CHAPTER 8

Factoring Polynomials

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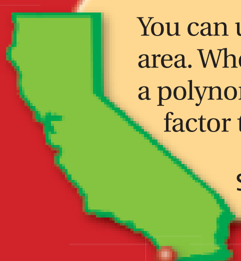
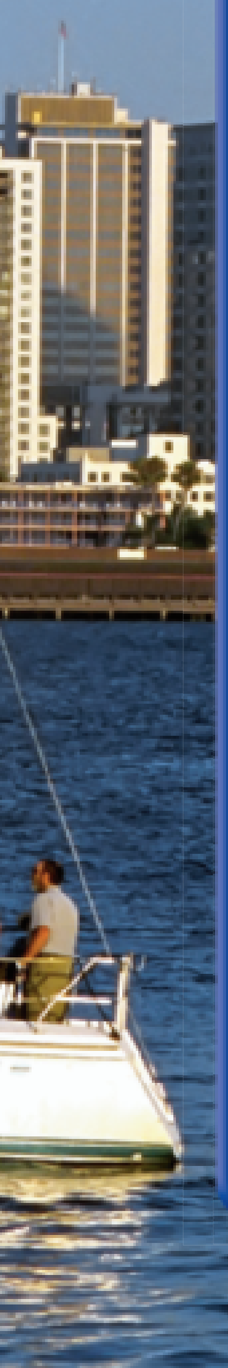


You can use polynomials to model area. When given the area of a sail as a polynomial, you can sometimes factor to find the sail's dimensions.

San Diego, CA



ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. binomial
2. composite number
3. factor
4. multiple
5. prime number

A. a whole number greater than 1 that has more than two whole-number factors

B. a polynomial with two terms

C. the product of any number and a whole number

D. a number written as the product of its prime factors

E. a whole number greater than 1 that has exactly two positive factors, itself and 1

F. a number that is multiplied by another number to get a product

Multiples

Write the first four multiples of each number.

6. 3 **7.** 4 **8.** 8 **9.** 15

Factors

Tell whether the second number is a factor of the first number.

10. 20, 5 **11.** 50, 6 **12.** 120, 8 **13.** 245, 7

Prime and Composite Numbers

Tell whether each number is prime or composite. If the number is composite, write it as the product of two numbers.

14. 2 **15.** 7 **16.** 10 **17.** 38

18. 115 **19.** 147 **20.** 151 **21.** 93

Multiply Monomials and Polynomials

Simplify.

22. $2(x+5)$ **23.** $3h(h+1)$ **24.** $xy(x^2 - xy^3)$ **25.** $6m(m^2 - 4m - 1)$

Multiply Binomials

Find each product.

26. $(x+3)(x+8)$ **27.** $(b-7)(b+1)$

28. $(2p-5)(p-1)$ **29.** $(3n+4)(2n+3)$

CHAPTER 8

Unpacking the Standards

The information below "unpacks" the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

California Standard	Academic Vocabulary	Chapter Concept
11.0 Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials. (Lessons **8-2, 8-3, 8-5, 8-6;** Labs **8-2, 8-3**)	**apply** use **technique** a way of doing something **common** shared among all members of a group	You learn several ways to rewrite polynomials as products. ***Example:*** $x^2 + 3x$ Both terms in the polynomial contain the common term x, so the polynomial can be factored. $x^2 + 3x = x(x + 3)$
25.1 Students use properties of numbers to construct simple, valid arguments (direct and **indirect**) for, or formulate counterexamples to, **claimed assertions.** (pp. 484–485)	**construct** make or prepare **valid** true and correct **assertion** a statement that is made without proof	You learn a new method, indirect proof, for proving a mathematical statement.

Reading and Writing Math

CHAPTER 8

Reading Strategy: Read a Lesson for Understanding

To help you learn new concepts, you should read each lesson with a purpose. As you read a lesson, make notes. Include the main ideas of the lesson and any questions you have. In class, listen for explanations of the vocabulary, clarification of the examples, and answers to your questions.

Reading Tips

California Standards

2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, **taking a root,** and raising to a fractional

The California Standards tell you the main standard covered in the lesson. The bold text indicates the specific part of the standard that the lesson focuses on.

If a power of 10 has a negative integer exponent, does that make the number negative?
How do I enter numbers written in scientific notation into my calculator?

Write down questions you have as you read the lesson.

EXAMPLE 1 Evaluating Powers of 10

Find the value of each power of 10.

A 10^{-3}

Start with 1 and move the decimal point three places to the left.

0. 0 0 1

0.001

Work through the examples and write down any questions you have.

CHECK IT OUT!

Practice what you've learned in the Check It Out sections.

Try This

Read Lesson 8-1 prior to your next class. Then answer the questions below.

1. What vocabulary, formulas, and symbols are new?
2. Which examples, if any, are unclear?
3. What questions do you have about the lesson?

8-1 Factors and Greatest Common Factors

California Standards

Preparation for 11.0 Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include **finding a common factor** for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

Vocabulary
prime factorization
greatest common factor

Who uses this?
Web site designers who sell electronic greeting cards can use greatest common factors to design their Web sites.
(See Example 4.)

The numbers that are multiplied to find a product are called *factors* of that product. A number is divisible by its factors.

Remember that a *prime number* is a whole number that has exactly two positive factors, itself and 1. The number 1 is not prime because it has only one factor.

You can use the factors of a number to write the number as a product. The number 12 can be factored several ways.

The order of the factors does not change the product, but there is only one example that cannot be factored further. The circled factorization is the **prime factorization** because all the factors are prime numbers. The prime factors can be written in any order, and, except for changes in the order, there is only one way to write the prime factorization of a number.

Factorizations of 12

$1 \cdot 12$

$2 \cdot 6$

$3 \cdot 4$

$1 \cdot 4 \cdot 3$

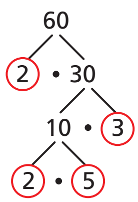
$2 \cdot 2 \cdot 3$

EXAMPLE 1 **Writing Prime Factorizations**

Write the prime factorization of 60.

Method 1 Factor tree

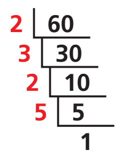
Choose any two factors of 60 to begin. Keep finding factors until each branch ends in a prime factor.



$60 = 2 \cdot 2 \cdot 5 \cdot 3$

Method 2 Ladder diagram

Choose a prime factor of 60 to begin. Keep dividing by prime factors until the quotient is 1.



$60 = 2 \cdot 3 \cdot 2 \cdot 5$

The prime factorization of 60 is $2 \cdot 2 \cdot 3 \cdot 5$ or $2^2 \cdot 3 \cdot 5$.

CHECK IT OUT! **Write the prime factorization of each number.**

1a. 40 **1b.** 33 **1c.** 49 **1d.** 19

Factors that are shared by two or more whole numbers are called common factors. The greatest of these common factors is called the **greatest common factor**, or GCF.

Factors of 12: 1, 2, 3, 4, 6, 12
Factors of 32: 1, 2, 4, 8, 16, 32

Common factors: 1, 2, 4

The greatest of the common factors is 4.

EXAMPLE 2 Finding the GCF of Numbers

Find the GCF of each pair of numbers.

A **24 and 60**

Method 1 List the factors.

factors of 24: 1, 2, 3, 4, 6, 8, 12, 24 — *List all the factors.*
factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 — *Circle the GCF.*

The GCF of 24 and 60 is 12.

B **18 and 27**

Method 2 Use prime factorization.

$$18 = 2 \cdot \boxed{3} \cdot \boxed{3}$$ *Write the prime factorization of each number.*

$$27 = \boxed{3} \cdot \boxed{3} \cdot 3$$ *Align the common factors.*

$$3 \cdot 3 = 9$$

The GCF of 18 and 27 is 9.

CHECK IT OUT! **Find the GCF of each pair of numbers.**

2a. 12 and 16 **2b.** 15 and 25

You can also find the GCF of monomials that include variables. To find the GCF of monomials, write the prime factorization of each coefficient and write all powers of variables as products. Then find the product of the common factors.

EXAMPLE 3 Finding the GCF of Monomials

Find the GCF of each pair of monomials.

A **$3x^3$ and $6x^2$**

$$3x^3 = \boxed{3} \cdot \boxed{x} \cdot \boxed{x} \cdot x$$ *Write the prime factorization of each coefficient and write powers as products.*

$$6x^2 = 2 \cdot \boxed{3} \cdot \boxed{x} \cdot \boxed{x}$$ *Align the common factors.*

$$3 \cdot x \cdot x = 3x^2$$ *Find the product of the common factors.*

The GCF of $3x^3$ and $6x^2$ is $3x^2$.

B **$4x^2$ and $5y^3$**

$$4x^2 = 2 \cdot 2 \cdot x \cdot x$$ *Write the prime factorization of each coefficient and write powers as products.*

$$5y^3 = 5 \cdot y \cdot y \cdot y$$ *Align the common factors.*

There are no common factors other than 1.

The GCF of $4x^2$ and $5y^3$ is 1.

Helpful Hint

If two terms contain the same variable raised to different powers, the GCF will contain that variable raised to the lower power.

CHECK IT OUT! **Find the GCF of each pair of monomials.**

3a. $18g^2$ and $27g^3$ **3b.** $16a^6$ and $9b$ **3c.** $8x$ and $7v^2$

EXAMPLE 4 *Technology Application*

Garrison is creating a Web page that offers electronic greeting cards. He has 24 special occasion designs and 42 birthday designs. The cards will be displayed with the same number of designs in each row. Special occasion and birthday designs will not appear in the same row. How many rows will there be if Garrison puts the greatest possible number of designs in each row?

The 24 special occasion designs and 42 birthday designs must be divided into groups of equal size. The number of designs in each row must be a common factor of 24 and 42.

factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

factors of 42: 1, 2, 3, 6, 7, 14, 21, 42

Find the common factors of 24 and 42.

The GCF of 24 and 42 is 6.

The greatest possible number of designs in each row is 6. Find the number of rows of each group of designs when there are 6 designs in each row.

$$\frac{24 \text{ special occasion designs}}{6 \text{ designs per row}} = 4 \text{ rows}$$

$$\frac{42 \text{ birthday designs}}{6 \text{ designs per row}} = 7 \text{ rows}$$

When the greatest possible number of designs is in each row, there are 11 rows in total.

CHECK IT OUT!

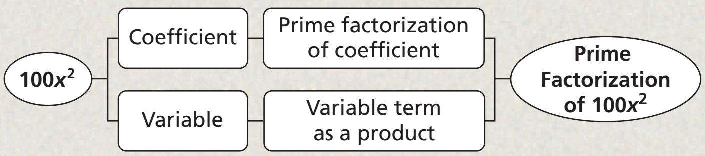
4. Adrianne is shopping for a CD storage unit. She has 36 CDs by pop music artists and 48 CDs by country music artists. She wants to put the same number of CDs on each shelf without putting pop music and country music CDs on the same shelf. If Adrianne puts the greatest possible number of CDs on each shelf, how many shelves does her storage unit need?

THINK AND DISCUSS

1. Describe two ways you can find the prime factorization of a number.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. Show how to write the prime factorization of $100x^2$ by filling in each box.



8-1 Exercises

California Standards Practice
10.0, Prep for 11.0, 15.0, 25.1

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Homework Help Online
KEYWORD: MA8CA 8-1
Parent Resources Online
KEYWORD: MA8CA Parent

GUIDED PRACTICE

1. Vocabulary Define the term *greatest common factor* in your own words.

SEE EXAMPLE 1 p. 478

Write the prime factorization of each number.

2. 20 **3.** 36 **4.** 27 **5.** 54

6. 96 **7.** 7 **8.** 100 **9.** 75

SEE EXAMPLE 2 p. 479

Find the GCF of each pair of numbers.

10. 12 and 60 **11.** 14 and 49 **12.** 55 and 121

13. 21 and 14 **14.** 13 and 40 **15.** 72 and 18

SEE EXAMPLE 3 p. 479

Find the GCF of each pair of monomials.

16. $15y^3$ and $-20y$ **17.** $6x^2$ and $5x^2$ **18.** $12r$ and $30r^2$

19. $2x^3$ and 6 **20.** $35a^2$ and $6a^3$ **21.** $13q^4$ and $2p^2$

SEE EXAMPLE 4 p. 480

22. Samantha is making beaded necklaces using 54 glass beads and 18 clay beads. She wants each necklace to have the same number of beads, but each necklace will have only one type of bead. If she puts the greatest possible number of beads on each necklace, how many necklaces can she make?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
23–30	1
31–36	2
37–42	3
43	4

Extra Practice
Skills Practice p. EP16
Application Practice p. EP31

Write the prime factorization of each number.

23. 18 **24.** 64 **25.** 12 **26.** 150

27. 17 **28.** 226 **29.** 49 **30.** 63

Find the GCF of each pair of numbers.

31. 36 and 63 **32.** 14 and 15 **33.** 30 and 40

34. 15 and 75 **35.** 18 and 22 **36.** 16 and 99

Find the GCF of each pair of monomials.

37. $9s$ and $63s^3$ **38.** $8a^2$ and 11 **39.** $-36w^3$ and $15w^2$

40. $5b^2$ and $3b$ **41.** $3x^2$ and $9x$ **42.** $-64n^4$ and $24n^2$

43. José is making fruit-filled tart shells for a party. He has 72 raspberries and 108 blueberries. The tarts will each have the same number of berries. Raspberries and blueberries will not be in the same tart. If he puts the greatest possible number of berries in each tart, how many tarts can he make?

Find the GCF of each pair of products.

44. $3 \cdot 5 \cdot t$ and $2 \cdot 2 \cdot 5 \cdot t \cdot t$ **45.** $-1 \cdot 2 \cdot 2 \cdot x \cdot x$ and $2 \cdot 2 \cdot 7 \cdot x \cdot x \cdot x$

46. $2 \cdot 2 \cdot 2 \cdot 11 \cdot x \cdot x \cdot x$ and $3 \cdot 11$ **47.** $2 \cdot 5 \cdot n \cdot n \cdot n$ and $-1 \cdot 2 \cdot 3 \cdot n$

48. Write About It Explain why the number 2 is the only even prime number.

49. Reasoning Show that the following statement is true or provide a counterexample to show it is false. If the GCF of two numbers is 1, then the two numbers are prime.

Music

DCI is a nonprofit organization that oversees drum and bugle corps performances and competitions for youths between the ages of 14 and 21.

50. Multi-Step Angelo is making a rectangular floor for a clubhouse with an area of 84 square feet. The length of each side of the floor is a whole number of feet.

a. What are the possible lengths and widths for Angelo's clubhouse floor?

b. What is the minimum perimeter for the clubhouse floor?

c. What is the maximum perimeter for the clubhouse floor?

51. Music The Cavaliers and the Blue Devils are two of the marching bands that are members of Drum Corps International (DCI). DCI bands are made up of percussionists, brass players, and color guard members who use flags and other props.

In 2004, there were 35 color guard members in the Cavaliers and 40 in the Blue Devils. The two color guards will march in rows with the same number of people in each row without mixing the guards together. If the greatest possible number of people are in each row, how many rows will there be?

For each set of numbers, determine which two numbers have a GCF greater than 1, and find that GCF.

52. 11, 12, 14

53. 8, 20, 63

54. 16, 21, 27

55. 32, 63, 105

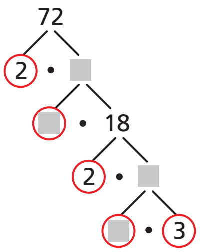
56. 25, 35, 54

57. 35, 54, 72

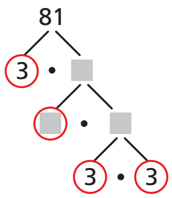
58. Number Sense The prime factorization of 24 is $2^3 \cdot 3$. Without performing any calculations or using a diagram, write the prime factorization of 48. Explain your reasoning.

Fill in each diagram. Then write the prime factorization of the number.

59.



60.



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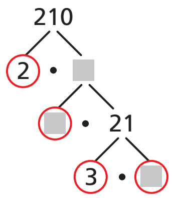

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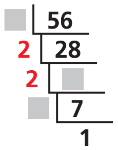

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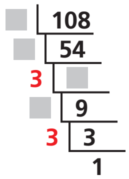

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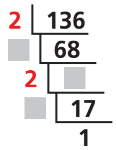

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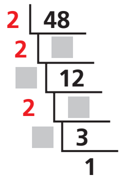

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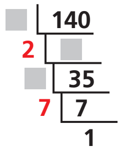
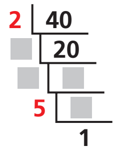


67.



68. Kate has $12a^2$ raffle tickets and Henry has $18a$ raffle tickets. Kate makes equal piles using all of her tickets. Henry makes equal piles using all of his tickets. Henry and Kate have the same number of piles.

a. Write an expression for the greatest number of piles that Kate and Henry can have.

b. How many tickets will be in each of Kate's piles? in each of Henry's?

CONCEPT CONNECTION

69. This problem will prepare you for the Concept Connection on page 512.

The equation for the motion of an object with constant acceleration is $d = vt + \frac{1}{2}at^2$ where d is distance traveled in feet, v is starting velocity in ft/s, a is acceleration in ft/s^2, and t is time in seconds.

a. A toy car begins with a velocity of 2 ft/s and accelerates at 2 ft/s^2. Write an expression for the distance the toy car travels after t seconds.

b. What is the GCF of the terms of your expression from part **a**?

Multiple Choice **For Exercises 70 and 71, choose the best answer.**

70. Which set of numbers has a GCF greater than 6?

Ⓐ 18, 24, 36 Ⓑ 30, 35, 40 Ⓒ 11, 29, 37 Ⓓ 16, 24, 48

71. The slope of a line is the GCF of 48 and 12. The y-intercept is the GCF of the slope and 8. Which equation describes the line?

Ⓐ $y = 12x + 4$ Ⓑ $y = 6x + 2$ Ⓒ $y = 4x + 4$ Ⓓ $y = 3x + 1$

72. **Extended Response** Patricia is making a dog pen in her back yard. The pen will be rectangular and have an area of 24 square feet. Draw and label a diagram that shows all possible whole-number dimensions for the pen. Find the perimeter of each rectangle you drew. Which dimensions should Patricia use in order to spend the least amount of money on fencing materials? Explain your reasoning.

CHALLENGE AND EXTEND

Find the GCF of each set.

73. $4n^3, 16n^2, 8n$

74. $27y^3, 18y^2, 81y$

75. $100, 25s^5, 50s$

76. $2p^4r, 8p^3r^2, 16p^2r^3$

77. $2x^3y, 8x^2y^2, 17xy^3$

78. $8a^4b^3, 4a^3b^3, 12a^2b^3$

79. **Geometry** The area of a triangle is 10 in^2. What are the possible whole-number dimensions for the base and height of the triangle?

80. **Number Sense** The GCF of three different numbers is 7. The sum of the three numbers is 105. What are the three numbers?

81. **Critical Thinking** Find three different *composite* numbers whose GCF is 1. (*Hint:* A composite number has factors other than 1 and itself.)

SPIRAL STANDARDS REVIEW 8.0, 10.0

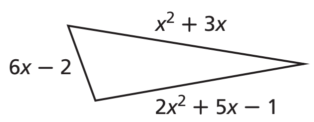
Identify which lines are parallel. *(Lesson 5-7)*

82. $y = 2x + 6;\ y = 5x + 2;\ y = 2x;\ y = 2$

83. $y = 4x - 2;\ y = 9;\ y = 9x;\ y = 8$

84. At a local grocery store, grapes cost \$2/lb and cherries cost \$3/lb. How many pounds of each should be used to make a 10 lb mixture that costs \$2.30/lb? *(Lesson 6-5)*

85. Write a simplified polynomial expression for the perimeter of the triangle. *(Lesson 7-7)*



LOGICAL REASONING

Use with Lesson 8-1

Indirect Proofs

In Chapter 1, you learned that some numbers are irrational. You now have enough knowledge to *prove* that some numbers are irrational by using deductive reasoning.

California Standards

25.1 Students use properties of numbers to construct simple, valid arguments (direct and **indirect**) **for,** or formulate counterexamples to, **claimed assertions.**

Example 1

Prove that if a prime number p is a factor of x^2, then p is also a factor of x.

Statements	Reasons
1. $x = p_1p_2p_3...p_n$ where $p_1, p_2, p_3, ..., p_n$ are prime.	Every number can be written as the product of primes.
2. $x^2 = x \cdot x = (p_1p_2p_3...p_n)(p_1p_2p_3...p_n)$	Substitute $p_1p_2p_3...p_n$ for x in Step 1.

The expression $(p_1p_2p_3...p_n)(p_1p_2p_3...p_n)$ contains only prime numbers, so it is the prime factorization of x^2. Since it contains only prime numbers that are factors of x, all prime numbers that are factors of x^2 are factors of x.

In other words, if a prime number is a factor of x^2, then it is also a factor of x.

Proving that some numbers are irrational often requires an *indirect proof*. In an **indirect proof,** first assume that the statement you want to prove is false—in other words, assume that the opposite of the statement is true.

Then use deductive reasoning to find a **contradiction**—two statements that cannot both be true at the same time.

An assumption that leads to a contradiction is false. Therefore, if you assume the opposite of a statement is true, and this leads to a contradiction, the assumption is false. This means the original statement must be true.

Try This

Write the opposite of each statement.

