2 \).
   
   The inequality contains the symbol \( < \). The solution region is below the boundary line. Press \( \text{ } \downarrow \text{ } \) to move the cursor to the left of \( Y_2 \). Press \( \text{ENTER} \) until the icon that looks like a region below a line appears. Press \( \text{GRAPH} \).

5. The solutions of the system are represented by the overlapping shaded regions. The points \((0, 0)\) and \((-1, 0)\) are in the shaded region.

**Check**

Test \((0, 0)\) in both inequalities.

\[
\begin{array}{ccc}
\text{y > 2x - 4} & \text{2.75y - x < 6} \\
0 > 2(0) - 4 & 2.75(0) - 0 < 6 \\
0 > -4 & 0 < 6
\end{array}
\]

Test \((-1, 0)\) in both inequalities.

\[
\begin{array}{ccc}
\text{y > 2x - 4} & \text{2.75y - x < 6} \\
2 > 2(-1) - 4 & 2.75(0) - (-1) < 6 \\
2 > -6 & 1 < 6
\end{array}
\]

**Try This**

Graph each system. Give two ordered pairs that are solutions.

1. \( \begin{cases} x + 5y > -10 \\ x - y < 4 \end{cases} \)
2. \( \begin{cases} y > x - 2 \\ y \leq x + 2 \end{cases} \)
3. \( \begin{cases} y > x - 2 \\ y \leq 3 \end{cases} \)
4. \( \begin{cases} y < x - 3 \\ y - 3 > x \end{cases} \)
Equations and Formulas

Bearable Sales  Gloria makes teddy bears. She dresses some as girl bears with dresses and bows and some as boy bears with bow ties. She is running low on supplies. She has only 100 eyes, 30 dresses, and 60 ties that can be used as bows on the girls and bow ties on the boys.

1. Write the inequalities that describe this situation. Let \( x \) represent the number of boy bears and \( y \) represent the number of girl bears.

2. Graph the inequalities and locate the region showing the number of boy and girl bears Gloria can make.

3. List at least three combinations of girl and boy bears that Gloria can make.

For 4 and 5, use the table.

4. Using the boundary line in your graph from Problem 2, copy and complete the table with the corresponding number of girl bears.

5. Gloria sells the bears for profit. She makes a profit of $8 for the girl bears and $5 for the boy bears. Use the table from Problem 4 to find the profit she makes for each given combination.

6. Which combination is the most profitable? Explain. Where does it lie on the graph?
**Quiz for Lessons 6-6 Through 6-7**

### 6-6 Solving Linear Inequalities

Tell whether the ordered pair is a solution of the inequality.

1. \((3, -2); y < -2x + 1\)
2. \((2, 1); y \geq 3x - 5\)
3. \((1, -6); y \leq 4x - 10\)

Graph the solutions of each linear inequality. Check your answers.

4. \(y \geq 4x - 3\)
5. \(3x - y < 5\)
6. \(2x + 3y < 9\)
7. \(y \leq -\frac{1}{2}x\)

8. Theo's mother has given him at most \$150 to buy clothes for school. The pants cost \$30 each and the shirts cost \$15 each. How many of each can he buy? Write a linear inequality to describe the situation. Graph the linear inequality and give three possible combinations of pants and shirts Theo could buy.

Write an inequality to represent each graph.

9. ![Graph 1](image1.png)
10. ![Graph 2](image2.png)
11. ![Graph 3](image3.png)

### 6-7 Solving Systems of Linear Inequalities

Tell whether the ordered pair is a solution of the given system.

12. \((-3, -1); \begin{cases} y > -2 \\ y < x + 4 \end{cases}\)
13. \((-3, 0); \begin{cases} y \leq x + 4 \\ y \geq -2x - 6 \end{cases}\)
14. \((0, 0); \begin{cases} y \geq 3x \\ 2x + y < -1 \end{cases}\)

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

15. \(\begin{cases} y > -2 \\ y < x + 3 \end{cases}\)
16. \(\begin{cases} x + y \leq 2 \\ 2x + y \geq -1 \end{cases}\)
17. \(\begin{cases} 2x - 5y \leq -5 \\ 3x + 2y < 10 \end{cases}\)

Graph each system of linear inequalities and describe the solutions.

