

# Rational Functions and Equations

## 10A Rational Functions and Expressions

- Lab Model Inverse Variation
- 10-1 Inverse Variation
- 10-2 Rational Functions
- 10-3 Simplifying Rational Expressions
- Lab Graph Rational Functions

### CONCEPT CONNECTION

## 10B Operations with Rational Expressions

- 10-4 Multiplying and Dividing Rational Expressions
- 10-5 Adding and Subtracting Rational Expressions
- Lab Model Polynomial Division
- 10-6 Dividing Polynomials
- 10-7 Solving Rational Equations
- 10-8 Applying Rational Equations

### CONCEPT CONNECTION



Chapter Project Online

KEYWORD: MA8CA ChProj

Ratios and rational expressions can be used to explore perspective.

**Golden Gate Bridge**  
San Francisco, CA

# ARE YOU READY?

## Vocabulary

Match each term on the left with a definition on the right.

- |                             |  |
|-----------------------------|--|
| 1. perfect-square trinomial | A. the greatest factor that is shared by two or more terms                                 |
| 2. greatest common factor   | B. a number, a variable, or a product of numbers and variables with whole-number exponents |
| 3. monomial                 | C. two numbers whose product is 1  |
| 4. polynomial               | D. a polynomial with three terms   |
| 5. reciprocals              | E. the sum or difference of monomials  |
|                             | F. a trinomial that is the result of squaring a binomial                                   |

## Simplify Fractions

Simplify.

- |                   |                     |                     |                     |
|-------------------|---------------------|---------------------|---------------------|
| 6. $\frac{12}{4}$ | 7. $\frac{100}{36}$ | 8. $\frac{240}{18}$ | 9. $\frac{121}{66}$ |
|-------------------|---------------------|---------------------|---------------------|

## Add and Subtract Fractions

Add or subtract.

- |                                 |                                 |   |  |
|---------------------------------|---------------------------------|---|--|
| 10. $\frac{1}{3} + \frac{1}{2}$ | 11. $\frac{7}{8} - \frac{1}{6}$ | 12. $\frac{3}{4} + \frac{2}{3} + \frac{1}{2}$ | 13. $\frac{5}{9} + \frac{1}{12} - \frac{1}{3}$ |
|---------------------------------|---------------------------------|---|--|

## Factor GCF from Polynomials

Factor each polynomial.

- |                 |                |                 |                 |
|-----------------|----------------|-----------------|-----------------|
| 14. $x^2 + 2x$  | 15. $x^2 + x$  | 16. $2x^2 + x$  | 17. $x^2 - x$   |
| 18. $3x^2 + 2x$ | 19. $4x^2 - 4$ | 20. $3x^2 - 6x$ | 21. $x^3 - x^2$ |

## Properties of Exponents

Simplify each expression.





- |                            |                        |                        |                           |
|----------------------------|------------------------|------------------------|---------------------------|
| 22. $4x \cdot 3x^2$        | 23. $-5 \cdot 2jk$     | 24. $-2a^3 \cdot 3a^4$ | 25. $3ab \cdot 4a^2b$     |
| 26. $2x \cdot 3y \cdot xy$ | 27. $a^2b \cdot 3ab^3$ | 28. $3rs \cdot 3rs^3$  | 29. $5m^2n^2 \cdot 4mn^2$ |

## Simplify Polynomial Expressions

Simplify each expression.

- |  |                           |
|--|---------------------------|
| 30. $4x - 2y - 8y$                       | 31. $2r - 4s + 3s - 8r$   |
| 32. $ab^2 - ab + 4ab^2 + 2a^2b + a^2b^2$ | 33. $3g(g - 4) + g^2 + g$ |

The information below “unpacks” the standards. The Academic Vocabulary is highlighted and defined to help you understand the language of the standards. Refer to the lessons listed after each standard for help with the math terms and phrases. The Chapter Concept shows how the standard is applied in this chapter.