1. Susie is an only child.
2. All squares have four sides.
3. The sum of two even numbers is always even.

Find the two statements in each set that cannot both be true at the same time.

4. *a* and *b* are opposites.
   a and *b* are both negative.
   a and *b* are integers.

5. Angles *A* and *B* are both acute.
   Angles *A* and *B* are adjacent.
   Angles *A* and *B* are supplementary.

Example 2

Use an indirect proof to show that all prime numbers have irrational square roots.

Statements	Reasons
1. Suppose there were a number p that is prime and $\sqrt{p}$ is rational.	Assume the opposite of the statement you want to prove. In other words, assume the statement you want to prove is false.
2. $\sqrt{p} = \frac{a}{b}$ for integers a and b, $b \neq 0$. Assume a and b have no common factors besides 1; in other words, $\frac{a}{b}$ is in simplest form.	Definition of rational number; every rational number can be written in simplest form.
3. $p = \left(\frac{a}{b}\right)^2$	Definition of square root
4. $p = \frac{a^2}{b^2}$	Power of a Quotient Property
5. $pb^2 = a^2$	Multiplication Property of Equality (Multiply both sides by b^2.)
6. p is a factor of a^2.	Definition of factor
7. p is a factor of a.	Example 1
8. a can be written as the product of p and some number n: $a = pn$	Definition of factor
9. $pb^2 = (pn)^2$	Substitute pn for a in Statement 5.
10. $pb^2 = p^2n^2$	Product of a Power Property
12. $b^2 = pn^2$	Division Property of Equality (Divide both sides by p, $p \neq 0$.)
13. p is a factor of b^2.	Definition of factor
14. p is a factor of b.	Example 1
15. p is a factor of both a and b.	Statements 7 and 14

Once you have proven a statement, you can use that statement in other proofs.

Statement 15 contradicts Statement 2. Therefore, the original statement, Statement 1, is false. There are no prime numbers that have rational square roots. In other words, all prime numbers have irrational square roots.

Try This

6. Complete the following indirect proof to show that there are infinitely many prime numbers.

Assume that there is a finite number of **a.** ___?___.

Then there is a prime number p that is the greatest prime number. Let n be the product of all prime numbers. In other words, $n = 2 \cdot 3 \cdot 5 \cdot \ldots \cdot p$. Then $n + 1 =$ **b.** ___?___. Since all numbers have a unique prime factorization, $n + 1$ must have a(n) **c.** ___?___.

However, 2 is not a factor of $n + 1$ because $n + 1$ is 1 greater than a multiple of **d.** ___?___. Also, 3 is not a factor of **e.** ___?___ because $n + 1$ is **f.** ___?___. There is no prime number that is a factor of $n + 1$ because $n + 1$ is 1 greater than a multiple of **g.** ___?___. So, $n + 1$ does not have a prime factorization. This is a contradiction.

Therefore, the original assumption is false and **h.** ___?___.

8-2 Algebra LAB

Model Factorization by GCF

You can use algebra tiles to write a polynomial as the product of its factors. This process is called factoring. Factoring is the reverse of multiplying.

Use with Lesson 8-2

KEY

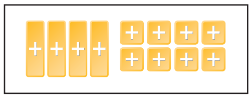
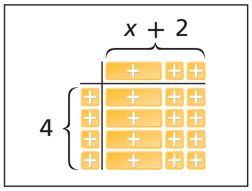
+ = 1   − = −1   + = x − = $-x$ + = x^2

California Standards

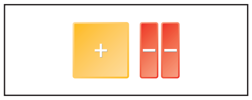
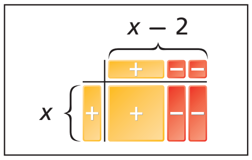
11.0 Students apply basic factoring techniques to second- and simple third-**degree polynomials.** These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

Activity

Use algebra tiles to factor $4x + 8$.

MODEL		ALGEBRA
	*Model $4x + 8$.*	$4x + 8$
	*Arrange the tiles into a rectangle. The total area represents $4x + 8$. The length and width represent the factors. The rectangle has a width of $x + 2$ and a length of 4.*	$4x + 8 = 4(x + 2)$

Use algebra tiles to factor $x^2 - 2x$.

MODEL		ALGEBRA
	*Model $x^2 - 2x$.*	$x^2 - 2x$
	*Arrange the tiles into a rectangle. The total area represents $x^2 - 2x$. The length and width represent the factors. The rectangle has a width of $x - 2$ and a length of x.*	$x^2 - 2x = x(x - 2)$

Try This

Use algebra tiles to factor each polynomial.

1. $3x + 9$ **2.** $2x + 8$ **3.** $4x - 12$ **4.** $3x - 12$

5. $2x^2 + 2x$ **6.** $x^2 + 4x$ **7.** $x^2 - 3x$ **8.** $2x^2 - 4x$

8-2 Factoring by GCF

California Standards

11.0 Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

Why learn this?

You can determine the dimensions of a solar panel by factoring an expression representing the panel's area. (See Example 2.)

Recall the Distributive Property: $ab + ac = a(b + c)$. The Distributive Property allows you to "factor" out the GCF of the terms in a polynomial.

A polynomial is fully factored when it is written as a product of monomials and polynomials whose terms have no common factors other than 1.

Fully Factored	$2(3x - 4)$	Neither 2 nor $3x - 4$ can be factored.
Not Fully Factored	$2(3x - 4x)$	$3x - 4x$ can be factored. The terms have a common factor of x.

EXAMPLE 1 Factoring by Using the GCF

Writing Math

Aligning common factors can help you find the greatest common factor of two or more terms.

Factor each polynomial. Check your answer.

A $4x^2 - 3x$

$$\begin{aligned} 4x^2 &= 2 \cdot 2 \cdot \boxed{x} \cdot x \\ 3x &= 3 \cdot \boxed{x} \\ &\quad x \end{aligned}$$

Find the GCF.

The GCF of $4x^2$ and $3x$ is x.

$4x(x) - 3(x)$ *Write terms as products using the GCF as a factor.*

$x(4x - 3)$ *Use the Distributive Property to factor out the GCF.*

Check $x(4x - 3)$ *Multiply to check your answer.*

$4x^2 - 3x$ ✓ *The product is the original polynomial.*

B $10y^3 + 20y^2 - 5y$

$$\begin{aligned} 10y^3 &= 2 \cdot \boxed{5} \cdot \boxed{y} \cdot y \cdot y \\ 20y^2 &= 2 \cdot 2 \cdot \boxed{5} \cdot \boxed{y} \cdot y \\ 5y &= \boxed{5} \cdot \boxed{y} \\ &\quad 5 \cdot y = 5y \end{aligned}$$

Find the GCF.

The GCF of $10y^3$, $20y^2$, and $5y$ is $5y$.

$2y^2(5y) + 4y(5y) - 1(5y)$ *Write terms as products using the GCF as a factor.*

$5y(2y^2 + 4y - 1)$ *Use the Distributive Property to factor out the GCF.*

Check $5y(2y^2 + 4y - 1)$ *Multiply to check your answer.*

$10y^3 + 20y^2 - 5y$ ✓ *The product is the original polynomial.*

Factor each polynomial. Check your answer.

C $-12x - 8x^2$

$-1(12x + 8x^2)$ *Both coefficients are negative. Factor out −1.*

$12x = \boxed{2} \cdot \boxed{2} \cdot 3 \cdot \boxed{x}$ *Find the GCF.*

$8x^2 = \boxed{2} \cdot \boxed{2} \cdot 2 \cdot \boxed{x} \cdot x$

$2 \cdot 2 \cdot \quad x = 4x$ *The GCF of $12x$ and $8x^2$ is $4x$.*

$-1[3(4x) + 2x(4x)]$ *Write each term as a product using the GCF.*

$-1[4x(3 + 2x)]$ *Use the Distributive Property to factor out the GCF.*

$-1(4x)(3 + 2x)$

$-4x(3 + 2x)$

Check

$-4x(3 + 2x) = -12x - 8x^2$ ✓ *Multiply to check your answer.*

D $5x^2 + 7$

$5x^2 = 5 \cdot x \cdot x$ *Find the GCF.*

$7 = 7$

$5x^2 + 7$ *There are no common factors other than 1.*

The polynomial cannot be factored further.

Caution!

When you factor out −1 as the first step, be sure to include it in all the other steps as well.

CHECK IT OUT! **Factor each polynomial. Check your answer.**

1a. $5b + 9b^3$ **1b.** $9d^2 - 8^2$

1c. $-18y^3 - 7y^2$ **1d.** $8x^4 + 4x^3 - 2x^2$

To write expressions for the length and width of a rectangle with area expressed by a polynomial, you need to write the polynomial as a product. You can write a polynomial as a product by factoring it.

EXAMPLE 2 *Science Application*

Mandy's calculator is powered by solar energy. The area of the solar panel is $(7x^2 + x)$ cm^2. Factor this polynomial to find possible expressions for the dimensions of the solar panel.

$A = 7x^2 + x$ *The GCF of $7x^2$ and x is x.*

$= 7x(x) + 1(x)$ *Write each term as a product using the GCF as a factor.*

$= x(7x + 1)$ *Use the Distributive Property to factor out the GCF.*

Possible expressions for the dimensions of the solar panel are x cm and $(7x + 1)$ cm.

CHECK IT OUT! **2. What if...?** The area of the solar panel on another calculator is $(2x^2 + 4x)$ cm^2. Factor this polynomial to find possible expressions for the dimensions of the solar panel.

Sometimes the GCF of terms is a binomial. This GCF is called a *common binomial factor*. You factor out a common binomial factor the same way you factor out a monomial factor.

EXAMPLE 3 Factoring Out a Common Binomial Factor

Factor each expression.

A $7(x-3)-2x(x-3)$

$7(x-3)-2x(x-3)$ — *The terms have a common binomial factor of $(x-3)$.*

$(x-3)(7-2x)$ — *Factor out $(x-3)$.*

B $-t(t^2+4)+(t^2+4)$

$-t(t^2+4)+(t^2+4)$ — *The terms have a common binomial factor of (t^2+4).*

$-t(t^2+4)+1(t^2+4)$ — *$(t^2+4)=1(t^2+4)$*

$(t^2+4)(-t+1)$ — *Factor out (t^2+4).*

C $9x(x+4)-5(4+x)$

$9x(x+4)-5(4+x)$ — *$(x+4)=(4+x)$, so the terms have a common binomial factor of $(x+4)$.*

$9x(x+4)-5(x+4)$

$(x+4)(9x-5)$ — *Factor out $(x+4)$.*

D $-3x^2(x+2)+4(x-7)$

$-3x^2(x+2)+4(x-7)$ — *There are no common factors.*

The expression cannot be factored.

CHECK IT OUT! **Factor each expression.**

3a. $4s(s+6)-5(s+6)$ **3b.** $7x(2x+3)+(2x+3)$

3c. $3x(y+4)-2y(x+4)$ **3d.** $5x(5x-2)-2(5x-2)$

You may be able to factor a polynomial by grouping. When a polynomial has four terms, you can sometimes make two groups and factor out the GCF from each group.

EXAMPLE 4 Factoring by Grouping

Factor each polynomial by grouping. Check your answer.

A $12a^3-9a^2+20a-15$

$(12a^3-9a^2)+(20a-15)$ — *Group terms that have a common number or variable as a factor.*

$3a^2(4a-3)+5(4a-3)$ — *Factor out the GCF of each group.*

$3a^2(4a-3)+5(4a-3)$ — *$(4a-3)$ is another common factor.*

$(4a-3)(3a^2+5)$ — *Factor out $(4a-3)$.*

Check $(4a-3)(3a^2+5)$ — *Multiply to check your solution.*

$4a(3a^2)+4a(5)-3(3a^2)-3(5)$

$12a^3+20a-9a^2-15$

$12a^3-9a^2+20a-15$ ✓ — *The product is the original polynomial.*

Factor each polynomial by grouping. Check your answer.

B $9x^3 + 18x^2 + x + 2$

$(9x^3 + 18x^2) + (x + 2)$ — *Group terms.*

$9x^2(x + 2) + 1(x + 2)$ — *Factor out the GCF of each group.*

$9x^2(x + 2) + 1(x + 2)$ — *$(x + 2)$ is a common factor.*

$(x + 2)(9x^2 + 1)$ — *Factor out $(x + 2)$.*

Check $(x + 2)(9x^2 + 1)$ — *Multiply to check your solution.*

$x(9x^2) + x(1) + 2(9x^2) + 2(1)$

$9x^3 + x + 18x^2 + 2$

$9x^3 + 18x^2 + x + 2$ ✓ — *The product is the original polynomial.*

CHECK IT OUT! **Factor each polynomial by grouping. Check your answer.**

4a. $6b^3 + 8b^2 + 9b + 12$ **4b.** $4r^3 + 24r + r^2 + 6$

Helpful Hint

If two quantities are opposites, their sum is 0.

$(5 - x) + (x - 5)$
$5 - x + x - 5$
$-x + x + 5 - 5$
$0 + 0$
0

Recognizing opposite binomials can help you factor polynomials. The binomials $(5 - x)$ and $(x - 5)$ are opposites. Notice $(5 - x)$ can be written as $-1(x - 5)$.

$-1(x - 5) = (-1)(x) + (-1)(-5)$ — *Distributive Property*

$= -x + 5$ — *Simplify.*

$= 5 - x$ — *Commutative Property of Addition*

So, $(5 - x) = -1(x - 5)$.

EXAMPLE 5 Factoring with Opposites

Factor $3x^3 - 15x^2 + 10 - 2x$.

$(3x^3 - 15x^2) + (10 - 2x)$ — *Group terms.*

$3x^2(x - 5) + 2(5 - x)$ — *Factor out the GCF of each group.*

$3x^2(x - 5) + 2(-1)(x - 5)$ — *Write $(5 - x)$ as $-1(x - 5)$.*

$3x^2(x - 5) - 2(x - 5)$ — *Simplify. $(x - 5)$ is a common factor.*

$(x - 5)(3x^2 - 2)$ — *Factor out $(x - 5)$.*

CHECK IT OUT! **Factor each polynomial. Check your answer.**

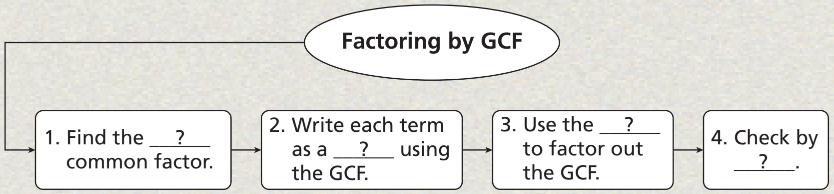
5a. $15x^2 - 10x^3 + 8x - 12$ **5b.** $8y - 8 - x + xy$

THINK AND DISCUSS

1. Explain how finding the GCF of monomials helps you factor a polynomial.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer.



8-2 Exercises

California Standards Practice
11.0, 25.1, 25.2

go.hrw.com
Homework Help Online
KEYWORD: MA8CA 8-2
Parent Resources Online
KEYWORD: MA8CA Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 487

Factor each polynomial. Check your answer.

1. $15a - 5a^2$ **2.** $10g^3 - 3g$ **3.** $-35x + 42$

4. $-4x^2 - 6x$ **5.** $12h^4 + 8h^2 - 6h$ **6.** $3x^2 - 9x + 3$

7. $9m^2 + m$ **8.** $14n^3 + 7n + 7n^2$ **9.** $36f + 18f^2 + 3$

SEE EXAMPLE 2 p. 488

10. Physical Science A model rocket is fired vertically into the air at 320 ft/s. The expression $-16t^2 + 320t$ gives the rocket's height after t seconds. Factor this expression.

SEE EXAMPLE 3 p. 489

Factor each expression.

11. $2b(b + 3) + 5(b + 3)$ **12.** $5(m - 2) - m(m - 2)$ **13.** $4(x - 3) - x(y + 2)$

SEE EXAMPLE 4 p. 489

Factor each polynomial by grouping. Check your answer.

14. $6x^3 + 4x^2 + 3x + 2$ **15.** $x^3 + 4x^2 + 2x + 8$ **16.** $10a^3 + 4a^2 + 5a + 2$

17. $7r^3 - 35r^2 + 6r - 30$ **18.** $2m^3 + 4m^2 + 6m + 12$ **19.** $4b^3 - 6b^2 + 10b - 15$

SEE EXAMPLE 5 p. 490

20. $6b^2 - 3b + 4 - 8b$ **21.** $2r^2 - 6r + 12 - 4r$ **22.** $6a^3 - 9a^2 - 12 + 8a$

23. $2m^3 - 6m^2 + 9 - 3m$ **24.** $3r - r^2 + 2r - 6$ **25.** $14q^2 - 21q + 6 - 4q$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
26–34	1
35	2
36–41	3
42–47	4
48–53	5

Extra Practice

Skills Practice p. EP16
Application Practice p. EP31

Factor each polynomial. Check your answer.

26. $36d^3 + 24$ **27.** $9y^2 + 45y$ **28.** $14x^3 + 63x^2 - 7x$

29. $-4d^4 + d^3 - 3d^2$ **30.** $-15f - 10f^2$ **31.** $-14x^4 + 5x^2$

32. $33d^3 + 22d + 11$ **33.** $21c^2 + 14c$ **34.** $-5g^3 - 15g^2$

35. Finance After t years, the amount of money in a savings account that earns simple interest is $P + Prt$, where P is the starting amount and r is the yearly interest rate. Factor this expression.

Factor each expression.

36. $-4x(x + 2) + 9(x + 2)$ **37.** $6a(a - 2) - 5b(b + 4)$ **38.** $5(3x - 2) + x(3x - 2)$

39. $-3(2 + b) + 4b(b + 2)$ **40.** $a(x - 3) + 2b(x - 3)$ **41.** $6y(y - 7) + (y - 7)$

Factor each polynomial by grouping. Check your answer.

42. $x^3 + 3x^2 + 5x + 15$ **43.** $2a^3 - 8a^2 + 3a - 12$ **44.** $10b^3 - 16b^2 + 25b - 40$

45. $n^3 - 2n^2 + 5n - 10$ **46.** $7x^3 + 2x^2 + 28x + 8$ **47.** $6x^3 + 18x^2 + x + 3$

48. $2d^3 - d^2 - 3 + 6d$ **49.** $2m^3 - 2m^2 + 3 - 3m$ **50.** $20 - 15x - 6x^2 + 8x$

51. $b^3 - 2b - 8 + 4b^2$ **52.** $5k^2 - k^3 + 3k - 15$ **53.** $6f^3 - 8f^2 + 20 - 15f$

54. Art Factor the expression for the area of the mural shown at right.

Fill in the missing part of each factorization.

55. $16v + 12v^2 = 4v(4 + \square)$

56. $15x - 25x^2 = 5x(3 - \square)$

57. $-16k^3 - 24k^2 = -8k^2(\square + 3)$

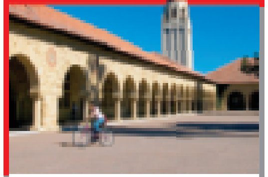
58. $-x - 10 = -1(\square + 10)$

Copy and complete the table.

	Polynomial	Number of Terms	Name	Completely Factored Form
	$3y + 3x + 9$	3	trinomial	$3(y + x + 3)$
59.	$x^2 + 5x$	$\square$	$\square$	$\square$
60.	$28c^2 - 49c$	$\square$	$\square$	$\square$
61.	$a^4 + a^3 + a^2$	$\square$	$\square$	$\square$
62.	$36 + 99r - 40r^2 - 110r^3$	$\square$	$\square$	$\square$

California LINK

Finance



About 76% of undergraduate students at Stanford University receive some form of financial aid to help with their college costs.

Source: Stanford University

63. Personal Finance The final amount of money in a certificate of deposit (CD) after n years can be represented by the expression Px^n, where P is the original amount contributed and x is the interest rate.

Year	Original Amount
2004	\$100.00
2005	\$200.00
2006	\$400.00

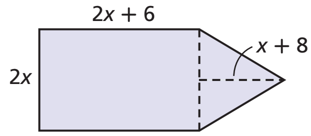
Justin's aunt purchased CDs to help him pay for college. The table shows the amount of the CD she purchased each year. In 2007, she will pay \$800.00 directly to the college.

a. Each CD has the same interest rate x. Write expressions for the value of the CDs purchased in 2004, 2005, and 2006 when Justin starts college in 2007.

b. Write a polynomial to represent the total value of the CDs purchased in 2004, 2005, and 2006 plus the amount paid to the college in 2007.

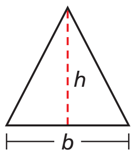
c. Factor the polynomial in part **c** by grouping. Evaluate the factored form of the polynomial when the interest rate is 1.09.

64. Write About It Describe how to find the area of the figure shown. Show each step and write your answer in factored form.



65. Critical Thinking Show two methods of factoring the expression $3a - 3b - 4a + 4b$.

66. Geometry The area of the triangle is represented by the expression $\frac{1}{2}(x^3 - 2x + 2x^2 - 4)$. The height of the triangle is $x + 2$. Write an expression for the base of the triangle. (*Hint:* The formula for the area of a triangle is $A = \frac{1}{2}bh$.)