18. \(\begin{cases} y \geq x + 1 \\ y \geq x - 4 \end{cases}\)
19. \(\begin{cases} y \geq 2x - 1 \\ y < 2x - 3 \end{cases}\)
20. \(\begin{cases} y < -3x + 5 \\ y > -3x - 2 \end{cases}\)

21. A grocer sells mangos for \$4/lb and apples for \$3/lb. The grocer starts with 45 lb of mangos and 50 lb of apples each day. The grocer's goal is to make at least \$300 by selling mangos and apples each day. Show and describe all possible combinations of mangos and apples that could be sold to meet the goal. List two possible combinations.
Complete the sentences below with vocabulary words from the list above.

1. A(n) ______ is a system that has exactly one solution.
2. A set of two or more linear equations that contain the same variable(s) is a(n) ______.
3. The ______ consists of all the ordered pairs that satisfy all the inequalities in the system.
4. A system consisting of equations of parallel lines with different y-intercepts is a(n) ______.
5. A(n) ______ consists of two intersecting lines.

Exercises

Tell whether the ordered pair is a solution of the given system.

6. (0, -5); \begin{align*}
y &= -6x + 5 \\
x - y &= 5
\end{align*}

7. (4, 3); \begin{align*}
x - 2y &= -2 \\
y &= \frac{1}{2}x + 1
\end{align*}

8. \begin{align*}
\left(\frac{3}{4}, \frac{7}{4}\right) \\
x + y &= 9 \\
2y &= 6x + 4
\end{align*}

9. (-1, -1); \begin{align*}
y &= -2x + 5 \\
3y &= 6x + 3
\end{align*}

Solve each system by graphing. Check your answer.

10. \begin{align*}
y &= 3x + 2 \\
y &= -2x - 3
\end{align*}

11. \begin{align*}
y &= -\frac{1}{3}x + 5 \\
2x - 2y &= -2
\end{align*}

12. Raheel is comparing the cost of two parking garages. Garage A charges a flat fee of $6 per car plus $0.50 per hour. Garage B charges a flat fee of $2 per car plus $1 per hour. After how many hours will the cost at garage A be the same as the cost at garage B? What will that cost be?
6-2 Solving Systems by Substitution (pp. 336–342)

**Example**

Solve \( \begin{align*}
2x - 3y &= -2 \\
y - 3x &= 10
\end{align*} \)

by substitution.

**Step 1**

\( y - 3x = 10 \)  
\( y = 3x + 10 \)

**Step 2**

\( 2x - 3y = -2 \)  
\( 2x - 3(3x + 10) = -2 \)

**Step 3**

\( 2x - 9x - 30 = -2 \)  
\( -7x = 28 \)  
\( x = -4 \)

**Step 4**

\( y - 3x = 10 \)  
\( y - 3(-4) = 10 \)  
\( y + 12 = 10 \)  
\( y = -2 \)

**Step 5**  
\( (-4, -2) \)

Write the solution as an ordered pair.

To check the solution, substitute \((-4, -2)\) into both equations in the system.

**Exercises**

Solve each system by substitution. Check your answer.

13. \( \begin{align*}
y &= x + 3 \\
y &= 2x + 12
\end{align*} \)

14. \( \begin{align*}
y &= -4x \\
y &= 2x - 3
\end{align*} \)

15. \( \begin{align*}
x + y &= 4 \\
x + y &= 3
\end{align*} \)

16. \( \begin{align*}
x + y &= -1 \\
y &= -2x + 3
\end{align*} \)

17. \( \begin{align*}
x &= y - 7 \\
y - 2x &= 8
\end{align*} \)

18. \( \begin{align*}
x &= y - 9 \\
3x - 4y &= -6
\end{align*} \)

19. The Nash family’s car needs repairs. Estimates for parts and labor from two garages are shown below.

<table>
<thead>
<tr>
<th>Garage</th>
<th>Parts ($)</th>
<th>Labor ($ per hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor Works</td>
<td>650</td>
<td>70</td>
</tr>
<tr>
<td>Jim’s Car Care</td>
<td>800</td>
<td>55</td>
</tr>
</tbody>
</table>

For how many hours of labor will the total cost of fixing the car be the same at both garages? What will that cost be? Which garage will be cheaper if the repairs require 8 hours of labor? Explain.