| California Standard   | Academic Vocabulary   | Chapter Concept   |
|---|---|---|
|  <b>10.0</b> Students add, subtract, multiply, and <b>divide monomials and polynomials</b> . Students solve <b>multistep problems, including word problems, by using these techniques</b> .<br>(Lab 10-6)<br>(Lesson 10-6)                             | <b>technique</b> a way of doing something   | You use long division to divide a polynomial by a binomial.   |
|  <b>12.0</b> Students simplify fractions with polynomials in the numerator and denominator by <b>factoring both and reducing them to the lowest terms</b> .<br>(Lesson 10-3, 10-6)   | <b>factoring</b> expressing a quantity as a product of two or more quantities<br><b>reduce to lowest terms</b> simplify | You use division to write a rational expression in simpler form.  |
|  <b>13.0</b> Students add, subtract, multiply, and divide rational expressions and functions. Students solve both <b>computationally and conceptually challenging problems by using these techniques</b> .<br>(Lessons 10-1, 10-2, 10-3, 10-4, 10-5) | <b>computational</b> having to do with numbers and operations<br><b>conceptual</b> having to do with general ideas      | You add and subtract rational expressions with like and unlike denominators. You also use the rules you learned to multiply and divide fractions to multiply and divide rational expressions. |
|  <b>15.0</b> Students apply algebraic techniques to solve rate problems, work problems, and <b>percent mixture problems</b> .<br>(Lesson 10-5, 10-7, 10-8)   | <b>percent mixture</b> a combination of parts that are expressed as percents of the whole                               | You learn how to solve rational equations so that you can solve real-world problems that involve rational expressions.  |

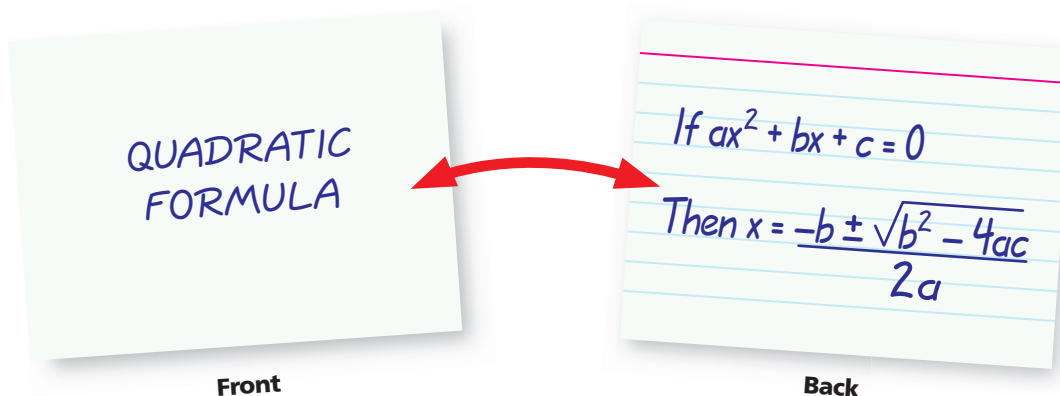
Standard 17.0 is also covered in this chapter. To see this standard unpacked, go to Chapter 4, p. 198.

## Study Strategy: Remember Formulas

In math, there are many formulas, properties, and rules that you should commit to memory.

**To memorize a formula**, create flash cards. Write the name of the formula on one side of a card. Write the formula on the other side of the card. You might also include a diagram or an example if helpful. Study your flash cards on a regular basis.

### Sample Flash Card



Knowing when and how to apply a mathematical formula is as important as memorizing the formula itself.

**To know what formula to apply**, read the problem carefully.

Solve each quadratic equation.

1.  $x^2 - 7x + 10 = 0$     2.  $3x^2 - 4x - 1 = 0$     3.  $x^2 - 10x + 25 = 0$

You can use the Quadratic Formula to solve all of these equations. But other solution methods may be simpler. The first problem can be easily solved by factoring. In the third problem, the left side of the equation is a perfect-square trinomial. This equation can be solved by factoring or by using square roots.

### Try This

Read each problem. Then write the formula(s) needed to solve it. What helped you identify the formula?

- Find the equation of the line with slope  $\frac{2}{3}$  that passes through the point (1, 1).
- The area of a rectangular pool is 120 square feet. The length is 1 foot less than twice the width. What is the perimeter of the pool?



# Model Inverse Variation

The relationship between the width and the length of a rectangle with a constant area is an inverse variation. In this activity, you will study this relationship by modeling rectangles with square tiles or grid paper.