67. Write About It Explain how you know when two binomials are opposites.

CONCEPT CONNECTION

68. This problem will prepare you for the Concept Connection on page 512.

a. What must be true about either a or b if $ab = 0$?

b. A toy car's distance in feet from the starting point is given by the equation $d = t(3 - t)$. Explain why $t(3 - t) = 0$ means that either $t = 0$ or $3 - t = 0$.

c. When $d = 0$, the car is at the starting point. Use the fact that $t = 0$ or $3 - t = 0$ when $d = 0$ to find the two times when the car is at the starting point.

69. **Reasoning** Fill in each blank with a property or definition that justifies the step.

$$\begin{aligned} 7x^3 + 2x + 21x^2 + 6 &= 7x^3 + 21x^2 + 2x + 6 && \textbf{a.}\ \underline{\quad ? \quad} \\ &= (7x^3 + 21x^2) + (2x + 6) && \textbf{b.}\ \underline{\quad ? \quad} \\ &= 7x^2(x+3) + 2(x+3) && \textbf{c.}\ \underline{\quad ? \quad} \\ &= (x+3)(7x^2+2) && \textbf{d.}\ \underline{\quad ? \quad} \end{aligned}$$

70. **ERROR ANALYSIS** Which factorization of $3n^3 - n^2$ is incorrect? Explain.

A

$3n^3 - n^2$

$n^2(3n) - n^2(0)$

$n^2(3n - 0)$

B

$3n^3 - n^2$

$n^2(3n) - n^2(1)$

$n^2(3n - 1)$

Multiple Choice **For Exercises 71–73, choose the best answer.**

71. Which is the complete factorization of $24x^3 - 12x^2$?

Ⓐ $6(4x^3 - 2x^2)$ Ⓑ $12(2x^3 - x^2)$ Ⓒ $12x(2x^2 - x)$ Ⓓ $12x^2(2x - 1)$

72. Which is NOT a factor of $18x^2 + 36x$?

Ⓐ 1 Ⓑ $4x$ Ⓒ $x + 2$ Ⓓ $18x$

73. The area of a rectangle is represented by the polynomial $x^2 + 3x - 6x - 18$. Which of the following could represent the length and width of the rectangle?

Ⓐ Length: $x + 3$; width: $x + 6$

Ⓑ Length: $x - 3$; width: $x - 6$

Ⓒ Length: $x + 3$; width: $x - 6$

Ⓓ Length: $x - 3$; width: $x + 6$

CHALLENGE AND EXTEND

Factor each polynomial. Check your answer.

74. $6ab^2 - 24a^2$

75. $-72a^2b^2 - 45ab$

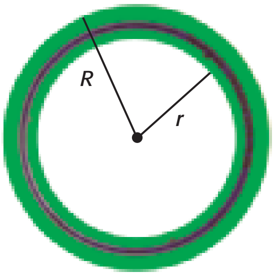
76. $-18a^2b^2 + 21ab$

77. $ab + bc + ad + cd$

78. $4y^2 + 8ay - y - 2a$

79. $x^3 - 4x^2 + 3x - 12$

80. **Geometry** The area between two concentric circles is called an *annulus*. The formula for area of an annulus is $A = \pi R^2 - \pi r^2$, where R is the radius of the larger circle and r is the radius of the smaller circle.



a. Factor the formula for area of an annulus by using the GCF.

b. Use the factored form to find the area of an annulus with $R = 12$ cm and $r = 5$ cm.

SPIRAL STANDARDS REVIEW

2.0, 8.0, 9.0, 25.1

81. The coordinates of the vertices of a quadrilateral are $A(-2, 5)$, $B(6, 5)$, $C(4, -3)$, and $D(-4, -3)$. Use slope to show that $ABCD$ is a parallelogram. *(Lesson 5-7)*

Solve each system by elimination. *(Lesson 6-3)*

82. $\begin{cases} x + 2y = 11 \\ 2x - y = 2 \end{cases}$

83. $\begin{cases} -2x + 2y = 14 \\ x - 3y = -19 \end{cases}$

84. $\begin{cases} 3x + 3y = 12 \\ x - 7y = -12 \end{cases}$

Simplify. *(Lesson 7-4)*

85. $\dfrac{5^6}{5^4}$

86. $\dfrac{4^5 \cdot 4^2}{4^6}$

87. $\dfrac{x^5y^3z}{y^3z^4}$

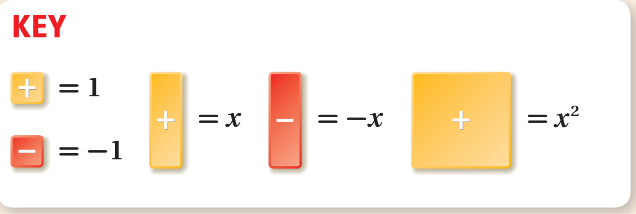
88. $\dfrac{x^8y^3z^7}{x^2y^4z}$

8-3 Algebra LAB

Model Factorization of $x^2 + bx + c$

You can use algebra tiles to express a trinomial as a product of two binomials. This is called factoring a trinomial.

Use with Lesson 8-3

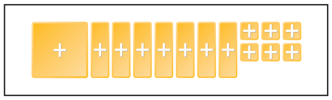
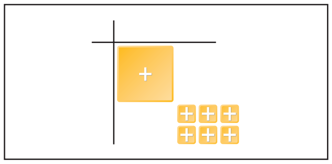
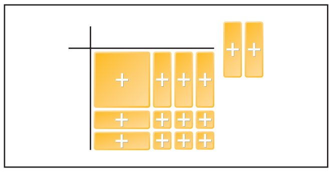
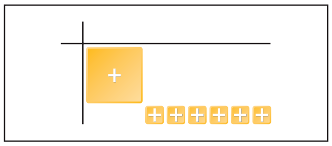
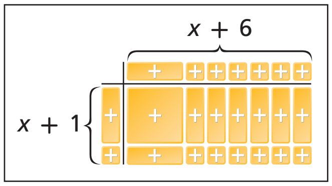


California Standards

11.0 Students apply basic factoring techniques to second- and simple third-**degree polynomials.** These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

Activity 1

Use algebra tiles to factor $x^2 + 7x + 6$.

MODEL		ALGEBRA
	*Model $x^2 + 7x + 6$.*	$x^2 + 7x + 6$
	*Try to arrange all of the tiles in a rectangle. Start by placing the x^2-tile in the upper left corner.* *Arrange the unit tiles in a rectangle so that the top left corner of this rectangle touches the bottom right corner of the x^2-tile.*	$x^2 + 7x + 6 \neq (x + 2)(x + 3)$
	*Arrange the x-tiles so that all the tiles together make one large rectangle.* *This arrangement does not work because two x-tiles are left over.*	
	*Rearrange the unit tiles to form another rectangle.*	$x^2 + 7x + 6 = (x + 1)(x + 6)$
	*Fill in the empty spaces with x-tiles. All 7 x-tiles fit. This is the correct arrangement.* *The total area represents the trinomial. The length and width represent the factors.*	

The rectangle has width $x + 1$ and length $x + 6$. So $x^2 + 7x + 6 = (x + 1)(x + 6)$.

Try This

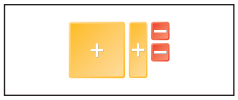
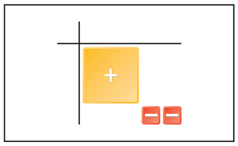
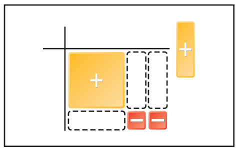
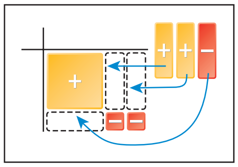
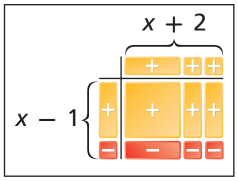
Use algebra tiles to factor each trinomial.

1. $x^2 + 2x + 1$ **2.** $x^2 + 3x + 2$ **3.** $x^2 + 6x + 5$ **4.** $x^2 + 6x + 9$

5. $x^2 + 5x + 4$ **6.** $x^2 + 6x + 8$ **7.** $x^2 + 5x + 6$ **8.** $x^2 + 8x + 12$

Activity 2

Use algebra tiles to factor $x^2 + x - 2$.

MODEL		ALGEBRA
	*Model $x^2 + x - 2$.*	$x^2 + x - 2$
	*Start by placing the x^2-tile in the upper left corner.* *Arrange the unit tiles in a rectangle so that the top left corner of this rectangle touches the bottom right corner of the x^2-tile.*	
	*To make a rectangle, you need to fill in the empty spaces, but there aren't enough x-tiles to fill in the empty spaces.*	
	*Add a zero pair. Arrange the x-tiles to complete the rectangle.* *The tiles in each row of the rectangle must be the same color.*	
	*The total area represents the trinomial. The length and width represent the factors.*	$x^2 + x - 2 = (x - 1)(x + 2)$

The rectangle has width $x - 1$ and length $x + 2$. So, $x^2 + x - 2 = (x - 1)(x + 2)$.

Try This

9. Why can you add one red $-x$-tile and one yellow x-tile?

Use algebra tiles to factor each polynomial.

10. $x^2 - x - 2$ **11.** $x^2 - 2x - 3$ **12.** $x^2 - 5x + 4$ **13.** $x^2 - 7x + 10$

14. $x^2 - 2x + 1$ **15.** $x^2 - 6x + 5$ **16.** $x^2 + 5x - 6$ **17.** $x^2 + 3x - 4$

18. $x^2 - x - 6$ **19.** $x^2 + 3x - 10$ **20.** $x^2 - 2x - 8$ **21.** $x^2 + x - 12$

8-3 Factoring $x^2 + bx + c$

California Standards

11.0 Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

Why learn this?

Factoring polynomials will help you find the dimensions of rectangular shapes, such as a fountain. (See Exercise 77.)

In Chapter 7, you learned how to multiply two binomials using the Distributive Property or the FOIL method. In this lesson, you will learn how to factor a trinomial into two binomials.

Notice that when you multiply $(x + 2)(x + 5)$, the constant term in the trinomial is the product of the constants in the binomials.

$$(x + 2)(x + 5) = x^2 + 7x + 10$$

Use this fact to factor some trinomials into binomial factors. Look for two integers (positive or negative) that are factors of the constant term in the trinomial. Write two binomials with those integers, and then multiply to check.

If no two factors of the constant term work, we say the trinomial is not factorable.

EXAMPLE 1 Factoring Trinomials

Factor $x^2 + 19x + 60$. Check your answer.

$(\square + \square)(\square + \square)$ *Write two sets of parentheses.*

$(x + \square)(x + \square)$ *The first term is x^2, so the variable terms have a coefficient of 1.*

The constant term in the trinomial is 60.

Try integer factors of 60 for the constant terms in the binomials.

$(x + 1)(x + 60) = x^2 + 61x + 60$ ✗

$(x + 2)(x + 30) = x^2 + 32x + 60$ ✗

$(x + 3)(x + 20) = x^2 + 23x + 60$ ✗

$(x + 4)(x + 15) = x^2 + 19x + 60$ ✓

The factors of $x^2 + 19x + 60$ are $(x + 4)$ and $(x + 15)$.

$x^2 + 19x + 60 = (x + 4)(x + 15)$

Check $(x + 4)(x + 15) = x^2 + 15x + 4x + 60$ *Use the FOIL method.*

$= x^2 + 19x + 60$ ✓ *The product is the original trinomial.*

Remember!

When you multiply two binomials, multiply:

First terms
Outer terms
Inner terms
Last terms

CHECK IT OUT! **Factor each trinomial. Check your answer.**

1a. $x^2 + 10x + 24$ **1b.** $x^2 + 7x + 12$

The method of factoring used in Example 1 can be made more efficient. Look at the product of $(x + a)$ and $(x + b)$.

$$(x+a)(x+b) = x^2 + ax + bx + ab$$
$$= x^2 + (a+b)x + ab$$

The coefficient of the middle term is the sum of a and b. The third term is the product of a and b.

Know it! Note

Factoring $x^2 + bx + c$

WORDS	EXAMPLE
To factor a quadratic trinomial of the form $x^2 + bx + c$, find two integer factors of c whose sum is b. If no such integers exist, we say the trinomial is not factorable.	To factor $x^2 + 9x + 18$, look for integer factors of 18 whose sum is 9.

Factors of 18	Sum	
1 and 18	19	✗
2 and 9	11	✗
3 and 6	9	✓

$x^2 + 9x + 18$
$(x + 3)(x + 6)$

When c is positive, its factors have the same sign. The sign of b tells you whether the factors are positive or negative. When b is positive, the factors are positive, and when b is negative, the factors are negative.

EXAMPLE 2 Factoring $x^2 + bx + c$ When c Is Positive

Factor each trinomial. Check your answer.

A $x^2 + 6x + 8$

$(x + \square)(x + \square)$ *$b = 6$ and $c = 8$; look for factors of 8 whose sum is 6.*

Factors of 8	Sum	
1 and 8	9	✗
2 and 4	6	✓

The factors needed are 2 and 4.

$(x + 2)(x + 4)$

Check $(x + 2)(x + 4) = x^2 + 4x + 2x + 8$ *Use the FOIL method.*

$= x^2 + 6x + 8$ ✓ *The product is the original trinomial.*

B $x^2 + 5x + 6$

$(x + \square)(x + \square)$ *$b = 5$ and $c = 6$; look for factors of 6 whose sum is 5.*

Factors of 6	Sum	
1 and 6	7	✗
2 and 3	5	✓

The factors needed are 2 and 3.

$(x + 2)(x + 3)$

Check $(x + 2)(x + 3) = x^2 + 3x + 2x + 6$ *Use the FOIL method.*

$= x^2 + 5x + 6$ ✓ *The product is the original trinomial.*

Factor each trinomial. Check your answer.

C $x^2 - 10x + 16$

$(x + \square)(x + \square)$ — *$b = -10$ and $c = 16$; look for factors of 16 whose sum is -10.*

Factors of 16	Sum	
-1 and -16	-17	✗
-2 and -8	-10	✓
-4 and -4	-8	✗

The factors needed are -2 and -8.

$(x - 2)(x - 8)$

Check $(x - 2)(x - 8) = x^2 - 8x - 2x + 16$ — *Use the FOIL method.*

$= x^2 - 10x + 16$ ✓ — *The product is the original trinomial.*

CHECK IT OUT! **Factor each trinomial. Check your answer.**

2a. $x^2 + 8x + 12$ **2b.** $x^2 - 5x + 6$

2c. $x^2 + 13x + 42$ **2d.** $x^2 - 13x + 40$

When c is negative, its factors have opposite signs. The sign of b tells you which factor is positive and which is negative. The factor with the greater absolute value has the same sign as b.

EXAMPLE 3 Factoring $x^2 + bx + c$ When c Is Negative

Factor each trinomial.

A $x^2 + 7x - 18$

$(x + \square)(x + \square)$ — *$b = 7$ and $c = -18$; look for factors of -18 whose sum is 7. The factor with the greater absolute value is positive.*

Factors of -18	Sum	
-1 and 18	17	✗
-2 and 9	7	✓
-3 and 6	3	✗

The factors needed are -2 and 9.

$(x - 2)(x + 9)$

B $x^2 - 5x - 24$

$(x + \square)(x + \square)$ — *$b = -5$ and $c = -24$; look for factors of -24 whose sum is -5. The factor with the greater absolute value is negative.*

Factors of -24	Sum	
1 and -24	-23	✗
2 and -12	-10	✗
3 and -8	-5	✓
4 and -6	-2	✗

The factors needed are 3 and -8.

$(x + 3)(x - 8)$

Helpful Hint

If you have trouble remembering the rules for which factor is positive and which is negative, you can try all the factor pairs and check their sums.

CHECK IT OUT! **Factor each trinomial. Check your answer.**

3a. $x^2 + 2x - 15$ **3b.** $x^2 - 6x + 8$ **3c.** $x^2 - 8x - 20$

A polynomial and the factored form of the polynomial are equivalent expressions. When you evaluate these two expressions for the same value of the variable, the results are the same.

EXAMPLE 4 Evaluating Polynomials

Factor $n^2 + 11n + 24$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3,$ and 4.

$n^2 + 11n + 24$

$(n + \square)(n + \square)$ *$b = 11$ and $c = 24$; look for factors of 24 whose sum is 11.*

Factors of 24	Sum	
1 and 24	25	✗
2 and 12	14	✗
3 and 8	11	✓
4 and 6	10	✗

The factors needed are 3 and 8.

$(n + 3)(n + 8)$

Evaluate the original polynomial and the factored form for $n = 0, 1, 2, 3,$ and 4.

n	$n^2 + 11n + 24$
0	$0^2 + 11(0) + 24 = 24$
1	$1^2 + 11(1) + 24 = 36$
2	$2^2 + 11(2) + 24 = 50$
3	$3^2 + 11(3) + 24 = 66$
4	$4^2 + 11(4) + 24 = 84$

n	$(n + 3)(n + 8)$
0	$(0 + 3)(0 + 8) = 24$
1	$(1 + 3)(1 + 8) = 36$
2	$(2 + 3)(2 + 8) = 50$
3	$(3 + 3)(3 + 8) = 66$
4	$(4 + 3)(4 + 8) = 84$

The original polynomial and the factored form have the same value for the given values of n.

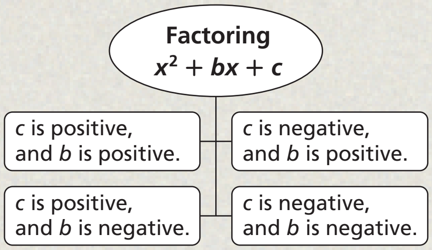
CHECK IT OUT!

4. Factor $n^2 - 7n + 10$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3,$ and 4.

THINK AND DISCUSS

1. Explain in your own words how to factor $x^2 + 9x + 14$. Show how to check your answer.
2. Explain how you can determine the signs of the factors of c when factoring a trinomial of the form $x^2 + bx + c$.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of a trinomial with the given properties and factor it.

Know it! Note



8-3 Exercises

California Standards Practice
10.0, 11.0, 15.0

go.hrw.com
Homework Help Online
KEYWORD: MA8CA 8-3
Parent Resources Online
KEYWORD: MA8CA Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 496

Factor each trinomial. Check your answer.

1. $x^2 + 13x + 36$
2. $x^2 + 11x + 24$
3. $x^2 + 14x + 40$
4. $x^2 + 5x + 4$
5. $x^2 + 6x + 5$
6. $x^2 + 8x + 15$

SEE EXAMPLE 2 p. 497

7. $x^2 + 10x + 16$
8. $x^2 + 4x + 3$
9. $x^2 - 11x + 24$
10. $x^2 - 9x + 14$
11. $x^2 - 7x + 6$
12. $x^2 + 15x + 44$

SEE EXAMPLE 3 p. 498

13. $x^2 + 6x - 27$
14. $x^2 - 6x - 7$
15. $x^2 - 4x - 45$
16. $x^2 - 3x - 18$
17. $x^2 - x - 2$
18. $x^2 + x - 30$

SEE EXAMPLE 4 p. 499

19. Factor $n^2 + 6n - 7$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3,$ and 4.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
20–25	1
26–31	2
32–37	3
38	4

Extra Practice

Skills Practice p. EP16
Application Practice p. EP31

Factor each trinomial. Check your answer.

20. $x^2 + 11x + 28$
21. $x^2 + 13x + 30$
22. $x^2 + 11x + 18$
23. $x^2 + 13x + 40$
24. $x^2 + 12x + 20$
25. $x^2 + 16x + 48$
26. $x^2 + 12x + 11$
27. $x^2 + 16x + 28$
28. $x^2 + 15x + 36$
29. $x^2 - 6x + 5$
30. $x^2 - 9x + 18$
31. $x^2 - 12x + 32$
32. $x^2 + x - 12$
33. $x^2 + 4x - 21$
34. $x^2 + 9x - 36$
35. $x^2 - 12x - 13$
36. $x^2 - 10x - 24$
37. $x^2 - 2x - 35$
38. Factor $n^2 - 12n - 45$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3,$ and 4.

Match each trinomial with its correct factorization.

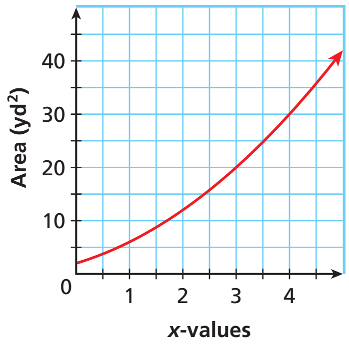
39. $x^2 + 3x - 10$ A. $(x - 2)(x - 5)$
40. $x^2 - 7x + 10$ B. $(x + 1)(x + 10)$
41. $x^2 - 9x - 10$ C. $(x - 2)(x + 5)$
42. $x^2 + 11x + 10$ D. $(x + 1)(x - 10)$

43. **Write About It** Compare multiplying binomials with factoring polynomials into binomial factors.

Factor each trinomial, if possible. Check your answer.

44. $x^2 + x - 20$
45. $x^2 - 11x + 18$
46. $x^2 - 4x - 21$
47. $x^2 + 10x + 9$
48. $x^2 - 12x + 32$
49. $x^2 + 13x + 42$
50. $x^2 - 7x - 12$
51. $x^2 + 11x + 18$
52. $x^2 - 6x - 27$
53. $x^2 + 5x - 24$
54. $x^2 - 10x + 21$
55. $x^2 + 4x - 45$
56. Factor $n^2 + 11n + 28$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3,$ and 4.