6-3 Solving Systems by Elimination (pp. 343–349)

**Example**

Solve \( \begin{align*}
2x - 3y &= -8 \\
x + 4y &= 7
\end{align*} \)

by elimination.

**Step 1**

\( \begin{align*}
2x - 3y &= -8 \\
x + 4y &= 7
\end{align*} \)

**Step 2**

\( \begin{align*}
2x - 3y &= -8 \\
-2x - 8y &= -14
\end{align*} \)

**Step 3**

\( 0x - 11y = -22 \)  
\( y = 2 \)

**Step 4**

\( 2x - 3y = -8 \)  
\( 2x - 3(2) = -8 \)  
\( 2x - 6 = -8 \)  
\( 2x = -2 \)  
\( x = -1 \)

**Step 5**  
\( (-1, 2) \)

Write the solution as an ordered pair.

To check the solution, substitute \((-1, 2)\) into both equations in the system.

**Exercises**

Solve each system by elimination. Check your answer.

20. \( \begin{align*}
4x + y &= -1 \\
2x - y &= -5
\end{align*} \)

21. \( \begin{align*}
x + 2y &= 1 \\
x + y &= 2
\end{align*} \)

22. \( \begin{align*}
x + y &= 12 \\
2x + 5y &= 27
\end{align*} \)

23. \( \begin{align*}
\frac{1}{3}x + 3y &= 9
\end{align*} \)

Solve each system by any method. Explain why you chose each method. Check your answer.

24. \( \begin{align*}
3x + y &= 2 \\
y &= -4x
\end{align*} \)

25. \( \begin{align*}
y &= \frac{1}{3}x - 6 \\
y &= -2x + 1
\end{align*} \)

26. \( \begin{align*}
2y &= -3x \\
y &= -2x + 2
\end{align*} \)

27. \( \begin{align*}
x - y &= 0 \\
3x + y &= 8
\end{align*} \)
6-4 Solving Special Systems (pp. 350–355)

**Example**

Classify the system. Give the number of solutions.

\[\begin{align*}
  y &= 3x + 4 \\
  6x - 2y &= -8
\end{align*}\]

Use the substitution method.

\[\begin{align*}
  6x - 2(3x + 4) &= -8 \\
  6x - 6x - 8 &= -8 \\
  -8 &= -8 \quad \text{True.}
\end{align*}\]

The equation is an identity. There are infinitely many solutions.

This system is **consistent** and **dependent**. The two lines are coincident because they have identical slopes and y-intercepts.

If the lines never intersect, the system is **inconsistent**. It has no solution. The system is consistent and independent when there is one solution.

6-5 Applying Systems (pp. 356–361)

**Example**

Against the wind, Devin skated 200 meters in 50 seconds. With the wind, he skated the same distance in 25 seconds. What is the rate at which Devin skated? What is the rate of the wind?

Solve the system

\[\begin{align*}
  50(d - w) &= 200 \\
  25(d + w) &= 200
\end{align*}\]

Step 1 \[\begin{align*}
  50d - 50w &= 200 \\
  2(25d + 25w) &= 200
\end{align*}\]

Step 2 \[\begin{align*}
  50d - 50w &= 200 \\
  50d + 50w &= 400
\end{align*}\]

Multiply each term by 2. Add.

\[\begin{align*}
  100d &= 600 \\
  d &= 6
\end{align*}\]

Simplify and solve for d.

Step 3 \[\begin{align*}
  50d - 50w &= 200 \\
  50w - 50w &= 200 \\
  300 - 50w &= 200 \\
  -300 &= 300
\end{align*}\]

Subtract 300 for d. Subtract 300 from both sides.

\[\begin{align*}
  -50w &= -100 \\
  w &= 2
\end{align*}\]

Simplify and solve for w.

Step 4 \( (6, 2) \) Write the solution as an ordered pair.

Devin’s skating rate is 6 m/s. The rate of the wind is 2 m/s.

**Exercises**

Classify each system. Give the number of solutions.