Use with Lesson 10-1



## California Standards

**Preparation for** **13.0** Students add, subtract, multiply, and divide **rational** expressions and **functions**. Students solve both computationally and conceptually challenging problems by using these techniques.

### Activity

Use 12 square tiles to form a rectangle with an area of 12 square units, or draw the rectangle on grid paper. Use a width of 1 unit and a length of 12 units.

Your rectangle should look like the one shown.



Using the same 12 square tiles, continue forming rectangles by changing the width and length until you have formed all the different rectangles you can that have an area of 12 square units. Copy and complete the table as you form each rectangle.

| Width ( $x$ ) | Length ( $y$ ) | Area ( $xy$ ) |
|---------------|----------------|---------------|
| 1             | 12             | 12            |
|               |                | 12            |
|               |                | 12            |
|               |                | 12            |
|               |                | 12            |
|               |                | 12            |

Plot the ordered pairs from the table on a graph. Draw a smooth curve through the points.

### Try This

1. Look at the table and graph above. What happens to the length as the width increases? Why?
2. This relationship between length and width is an example of an *inverse variation*. Why do you think it is called that?
3. For each point, what does  $xy$  equal? Complete the equation  $xy = \square$ . Solve this equation for  $y$ .
4. Form all the different rectangles that have an area of 24 square units. Record their widths and lengths in a table. Graph your results. Write an equation relating the width  $x$  and length  $y$ .
5. **Make a Conjecture** Using the equations you wrote in 3 and 4, what do you think the equation of any inverse variation might look like when solved for  $y$ ?

# 10-1

# Inverse Variation



California Standards

**Preparation for 13.0**

Students add, subtract, multiply, and divide **rational** expressions and **functions**. Students solve both computationally and conceptually challenging problems by using these techniques.

Also covered: **17.0**

**Vocabulary**

inverse variation

**Why learn this?**

Inverse variation can be used to find the frequency at which a guitar string vibrates. (See Example 3.)



A relationship that can be written in the form  $y = \frac{k}{x}$ , where  $k$  is a nonzero constant and  $x \neq 0$ , is an **inverse variation**. The constant  $k$  is the constant of variation.

Multiplying both sides of  $y = \frac{k}{x}$  by  $x$  gives  $xy = k$ . So, for any inverse variation, the product of  $x$  and  $y$  is a nonzero constant.



**Inverse Variations**

| WORDS                                  | NUMBERS           | ALGEBRA                 |
|--|-------------------|-------------------------|
| $y$ varies inversely as $x$ .          | $y = \frac{3}{x}$ | $y = \frac{k}{x}$       |
| $y$ is inversely proportional to $x$ . | $xy = 3$          | $xy = k$ ( $k \neq 0$ ) |

There are two methods to determine whether a relationship between data is an inverse variation. You can write a function rule in  $y = \frac{k}{x}$  form, or you can check whether  $xy$  is constant for each ordered pair.

**EXAMPLE 1 Identifying an Inverse Variation**

Tell whether each relationship is an inverse variation. Explain.

**A**

| $x$ | $y$ |
|-----|-----|
| 1   | 20  |
| 2   | 10  |
| 4   | 5   |

**Method 1** Write a function rule.

$y = \frac{20}{x}$  *Can write in  $y = \frac{k}{x}$  form.*

The relationship is an inverse variation.

**Method 2** Find  $xy$  for each ordered pair.

$1(20) = 20, 2(10) = 20, 4(5) = 20$

The product  $xy$  is constant, so the relationship is an inverse variation.

**B**

| $x$ | $y$ |
|-----|-----|
| 2   | 6   |
| 3   | 9   |
| 6   | 18  |

**Method 1** Write a function rule.

$y = 3x$  *Cannot write in  $y = \frac{k}{x}$  form.*

The relationship is not an inverse variation.

**Method 2** Find  $xy$  for each ordered pair.

$2(6) = 12, 3(9) = 27, 6(18) = 108$

The product  $xy$  is not constant, so the relationship is not an inverse variation.

Tell whether each relationship is an inverse variation. Explain.