57. **Estimation** The graph shows the areas of rectangles with dimensions $(x + 1)$ yards and $(x + 2)$ yards. Estimate the value of x for a rectangle with area 9 square yards.



58. **Geometry** The area of a rectangle in square feet can be represented by $x^2 + 8x + 12$. The length is $(x + 6)$ ft. What is the width of the rectangle?

59. **Remodeling** A homeowner wants to enlarge a closet that has an area of $(x^2 + 3x + 2)$ ft^2. The length is $(x + 2)$ ft. After construction, the area will be $(x^2 + 8x + 15)$ ft^2 with a length of $(x + 3)$ ft.

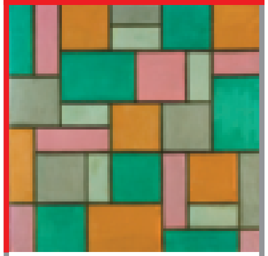
a. Find the dimensions of the closet before construction.

b. Find the dimensions of the closet after construction.

c. By how many feet will the length and width increase after construction?

LINK

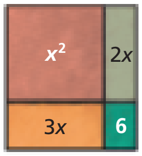
Art



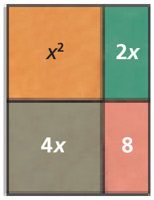
The Dutch painter Theo van Doesburg (1883–1931) is most famous for his paintings composed of lines and rectangles, such as the one shown above.

Art Write the polynomial modeled and then factor.

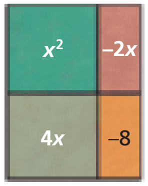
60.



61.



62.



Copy and complete the table.

	$x^2 + bx + c$	Sign of c	Binomial Factors	Signs of Numbers in Binomials
	$x^2 + 4x + 3$	Positive	$(x + 1)(x + 3)$	Both positive
63.	$x^2 - 4x + 3$		$(x \;\square\; 1)(x \;\square\; 3)$	
64.	$x^2 + 2x - 3$		$(x \;\square\; 1)(x \;\square\; 3)$	
65.	$x^2 - 2x - 3$		$(x \;\square\; 1)(x \;\square\; 3)$	

66. **Geometry** A rectangle has area $x^2 + 6x + 8$. The length is $x + 4$. Find the width of the rectangle. Could the rectangle be a square? Explain why or why not.

CONCEPT CONNECTION

67. This problem will prepare you for the Concept Connection on page 512.

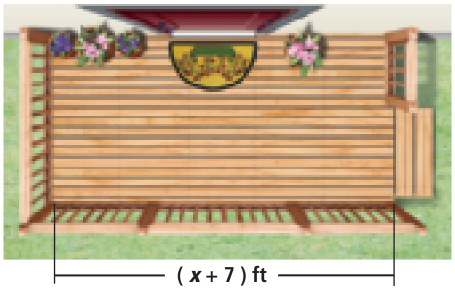
The equation for the motion of an object with constant acceleration is $d = vt + \frac{1}{2}at^2$ where d is distance traveled in feet, v is starting velocity in feet per second, a is acceleration in feet per second squared, and t is time in seconds.

a. Janna has two toy race cars on a track. One starts with a velocity of 0 ft/s and accelerates at 2 ft/s^2. Write an equation for the distance the car travels in time t.

b. The second car travels at a constant speed of 4 ft/s. Write an equation for the distance the second car travels in time t. (*Hint:* When speed is constant, the acceleration is 0 ft/s^2.)

c. By setting the equations equal to each other you can determine when the cars have traveled the same distance: $t^2 = 4t$. This can be written as $t^2 - 4t = 0$. Factor the left side of the equation.

68. **Construction** The length of a platform is $(x+7)$ ft. The area of the platform is $(x^2+9x+14)$ ft^2. Find the width of the platform. Check your answer.



Reasoning **Tell whether each statement is true or false. If false, explain.**

69. The third term in a factorable trinomial is equal to the product of the constants in its binomial factors.

70. The constants in the binomial factors of x^2+x-2 are both negative.

71. The correct factorization of x^2-3x-4 is $(x+4)(x-1)$.

72. All trinomials of the form x^2+bx+c can be factored.

Fill in the missing part of each factorization.

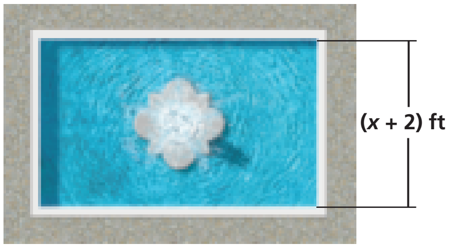
73. $x^2-6x+8=(x-2)(x-\square)$

74. $x^2-2x-8=(x+2)(x-\square)$

75. $x^2+2x-8=(x-2)(x+\square)$

76. $x^2+6x+8=(x+2)(x+\square)$

77. **Construction** The area of a rectangular fountain is $(x^2+12x+20)$ ft^2. The width is $(x+2)$ ft.
   a. Find the length of the fountain.
   b. A 2-foot walkway is built around the fountain. Find the dimensions of the outside border of the walkway.
   c. Find the total area covered by the fountain and walkway.



78. **Critical Thinking** Find all possible values of b so that x^2+bx+6 can be factored into binomial factors.

Multiple Choice **For Exercises 79–81, choose the best answer.**

79. Which is the correct factorization of $x^2-10x-24$?

(A) $(x-4)(x-6)$ (C) $(x-2)(x+12)$

(B) $(x+4)(x-6)$ (D) $(x+2)(x-12)$

80. Which value of b would make $x^2+bx-20$ factorable?

(A) 9 (B) 12 (C) 19 (D) 21

81. Which value of b would NOT make $x^2+bx-36$ factorable?

(A) 5 (B) 9 (C) 15 (D) 16

82. **Short Response** What are the factors of $x^2+2x-24$? Show and explain each step of factoring the polynomial.

CHALLENGE AND EXTEND

Factor each trinomial. Check your answer.

83. x^4+18x^2+81

84. y^4-5y^2-24

85. d^4+22d^2+21

86. $(u+v)^2+2(u+v)-3$

87. $(de)^2-(de)-20$

88. $(m-n)^2-4(m-n)-45$

89. Find all possible values of b such that, when $x^2 + bx + 28$ is factored, both constants in the binomials are positive.

90. Find all possible values of b such that, when $x^2 + bx + 32$ is factored, both constants in the binomials are negative.

91. **Landscaping** The area of Beth's rectangular garden is $(x^2 + 13x + 42)$ ft^2. The width is $(x + 6)$ ft.

Item	Cost
Fertilizer	\$0.28/ft^2
Fencing	\$2.00/ft

a. What is the length of the garden?
b. Find the perimeter in terms of x.
c. Find the cost to fence the garden when x is 5.
d. Find the cost of fertilizer when x is 5.
e. Find the total cost to fence and fertilize Beth's garden when x is 5.

SPIRAL STANDARDS REVIEW

2.0, 9.0, 11.0

Solve each system by substitution. *(Lesson 6-4)*

92. $\begin{cases} 3x + y = 13 \\ x - 3y = 1 \end{cases}$

93. $\begin{cases} 2x - y = 1 \\ x - y = -2 \end{cases}$

94. $\begin{cases} -x - y = -2 \\ x - 2y = 20 \end{cases}$

Simplify. *(Lesson 7-3)*

95. x^3x^2

96. $m^8n^3m^{-12}$

97. $(t^4)^3$

98. $(-2xy^3)^5$

Factor each polynomial by grouping. *(Lesson 8-2)*

99. $x^3 + 2x^2 + 5x + 10$

100. $2n^3 - 8n^2 - 3n + 12$

101. $2p^4 - 4p^3 + 7p - 14$

102. $x^3 - 4x^2 + x - 4$

Career Path

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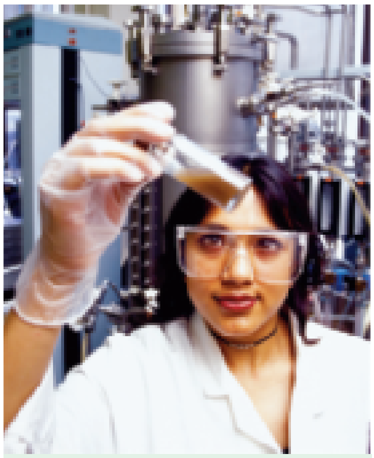
Career Resources Online
KEYWORD: MA8CA Career



Jessica Rubino
Environmental Sciences major

Q: What math classes did you take in high school?
A: Algebra 1, Algebra 2, and Geometry

Q: What college math classes have you taken?
A: I took several computer modeling and programming classes as well as Statistics and Probability.

Q: How is math used in some of your projects?
A: Computer applications help me analyze data collected from a local waste disposal site. I used my mathematical knowledge to make recommendations on how to preserve surrounding water supplies.

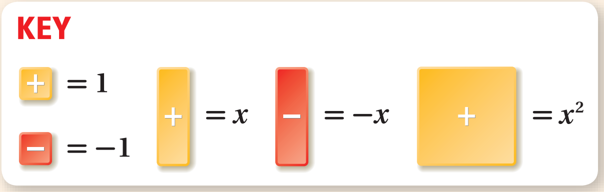
Q: What plans do you have for the future?
A: I enjoy my studies in the area of water pollution. I would also like to research more efficient uses of natural energy resources.

8-4 Algebra Lab

Model Factorization of $ax^2 + bx + c$

You can use algebra tiles to factor a trinomial whose lead coefficient is not 1.

Use with Lesson 8-4

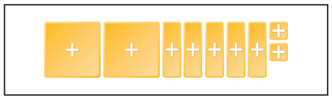
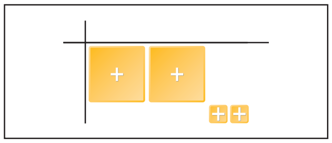
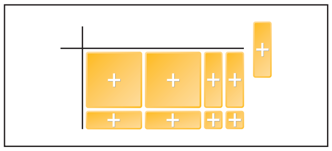
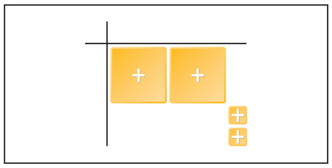
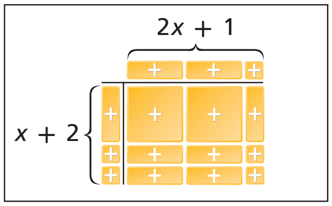


California Standards

11.0 Students apply basic factoring techniques to second- and simple third-**degree polynomials.** These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

Activity 1

Use algebra tiles to factor $2x^2 + 5x + 2$.

MODEL		ALGEBRA
	*Model $2x^2 + 5x + 2$.*	$2x^2 + 5x + 2$
 	*Try to arrange all of the tiles in a rectangle. Place the x^2-tiles in the upper left corner.* *Arrange the unit tiles in a rectangle so that the top left corner of this rectangle touches the bottom right corner of the second x^2-tile.* *Arrange the x-tiles so that all the tiles together make one large rectangle.* *This does not work. One x-tile is left over.*	$2x^2 + 5x + 2 \neq (x + 1)(2x + 2)$
	*Rearrange the unit tiles to form another rectangle.*	
	*Fill in the empty spaces with x-tiles. All 5 x-tiles fit. This is the correct arrangement.* *The total area represents the trinomial $2x^2 + 5x + 2$. The length $2x + 1$ and width $x + 2$ represent the factors.*	$2x^2 + 5x + 2 = (x + 2)(2x + 1)$

Try This

Use algebra tiles to factor each trinomial.

1. $3x^2 + 7x + 4$ **2.** $3x^2 + 4x - 4$ **3.** $2x^2 - x - 1$ **4.** $4x^2 - 8x + 3$

8-4

Factoring $ax^2 + bx + c$

California Standards

11.0 Students apply basic factoring techniques to second- and simple third-degree **polynomials.** These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

Why learn this?

The height of a football that has been kicked can be modeled by a factored polynomial. (See Exercise 75.)

In the previous lesson you factored trinomials of the form $x^2 + bx + c$. Now you will factor trinomials of the form $ax^2 + bx + c$, where $a \neq 0$ or 1.

When you multiply $(3x + 2)(2x + 5)$, the coefficient of the x^2-term is the product of the coefficients of the x-terms. Also, the constant term in the trinomial is the product of the constants in the binomials.

$$(3x + 2)(2x + 5) = 6x^2 + 19x + 10$$

To factor a trinomial like $ax^2 + bx + c$ into its binomial factors, first write two sets of parentheses: $(\square x + \square)(\square x + \square)$.

Write two integers that are factors of a next to the x's and two integers that are factors of c in the other blanks. Then multiply to see if the product is the original trinomial. If there are not two such integers, the trinomial is not factorable.

EXAMPLE 1 Factoring $ax^2 + bx + c$

Factor $4x^2 + 16x + 15$. Check your answer.

$(\square x + \square)(\square x + \square)$ *The first term is $4x^2$, so at least one variable term has a coefficient other than 1.*

The coefficient of the x^2-term is 4. The constant term in the trinomial is 15.

Try integer factors of 4 for the coefficients and integer factors of 15 for the constant terms.

$(1x + 15)(4x + 1) = 4x^2 + 61x + 15$ ✗

$(1x + 5)(4x + 3) = 4x^2 + 23x + 15$ ✗

$(1x + 3)(4x + 5) = 4x^2 + 17x + 15$ ✗

$(1x + 1)(4x + 15) = 4x^2 + 19x + 15$ ✗

$(2x + 15)(2x + 1) = 4x^2 + 32x + 15$ ✗

$(2x + 5)(2x + 3) = 4x^2 + 16x + 15$ ✓

Check $(2x + 5)(2x + 3) = 4x^2 + 10x + 6x + 15$ *Use the FOIL method.*

$= 4x^2 + 16x + 15$ ✓ *The product is the original trinomial.*

CHECK IT OUT! **Factor each trinomial. Check your answer.**

1a. $6x^2 + 11x + 3$ **1b.** $3x^2 - 2x - 8$

So, to factor $ax^2 + bx + c$, check the factors of a and the factors of c in the binomials. The sum of the products of the outer and inner terms should be b.

Product $= a$ Product $= c$

$$(\square x + \square)(\square x + \square) = ax^2 + bx + c$$

Sum of outer and inner products $= b$

Since you need to check all the factors of a and all the factors of c, it may be helpful to make a table. Then check the products of the outer and inner terms to see if the sum is b. You can multiply the binomials to check your answer.

EXAMPLE 2 Factoring $ax^2 + bx + c$ When c Is Positive

Factor each trinomial. Check your answer.

A $2x^2 + 11x + 12$

$(\square x + \square)(\square x + \square)$ *$a = 2$ and $c = 12$; Outer + Inner = 11*

Factors of 2	Factors of 12	Outer + Inner
1 and 2	1 and 12	$1(12) + 2(1) = 14$ ✗
1 and 2	12 and 1	$1(1) + 2(12) = 25$ ✗
1 and 2	2 and 6	$1(6) + 2(2) = 10$ ✗
1 and 2	6 and 2	$1(2) + 2(6) = 14$ ✗
1 and 2	3 and 4	$1(4) + 2(3) = 10$ ✗
1 and 2	4 and 3	$1(3) + 2(4) = 11$ ✓

$(x + 4)(2x + 3)$

Check $(x + 4)(2x + 3) = 2x^2 + 3x + 8x + 12$ *Use the FOIL method.*
$= 2x^2 + 11x + 12$ ✓

Remember!

When b is negative and c is positive, the factors of c are both negative.

B $5x^2 - 14x + 8$

$(\square x + \square)(\square x + \square)$ *$a = 5$ and $c = 8$; Outer + Inner = −14*

Factors of 5	Factors of 8	Outer + Inner
1 and 5	−1 and −8	$1(-8) + 5(-1) = -13$ ✗
1 and 5	−8 and −1	$1(-1) + 5(-8) = -41$ ✗
1 and 5	−2 and −4	$1(-4) + 5(-2) = -14$ ✓

$(x - 2)(5x - 4)$

Check $(x - 2)(5x - 4) = 5x^2 - 4x - 10x + 8$ *Use the FOIL method.*
$= 5x^2 - 14x + 8$ ✓

CHECK IT OUT!

Factor each trinomial. Check your answer.

2a. $6x^2 + 17x + 5$ **2b.** $9x^2 - 15x + 4$ **2c.** $3x^2 + 13x + 12$

When c is negative, one factor of c will be positive and the other factor will be negative. Only some of the factors are shown in the examples, but you may need to check all of the possibilities.

EXAMPLE 3 Factoring $ax^2 + bx + c$ When c Is Negative

Factor each trinomial. Check your answer.

A $4y^2 + 7y - 2$

$(\square y + \square)(\square y + \square)$ *$a = 4$ and $c = -2$; Outer + Inner = 7*

Factors of 4	Factors of −2	Outer + Inner
1 and 4	1 and −2	$1(-2) + (4)1 = 2$ ✗
1 and 4	−1 and 2	$(1)2 + 4(-1) = -2$ ✗
1 and 4	2 and −1	$1(-1) + (4)2 = 7$ ✓

$(y + 2)(4y - 1)$

Check $(y + 2)(4y - 1) = 4y^2 - y + 8y - 2$ *Use the FOIL method.*

$= 4y^2 + 7y - 2$ ✓

B $4x^2 + 19x - 5$

$(\square x + \square)(\square x + \square)$ *$a = 4$ and $c = -5$; Outer + Inner = 19*

Factors of 4	Factors of −5	Outer + Inner
1 and 4	1 and −5	$1(-5) + (4)1 = -1$ ✗
1 and 4	−1 and 5	$(1)5 + 4(-1) = 1$ ✗
1 and 4	5 and −1	$1(-1) + (4)5 = 19$ ✓

$(x + 5)(4x - 1)$

Check $(x + 5)(4x - 1) = 4x^2 - x + 20x - 5$ *Use the FOIL method.*

$= 4x^2 + 19x - 5$ ✓

C $2x^2 - 7x - 15$

$(\square x + \square)(\square x + \square)$ *$a = 2$ and $c = -15$; Outer + Inner = −7*

Factors of 2	Factors of −15	Outer + Inner
1 and 2	1 and −15	$1(-15) + (2)1 = -13$ ✗
1 and 2	−1 and 15	$(1)15 + 2(-1) = 13$ ✗
1 and 2	3 and −5	$1(-5) + (2)3 = 1$ ✗
1 and 2	−3 and 5	$(1)5 + 2(-3) = -1$ ✗
1 and 2	5 and −3	$1(-3) + (2)5 = 7$ ✗
1 and 2	−5 and 3	$(1)3 + 2(-5) = -7$ ✓

$(x - 5)(2x + 3)$

Check $(x - 5)(2x + 3) = 2x^2 + 3x - 10x - 15$ *Use the FOIL method.*

$= 2x^2 - 7x - 15$ ✓

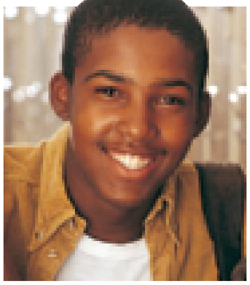
CHECK IT OUT! **Factor each trinomial. Check your answer.**

3a. $6x^2 + 7x - 3$

3b. $4n^2 - n - 3$

Student to Student

Factoring $ax^2 + bx + c$



Reggie Wilson
Franklin High School

I like to use a box to help me factor trinomials. I look for factors of ac that add to b. Then I arrange the terms in a box and factor.

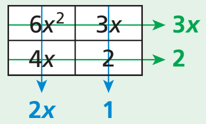
To factor $6x^2 + 7x + 2$, first I find the factors I need.

$ac = 2(6) = 12 \quad b = 7$

Factors of 12	*Sum*
1 and 12	13
2 and 6	8
3 and 4	7

Then I rewrite the trinomial as $6x^2 + 3x + 4x + 2$.

Now I arrange $6x^2 + 3x + 4x + 2$ in a box and factor out the common factors from each row and column.



The factors are $(2x + 1)$ and $(3x + 2)$.

When the leading coefficient is negative, factor out -1 from each term before using other factoring methods.

EXAMPLE 4 Factoring $ax^2 + bx + c$ When a Is Negative

Factor $-2x^2 - 15x - 7$.

$-1(2x^2 + 15x + 7)$ *Factor out -1.*

$-1(\square x + \square)(\square x + \square)$ *$a = 2$ and $c = 7$; Outer + Inner = 15*

Factors of 2	Factors of 7	Outer + Inner	
1 and 2	1 and 7	$(1)7 + (2)1 = 9$	✗
1 and 2	7 and 1	$(1)1 + (2)7 = 15$	✓

$(x + 7)(2x + 1)$

$-1(x + 7)(2x + 1)$

Caution!

When you factor out -1 in an early step, you must carry it through the rest of the steps and into the answer.

CHECK IT OUT! **Factor each trinomial. Check your answer.**

4a. $-6x^2 - 17x - 12$ **4b.** $-3x^2 - 17x - 10$

THINK AND DISCUSS

1. Let a, b, and c be positive. If $ax^2 + bx + c$ is the product of two binomials, what do you know about the signs of the numbers in the binomials?

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. Write each of the following trinomials in the appropriate box and factor each one.