28. \[\begin{align*}
  y &= \frac{1}{2}x + 2 \\
  y &= \frac{1}{4}x - 8
\end{align*}\]

29. \[\begin{align*}
  y &= 3x - 7 \\
  y &= 3x + 2
\end{align*}\]

30. \[\begin{align*}
  2x + y &= 2 \\
  y &= -2x
\end{align*}\]

31. \[\begin{align*}
  -3x - y &= -5 \\
  y &= -3x - 5
\end{align*}\]

32. \[\begin{align*}
  2x + 3y &= 1 \\
  3x + 2y &= 1
\end{align*}\]

33. \[\begin{align*}
  x + \frac{1}{2}y &= 3 \\
  2x &= 6 - y
\end{align*}\]

34. The two parallel lines graphed below represent a system of equations. Classify the system and give the number of solutions.

35. Gena walked 160 feet in 40 seconds on a moving walkway. Against the walkway, she was able to walk the same distance in 80 seconds. What is the rate at which Gena walked, and what is the rate of the walkway?

36. Blake rows his boat against a current 90 yards in 15 minutes. With a current, he rows 90 yards in 9 minutes. What is the rate at which Blake rows, and what is the rate of the current?

37. Cole has a solution that is 20% water and another solution that is 60% water. He wants to mix these to make a 40 mL solution that is 30% water. How many mL of each solution should Cole mix together?

38. The sum of the digits of a two-digit number is 11. When the digits are reversed, the new number is 63 more than the original number. What is the original number?
6-6  Solving Linear Inequalities  (pp. 364–370)

**Example**

Graph the solutions of $x - 2y < 6$.

**Step 1** Solve the inequality for $y$.

\[
\begin{align*}
x - 2y &< 6 \\
-2y &< -x + 6 \\
y &> \frac{1}{2} x - 3
\end{align*}
\]

**Step 2** Graph $y = \frac{1}{2} x - 3$.

Use a dashed line for $>$.

**Step 3** The inequality is $>$, so shade above the boundary line.

**Check** Substitute $(0, 0)$ for $(x, y)$ because it is not on the boundary line.

\[
\begin{align*}
x - 2y &< 6 \\
0 - 2(0) &< 6 \\
0 &< 6 \checkmark
\end{align*}
\]

$(0, 0)$ satisfies the inequality, so the graph is shaded correctly.

6-7  Solving Systems of Linear Inequalities  (pp. 371–376)

**Example**

Graph $\begin{cases} y < -x + 5 \\ y \geq 2x - 3 \end{cases}$. Give two ordered pairs that are solutions and two that are not solutions.

Graph both inequalities.

The solutions of the system are represented by the overlapping shaded regions.

The points $(0, 0)$ and $(-2, 2)$ are solutions of the system.

The points $(3, -2)$ and $(4, 4)$ are not solutions.

**Exercises**

Tell whether the ordered pair is a solution of the given system.

50. $(3, 3); \begin{cases} y > -2x + 9 \\ y \geq x \end{cases}$

51. $(-1, 0); \begin{cases} 2x - y > -5 \\ y \leq -3x - 3 \end{cases}$

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

52. $\begin{cases} y \geq x + 4 \\ y > 6x - 3 \end{cases}$

53. $\begin{cases} y \leq -2x + 8 \\ y > 3x - 5 \end{cases}$

54. $\begin{cases} -x + 2y > 6 \\ x + y < 4 \end{cases}$

55. $\begin{cases} x - y > 7 \\ x + 3y \leq 15 \end{cases}$

Graph each system of linear inequalities.

56. $\begin{cases} y > -x - 6 \\ y < -x + 5 \end{cases}$

57. $\begin{cases} 4x + 2y \geq 10 \\ 6x + 3y < -9 \end{cases}$
Tell whether the ordered pair is a solution of the given system.

1. \((1, -4); \begin{cases} y = -4x \\ y = 2x - 2 \end{cases} \)

2. \((0, -1); \begin{cases} 3x - y = 1 \\ x + 5y = -5 \end{cases} \)

3. \((3, 2); \begin{cases} x - 2y = -1 \\ -3x + 2y = 5 \end{cases} \)

Solve each system by graphing.

4. \(\begin{cases} y = x - 3 \\ y = -2x - 3 \end{cases} \)

5. \(\begin{cases} 2x + y = -8 \\ y = \frac{1}{3}x - 1 \end{cases} \)

6. \(\begin{cases} y = -x + 4 \\ x = y + 2 \end{cases} \)

Solve each system by substitution.