**C**  $5xy = -21$

$$\frac{5xy}{5} = \frac{-21}{5}$$

$$xy = \frac{-21}{5}$$

$xy$  equals the constant  $\frac{-21}{5}$ , so the relationship is an inverse variation.

Find  $xy$ . Since  $xy$  is multiplied by 5, divide both sides by 5 to undo the multiplication.

Simplify.



Tell whether each relationship is an inverse variation. Explain.

1a.

| x   | y   |
|-----|-----|
| -12 | 24  |
| 1   | -2  |
| 8   | -16 |

1b.

| x  | y   |
|----|-----|
| 3  | 3   |
| 9  | 1   |
| 18 | 0.5 |

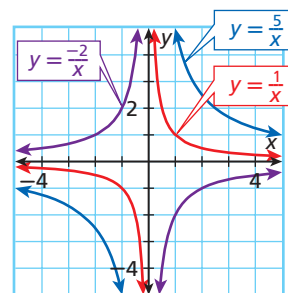
1c.  $2x + y = 10$

### Helpful Hint

Since  $k$  is a nonzero constant,  $xy \neq 0$ . Therefore, neither  $x$  nor  $y$  can equal 0, and the graph will not intersect the  $x$ - or  $y$ -axes.

An inverse variation can also be identified by its graph. Some inverse variation graphs are shown. Notice that each graph has two parts that are not connected.

Also notice that none of the graphs contain  $(0, 0)$ . In other words,  $(0, 0)$  can never be a solution of an inverse variation equation.



## EXAMPLE 2 Graphing an Inverse Variation

Write and graph the inverse variation in which  $y = 2$  when  $x = 4$ .

Step 1 Find  $k$ .

$$\begin{aligned} k &= xy && \text{Write the rule for constant of variation.} \\ &= 4(2) && \text{Substitute 4 for } x \text{ and 2 for } y. \\ &= 8 \end{aligned}$$

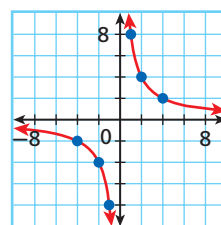
Step 2 Use the value of  $k$  to write an inverse variation equation.

$$\begin{aligned} y &= \frac{k}{x} && \text{Write the rule for inverse variation.} \\ y &= \frac{8}{x} && \text{Substitute 8 for } k. \end{aligned}$$

Step 3 Use the equation to make a table of values.

|   |    |    |    |   |   |   |
|---|----|----|----|---|---|---|
| x | -4 | -2 | -1 | 1 | 2 | 4 |
| y | -2 | -4 | -8 | 8 | 4 | 2 |

Step 4 Plot the points and connect them with smooth curves.



2. Write and graph the inverse variation in which  $y = \frac{1}{2}$  when  $x = 10$ .

**EXAMPLE 3 Music Application**

The inverse variation  $xy = 2400$  relates the vibration frequency  $y$  in hertz (Hz) to the length  $x$  in centimeters of a guitar string. Determine a reasonable domain and range, and then graph this inverse variation.

**Step 1** Solve the function for  $y$ .

$$xy = 2400$$

$$y = \frac{2400}{x} \quad \text{Divide both sides by } x.$$

**Step 2** Decide on a reasonable domain and range.

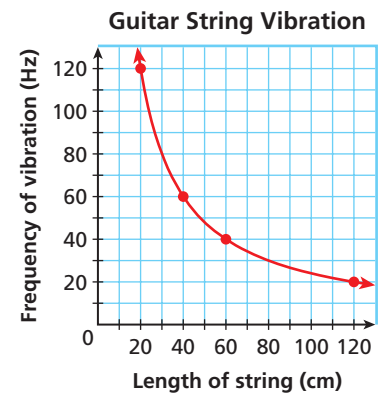
$$x > 0 \quad \text{Length is never negative and } x \neq 0.$$

$$y > 0 \quad \text{Because } x \text{ and } xy \text{ are both positive, } y \text{ is also positive.}$$

**Step 3** Use values of the domain to generate reasonable ordered pairs.

|     |     |    |    |     |
|-----|-----|----|----|-----|
| $x$ | 20  | 40 | 60 | 120 |
| $y$ | 120 | 60 | 40 | 20  |

**Step 4** Plot the points. Connect them with a smooth curve.

**Remember!**

Recall that sometimes domain and range are restricted in real-world situations.