$3x^2 + 10x - 8 \quad 3x^2 + 10x + 8$

$3x^2 - 10x + 8 \quad 3x^2 - 10x - 8$

Factoring $ax^2 + bx + c$	
$c > 0$	
$b > 0$	**$b < 0$**
$c < 0$	
$b < 0$	**$b > 0$**

8-4 Exercises

California Standards Practice
11.0, 15.0, 25.2

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Homework Help Online
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KEYWORD: MA8CA Parent

GUIDED PRACTICE

Factor each trinomial. Check your answer.

SEE EXAMPLE 1
p. 505

1. $2x^2 + 9x + 10$
2. $5x^2 + 31x + 6$
3. $5x^2 + 7x - 6$
4. $6x^2 + 37x + 6$
5. $3x^2 - 14x - 24$
6. $6x^2 + x - 2$

SEE EXAMPLE 2
p. 506

7. $5x^2 + 11x + 2$
8. $2x^2 + 11x + 5$
9. $4x^2 - 9x + 5$
10. $2y^2 - 11y + 14$
11. $5x^2 + 9x + 4$
12. $3x^2 + 7x + 2$

SEE EXAMPLE 3
p. 507

13. $4a^2 + 8a - 5$
14. $15x^2 + 4x - 3$
15. $2x^2 + x - 6$
16. $6n^2 - 11n - 10$
17. $10x^2 - 9x - 1$
18. $7x^2 - 3x - 10$

SEE EXAMPLE 4
p. 508

19. $-2x^2 + 5x + 12$
20. $-4n^2 - 16n + 9$
21. $-5x^2 + 7x + 6$
22. $-6x^2 + 13x - 2$
23. $-4x^2 - 8x + 5$
24. $-5x^2 + x + 18$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
25–33	1
34–42	2
43–48	3
49–51	4

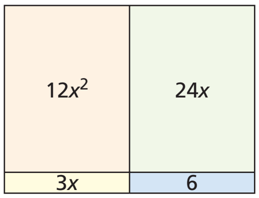
Extra Practice

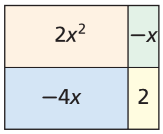
Skills Practice p. EP17
Application Practice p. EP31

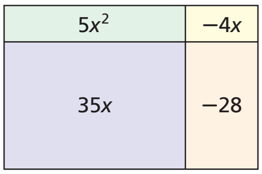
Factor each trinomial. Check your answer.

25. $9x^2 + 9x + 2$
26. $2x^2 + 7x + 5$
27. $3n^2 + 8n + 4$
28. $10d^2 + 17d + 7$
29. $4c^2 - 17c + 15$
30. $6x^2 + 14x + 4$
31. $8x^2 + 22x + 5$
32. $6x^2 - 13x + 6$
33. $5x^2 + 9x - 18$
34. $6x^2 + 23x + 7$
35. $10n^2 - 17n + 7$
36. $3x^2 + 11x + 6$
37. $7x^2 + 15x + 2$
38. $3n^2 + 4n + 1$
39. $3x^2 - 19x + 20$
40. $6x^2 + 11x + 4$
41. $4x^2 - 31x + 21$
42. $10x^2 + 31x + 15$
43. $12y^2 + 17y - 5$
44. $3x^2 + 10x - 8$
45. $4x^2 + 4x - 3$
46. $2n^2 - 7n - 4$
47. $3x^2 - 4x - 15$
48. $3n^2 - n - 4$
49. $-4x^2 - 4x + 15$
50. $-3x^2 + 16x - 16$
51. $-3x^2 - x + 2$

Geometry For Exercises 52–54, write the polynomial modeled and then factor.

52.


53.


54.


Factor each trinomial, if possible. Check your answer.

55. $9n^2 + 17n + 8$
56. $2x^2 - 7x - 4$
57. $4x^2 - 12x + 5$
58. $5x^2 - 4x + 12$
59. $3x^2 + 14x + 16$
60. $-3x^2 - 11x + 4$
61. $6x^2 - x - 12$
62. $10a^2 + 11a + 3$
63. $4x^2 - 12x + 9$
64. $-6x^2 - 11x + 2$
65. $12x^2 - 8x + 1$
66. $-8x^2 - 7x + 1$
67. $15x^2 + 23x + 8$
68. $8x^2 - 4x - 4$
69. $9x^2 - x + 2$

70. **Geometry** The area of a rectangle is $6x^2 + 11x + 5$ cm^2. The width is $(x + 1)$ cm. What is the length of the rectangle?

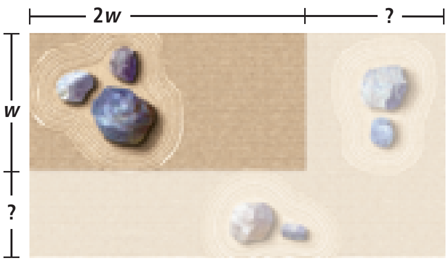
71. **Write About It** Write a paragraph describing how to factor $6x^2 + 13x + 6$. Show each step you would take and explain your steps.

Complete each factorization.

72.
$$\begin{aligned} &8x^2 + 18x - 5 \\ &8x^2 + 20x - 2x - 5 \\ &(8x^2 + 20x) - (2x + 5) \\ &\square(\square + \square) - \square(2x + 5) \\ &(\square - \square)(2x + 5) \end{aligned}$$

73.
$$\begin{aligned} &4x^2 + 9x + 2 \\ &4x^2 + 8x + x + 2 \\ &(4x^2 + 8x) + (x + 2) \\ &\square(\square + \square) + \square(x + 2) \\ &(\square + \square)(x + 2) \end{aligned}$$

74. **Gardening** The length of Rebecca's rectangular garden was two times the width w. Rebecca increased the length and width of the garden so that the area of the new garden is $(2w^2 + 7w + 6)$ square yards. By how much did Rebecca increase the length and the width of the garden?



75. **Physical Science** The height of a football that has been thrown or kicked can be described by the expression $-16t^2 + vt + h$ where t is the time in seconds, v is the initial upward velocity, and h is the initial height in feet.

a. Write an expression for the height of a football at time t when the initial upward velocity is 20 feet per second and the initial height is 6 feet.

b. Factor your expression from part **a.** Check your answer.

c. Find the height of the football after 1 second.

76. **ERROR ANALYSIS** A student attempted to factor $2x^2 + 11x + 12$ as shown. Find and explain the error.

$2x^2 + 11x + 12$

Factors of 12	Sum	
1 and 12	13	✓
2 and 6	8	✗
3 and 4	7	✗

$(2x + 1)(x + 12)$

Concept Connection

77. This problem will prepare you for the Concept Connection on page 512.

The equation $d = 2t^2$ gives the distance from the start point of a toy boat that starts at rest and accelerates at 4 cm/s^2. The equation $d = 10t - 8$ gives the distance from the start point of a second boat that starts at rest 8 cm behind the first boat and travels at a constant rate of 10 cm/s.

a. By setting the equations equal to each other, you can determine when the cars are the same distance from the start point: $2t^2 = 10t - 8$. Use properties of algebra to collect all terms on the left side of the equation, leaving 0 on the right side.

b. Factor the expression on the left side of the equation.

c. The boats are the same distance from the start point at $t = 1$ and $t = 4$. Explain how the factors you found in part **b** are related to these two times.

Match each trinomial with its correct factorization.

78. $6x^2 - 29x - 5$ **A.** $(x + 5)(6x + 1)$

79. $6x^2 - 31x + 5$ **B.** $(x - 5)(6x - 1)$

80. $6x^2 + 31x + 5$ **C.** $(x + 5)(6x - 1)$

81. $6x^2 + 29x - 5$ **D.** $(x - 5)(6x + 1)$

82. **Reasoning** A quadratic trinomial $ax^2 + bx + c$ has $a > 0$ and can be factored into the product of two binomials.

a. Explain what you know about the signs of the constants in the factors if $c > 0$.

b. Explain what you know about the signs of the constants in the factors if $c < 0$.

Multiple Choice **For Exercises 83–86, choose the best answer.**

83. What value of b would make $3x^2 + bx - 8$ factorable?

(A) 3 (B) 10 (C) 11 (D) 25

84. Which product of binomials is represented by the model?

(A) $(x + 4)(3x + 5)$ (C) $(x + 3)(5x + 4)$

(B) $(x + 4)(5x + 3)$ (D) $(x + 5)(3x + 4)$

$5x^2$	$4x$
$15x$	12

85. Which binomial is a factor of $24x^2 - 49x + 2$?

(A) $x - 2$ (B) $x - 1$ (C) $x + 1$ (D) $x + 2$

86. Which value of c would make $2x^2 + x + c$ NOT factorable?

(A) -15 (B) -9 (C) -6 (D) -1

CHALLENGE AND EXTEND

Factor each trinomial. Check your answer.

87. $1 + 4x + 4x^2$ **88.** $1 - 14x + 49x^2$ **89.** $1 + 18x + 81x^2$

90. $25 + 30x + 9x^2$ **91.** $4 + 20x + 25x^2$ **92.** $4 - 12x + 9x^2$

93. Find all possible values of b such that $3x^2 + bx + 2$ can be factored.

94. Find all possible values of b such that $3x^2 + bx - 2$ can be factored.

95. Find all possible values of b such that $5x^2 + bx + 1$ can be factored.

SPIRAL STANDARDS REVIEW 6.0, 9.0, 11.0, 17.0

96. Archie makes \$12 per hour and is paid for whole numbers of hours. The function $f(x) = 12x$ gives the amount of money that Archie makes in x hours. Graph this function and give its domain and range. *(Lesson 5-1)*

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions. *(Lesson 6-6)*

97. $\begin{cases} y < -2x + 1 \\ y > 3x - 5 \end{cases}$ **98.** $\begin{cases} y \geq -x + 2 \\ y \leq x - 3 \end{cases}$ **99.** $\begin{cases} y \leq -4x \\ y > 2x - 6 \end{cases}$

Factor each trinomial. Check your answer. *(Lesson 8-3)*

100. $x^2 + 6x + 8$ **101.** $x^2 - 8x - 9$ **102.** $x^2 - 8x + 12$

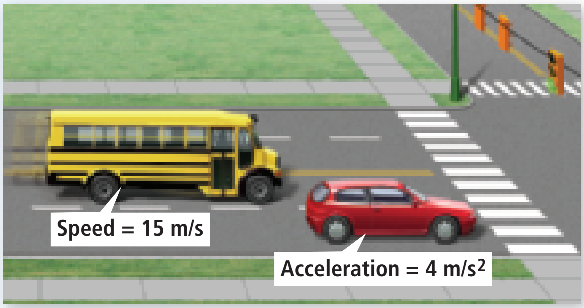
CHAPTER 8

CONCEPT CONNECTION

SECTION 8A

Factoring

Red Light, Green Light The equation for the motion of an object with constant acceleration is $d = vt + \frac{1}{2}at^2$ where d is distance traveled in meters, v is starting velocity in m/s, a is acceleration in m/s^2, and t is time in seconds.



1. A car is stopped at a traffic light. The light changes to green and the driver starts to drive, accelerating at a rate of 4 m/s^2. Write an equation for the distance the car travels in time t.
2. A bus is traveling at a speed of 15 m/s. The driver approaches the same traffic light in another traffic lane. He does not brake, and continues at the same speed. Write an equation for the distance the bus travels in time t. (*Hint:* At a constant speed, the acceleration is 0 m/s^2.)
3. Set the equations equal to each other so you can determine when the car and bus are the same distance from the intersection. Collect all the terms on the left side of this new equation, leaving 0 on the right side. Factor the expression on the left side of the equation. Check your answer.
4. Let $t = 0$ be the point at which the car is just starting to drive and the bus is even with the car. Find the other time when the vehicles will be the same distance from the intersection.
5. What distance will the two vehicles have traveled when they are again at the same distance from the intersection?
6. A truck traveling at 16 m/s is 24 meters behind the bus at $t = 0$. The equation $d = -24 + 16t$ gives the position of the truck. At what time will the truck be the same distance from the intersection as the bus? What will that distance be?

READY TO GO ON?

Quiz for Lessons 8-1 Through 8-4

8-1 Factors and Greatest Common Factors

Write the prime factorization of each number.

1. 54 **2.** 42 **3.** 50 **4.** 120 **5.** 44 **6.** 78

Find the GCF of each pair of monomials.

7. $6p^3$ and $2p$ **8.** $12x^3$ and $18x^4$ **9.** -15 and $20s^4$ **10.** $3a$ and $4b^2$

11. Brent is making a wooden display case for his baseball collection. He has 24 balls from American League games and 30 balls from National League games. He wants to display the same number of baseballs in each row and does not want to put American League baseballs in the same row as National League baseballs. How many rows will Brent need in the display case to put the greatest number of baseballs possible in each row?

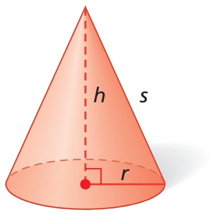
8-2 Factoring by GCF

Factor each polynomial. Check your answer.

12. $2d^3 + 4d$ **13.** $m^2 - 8m^5$

14. $12x^4 - 8x^3 - 4x^2$ **15.** $3k^2 + 6k - 3$

16. The surface area of a cone can be found using the expression $s\pi r + \pi r^2$, where s represents the slant height and r represents the radius of the base. Factor this expression.



Factor each polynomial by grouping. Check your answer.

17. $w^3 - 4w^2 + w - 4$ **18.** $3x^3 + 6x^2 - 4x - 8$

19. $2p^3 - 6p^2 + 15 - 5p$ **20.** $n^3 - 6n^2 + 5n - 30$

8-3 Factoring $x^2 + bx + c$

Factor each trinomial. Check your answer.

21. $n^2 + 9n + 20$ **22.** $d^2 - 6d - 7$ **23.** $x^2 - 6x + 8$

24. $y^2 + 7y - 30$ **25.** $k^2 - 6k + 5$ **26.** $c^2 - 10c + 24$

27. Simplify and factor the polynomial $n(n + 3) - 4$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3,$ and 4.

8-4 Factoring $ax^2 + bx + c$

Factor each trinomial. Check your answer.

28. $2x^2 + 11x + 5$ **29.** $3n^2 + 16n + 21$ **30.** $5y^2 - 7y - 6$

31. $4g^2 - 10g + 6$ **32.** $6p^2 - 18p - 24$ **33.** $12d^2 + 7d - 12$

34. The area of a rectangle is $(8x^2 + 8x + 2)$ cm^2. The width is $(2x + 1)$ cm. What is the length of the rectangle?

8-5 Factoring Special Products

California Standards

11.0 Students apply basic factoring techniques to second- and simple third-**degree polynomials. These techniques include** finding a common factor for all terms in a polynomial, **recognizing the difference of two squares, and recognizing perfect squares of binomials.**

Who uses this?

Urban planners can use the area of a square park to find its length and width. (See Example 2.)

You studied the patterns of some special products of binomials in Chapter 7. You can use those patterns to factor certain polynomials.

A trinomial is a perfect square if:

- The **first** and **last** terms are perfect squares.
- The **middle** term is two times one factor from the first term and one factor from the last term.

$$9x^2 \quad + \quad 12x \quad + \quad 4$$

$$3x \cdot 3x \qquad 2(3x \cdot 2) \qquad 2 \cdot 2$$

Know it! Note

Factoring Perfect-Square Trinomials

PERFECT-SQUARE TRINOMIAL	EXAMPLES
$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$	$x^2 + 6x + 9 = (x + 3)(x + 3) = (x + 3)^2$
$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$	$x^2 - 2x + 1 = (x - 1)(x - 1) = (x - 1)^2$

EXAMPLE 1

Recognizing and Factoring Perfect-Square Trinomials

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

A $x^2 + 12x + 36$

$$x^2 + 12x + 36$$

$$x \cdot x \qquad 2(x \cdot 6) \qquad 6 \cdot 6$$

The trinomial is a perfect square. Factor.

Method 1 Factor.

$x^2 + 12x + 36$

Factors of 36	Sum	
1 and 36	37	✗
2 and 18	20	✗
3 and 12	15	✗
4 and 9	13	✗
6 and 6	12	✓

$(x + 6)(x + 6)$

Method 2 Use the rule.

$x^2 + 12x + 36$ — *$a = x$, $b = 6$*

$x^2 + 2(x)(6) + 6^2$ — *Write the trinomial as $a^2 + 2ab + b^2$.*

$(x + 6)^2$ — *Write the trinomial as $(a + b)^2$.*

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

B $4x^2 - 12x + 9$

$4x^2 - 12x + 9$

$2x \cdot 2x \quad 2(2x \cdot 3) \quad 3 \cdot 3$ *The trinomial is a perfect square. Factor.*

$4x^2 - 12x + 9$ *$a = 2x$, $b = 3$*

$(2x)^2 - 2(2x)(3) + 3^2$ *$a^2 - 2ab + b^2$*

$(2x - 3)^2$ *$(a - b)^2$*

C $x^2 + 9x + 16$

$x^2 + 9x + 16$

$x \cdot x \quad 2(x \cdot 4) \quad 4 \cdot 4$ *$2(x \cdot 4) \neq 9x$*

$x^2 + 9x + 16$ is not a perfect-square trinomial because $9x \neq 2(x \cdot 4)$.

Remember!

You can check your answer by using the FOIL method.

For Example 1B,

$(2x - 3)^2 =$

$(2x - 3)(2x - 3) =$

$4x^2 - 6x - 6x + 9 =$

$4x^2 - 12x + 9$

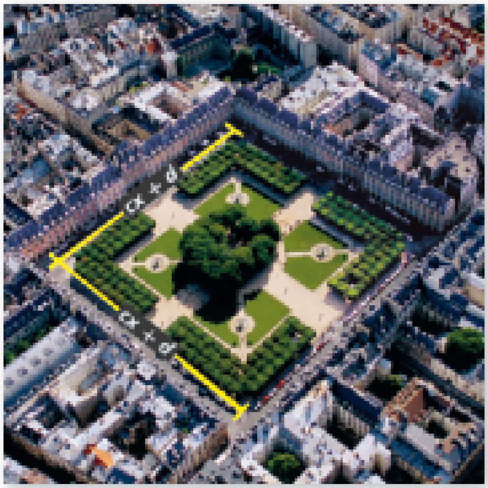
CHECK IT OUT! Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

1a. $x^2 + 4x + 4$ **1b.** $x^2 - 14x + 49$ **1c.** $9x^2 - 6x + 4$

EXAMPLE 2 *Problem-Solving Application*

PROBLEM SOLVING

The park in the center of the Place des Vosges in Paris, France, is in the shape of a square. The area of the park is $(25x^2 + 70x + 49)$ ft². The side length of the park is in the form $cx + d$, where c and d are whole numbers. Find an expression in terms of x for the perimeter of the park. Find the perimeter when $x = 8$ ft.



1 Understand the Problem

The **answer** will be an expression for the perimeter of the park and the value of the expression when $x = 8$.

List the **important information:**

- The park is a square with area $(25x^2 + 70x + 49)$ft².
- The side length of the park is in the form $cx + d$, where c and d are whole numbers.

2 Make a Plan

The formula for the area of a square is area = $(\text{side})^2$.

Factor $25x^2 + 70x + 49$ to find the side length of the park. Write a formula for the perimeter of the park, and evaluate the expression for $x = 8$.

3 Solve

$$25x^2 + 70x + 49 \qquad a = 5x,\ b = 7$$
$$(5x)^2 + 2(5x)(7) + 7^2 \qquad \textit{Write the trinomial as } a^2 + 2ab + b^2.$$
$$(5x + 7)^2 \qquad \textit{Write the trinomial as } (a + b)^2.$$

$$25x^2 + 70x + 49 = (5x + 7)(5x + 7)$$

Each side length of the park is $(5x + 7)$ ft.

Write a formula for the perimeter of the park.

$$\begin{aligned} P &= 4s && \textit{Write the formula for the perimeter of a square.} \\ &= 4(5x + 7) && \textit{Substitute the side length for s.} \\ &= 20x + 28 && \textit{Distribute 4.} \end{aligned}$$

An expression for the perimeter of the park in feet is $20x + 28$.

Evaluate the expression when $x = 8$.

$$\begin{aligned} P &= 20x + 28 \\ &= 20(8) + 28 && \textit{Substitute 8 for x.} \\ &= 188 \end{aligned}$$

When $x = 8$ ft, the perimeter of the park is 188 ft.

4 Look Back

For a square with a perimeter of 188 ft, the side length is $\frac{188}{4} = 47$ ft and the area is $47^2 = 2209$ ft^2.

Evaluate $25x^2 + 70x + 49$ for $x = 8$:

$$25(8)^2 + 70(8) + 49$$
$$1600 + 560 + 49$$
$$2209 \checkmark$$

CHECK IT OUT!

2. **What if...?** A company produces square sheets of aluminum, each of which has an area of $(9x^2 + 6x + 1)$ m^2. The side length of each sheet is in the form $cx + d$, where c and d are whole numbers. Find an expression in terms of x for the perimeter of a sheet. Find the perimeter when $x = 3$ m.

In Chapter 7 you learned that the difference of two squares has the form $a^2 - b^2$. The difference of two squares can be written as the product $(a + b)(a - b)$. You can use this pattern to factor some polynomials.