7. \(\begin{cases} y = -6 \\ y = -2x - 2 \end{cases} \)

8. \(\begin{cases} -x + y = -4 \\ y = 2x - 11 \end{cases} \)

9. \(\begin{cases} x - 3y = 3 \\ 2x = 3y \end{cases} \)

10. The costs for services at two kennels are shown in the table. Joslyn plans to board her dog and have him bathed once during his stay. For what number of days will the cost for boarding and bathing her dog at each kennel be the same? What will that cost be? If Joslyn plans a week-long vacation, which is the cheaper service? Explain.

<table>
<thead>
<tr>
<th>Kennel Costs</th>
<th>Boarding ($ per day)</th>
<th>Bathing ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pet Care</td>
<td>30</td>
<td>15</td>
</tr>
<tr>
<td>Fido’s</td>
<td>28</td>
<td>27</td>
</tr>
</tbody>
</table>

Solve each system by elimination.

11. \(\begin{cases} 3x - y = 7 \\ 2x + y = 3 \end{cases} \)

12. \(\begin{cases} 4x + y = 0 \\ x + y = -3 \end{cases} \)

13. \(\begin{cases} 2x + y = 3 \\ x - 2y = -1 \end{cases} \)

Classify each system. Give the number of solutions.

14. \(\begin{cases} y = 6x - 1 \\ 6x - y = 1 \end{cases} \)

15. \(\begin{cases} y = -3x - 3 \\ 3x + y = 3 \end{cases} \)

16. \(\begin{cases} 2x - y = 1 \\ -4x + y = 1 \end{cases} \)

17. The sum of the digits of a two-digit number is 13. When the digits are reversed, the new number is 27 less than the original number. What is the original number?

Graph the solutions of each linear inequality.

18. \(y < 2x - 5\)

19. \(-y \geq 8\)

20. \(y > \frac{1}{3}x\)

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

21. \(\begin{cases} y > \frac{1}{2}x - 5 \\ y \leq 4x - 1 \end{cases} \)

22. \(\begin{cases} y > -x + 4 \\ 3x - y > 3 \end{cases} \)

23. \(\begin{cases} y \geq 2x \\ y - 2x < 6 \end{cases} \)

24. Ezra and Tava sold at least 150 coupon books. Ezra sold at most 30 books more than twice the number Tava sold. Show and describe all possible combinations of the numbers of coupon books Ezra and Tava sold. List two possible combinations.
FOCUS ON ACT

Four scores are reported for the ACT Mathematics Test: one score based on all 60 problems and one for each content area. The three content areas are: Pre-Algebra/Elementary Algebra, Intermediate Algebra/Coordinate Geometry, and Plane Geometry/Trigonometry.

You may want to time yourself as you take this practice test. It should take you about 5 minutes to complete.

1. Which system of inequalities is represented by the graph?

(A) \[
\begin{align*}
-x + 2y &< 6 \\
2x + y &> -4
\end{align*}
\]

(B) \[
\begin{align*}
x - 2y &\leq 6 \\
2x - y &\geq 4
\end{align*}
\]

(C) \[
\begin{align*}
-x + 2y &\leq 6 \\
2x + y &\geq 4
\end{align*}
\]

(D) \[
\begin{align*}
-x + 2y &\leq 6 \\
2x + y &> -4
\end{align*}
\]

(E) \[
\begin{align*}
x - 2y &\leq 6 \\
2x - y &> 4
\end{align*}
\]

2. What is the solution for $y$ in the given system?

\[
\begin{align*}
4x + 3y &= 1 \\
-4x + 3y &= -7
\end{align*}
\]

(F) -1

(G) 0

(H) 1

(J) 2

(K) 6

3. Wireless phone company A charges $20 per month plus $0.12 per minute. Wireless phone company B charges $50 per month plus $0.06 per minute. For how many minutes of calls will the monthly bills be the same?

(A) 80 minutes

(B) 100 minutes

(C) 160 minutes

(D) 250 minutes

(E) 500 minutes

4. Which of the following systems of equations does NOT have a solution?

(F) \[
\begin{align*}
x + 5y &= 30 \\
-4x + 5y &= 10
\end{align*}
\]

(G) \[
\begin{align*}
x + 5y &= -30 \\
-4x + 5y &= 10
\end{align*}
\]

(H) \[
\begin{align*}
x + 5y &= -30 \\
-4x + 5y &= -10
\end{align*}
\]

(J) \[
\begin{align*}
-4x + 5y &= -10 \\
-8x + 10y &= -20
\end{align*}
\]

(K) \[
\begin{align*}
-4x + 5y &= -10 \\
-4x + 5y &= -30
\end{align*}
\]
Any Question Type: Read the Problem for Understanding

Standardized test questions may vary in format including multiple choice, gridded response, and short or extended response. No matter what format the test uses, read each question carefully and critically. Do not rush. Be sure you completely understand what you are asked to do and what your response should include.