3. The inverse variation  $xy = 100$  represents the relationship between the pressure  $x$  in atmospheres (atm) and the volume  $y$  in  $\text{mm}^3$  of a certain gas. Determine a reasonable domain and range, and then graph this inverse variation.

The fact that  $xy = k$  is the same for every ordered pair in any inverse variation can help you find missing values in the relationship.

**Product Rule for Inverse Variation**

If  $(x_1, y_1)$  and  $(x_2, y_2)$  are solutions of an inverse variation, then  $x_1y_1 = x_2y_2$ .

**EXAMPLE 4 Using the Product Rule**

Let  $x_1 = 3$ ,  $y_1 = 2$ , and  $y_2 = 6$ . Let  $y$  vary inversely as  $x$ . Find  $x_2$ .

$$x_1y_1 = x_2y_2 \quad \text{Write the Product Rule for Inverse Variation.}$$

$$(3)(2) = x_2(6) \quad \text{Substitute 3 for } x_1, 2 \text{ for } y_1, \text{ and 6 for } y_2.$$

$$6 = 6x_2 \quad \text{Simplify.}$$

$$\frac{6}{6} = \frac{6x_2}{6} \quad \text{Solve for } x_2 \text{ by dividing both sides by 6.}$$

$$1 = x_2 \quad \text{Simplify.}$$

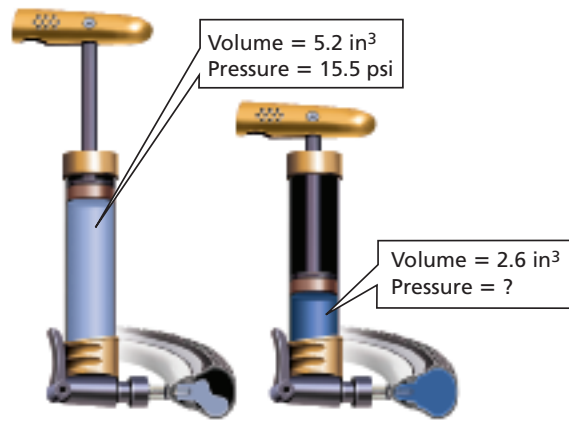


4. Let  $x_1 = 2$ ,  $y_1 = -6$ , and  $x_2 = -4$ . Let  $y$  vary inversely as  $x$ . Find  $y_2$ .



**EXAMPLE 5****Physical Science Application**

Boyle's law states that the pressure of a quantity of gas  $x$  varies inversely as the volume of the gas  $y$ . The volume of air inside a bicycle pump is  $5.2 \text{ in}^3$ , and the pressure is 15.5 pounds per square inch (psi). Assuming no air escapes, what is the pressure of the air inside the pump after the handle is pushed in and the air is compressed to a volume of  $2.6 \text{ in}^3$ ?

**Reading Math**

In Example 5,  $x_1$  and  $y_1$  represent volume and pressure **before** the handle is pushed in, and  $x_2$  and  $y_2$  represent volume and pressure **after** the handle is pushed in.

$$x_1 y_1 = x_2 y_2$$

$$(5.2)(15.5) = (2.6)y_2$$

$$80.6 = 2.6y_2$$

$$\frac{80.6}{2.6} = \frac{2.6y_2}{2.6}$$

$$31 = y_2$$

Use the Product Rule for Inverse Variation.

Substitute 5.2 for  $x_1$ , 15.5 for  $y_1$ , and 2.6 for  $x_2$ .

Simplify.

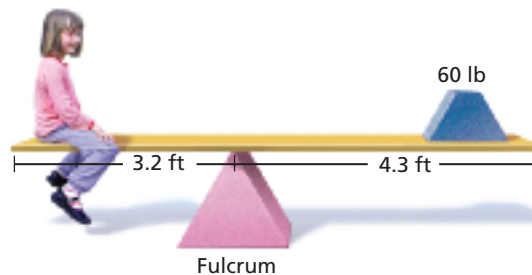
Solve for  $y_2$  by dividing both sides by 2.6.

Simplify.