A polynomial is a difference of two squares if:

- There are two terms, one subtracted from the other.
- Both terms are perfect squares.

$$\underbrace{4x^2}_{2x \cdot 2x} - \underbrace{9}_{3 \cdot 3}$$

Know it! Note

Factoring a Difference of Two Squares

DIFFERENCE OF TWO SQUARES	EXAMPLE
$a^2 - b^2 = (a + b)(a - b)$	$x^2 - 9 = (x + 3)(x - 3)$

EXAMPLE 3 Recognizing and Factoring the Difference of Two Squares

Reading Math

Recognize a difference of two squares: the coefficients of variable terms are perfect squares, powers on variable terms are even, and constants are perfect squares.

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

A $x^2 - 81$

$x^2 - 81$

$x \cdot x \qquad 9 \cdot 9$ — *The polynomial is a difference of two squares.*

$x^2 - 9^2$ — *$a = x$, $b = 9$*

$(x + 9)(x - 9)$ — *Write the polynomial as $(a + b)(a - b)$.*

$x^2 - 81 = (x + 9)(x - 9)$

B $9p^4 - 16q^2$

$9p^4 - 16q^2$

$3p^2 \cdot 3p^2 \qquad 4q \cdot 4q$ — *The polynomial is a difference of two squares.*

$(3p^2)^2 - (4q)^2$ — *$a = 3p^2$, $b = 4q$*

$(3p^2 + 4q)(3p^2 - 4q)$ — *Write the polynomial as $(a + b)(a - b)$.*

$9p^4 - 16q^2 = (3p^2 + 4q)(3p^2 - 4q)$

C $x^6 - 7y^2$

$x^6 - 7y^2$

$x^3 \cdot x^3$ — *$7y^2$ is not a perfect square.*

$x^6 - 7y^2$ is not the difference of two squares because $7y^2$ is not a perfect square.

CHECK IT OUT! **Determine whether the binomial is a difference of two squares. If so, factor. If not, explain.**

3a. $1 - 4x^2$ **3b.** $p^8 - 49q^6$ **3c.** $16x^2 - 4y^5$

THINK AND DISCUSS

1. The binomial $1 - x^4$ is a difference of two squares. Use the rule to identify a and b in $1 - x^4$.

2. The polynomial $x^2 + 8x + 16$ is a perfect-square trinomial. Use the rule to identify a and b in $x^2 + 8x + 16$.

3. GET ORGANIZED Copy and complete the graphic organizer. Write an example of each type of special product and factor it.

Know it! Note

Special Product	Factored Form
Perfect-square trinomial with positive coefficient of middle term	
Perfect-square trinomial with negative coefficient of middle term	
Difference of two squares	

8-5 Exercises

California Standards Practice
10.0, 11.0, 25.2

go.hrw.com
Homework Help Online
KEYWORD: MA8CA 8-5
Parent Resources Online
KEYWORD: MA8CA Parent

GUIDED PRACTICE

SEE EXAMPLE 1
p. 514

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

1. $x^2 - 4x + 4$ **2.** $x^2 - 4x - 4$ **3.** $9x^2 - 12x + 4$

4. $x^2 + 2x + 1$ **5.** $x^2 - 6x + 9$ **6.** $x^2 - 6x - 9$

SEE EXAMPLE 2
p. 515

7. City Planning A city purchases a rectangular plot of land with an area of $(x^2 + 24x + 144)$ yd^2 for a park. The dimensions of the plot are of the form $ax + b$, where a and b are whole numbers. Find an expression for the perimeter of the park. Find the perimeter when $x = 10$ yd.

SEE EXAMPLE 3
p. 517

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

8. $1 - 4x^2$ **9.** $s^2 - 4^2$ **10.** $81x^2 - 1$

11. $4x^4 - 9y^2$ **12.** $x^8 - 50$ **13.** $x^6 - 9$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–19	1
20	2
21–26	3

Extra Practice
Skills Practice p. EP17
Application Practice p. EP31

Determine whether the trinomial is a perfect square. If so, factor. If not, explain.

14. $4x^2 - 4x + 1$ **15.** $4x^2 - 4x - 1$ **16.** $36x^2 - 12x + 1$

17. $25x^2 + 10x + 4$ **18.** $9x^2 + 18x + 9$ **19.** $16x^2 - 40x + 25$

20. Measurement You are given a sheet of paper and told to cut out a rectangular piece with an area of $(4x^2 - 44x + 121)$ mm^2. The dimensions of the rectangle have the form $ax - b$, where a and b are whole numbers. Find an expression for the perimeter of the rectangle you cut out. Find the perimeter when $x = 41$ mm.

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

21. $1^2 - 4x^2$ **22.** $25m^2 - 16n^2$ **23.** $4x - 9y$

24. $49p^{12} - 9q^6$ **25.** $9^2 - 100x^4$ **26.** $x^3 - y^3$

Find the missing term in each perfect-square trinomial.

27. $x^2 + 14x + \square$ **28.** $9x^2 + \square + 25$ **29.** $\square - 36y + 81$

Factor each polynomial using the rule for perfect-square trinomials or the rule for a difference of two squares. Tell which rule you used and check your answer.

30. $x^2 - 8x + 16$ **31.** $100x^2 - 81y^2$ **32.** $36x^2 + 24x + 4$

33. $4r^6 - 25s^6$ **34.** $49x^2 - 70x + 25$ **35.** $x^{14} - 144$

36. Write About It What is similar about a perfect-square trinomial and a difference of two squares? What is different?

37. Critical Thinking Describe two ways to create a perfect-square trinomial.

38. For what value of b would $(x + b)(x + b)$ be the factored form of $x^2 - 22x + 121$?

39. For what value of c are the factors of $x^2 + cx + 256$ the same?

CONCEPT CONNECTION

40. This problem will prepare you for the Concept Connection on page 528.

Juanita designed a vegetable garden in the shape of a square and purchased fencing for that design. Then she decided to change the design to a rectangle.

a. The square garden had an area of x^2 ft^2. The area of the rectangular garden is $(x^2 - 25)$ ft^2. Factor the expression for the area of the rectangular garden.

b. The rectangular garden must have the same perimeter as the square garden, so Juanita added a number of feet to the length and subtracted the same number of feet from the width. Use your factors from part **a** to determine how many feet were added to the length and subtracted from the width.

c. If the original length of the square garden was 8 feet, what are the length and width of the new garden?

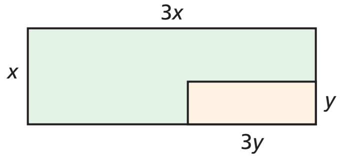
41. **Multi-Step** The area of a square is represented by $25z^2 - 40z + 16$.

a. What expression represents the length of a side of the square?

b. What expression represents the perimeter of the square?

c. What are the length of a side, the perimeter, and the area of the square when $z = 3$?

42. **Multi-Step** A small rectangle is drawn inside a larger rectangle as shown.



a. What is the area of each rectangle?

b. What is the area of the green region?

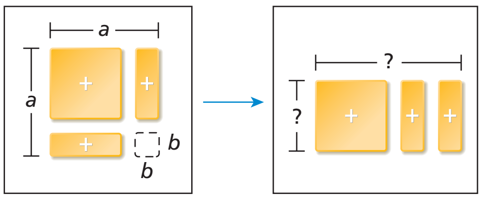
c. Factor the expression for the area of the green region. (*Hint:* First factor out the common factor of 3 and then factor the binomial.)

43. Evaluate each expression for the values of x.

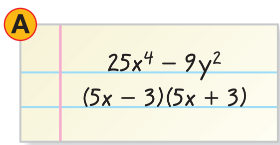
	x	$x^2 + 10x + 25$	$(x + 5)^2$	$(x - 5)^2$	$x^2 - 10x + 25$	$x^2 - 25$
a.	-5					
b.	-1					
c.	0					
d.	1					
e.	5					

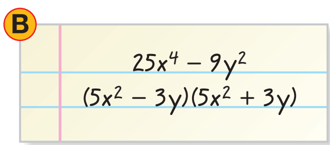
44. In the table above, which columns have equivalent values? Explain why.

45. **Geometry** A model for the difference of two squares is shown below. Copy and complete the second figure by writing the missing labels.



46. **ERROR ANALYSIS** Two students factored $25x^4 - 9y^2$. Which is incorrect? Explain the error.





Multiple Choice **For Exercises 47 and 48, choose the best answer.**

47. A polynomial expression is evaluated for the x- and y-values shown in the table. Which expression was evaluated to give the values shown in the third column?

x	y	Value of Expression
0	0	0
–1	–1	0
1	1	0
1	–1	4

(A) $x^2 - y^2$

(B) $x^2 + 2xy + y^2$

(C) $x^2 - 2xy + y^2$

(D) None of the above

48. The area of a square is $4x^2 + 20x + 25$. Which expression can also be used to model the area of the square?

(A) $(2x - 5)(5 - 2x)$

(B) $(2x + 5)(2x - 5)$

(C) $(2x - 5)^2$

(D) $(2x + 5)^2$

49. **Gridded Response** Evaluate the polynomial expression $x^2 - 18x + 81$ for $x = 10$.

CHALLENGE AND EXTEND

50. The binomial $81x^4 - 16$ can be factored using the rule for a difference of two squares.

a. Fill in the factorization: $81x^4 - 16$

$$(9x^2 + \square)(\square - \square)$$

b. One binomial from part **a** can be further factored. Identify the binomial and factor it. What is the complete factorization of $81x^4 - 16$?

c. Write your own binomial that can be factored twice as the difference of two squares.

51. The expression $4 - (v + 2)^2$ is the difference of two squares, because it fits the rule $a^2 - b^2$.

a. Identify a and b in the expression.

b. Factor and simplify $4 - (v + 2)^2$.

The *difference of cubes* is an expression of the form $a^3 - b^3$. It can be factored according to the rule $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$. For each binomial, identify a and b, and factor using the rule. Check your answer.

52. $x^3 - 1$ **53.** $27y^3 - 64$ **54.** $n^6 - 8$

SPIRAL STANDARDS REVIEW

10.0, 11.0, 16.0, 17.0, 18.0

Find the domain and range for each relation and tell whether the relation is a function. *(Lesson 4-2)*

55. $\{(5, 2), (4, 1), (3, 0), (2, -1)\}$

56. $\{(-3, 6), (-1, 6), (1, 6), (3, 6)\}$

57. $\{(2, -8), (2, -2), (2, 4), (2, 10)\}$

58. $\{(-2, 4), (-1, 1), (0, 0), (1, 1)\}$

Multiply. *(Lesson 7-7)*

59. $2a(3a^2 + 7a - 5)$ **60.** $(x + 3)(x - 8)$ **61.** $(t - 4)^2$

Factor each trinomial. Check your answer. *(Lesson 8-3)*

62. $x^2 + 3x - 10$ **63.** $x^2 - x - 12$ **64.** $x^2 + 7x + 8$

Connecting Algebra to Number Theory

Mental Math

Recognizing patterns of special products can help you perform calculations mentally.

Remember these special products that you studied in Chapter 7 and in Lesson 8-5.

Patterns of Special Products	
Difference of Two Squares	$(a+b)(a-b)=a^2-b^2$
Perfect-Square Trinomial	$(a+b)^2=a^2+2ab+b^2$ $(a-b)^2=a^2-2ab+b^2$

Example 1

Simplify $17^2 - 7^2$.

This expression is a difference of two squares with $a = 17$ and $b = 7$.

$$a^2 - b^2 = (a+b)(a-b) \quad \text{Write the rule for a difference of two squares.}$$
$$17^2 - 7^2 = (17+7)(17-7) \quad \text{Substitute 17 for a and 7 for b.}$$
$$= (24)(10) \quad \text{Simplify each group.}$$
$$= 240$$

Example 2

Simplify $14^2 + 2(14)(6) + 6^2$.

This expression is a perfect-square trinomial with $a = 14$ and $b = 6$.

$$a^2 + 2ab + b^2 = (a+b)^2 \quad \text{Write the rule for a perfect-square trinomial.}$$
$$14^2 + 2(14)(6) + 6^2 = (14+6)^2 \quad \text{Substitute 14 for a and 6 for b.}$$
$$= (20)^2 \quad \text{Simplify.}$$
$$= 400$$

Try This

Simplify each expression using the rules for special products.

1. $18^2 - 12^2$

2. $11^2 + 2(11)(14) + 14^2$

3. $22^2 - 18^2$

4. $38^2 - 2(38)(27) + 27^2$

5. $29^2 - 2(29)(17) + 17^2$

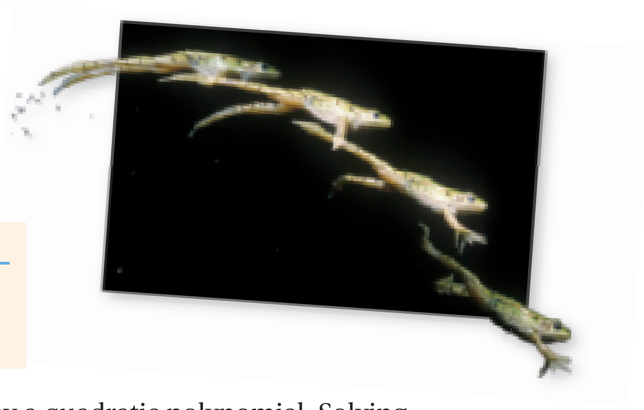
6. $55^2 + 2(55)(45) + 45^2$

7. $14^2 - 9^2$

8. $13^2 - 12^2$

9. $14^2 + 2(14)(16) + 16^2$

8-6 Choosing a Factoring Method



California Standards

11.0 Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

Why learn this?

You can factor polynomials to model the height of a leaping person or animal. (See Exercise 42.)

The height of a leaping frog can be modeled by a quadratic polynomial. Solving an equation that involves the polynomial may require factoring the polynomial.

Recall that a polynomial is fully or completely factored when it is written as a product of monomials and polynomials whose terms have no common factors other than 1.

EXAMPLE 1 Determining Whether an Expression Is Completely Factored

Tell whether each expression is completely factored. If not, factor it.

A $2x(x^2 + 4)$

$2x(x^2 + 4)$ — *Neither $2x$ nor $x^2 + 4$ can be factored further.*

$2x(x^2 + 4)$ is completely factored.

B $(2x + 6)(x + 5)$

$(2x + 6)(x + 5)$ — *$2x + 6$ can be factored further.*

$2(x + 3)(x + 5)$ — *Factor out 2, the GCF of $2x$ and 6.*

$2(x + 3)(x + 5)$ is completely factored.

C $2n(n^2 + 4n - 21)$

$2n(n^2 + 4n - 21)$ — *$n^2 + 4n - 21$ can be factored further.*

$2n(n + 7)(n - 3)$ — *Factor $n^2 + 4n - 21$.*

$2n(n + 7)(n - 3)$ is completely factored.

Caution!

$x^2 + 4$ is a *sum* of squares, and cannot be factored.

CHECK IT OUT! Tell whether each expression is completely factored. If not, factor it.

1a. $5x^2(x - 1)$ **1b.** $(4x + 4)(x + 1)$

To factor a polynomial completely, you may need to use more than one factoring method. Use the steps below to factor a polynomial completely.

Factoring Polynomials	
Step 1	Check for a greatest common factor.
Step 2	Check for a pattern that fits the difference of two squares or a perfect-square trinomial.
Step 3	To factor $x^2 + bx + c$, look for two integers whose sum is b and whose product is c. To factor $ax^2 + bx + c$, check integer factors of a and c in the binomial factors. The sum of the products of the outer and inner terms should be b.
Step 4	Check for common factors.

EXAMPLE 2 Factoring by GCF and Recognizing Patterns

Factor $-2xy^2 + 16xy - 32x$ completely. Check your answer.

$-2xy^2 + 16xy - 32x$

$-2x(y^2 - 8y + 16)$ — *Factor out the GCF. $y^2 - 8y + 16$ is a perfect-square trinomial of the form $a^2 - 2ab + b^2$.*

$-2x(y - 4)^2$ — *$a = y$, $b = 4$*

Check $-2x(y-4)^2 = -2x(y^2 - 8y + 16)$
$= -2xy^2 + 16xy - 32x$ ✓

CHECK IT OUT! **Factor each polynomial completely. Check your answer.**

2a. $4x^3 + 16x^2 + 16x$ **2b.** $2x^2y - 2y^3$

EXAMPLE 3 Factoring by Multiple Methods

Factor each polynomial completely.

A $2x^2 + 5x + 4$

$2x^2 + 5x + 4$ — *The GCF is 1 and there is no pattern.*

$(\square x + \square)(\square x + \square)$ — *$a = 2$ and $c = 4$; Outer + Inner = 5*

Factors of 2	Factors of 4	Outer + Inner	
1 and 2	1 and 4	$(1)4 + (2)1 = 6$	✗
1 and 2	4 and 1	$(1)1 + (2)4 = 9$	✗
1 and 2	2 and 2	$(1)2 + (2)2 = 6$	✗

$2x^2 + 5x + 4$ cannot be factored any further. It is factored completely.

Remember!

For a polynomial of the form $ax^2 + bx + c$, if there are no integers whose sum is b and whose product is ac, then the polynomial is said to be unfactorable.

B $3n^4 - 15n^3 + 12n^2$

$3n^2(n^2 - 5n + 4)$ — *Factor out the GCF. There is no pattern.*

$(n + \square)(n + \square)$ — *$b = -5$ and $c = 4$; look for integer factors of 4 whose sum is -5.*

Factors of 4	Sum	
-1 and -4	-5	✓
-2 and -2	-4	✗

The factors needed are -1 and -4.

$3n^2(n - 1)(n - 4)$

C $4x^3 + 18x^2 + 20x$

$2x(2x^2 + 9x + 10)$ — *Factor out the GCF. There is no pattern.*

$(\square x + \square)(\square x + \square)$ — *$a = 2$ and $c = 10$; Outer + Inner = 9*

Factors of 2	Factors of 10	Outer + Inner	
1 and 2	1 and 10	$(1)10 + (2)1 = 12$	✗
1 and 2	10 and 1	$(1)1 + (2)10 = 21$	✗
1 and 2	2 and 5	$(1)5 + (2)2 = 9$	✓

$(x + 2)(2x + 5)$

$2x(x + 2)(2x + 5)$

D $p^5 - p$

$p(p^4 - 1)$	*Factor out the GCF.*
$p(p^2 + 1)(p^2 - 1)$	*$p^4 - 1$ is a difference of two squares.*
$p(p^2 + 1)(p + 1)(p - 1)$	*$p^2 - 1$ is a difference of two squares.*

CHECK IT OUT! **Factor each polynomial completely. Check your answer.**

3a. $3x^2 + 7x + 4$ **3b.** $2p^5 + 10p^4 - 12p^3$

3c. $9q^6 + 30q^5 + 24q^4$ **3d.** $2x^4 + 18$

Know it! Note

Methods to Factor Polynomials

Any Polynomial—Look for the greatest common factor.	
$ab - ac = a(b - c)$	$6x^2y + 10xy^2 = 2xy(3x + 5y)$
Binomials—Look for a difference of two squares.	
$a^2 - b^2 = (a + b)(a - b)$	$x^2 - 9y^2 = (x + 3y)(x - 3y)$
Trinomials—Look for perfect-square trinomials and other factorable trinomials.	
$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	$x^2 + 4x + 4 = (x + 2)^2$ $x^2 - 2x + 1 = (x - 1)^2$
$x^2 + bx + c = (x + \square)(x + \square)$ $ax^2 + bx + c = (\square x + \square)(\square x + \square)$	$x^2 + 3x + 2 = (x + 1)(x + 2)$ $6x^2 + 7x + 2 = (2x + 1)(3x + 2)$
Polynomials of Four or More Terms—Factor by grouping.	
$ax + bx + ay + by = x(a + b) + y(a + b)$ $= (x + y)(a + b)$	$2x^3 + 4x^2 + x + 2 = (2x^3 + 4x^2) + (x + 2)$ $= 2x^2(x + 2) + 1(x + 2)$ $= (x + 2)(2x^2 + 1)$

THINK AND DISCUSS

1. Give an expression that includes a polynomial that is not completely factored.

2. Give an example of an unfactorable binomial and an unfactorable trinomial.

Know it! Note

3. GET ORGANIZED Copy the graphic organizer. Draw an arrow from each expression to the method you would use to factor it.

Factoring Methods

Polynomial	Method
1. $16x^4 - 25y^8$	**A.** Factoring out the GCF
2. $x^2 + 10x + 25$	**B.** Factoring by grouping
3. $9t^2 + 27t + 18t^4$	**C.** Unfactorable
4. $a^2 + 3a - 7a - 21$	**D.** Difference of two squares
5. $100b^2 + 81$	**E.** Perfect-square trinomial

8-6 Exercises

California Standards Practice
11.0, 25.2

go.hrw.com
Homework Help Online
KEYWORD: MA8CA 8-6
Parent Resources Online
KEYWORD: MA8CA Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 522

Tell whether each expression is completely factored. If not, factor it.

1. $3x(9x^2 + 1)$
2. $2(4x^3 - 3x^2 - 8x)$
3. $2k^2(4 - k^3)$
4. $(2x + 3)(3x - 5)$
5. $4(4p^4 - 1)$
6. $a(a^3 + 2ab + b^2)$

SEE EXAMPLE 2 p. 523

Factor each polynomial completely. Check your answer.