Extended Response
An interior decorator charges a consultation fee of $50 plus $12 per hour. Another interior decorator charges a consultation fee of $5 plus $22 per hour. Write a system of equations to find the amount of time for which the cost of both decorators will be the same. Graph the system. After how many hours will the cost be the same for both decorators? What will the cost be?

Read the problem again.

What information are you given?
the consultation fees and hourly rates of two decorators

What are you asked to do?
1. Write a system of equations.
2. Graph the system.
3. Interpret the solution to the system.

What should your response include?
1. a system of equations with variables defined
2. a graph of the system
3. the time when the cost is the same for both decorators
4. the cost at that time
Read each test item and answer the questions that follow.

**Item A**
Short Response Which value of $b$ will make the lines intersect at the point $(-2, 14)$?

\[
\begin{align*}
  y &= -6x + 2 \\
  y &= 4x + b
\end{align*}
\]

1. What information are you given?
2. What are you asked to do?
3. Ming’s answer to this test problem was $y = 4x + 22$. Did Ming answer correctly? Explain.

**Item B**
Extended Response Solve the system by using elimination. Explain how you can check your solution algebraically and graphically.

\[
\begin{align*}
  4x + 10y &= -48 \\
  6x - 10y &= 28
\end{align*}
\]

4. What method does the problem ask you to use to solve the system of equations?
5. What methods does the problem ask you to use to check your solution?
6. How many parts are there to this problem? List what needs to be included in your response.

**Item C**
Gridded Response What is the $x$-coordinate of the solution to this system?

\[
\begin{align*}
  y &= 6x + 9 \\
  y &= 12x - 15
\end{align*}
\]

7. What question is being asked?
8. A student correctly found the solution of the system to be $(4, 33)$. What should the student mark on the grid so that the answer is correct?

**Item D**
Short Response Write an inequality to represent the graph below. Give a real-world situation that this inequality could describe.

[Graph of a linear inequality]

9. As part of his answer, a student wrote the following response:

The point $(1, 5)$ is not a solution to the inequality because it lies on the line, but $(2, 12)$ is a solution because it lies above the line.

Is his response appropriate? Explain.
10. What should the response include so that it answers all parts of the problem?

**Item E**
Multiple Choice Taylor bikes 50 miles per week and increases her distance by 2 miles each week. Josie bikes 30 miles per week and increases her distance by 10 miles each week. In how many weeks will Taylor and Josie be biking the same distance?

\[
\begin{align*}
  A & \quad 2.5 \text{ weeks} \\
  B & \quad 7.5 \text{ weeks} \\
  C & \quad 55 \text{ weeks} \\
  D & \quad 110 \text{ weeks}
\end{align*}
\]

11. What question is being asked?
12. Carson incorrectly selected option C as his answer. What question did he most likely answer?
Multiple Choice

1. What is the $x$-intercept of $3x + 2y = -6$?
   - A) $-3$
   - B) $-2$
   - C) $2$
   - D) $3$

2. Which of the problems below could be solved by finding the solution of this system?
   \[
   \begin{align*}
   2x + 2y &= 56 \\
   y &= \frac{1}{3}x
   \end{align*}
   \]
   - A) The area of a rectangle is 56. The width is one-third the length. Find the length of the rectangle.
   - B) The area of a rectangle is 56. The length is one-third the perimeter. Find the length of the rectangle.
   - C) The perimeter of a rectangle is 56. The length is one-third more than the width. Find the length of the rectangle.
   - D) The perimeter of a rectangle is 56. The width is one-third the length. Find the length of the rectangle.