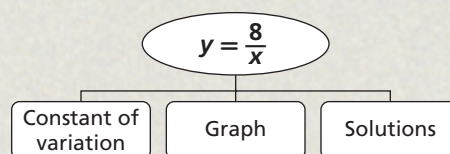
The pressure after the handle is pushed in is 31 psi.



5. On a balanced lever, weight varies inversely as the distance from the fulcrum to the weight. The diagram shows a balanced lever. How much does the child weigh?

**THINK AND DISCUSS**

- Name two ways you can identify an inverse variation.
- GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of the parts of the given inverse variation.





## GUIDED PRACTICE

1. **Vocabulary** Describe the graph of an *inverse variation*.

## SEE EXAMPLE 1

p. 627

- Tell whether each relationship is an inverse variation. Explain.

2.

| x | y |
|---|---|
| 1 | 8 |
| 4 | 2 |
| 2 | 4 |

3.

| x             | y  |
|---------------|----|
| $\frac{1}{6}$ | 1  |
| $\frac{1}{3}$ | 2  |
| 2             | 12 |

4.  $x + y = 8$

5.  $4xy = 3$

## SEE EXAMPLE 2

p. 628

6. Write and graph the inverse variation in which  $y = 2$  when  $x = 2$ .

7. Write and graph the inverse variation in which  $y = 6$  when  $x = -1$ .

## SEE EXAMPLE 3

p. 629

8. **Travel** The inverse variation  $xy = 30$  relates the constant speed  $x$  in mi/h to the time  $y$  in hours that it takes to travel 30 miles. Determine a reasonable domain and range, and then graph this inverse variation.

## SEE EXAMPLE 4

p. 629

9. Let  $x_1 = 3$ ,  $y_1 = 12$ , and  $x_2 = 9$ . Let  $y$  vary inversely as  $x$ . Find  $y_2$ .

10. Let  $x_1 = 1$ ,  $y_1 = 4$ , and  $y_2 = 16$ . Let  $y$  vary inversely as  $x$ . Find  $x_2$ .

## SEE EXAMPLE 5

p. 630

11. **Mechanics** The rotational speed of a gear varies inversely as the number of teeth on the gear. A gear with 12 teeth has a rotational speed of 60 rpm. How many teeth are on a gear that has a rotational speed of 45 rpm?

## PRACTICE AND PROBLEM SOLVING

## Independent Practice

| For Exercises | See Example |
|---------------|-------------|
| 12–15         | 1           |
| 16–17         | 2           |
| 18            | 3           |
| 19–20         | 4           |
| 21            | 5           |

## Extra Practice

Skills Practice p. EP20

Application Practice p. EP33

- Tell whether each relationship is an inverse variation. Explain.

12.

| x  | y  |
|----|----|
| 3  | -3 |
| -5 | 5  |
| 7  | -7 |

13.

| x   | y    |
|-----|------|
| 2   | 5    |
| 0.5 | 20   |
| 8   | 1.25 |

14.  $x = \frac{13}{y}$

15.  $y = 5x$

16. Write and graph the inverse variation in which  $y = -2$  when  $x = 5$ .

17. Write and graph the inverse variation in which  $y = -6$  when  $x = -\frac{1}{3}$ .

18. **Engineering** The inverse variation  $xy = 12$  relates the current  $x$  in amps to the resistance  $y$  in ohms of a circuit attached to a 12-volt battery. Determine a reasonable domain and range, and then graph this inverse variation.

19. Let  $x_1 = -3$ ,  $y_1 = -4$ , and  $y_2 = 6$ . Let  $y$  vary inversely as  $x$ . Find  $x_2$ .

20. Let  $x_1 = 7$ ,  $y_1 = 9$ , and  $x_2 = 6$ . Let  $y$  vary inversely as  $x$ . Find  $y_2$ .



## Winter Sports



Snowshoes were originally made of wooden frames strung with animal intestines. Modern snowshoes are made with steel frames and may have cleats for gripping the snow.