7. $3x^5 - 12x^3$
8. $4x^3 + 8x^2 + 4x$
9. $8pq^2 + 8pq + 2p$
10. $18rs^2 - 2r$
11. $mn^5 - m^3n$
12. $2x^2y - 20xy + 50y$

SEE EXAMPLE 3 p. 523

13. $6x^4 - 3x^3 - 9x^2$
14. $3y^2 + 14y + 4$
15. $p^5 + 3p^3 + p^2 + 3$
16. $7x^5 + 21x^4 - 28x^3$
17. $2z^2 + 11z + 6$
18. $9p^2 - q^2 + 3p + q$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
19–24	1
25–30	2
31–36	3

Extra Practice
Skills Practice p. EP17
Application Practice p. EP31

Tell whether each expression is completely factored. If not, factor it.

19. $2x(y^3 - 4y^2 + 5y)$
20. $2r(25r^6 - 36)$
21. $3n^2(n^2 - 25)$
22. $2m(m + 1)(m + 4)$
23. $2y^2(4x^2 + 9)$
24. $4(7g + 9h^2)$

Factor each polynomial completely. Check your answer.

25. $-4x^3 + 24x^2 - 36x$
26. $24r^2 - 6r^4$
27. $5d^2 - 60d + 135$
28. $4y^8 + 36y^7 + 81y^6$
29. $98x^3 - 50xy^2$
30. $4x^3y - 4x^2y - 8xy$
31. $5x^2 - 10x + 14$
32. $121x^2 + 36y^2$
33. $p^4 - 16$
34. $4m^6 - 30m^5 + 36m^4$
35. $2k^3 + 3k^2 + 6k + 9$
36. $ab^4 - 16a$

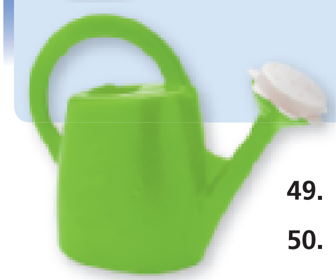
Write an expression for each situation. Factor your expression.

37. the square of Ella's age plus 12 times Ella's age plus 36
38. the square of the distance from point A to point B minus 81
39. the square of the number of seconds Bob can hold his breath minus 16 times the number of seconds plus 28
40. three times the square of apples on a tree minus 22 times the number of apples plus 35
41. the square of Beth's score minus 49
42. **Physical Science** The height in meters of a ballet dancer's center of mass when she leaps can be modeled by the polynomial $-5t^2 + 30t + 1$, where t is time in seconds after the jump. Tell whether the polynomial is fully factored when written as $-1(5t^2 - 30t - 1)$. Explain.
43. **Write About It** When asked to factor a polynomial completely, you first determine that the terms in the polynomial do not share any common factors. What would be your next step?

Factor and simplify each expression. Check your answer.

44. $12(x + 1)^2 + 60(x + 1) + 75$
45. $(2x + 3)^2 - (x - 4)^2$
46. $45x(x - 2)^2 + 60x(x - 2) + 20x$
47. $(3x - 5)^2 - (y + 2)^2$

CONCEPT CONNECTION



48. This problem will prepare you for the Concept Connection on page 528.

a. The area of a Marci's rectangular flower garden is $(x^2 + 2x - 15)$ ft^2. Factor this expression for area. Check your answer.

b. Draw a diagram of the garden and label the length and width with your factors from part **a.**

c. Find the length and width of the flower garden if $x = 7$ ft.

49. **Critical Thinking** Show two methods of factoring $4x^2 - 100$.

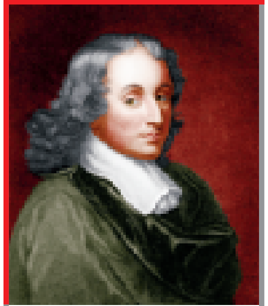
50. **Estimation** Estimate the value of $2x^2 + 5xy + 3y^2$ when $x = -10.1$ and $y = 10.05$. (*Hint:* Factor the expression first.)

51. **ERROR ANALYSIS** Examine the factorization shown. Explain why the factorization is incorrect.

$12x^2 - 12x - 3$
$3(4x^2 - 4x - 1)$
$3(2x - 1)(2x - 1)$

LINK

Math History



Blaise Pascal was a French mathematician who lived in the 1600s.

Math History **Use the following information for Exercises 52–54.**

The triangle at right is called *Pascal's Triangle*. The triangle starts with 1 and each of the other numbers in the triangle is the sum of the two numbers in the row above it.

0						1						
1					1		1					
2				1		2		1				
3			1		3		3		1			
4		1		4		6		4		1		
5	1		5		10		10		5		1	

Pascal's Triangle can be used to write the product of a binomial raised to an integer power. The numbers in each row give you the coefficients of each term in the product.

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

The numbers in row 3 are 1, 3, 3, 1. These are the coefficients of the terms in the product $(a + b)^3$. The power of a decreases in each term and the power of b increases in each term.

Use the patterns you see in Pascal's Triangle to write the power of the binomial $a + b$ given by each product.

52. $a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6 = (a + b)^{\square}$

53. $a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + b^8 = (a + b)^{\square}$

54. $a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7 = (a + b)^{\square}$

Multiple Choice **For Exercises 55–57, choose the best answer.**

55. Which expression equals $6x^2 + 7x - 10$?

Ⓐ $(6x + 2)(x - 5)$
Ⓑ $(2x + 5)(3x - 2)$
Ⓒ $(x + 2)(6x - 5)$
Ⓓ $(3x + 2)(2x - 5)$

56. What is the complete factorization of $16x^{12} - 256$?

Ⓐ $16(x^6 + 4)(x^6 - 4)$
Ⓑ $(4x^6 + 16)(4x^6 - 16)$
Ⓒ $16(x^6 + 4)(x^3 + 2)(x^3 - 2)$
Ⓓ $(4x^6 + 16)(2x^3 + 4)(2x^3 - 4)$

57. Which of the expressions below represents the fifth step of the factorization?

Step 1: $40a^3 - 60a^2 - 10a + 15$

Step 2: $5(8a^3 - 12a^2 - 2a + 3)$

Step 3: $5[(8a^3 - 12a^2) - (2a - 3)]$

Step 4: $5[4a^2(2a - 3) - 1(2a - 3)]$

Step 5:

Step 6: $5(2a - 3)(2a + 1)(2a - 1)$

Ⓐ $5(2a - 3)(2a + 3)(4a^2 - 1)$

Ⓑ $5(2a - 3)(4a^2 + 1)$

Ⓒ $5(2a - 3)(4a^2 - 1)$

Ⓓ $5(2a - 3)(2a - 3)(4a^2 - 1)$

58. Short Response Use the polynomial $8x^3 + 24x^2 + 18x$ for the following.

a. Factor the polynomial. Explain each step and tell whether you used any rules for special products.

b. Explain another set of steps that could be used to factor the polynomial.

CHALLENGE AND EXTEND

$V = \pi r^2 h$

59. Geometry The volume of the cylinder shown is represented by the expression $72\pi p^3 + 48\pi p^2 + 8\pi p$. The height of the cylinder is $8p$.

a. Factor the expression for volume.

b. What expression represents the radius of the cylinder?

c. If the radius is 4 cm, what are the height and volume of the cylinder?

Factor. Check your answer.

60. $g^7 + g^3 + g^5 + g^4$

61. $h^2 + h^8 + h^6 + h^4$

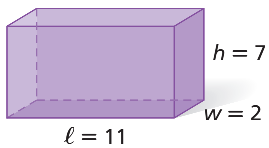
62. $x^{n+2} + x^{n+1} + x^n$

63. $x^{n+5} + x^{n+4} + x^{n+3}$

64. Geometry The rectangular prism has the dimensions shown.

a. Write expressions for the height and length of the prism using w.

b. Write a polynomial that represents the volume of the prism using w.



SPIRAL STANDARDS REVIEW

8.0, 11.0, 16.0, 17.0. 18.0

Give the domain and range of the relation. Tell whether the relation is a function. *(Lesson 4-2)*

65.

x	−1	0	1	2
y	−2	1	4	7

66.

x	1	2	3	1
y	−2	−1	0	−2

Identify which lines are perpendicular. *(Lesson 5-7)*

67. $y = -5x + 4;\ y = \frac{1}{5}x + 2;\ y = 5;\ y = 0$

68. $y = -x + 3;\ y = 8x;\ y = -\frac{1}{8}x + 5;\ y = x$

Factor each trinomial. Check your answer. *(Lesson 8-4)*

69. $2x^2 + 13x + 15$

70. $4x^2 + 4x - 3$

71. $6x^2 - 11x - 10$

CHAPTER 8

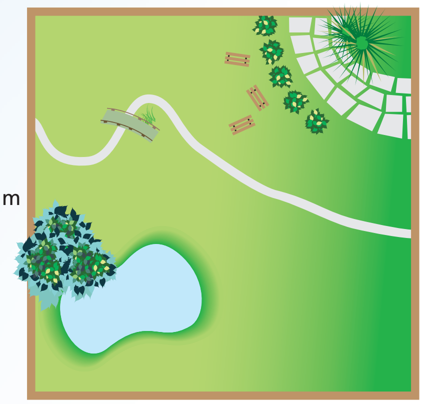
CONCEPT CONNECTION

SECTION 8B

Factoring

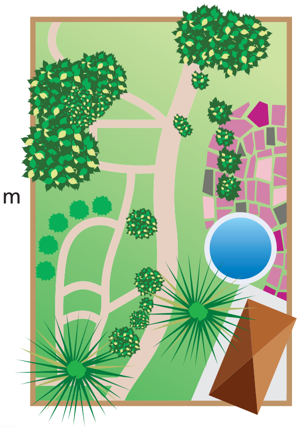
Shaping the Environment The Environmental Awareness Club is going to plant a garden on the front lawn of the school. Henry suggests a garden in the shape of a square. Theona suggests a rectangular shape.

1. Henry's plans include a square garden with an area of $(x^2 + 12x + 36)$ m^2. Write expressions for the length and width of the square garden.



2. A drawing of the square garden shows a length of 12 m. What is the width of the square garden? What is the value of x? What is the total area of the square garden?

3. Theona's plans include a rectangular garden with an area of $(x^2 + 14x + 24)$ m^2. Write expressions for the length and width of the rectangular garden.



4. A drawing of the rectangular garden shows that the length is 6 m longer than the length of the square garden. What is the width of the rectangular garden? How much shorter is the width of the rectangular garden than the square garden?

5. Find the perimeter of each garden in terms of x.

6. Which plan should the club choose if they want the garden that covers the most area? Which plan should the club choose if they want the garden that requires the least fencing around it? Explain your reasoning.

READY TO GO ON?

CHAPTER 8

SECTION 8B

Quiz for Lessons 8-5 Through 8-6

8-5 Factoring Special Products

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

1. $x^2 + 8x + 16$ **2.** $4x^2 - 20x + 25$ **3.** $x^2 + 3x + 9$

4. $2x^2 - 4x + 4$ **5.** $9x^2 - 12x + 4$ **6.** $x^2 - 12x - 36$

7. An architect is designing rectangular windows with an area of $(x^2 + 20x + 100)$ ft^2. The dimensions of the windows are of the form $ax + b$, where a and b are whole numbers. Find an expression for the perimeter of the windows. Find the perimeter of a window when $x = 4$ ft.

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

8. $x^2 - 121$ **9.** $4t^2 - 20$ **10.** $1 - 9y^4$

11. $25m^2 - 4m^6$ **12.** $16x^2 + 49$ **13.** $r^4 - t^2$

14. The area of a square is $(36d^2 - 36d + 9)$ in^2.

a. What expression represents the length of a side of the square?

b. What expression represents the perimeter of the square?

c. What are the length of a side, the perimeter, and the area of the square when $d = 2$ in.?

8-6 Choosing a Factoring Method

Tell whether each expression is completely factored. If not, factor it.

15. $5(x^2 + 3x + 1)$ **16.** $6x(5x^2 - x)$

17. $3t(t^4 - 9)$ **18.** $2(m^2 - 10m + 25)$

19. $3(2y^2 - 5)(y + 1)$ **20.** $(2n + 6)(n - 4)$

Factor each polynomial completely. Check your answer.

21. $3x^3 - 12x^2 + 12x$ **22.** $16m^3 - 4m$ **23.** $5x^3y - 45xy$

24. $3t^2 + 5t - 1$ **25.** $3c^2 + 12c - 63$ **26.** $x^5 - 81x$

Write an expression for each situation. Then factor your expression.

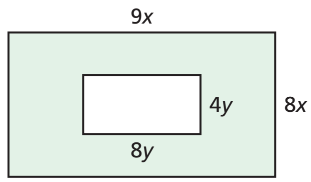
27. the difference of the square of a board's length and 36

28. the square of Michael's age minus 8 times Michael's age plus 16

29. two times the square of a car's speed plus 2 times the car's speed minus 12

30. three times the cube of Jessie's height plus 3 times the square of Jessie's height minus 6 times Jessie's height

31. Write an expression for the area of the shaded region. Then factor the expression.



CHAPTER 8 Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. A number written as a product so that each of its factors has no factors other than 1 and itself is the ___?___.

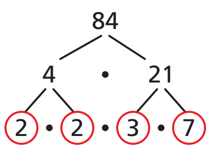
2. The ___?___ of two monomials is the greatest of the factors that the monomials share.

8-1 Factors and Greatest Common Factors *(pp. 478–483)*

Prep for **11.0**

EXAMPLES

■ **Write the prime factorization of 84.**



Write as a product.
Continue until all factors are prime.

■ **Write the prime factorization of 75.**

$$\begin{array}{r|l} 3 & 75 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

Keep dividing by prime factors until the quotient is 1.

$75 = 3 \cdot 5 \cdot 5 = 3 \cdot 5^2$

■ **Find the GCF of 36 and 90.**

$36 = 2 \cdot \boxed{2} \cdot \boxed{3} \cdot \boxed{3}$

$90 = \boxed{2} \cdot \boxed{3} \cdot \boxed{3} \cdot 5$

$2 \cdot 3 \cdot 3 = 18$

Write the prime factorization of each number.
Find the product of the common factors.

The GCF of 36 and 90 is 18.

■ **Find the GCF of $10x^5$ and $4x^2$.**

$10x^5 = \boxed{2} \cdot 5 \cdot \boxed{x} \cdot \boxed{x} \cdot x \cdot x \cdot x$

$4x^2 = \boxed{2} \cdot 2 \cdot \boxed{x} \cdot \boxed{x}$

$2 \cdot x \cdot x = 2x^2$

Write the prime factorization of each coefficient.
Write powers as products.
Find the product of the common factors.

The GCF of $10x^5$ and $4x^2$ is $2x^2$.

EXERCISES

Write the prime factorization of each number.

3. 12 **4.** 20

5. 32 **6.** 23

7. 40 **8.** 64

9. 66 **10.** 114

Find the GCF of each pair of numbers.

11. 15 and 50

12. 36 and 132

13. 29 and 30

14. 54 and 81

15. 20 and 48

Find the GCF of each pair of monomials.

16. $9m$ and 3

17. $4x$ and $2x^2$

18. $-18b^4$ and $27b^2$

19. $100r$ and $25r^5$

20. A hardware store carries 42 types of boxed nails and 36 types of boxed screws. The store manager wants to build a rack so that he can display the hardware in rows. He wants to put the same number of boxes in each row, but he wants no row to contain both nails and screws. What is the greatest number of boxes that he can display in one row? How many rows will there be if the manager puts the greatest number of boxes in each row?

8-2 Factoring by GCF *(pp. 487–493)*

11.0

EXAMPLES

- **Factor $3t^3 - 9t^2$. Check your answer.**

$3t^3 = 3 \cdot t \cdot t \cdot t$
$9t^2 = 3 \cdot 3 \cdot t \cdot t$ *Find the GCF.*

GCF: $3 \cdot t \cdot t = 3t^2$

$3t^3 - 9t^2 = 3t^2(t) - 3t^2(3)$
$= 3t^2(t - 3)$ *Factor out the GCF.*

Check $3t^2(t-3) = 3t^3 - 9t^2$ ✓

- **Factor $-12s - 6s^3$. Check your answer.**

$-1(12s + 6s^3)$ *Factor out −1.*

$12s = 2 \cdot 2 \cdot 3 \cdot s$
$6s^3 = 2 \cdot \quad 3 \cdot s \cdot s \cdot s$ *Find the GCF.*

GCF: $2 \cdot 3 \cdot s = 6s$

$-1(12s + 6s^3)$

$1[(6s)(2) + (6s)(s^2)]$

$-1[(6s)(2 + s^2)]$

$-6s(2 + s^2)$ *Factor out the GCF.*

Check $-6s(2 + s^2) = -12s - 6s^3$ ✓

- **Factor $5(x - 7) + 3x(x - 7)$.**

$5(x - 7) + 3x(x - 7)$ *The terms have a common factor of $(x - 7)$.*

$(x - 7)(5 + 3x)$ *Factor out $(x - 7)$.*

- **Factor $6b^3 + 8b + 15b^2 + 20$ by grouping.**

$(6b^3 + 8b) + (15b^2 + 20)$ *Group terms that have a common factor.*

$2b(3b^2 + 4) + 5(3b^2 + 4)$ *Factor each group.*

$(3b^2 + 4)(2b + 5)$ *Factor out $(3b^2 + 4)$.*

- **Factor $2m^3 - 6m^2 + 15 - 5m$. Check your answer.**

$(2m^3 - 6m^2) + (15 - 5m)$ *Group terms.*
$2m^2(m - 3) + 5(3 - m)$ *Factor each group.*

$2m^2(m - 3) + 5(-1)(m - 3)$ *Rewrite $(3 - m)$ as $(-1)(m - 3)$.*

$2m^2(m - 3) - 5(m - 3)$ *Simplify.*

$(m - 3)(2m^2 - 5)$ *Factor out $(m - 3)$.*

Check $(m - 3)(2m^2 - 5)$

$2m^3 - 5m - 6m^2 + 15$
$2m^3 - 6m^2 + 15 - 5m$ ✓

EXERCISES

Factor each polynomial. Check your answer.

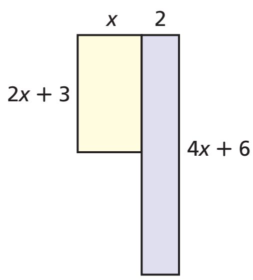
21. $5x - 15x^3$
22. $-16b + 32$
23. $-14v - 21$
24. $4a^2 - 12a - 8$
25. $5g^5 - 10g^3 - 15g$
26. $40p^2 - 10p + 30$
27. A civil engineer needs the area of a rectangular lot to be $(6x^2 + 5x)$ ft^2. Factor this polynomial to find expressions for the dimensions of the lot.

Factor each expression.

28. $2x(x - 4) + 9(x - 4)$
29. $t(3t + 5) - 6(3t + 5)$
30. $5(6 - n) - 3n(6 - n)$
31. $b(b + 4) + 2(b + 4)$
32. $x^2(x - 3) + 7(x - 3)$

Factor each polynomial by grouping. Check your answer.

33. $n^3 + n - 4n^2 - 4$
34. $6b^2 - 8b + 15b - 20$
35. $2h^3 - 7h + 14h^2 - 49$
36. $3t^2 + 18t + t + 6$
37. $10m^3 + 15m^2 - 2m - 3$
38. $8p^3 + 4p - 6p^2 - 3$
39. $5r - 10 + 2r - r^2$
40. $b^3 - 5b + 15 - 3b^2$
41. $6t - t^3 - 4t^2 + 24$
42. $12h - 3h^2 + h - 4$
43. $d - d^2 + d - 1$
44. $6b - 5b^2 + 10b - 12$
45. $5t - t^2 - t + 5$
46. $8b^2 - 2b^3 - 5b + 20$
47. $3r - 3r^2 - 1 + r$
48. Write an expression for the area of each of the two rectangles shown. Then write and factor an expression for the combined area.



8-3 Factoring $x^2 + bx + c$ (pp. 496–503)

11.0

EXAMPLES

Factor each trinomial. Check your answer.

- $x^2 + 14x + 45$

$(x + \square)(x + \square)$ *Look for factors of 45 whose sum is 14.*

$(x + 9)(x + 5)$

Check $(x + 9)(x + 5) = x^2 + 5x + 9x + 45$

$= x^2 + 14x + 45$ ✓

- $x^2 + 6x - 27$

$(x + \square)(x - \square)$ *Look for factors of −27 whose sum is 6.*

$(x + 9)(x - 3)$

Check $(x + 9)(x - 3) = x^2 - 3x + 9x - 27$

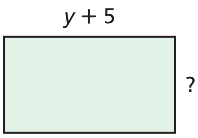
$= x^2 + 6x - 27$ ✓

EXERCISES

Factor each trinomial. Check your answer.

49. $x^2 + 6x + 5$
50. $x^2 + 6x + 8$
51. $x^2 + 8x + 15$
52. $x^2 - 8x + 12$
53. $x^2 + 10x + 25$
54. $x^2 - 13x + 22$
55. $x^2 + 24x + 80$
56. $x^2 - 26x + 120$
57. $x^2 + 5x - 84$
58. $x^2 - 5x - 24$
59. $x^2 - 3x - 28$
60. $x^2 + 4x - 5$
61. $x^2 + x - 6$
62. $x^2 + x - 20$
63. $x^2 - 2x - 48$
64. $x^2 - 5x - 36$
65. $x^2 - 6x - 72$
66. $x^2 - 3x - 70$
67. $x^2 + 14x - 120$
68. $x^2 + 6x - 7$

69. The rectangle shown has an area of $(y^2 + 8y + 15)$ m^2. What is the width of the rectangle?



8-4 Factoring $ax^2 + bx + c$ (pp. 505–511)

11.0

EXAMPLES

Factor each trinomial.