3. What is the slope of a line perpendicular to a line that passes through $(3, 8)$ and $(1, -4)$?
   - A) $-\frac{1}{6}$
   - B) $-\frac{1}{2}$
   - C) $2$
   - D) $6$

4. Which inequality is graphed below?
   - A) $-x > -3$
   - B) $-y > -3$
   - C) $2x < -6$
   - D) $3y < 9$

5. A chemist has a bottle of a 10% acid solution and a bottle of a 30% acid solution. He mixes the solutions together to get 500 mL of a 25% acid solution. How much of the 30% solution did he use?
   - A) 125 mL
   - B) 150 mL
   - C) 375 mL
   - D) 450 mL

6. Which ordered pair is NOT a solution of the system graphed below?
   - A) $(0, 0)$
   - B) $(0, 3)$
   - C) $(1, 1)$
   - D) $(2, 1)$

7. Which of the following best classifies a system of linear equations whose graph is two intersecting lines?
   - A) inconsistent and dependent
   - B) inconsistent and independent
   - C) consistent and dependent
   - D) consistent and independent

8. Which ordered pair is a solution of this system?
   \[
   \begin{align*}
   2x - y &= -2 \\
   \frac{1}{3}y &= x
   \end{align*}
   \]
   - A) $(0, 2)$
   - B) $(1, 3)$
   - C) $(2, 6)$
   - D) $(3, 8)$

9. Where does the graph of $5x - 10y = 30$ cross the $y$-axis?
   - A) $(0, -3)$
   - B) $\left(0, \frac{1}{2}\right)$
   - C) $(6, 0)$
   - D) $(0, -6)$
Most standardized tests allow you to write in your test booklet. Cross out each answer choice you eliminate. This may keep you from accidentally marking an answer other than the one you think is right. However, don’t draw in your math book!

10. Hillary needs markers and poster board for a project. The markers are $0.79 each and the poster board is $1.89 per sheet. She needs at least 4 sheets of poster board. Hillary has $15 to spend on project materials. Which system models this information?

\[
\begin{cases}
p \geq 4 \\
0.79m + 1.89p \leq 15
\end{cases}
\]

11. How many different values of x are solutions of \(|x - 2| + 5 = 5|?\)

\[
\begin{array}{l}
\text{A} \quad 0 \\
\text{B} \quad 1 \\
\text{C} \quad 2 \\
\text{D} \quad 3
\end{array}
\]

**Gridded Response**

12. Kendra graphs the line shown below. Then she finds the equation of the line passing through \((-2, 2)\) that is perpendicular to this line. She writes the equation in slope-intercept form, \(y = mx + b\). What is the value of \(b\)?

13. What value of \(y\) will make the line passing through \((4, -4)\) and \((-8, y)\) have a slope of \(-\frac{1}{2}\)?

14. What value of \(k\) will make the system \(y - 5x = -1\) and \(y = kx + 3\) inconsistent?

15. What is the domain and range of the function shown in the graph?

16. Graph \(y > \frac{x}{3} - 1\) on a coordinate plane. Name one point that is a solution of the inequality.

17. Marc and his brother Ty start saving money at the same time. Marc has $145 and will add $10 to his savings every week. Ty has $20 and will add $15 to his savings every week. After how many weeks will Marc and Ty have the same amount saved? What is that amount? Show your work.

18. A movie producer is looking for extras to act as office employees in his next movie. The producer needs extras that are at least 40 years old but less than 70 years old. They should be at least 60 inches tall but less than 75 inches tall. Graph all the possible combinations of ages and heights for extras that match the producer’s needs. Let \(x\) represent age and \(y\) represent height. Show your work.

19. Graph the system

\[
\begin{cases}
y < -2x + 3 \\
y \geq 6x + 6
\end{cases}
\]

a. Is \((0, 0)\) a solution of the system you graphed? Explain why or why not.

b. Is \((-4, 5)\) a solution of the system you graphed? Explain why or why not.

**Extended Response**

20. Every year, Erin knits scarves and sells them at the craft fair. This year she used $6 worth of yarn for each scarf. She also paid $50 to rent a table at the fair. She sold every scarf for $10.

a. Write a system of linear equations to represent the amount Erin spent and the amount she collected. Tell what your variables represent. Tell what each equation in the system represents.

b. Use any method to solve the system you wrote in part a. Show your work. How many scarves did Erin need to sell to make a profit? Explain.

c. Describe two ways you could check your solution to part b. Check your solution by using one of those ways. Show your work.