21. **Home Economics** The length of fabric that June can afford varies inversely as the price per yard of the fabric. June can afford exactly 5 yards of fabric that costs \$10.50 per yard. How many yards of fabric that costs \$4.25 per yard can June buy? (Assume that she can only buy whole yards.)
22. **Winter Sports** When a person is snowshoeing, the pressure on the top of the snow in psi varies inversely as the area of the bottom of the snowshoe in square inches. The constant of variation is the weight of the person wearing the snowshoes in pounds.
- Helen weighs 120 pounds. About how much pressure does she put on top of the snow if she wears snowshoes that cover  $360 \text{ in}^2$ ?
  - Max weighs 207 pounds. If he exerts 0.4 psi of pressure on top of the snow, what is the area of the bottom of his snowshoes in square inches?

Determine if each equation represents a direct variation, an inverse variation, or neither. Find the constant of variation when one exists.

23.  $y = 8x$       24.  $y = \frac{14}{x}$       25.  $y = \frac{1}{3}x - 2$       26.  $y = \frac{1}{5}x$
27.  $y = 4\frac{3}{x}$       28.  $y = \frac{x}{2} + 7$       29.  $y = \frac{15}{x}$       30.  $y = 5x$
31. **Multi-Step** A track team is competing in a 10 km race. The distance will be evenly divided among the team members. Write an equation that represents the distance  $d$  each runner will run if there are  $n$  runners. Does this represent a direct variation, inverse variation, or neither?

Determine whether each data set represents a direct variation, an inverse variation, or neither.

32. 



|   |   |    |    |
|---|---|----|----|
| x | 2 | 4  | 8  |
| y | 5 | 10 | 20 |

      33. 

|   |   |    |    |
|---|---|----|----|
| x | 6 | 12 | 15 |
| y | 6 | 8  | 9  |

      34. 

|   |    |   |   |
|---|----|---|---|
| x | 1  | 2 | 3 |
| y | 12 | 6 | 4 |

35. **Multi-Step** Your club awards one student a \$2000 scholarship each year, and each member contributes an equal amount. Your contribution  $y$  depends on the number of members  $x$ . Write and graph an inverse variation equation that represents this situation. What are a reasonable domain and range?
36. **Estimation** Estimate the value of  $y$  if  $y$  is inversely proportional to  $x$ ,  $x = 4$ , and the constant of variation is  $6\pi$ .
37. Why will the point  $(0, 0)$  never be a solution to an inverse variation?
-  38. **Write About It** Explain how to write an inverse variation equation of the form  $y = \frac{k}{x}$  when values of  $x$  and  $y$  are known.
-  39. **Write About It** List all the mathematical terms you know that contain the word *inverse*. How are these terms all similar? How is *inverse variation* similar to these terms?

## CONCEPT CONNECTION



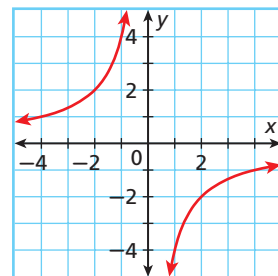
40. This problem will prepare you for the Concept Connection on page 650. The total number of workdays it takes to build the frame of a house varies inversely as the number of people working in a crew. Let  $x$  be the number of people in the crew, and let  $y$  be the number of workdays.
- Find the constant of variation when  $y = 75$  and  $x = 2$ .
  - Write the rule for the inverse of variation equation.
  - Graph the equation of this inverse variation.

**Multiple Choice** For Exercises 41–43, choose the best answer.

41. Which equation best represents the graph?

(A)  $y = -\frac{1}{4}x$       (C)  $y = -\frac{4}{x}$

(B)  $y = \frac{1}{4}x$       (D)  $y = \frac{4}{x}$



42. Determine the constant of variation if  $y$  varies inversely as  $x$  and  $y = 2$  when  $x = 7$ .

(A)  $\frac{2}{7}$       (B)  $\frac{7}{2}$       (C) 3.5      (D) 14

43. Which of the following relationships does NOT represent an inverse variation?

(A) 

|     |    |   |   |
|-----|----|---|---|
| $x$ | 2  | 4 | 5 |
| $y$ | 10 | 5 | 4 |

(C) 

|     |   |    |    |
|-----|---|----|----|
| $x$ | 2 | 4  | 5  |
| $y$ | 8 | 16 | 20 |