- $6x^2 + 17x + 5$

$(\square x + \square)(\square x + \square)$ *$a = 6$ and $c = 5$; Outer + Inner = 17*

Factors of 6	Factors of 5	Outer + Inner
1 and 6	5 and 1	$(1)1 + (6)5 = 31$
2 and 3	1 and 5	$(2)5 + (3)1 = 13$
2 and 3	5 and 1	$(2)1 + (3)5 = 17$

$(2x + 5)(3x + 1)$

- $2n^2 - n - 10$

$(\square n + \square)(\square n + \square)$ *$a = 2$ and $c = -10$; Outer + Inner = −1*

Factors of 2	Factors of −10	Outer + Inner
1 and 2	1 and −10	$1(-10) + 2(1) = -8$
1 and 2	−1 and 10	$1(10) + 2(-1) = 8$
1 and 2	2 and −5	$1(-5) + 2(2) = -1$

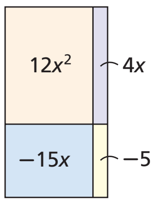
$(1n + 2)(2n - 5) = (n + 2)(2n - 5)$

EXERCISES

Factor each trinomial. Check your answer.

70. $2x^2 + 11x + 5$
71. $3x^2 + 10x + 7$
72. $2x^2 - 3x + 1$
73. $3x^2 + 8x + 4$
74. $5x^2 + 28x + 15$
75. $6x^2 - 19x + 15$
76. $4x^2 + 13x + 10$
77. $3x^2 + 10x + 8$
78. $7x^2 - 37x + 10$
79. $9x^2 + 18x + 8$
80. $2x^2 - x - 1$
81. $3x^2 - 11x - 4$
82. $2x^2 - 11x + 5$
83. $7x^2 - 19x - 6$
84. $5x^2 - 9x - 2$
85. $-6x^2 - x + 2$
86. $6x^2 - x - 5$
87. $6x^2 + 17x - 14$
88. $-4x^2 + 8x + 5$
89. $-10x^2 + 11x + 6$

90. Write the polynomial modeled and then factor.



8-5 Factoring Special Products *(pp. 514–520)*

11.0

EXAMPLES

- **Determine whether $x^2 + 18x + 81$ is a perfect square. If so, factor. If not, explain.**

$$x^2 + 18x + 81$$

$$x \cdot x \quad 2(x \cdot 9) \quad 9 \cdot 9$$

The trinomial is of the form $a^2 + 2ab + b^2$, so it is a perfect-square trinomial.

$$x^2 + 18x + 81 = (x + 9)^2$$

- **Determine whether $49x^4 - 25y^6$ is a difference of two squares. If so, factor. If not, explain.**

$$49x^4 - 25y^6$$

$$7x^2 \cdot 7x^2 \quad 5y^3 \cdot 5y^3$$

The binomial is a difference of two squares.

$$(7x^2)^2 - (5y^3)^2$$ *$a = 7x^2$, $b = 5x^3$*

$$(7x^2 + 5y^3)(7x^2 - 5y^3)$$ *Write the binomial as $(a + b)(a - b)$.*

$$49x^4 - 25y^6 = (7x^2 + 5y^3)(7x^2 - 5y^3)$$

EXERCISES

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

91. $x^2 + 12x + 36$
92. $x^2 + 5x + 25$
93. $4x^2 - 2x + 1$
94. $9x^2 + 12x + 4$
95. $16x^2 + 8x + 4$
96. $x^2 + 14x + 49$

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

97. $100x^2 - 81$
98. $x^2 - 2$
99. $5x^4 - 10y^6$
100. $(-12)^2 - (x^3)^2$
101. $121b^2 + 9c^8$
102. $100p^2 - 25q^2$

Factor each polynomial using the pattern of perfect-square trinomials or the difference of two squares. Tell which pattern you used and check your answer.

103. $x^2 - 25$
104. $x^2 + 20x + 100$
105. $j^2 - k^4$
106. $9x^2 - 42x + 49$
107. $81x^2 + 144x + 64$
108. $16b^4 - 121c^6$

8-6 Choosing a Factoring Method *(pp. 522–527)*

11.0

EXAMPLES

- **Tell whether $(3x - 9)(x + 4)$ is completely factored. If not, factor it.**

$(3x - 9)(x + 4)$ — *$3x - 9$ can be factored.*
$3(x - 3)(x + 4)$ — *Factor out 3, the GCF of $3x$ and 9.*

- $3ab^2 - 48a$

$3a(b^2 - 16)$ — *Factor out the GCF.*
$3a(b + 4)(b - 4)$ — *Factor the difference of two squares.*

Check $3a(b + 4)(b - 4) = 3a(b^2 - 16)$
$= 3ab^2 - 48a$ ✓

- $2m^3 + 4m^2 - 48m$

$2m(m^2 + 2m - 24)$ — *Factor out the GCF.*
$2m(m - 4)(m + 6)$ — *Factor the trinomial.*

Check $2m(m - 4)(m + 6)$
$2m(m^2 + 2m - 24)$
$2m^3 + 4m^2 - 48m$ ✓

EXERCISES

Tell whether each polynomial is completely factored. If not, factor it.

109. $4x^2 + 10x + 6 = (4x + 6)(x + 1)$
110. $3y^2 + 75 = 3(y^2 + 25)$
111. $b^4 - 81 = (b^2 + 9)(b^2 - 9)$
112. $x^2 - 6x + 9 = (x - 3)^2$

Factor each polynomial completely. Check your answer.

113. $4x^2 - 64$
114. $3b^5 - 6b^4 - 24b^3$
115. $a^4b^3 - a^2b^5$
116. $t^{20} - t^4$
117. $5x^2 + 20x + 15$
118. $2x^4 - 50x^2$
119. $8t + 32 + 2st + 8s$
120. $25m^3 - 90m^2 - 40m$
121. $32x^4 - 48x^3 + 8x^2 - 12x$
122. $6s^4t + 12s^3t^2 + 6s^2t^3$
123. $10m^3 + 4m^2 - 90m - 36$

CHAPTER 8

CHAPTER TEST

Find the GCF of each pair of monomials.

1. $3t^4$ and $8t^2$

2. $2y^3$ and $-12y$

3. $15n^5$ and $9n^4$

4. Write the prime factorization of 360.

5. A coin collector is arranging a display of three types of nickels. The types of nickels and number of each type are shown in the table. The collector wants to arrange them in rows with the same number in each row without having different types in the same row. How many rows will she need if she puts the greatest possible number of nickels in each row?

Type of Nickel	Number of Nickels
Liberty	16
Buffalo	24
Jefferson	40

Factor each expression.

6. $24m^2 + 4m^3$

7. $9x^5 - 12x$

8. $-2r^4 - 6$

9. $3(c - 5) + 4c(c - 5)$

10. $10x^3 + 4x - 25x^2 - 10$

11. $4y^3 - 4y^2 - 3 + 3y$

12. A model rocket is shot vertically from a deck into the air at a speed of 50 m/s. The expression $-5t^2 + 50t + 5$ gives the approximate height of the rocket after t seconds. Factor this expression.

Factor each trinomial.

13. $x^2 + 6x + 5$

14. $x^2 - 4x - 21$

15. $x^2 - 8x + 15$

16. $2x^2 + 9x + 7$

17. $2x^2 + 9x - 18$

18. $-3x^2 - 2x + 8$

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

19. $a^2 + 14a + 49$

20. $2x^2 + 10x + 25$

21. $9t^2 - 6t + 1$

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

22. $b^2 - 16$

23. $25y^2 - 10$

24. $9a^2 - b^{10}$

25. A company is producing rectangular sheets of plastic. Each has an area of $(9x^2 + 30x + 25)$ ft². The dimensions of each sheet are of the form $ax + b$, where a and b are whole numbers. Find an expression for the perimeter of a sheet. Find the perimeter when $x = 4$ ft.

Tell whether each expression is completely factored. If not, factor it.

26. $(6x - 3)(x + 5)$

27. $(v^5 + 10)(v^5 - 10)$

28. $(2b + 3)(3b - 2)$

Factor each polynomial completely.

29. $8x^3 + 72x^2 + 160x$

30. $3x^5 - 27x^3$

31. $8x^3 + 64x^2 - 20x - 160$

32. $cd^4 - c^7d^6$

33. $100x^2 - 80x + 16$

34. $7m^8 - 7$

COLLEGE ENTRANCE EXAM PRACTICE

CHAPTER 8

FOCUS ON ACT

The ACT Mathematics test booklet usually has writing space for scratch work. If not, the administrator of the test should have blank paper for you to use. The scratch work is for your use only. Be sure to transfer your final answer to the answer sheet.

HOT TIP! If you are unsure how to solve a problem, look through the answer choices. They may provide you with a clue to the solution method. It may take longer to work backward from the answer choices, so make sure you monitor your time.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. What is the value of $c^2 - d^2$ if $c + d = 7$ and $c - d = -2$?

(A) -14

(B) -5

(C) 5

(D) 14

(E) 45

2. Which of the following is the complete factorization of $6a^3b + 3a^2b^3$?

(F) $6a^3b^3$

(G) $9a^5b^4$

(H) $3ab(2a^2 + ab^2)$

(J) $3a^2b(2a + b^2)$

(K) $(6a^3b)(3a^2b^3)$

3. Which of the following is a factor of $x^2 + 3x - 18$?

(A) $x + 2$

(B) $x + 3$

(C) $x + 6$

(D) $x + 9$

(E) $x + 18$

4. The binomial $x - 3$ is NOT a factor of which of the following trinomials?

(F) $2x^2 - x - 3$

(G) $2x^2 - 5x - 3$

(H) $2x^2 - 8x + 6$

(J) $3x^2 - 6x - 9$

(K) $3x^2 - 10x + 3$

5. For what value of n is $4x^2 + 20x + n^2 = (2x + n)^2$ true for any real number x?

(A) 4

(B) 5

(C) 8

(D) 10

(E) 25

6. What is the factored form of $x^2 + \frac{2x}{3} + \frac{x}{2} + \frac{2}{6}$?

(F) $\left(x + \frac{1}{3}\right)\left(x + \frac{1}{2}\right)$

(G) $\left(x + \frac{1}{2}\right)\left(x + \frac{2}{3}\right)$

(H) $\left(x + \frac{2}{3}\right)\left(x + \frac{1}{6}\right)$

(J) $(x + 2)\left(x + \frac{1}{3}\right)$

(K) $\left(x + \frac{1}{3}\right)\left(x + \frac{2}{3}\right)$

CHAPTER 8

STRATEGIES FOR SUCCESS

Any Question Type: Translate Words to Math

When reading a word problem, look for actions and context clues to help you translate the words into a mathematical equation or expression.

Some actions, such as those shown in this table, imply certain mathematical operations.

Action	Math Operation
Combining, increasing	Addition
Decreasing, reducing	Subtraction
Increasing or decreasing by a factor	Multiplication
Separating	Division

EXAMPLE 1

Short Response **The polynomial $x^2 + 7x + 12$ represents the area of a rectangle in square meters. The width is $(x + 3)$ meters. Find the combined measure of the length and the width.**

Use actions and context clues to translate the words into equations.

$x^2 + 7x + 12$ represents the area of a rectangle in square meters.

$x^2 + 7x + 12 \quad = \quad A$

The width is $(x + 3)$ meters.

$w \quad = (x + 3)$

Find the combined measure of the length and the width.

$m \quad = \quad \ell \quad + \quad w$

Now use the equations to solve the problem.

$A = \ell w$	*Write the formula for area of a rectangle.*
$x^2 + 7x + 12 = \ell(x + 3)$	*Substitute $x^2 + 7x + 12$ for A and $(x + 3)$ for w.*
$(x + \boxed{?})(x + 3)$	*Factor $x^2 + 7x + 12$ to find an expression for the length.*
$(x + 4)(x + 3)$	*$3(4) = 12$; $3 + 4 = 7$*

The length is $(x + 4)$.

$m = \ell + w$	*Write the equation for the combined measure of the length and width.*
$m = (x + 4) + (x + 3)$	*Substitute $(x + 4)$ for ℓ and $(x + 3)$ for w.*
$m = 2x + 7$	*Combine like terms.*

The combined measure of the length and width is $(2x + 7)$ meters.

HOT TIP! Sometimes you cannot write an expression or equation in the order that the actions appear. For example, the expression "4 years younger than Maria" is written mathematically as $m - 4$.

Read each test item and answer the questions that follow.

Item A

Short Response The width of Alvin's rectangular mural is 6 times the length x. Alvin plans to make a new mural with an area of $(6x^2 - 24x + 24)$ square meters. By how much did Alvin decrease the area of the mural? Show your work.

1. What important words or context clues are in the first sentence of the test item? Use these clues to write an expression that represents the width of the rectangle.
2. Write an equation to represent the area of Alvin's first mural.
3. What math operation does the action *decrease* represent?

Item B

Multiple Choice Which factored expression represents the phrase shown below?

the square of the number of hours it takes to empty a cistern minus 20 times the number of hours plus 64

Ⓐ $(h - 16)(h - 4)$

Ⓑ $(h^2 - 20)(h - 64)$

Ⓒ $(h - 8)(h - 8)$

Ⓓ $(h - 16)(h + 4)$

4. Which word in the phrase tells you to use an exponent in your expression?
5. What is the unknown value in the expression? Define a variable to represent this value.
6. Identify other action words and the mathematical operation phrase each one represents.

Item C

Multiple Choice A company owns two packaging plants. The polynomial $0.05x^2 + 16x - 9400$ models one plant's profit, where x is the number of units packaged. The polynomial $-0.01x^2 + 17x - 5400$ models the other plant's profit. If x is 25,000, what is the total profit of both plants?

Ⓐ −$5,830,300

Ⓑ $25,810,200

Ⓒ $31,640,500

Ⓓ $37,471,000

7. What mathematical symbol does the action *models* represent?
8. Write an equation for each plant that can be used to determine its profit P.
9. What mathematical operation does the term "total profit" represent?

Item D

Gridded Response One of the bases of a trapezoid is 12 meters greater than its height. The other base is 4 meters less than its height. Find the area of the trapezoid when the height is 6 meters.

10. Identify the unknown dimension, and assign it a variable.
11. A student is unsure how many bases a trapezoid has. Identify the context clues that can help this student.
12. Make a list of the actions in the problem, and link each word to its mathematical meaning.
13. Write an expression for each base of the trapezoid.

CHAPTER 8

MASTERING THE STANDARDS

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CUMULATIVE ASSESSMENT, CHAPTERS 1–8

Multiple Choice

1. A rectangle has an area of $(x^2 + 5x - 24)$ square units. Which of the following are possible expressions for the length and the width of the rectangle?

Ⓐ Length: $(x - 24)$ units; width: $(x + 1)$ units

Ⓑ Length: $(x - 4)$ units; width: $(x + 6)$ units

Ⓒ Length: $(x - 3)$ units; width: $(x + 8)$ units

Ⓓ Length: $(x + 12)$ units; width: $(x - 2)$ units

2. Which property of real numbers is used to transform the equation in Step 1 into the equation in Step 2?

Step 1: $4(x - 5) + 8 = 88$
Step 2: $4x - 20 + 8 = 88$
Step 3: $4x - 12 = 88$
Step 4: $4x = 100$
Step 5: $4x = 25$

Ⓐ Commutative Property of Multiplication

Ⓑ Associative Property of Multiplication

Ⓒ Multiplication Property

Ⓓ Distributive Property

3. If $\frac{2}{3}x - 9 = 3$, what is the value of the expression $8x - 3$?

Ⓐ -75 Ⓒ 61

Ⓑ -35 Ⓓ 141

4. Carlos and Bonita were just hired at a manufacturing plant. Carlos will earn \$12.50 per hour. He will receive a hiring bonus of \$300. Bonita will not get a hiring bonus, but she will earn \$14.50 per hour. Which equation can you use to determine the number of hours h when both employees will have earned the same total amount?

Ⓐ $300 + 14.50h = 12.50h$

Ⓑ $14.50h + 300 = 12.50h$

Ⓒ $14.50h + 12.50h = 300$

Ⓓ $300 + 12.50h = 14.50h$

5. Which of the following expressions is equivalent to $x^2 - 8x + 16$?

Ⓐ $(x + 4)^2$ Ⓒ $(x + 8)(x + 2)$

Ⓑ $(x + 4)(x - 4)$ Ⓓ $(x - 4)^2$

6. Michael claims that the whole numbers are closed under subtraction. Stephanie disagrees. Which equation can Stephanie use as a counterexample to show that Michael's claim is false?

Ⓐ $6 - 4 = 2$

Ⓑ $-3 - 8 = -11$

Ⓒ $4 - 5 = -1$

Ⓓ $7 - (-2) = 9$

7. What is the value of y if the line through $(1, -1)$ and $(2, 2)$ is parallel to the line through $(-2, 1)$ and $(-1, y)$?

Ⓐ -8 Ⓒ 3

Ⓑ -2 Ⓓ 4

8. Which of the following shows the complete factorization of $2x^3 + 4x^2 - 6x$?

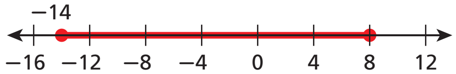
Ⓐ $(2x^2 - 2x)(x + 3)$

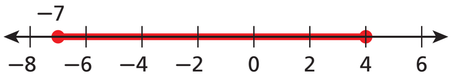
Ⓑ $2x(x^2 + 2x - 3)$

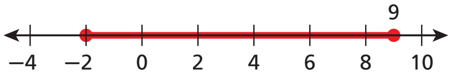
Ⓒ $2x(x - 1)(x + 3)$

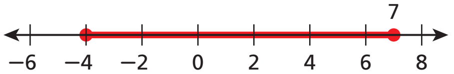
Ⓓ $2(x^3 + 2x^2 - 3x)$

9. Which graph shows the solution set of the compound inequality $-9 \le 5 - 2x \le 13$?

Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

Read problems, graphs, and diagrams carefully. If you are allowed to write in your test booklet, underline or circle important words, labels, or other information given in the problem.

10. Which point lies on the graph of both functions?

$f(x) = 2x - 10$
$g(x) = 10 - 2x$

Ⓐ $(5, 0)$ Ⓒ $(0, 0)$

Ⓑ $(1, -8)$ Ⓓ $(2, 6)$

11. Hayley plans to solve the system of equations below.

$$\begin{cases} x + 3y = 8 \\ 5x - y = 8 \end{cases}$$

Which of the following does NOT show an equation Hayley can use to solve the system of equations?

Ⓐ $x + 3(5x - 8) = 8$

Ⓑ $5(8 - 3y) - y = 8$

Ⓒ $x = 8 - 3y$

Ⓓ $5x - (-x + 8) = 8$

12. Which value of b would make $x^2 + bx - 2$ factorable?

Ⓐ -2 Ⓒ 0

Ⓑ -1 Ⓓ 3

13. Which expression is equivalent to $xy \cdot \left(x^3y^{\frac{1}{2}}\right)^4$?

Ⓐ $x^4y^{\frac{3}{2}}$ Ⓒ $x^{13}y^3$

Ⓑ $x^{16}y^6$ Ⓓ $x^8y^{\frac{11}{2}}$

Gridded Response

14. The complete factorization of $-12x^3 + 14x^2 + 6x$ is $-2x(ax + 1)(2x - 3)$. What is the value of a?

15. The expression $x^2 + x + b$ is a perfect-square trinomial. What is the value of b?

16. What is the slope of a line that is perpendicular to the line described by $2y + 5x = 6$?

17. The point $(3, k)$ lies on the line $3x - 4y = 7$. What is the value of k?

Short Response

18. The area of a certain circle is $\pi(9x^2 + 6x + 1)$ square centimeters. Find an expression for the length of the circle's radius. Explain how you found your answer.

19. A rectangle has an area of $(x^2 - 25)$ square feet.

a. Use factoring to write possible expressions for the length and width of the rectangle.

b. Use your expressions from part **a** to write an expression for the perimeter of the rectangle. Simplify the expression.

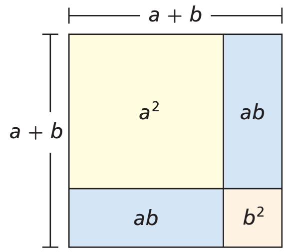
c. Use your expressions from parts **a** and **b** to find the perimeter and the area of the rectangle when $x = 10$ feet. Show your work.

20. Write the numbers 57,000,000,000 and 19,000 in scientific notation. Then show how to divide 57,000,000,000 by 19,000 using properties of exponents.

21. Show that you can factor the expression $x^2y - 12 + 3y - 4x^2$ by grouping in two different ways.

Extended Response

22. The diagram below can be used to show that the expression $(a + b)^2$ is equivalent to the expression $a^2 + 2ab + b^2$.



a. Make a diagram similar to the one above to model the expression $(a + b + c)^2$. Label each distinct area.

b. Use the labels from your diagram to write an expression equivalent to $(a + b + c)^2$.

c. Show that your expression in part **b** is equivalent to $(a + b + c)^2$ by evaluating each expression for $a = 4$, $b = 2$, and $c = 1$.

d. Factor $x^2 + y^2 + 9 + 2xy + 6x + 6y$. Show or explain how you found your answer.