(B)  $y = \frac{17.5}{x}$

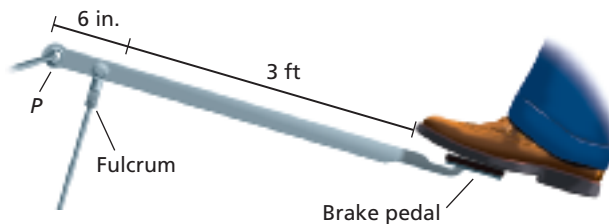
(D)  $\frac{11}{2} = xy$

44. **Gridded Response** At a carnival, the number of tickets Brad can buy is inversely proportional to the price of the tickets. He can afford 12 tickets that cost \$2.50 each. How many tickets can Brad buy if each costs \$3.00?

## CHALLENGE AND EXTEND

45. The definition of inverse variation says that  $k$  is a nonzero constant. What function would  $y = \frac{k}{x}$  represent if  $k = 0$ ?

46. **Mechanics** A part of a car's braking system uses a lever to multiply the force applied to the brake pedal. The force at the end of a lever varies inversely with the distance from the fulcrum. Point  $P$  is the end of the lever. A force of 2 lb is applied to the brake pedal. What is the force created at the point  $P$ ?



47. **Communication** The strength of a radio signal varies inversely with the square of the distance from the transmitter. A signal has a strength of 2000 watts when it is 4 kilometers from the transmitter. What is the strength of the signal 6 kilometers from the transmitter?



## SPIRAL STANDARDS REVIEW

🔑 14.0, 16.0, 17.0, 18.0

Find the domain and range for each relation. Tell whether the relation is a function.

(Lesson 4-2)

48.  $\{(-2, -4), (-2, -2), (-2, 0), (-2, 2)\}$       49.  $\{(-4, 5), (-2, 3), (0, 1), (2, 3), (4, 5)\}$

Solve by completing the square. (Lesson 9-7)

50.  $x^2 + 12x = 45$

51.  $d^2 - 6d - 7 = 0$

52.  $2y^2 + 6y = -\frac{5}{2}$

53. Find the number of solutions of  $4x^2 + 3x - 6 = 0$  using the discriminant.

(Lesson 9-9)

# 10-2

# Rational Functions



## California Standards

**13.0** Students add, subtract, multiply, and divide **rational** expressions and **functions**. Students solve both computationally and conceptually challenging problems by using these techniques.

Also covered: **17.0**

### Who uses this?

Gemologists can use rational functions to maximize reflected light. (See Example 4.)

A **rational function** is a function whose rule is a quotient of polynomials. The inverse variations you studied in the previous lesson are a special type of rational function.

$$\text{Rational functions: } y = \frac{2}{x}, y = \frac{3}{4 - 2x}, y = \frac{1}{x^2}$$

For any function involving  $x$  and  $y$ , an **excluded value** is any  $x$ -value that makes the function value  $y$  undefined. For a rational function, an excluded value is any value that makes the denominator equal 0.

### EXAMPLE 1 Identifying Excluded Values

#### Vocabulary

rational function  
excluded value  
discontinuous function  
asymptote

Identify any excluded values for each rational function.

**A**  $y = \frac{8}{x}$

$$x = 0$$

Set the denominator equal to 0.

The excluded value is 0.

**B**  $y = \frac{3}{x + 3}$

$$x + 3 = 0$$

Set the denominator equal to 0.

$$x = -3$$

Solve for  $x$ .

The excluded value is  $-3$ .

**C**  $y = \frac{4}{x^2 + 1}$

$$x^2 + 1 = 0$$

Set the denominator equal to 0.

$$x^2 = -1$$

This equation has no real solutions, so the function has no excluded values.



Identify any excluded values for each rational function.

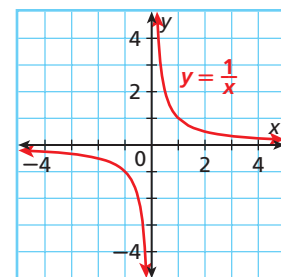
1a.  $y = \frac{10}{x}$

1b.  $y = \frac{4}{x - 1}$

1c.  $y = -\frac{5}{x + 4}$

Many rational functions are **discontinuous functions**, meaning their graphs contain one or more jumps, breaks, or holes. This occurs at an excluded value.

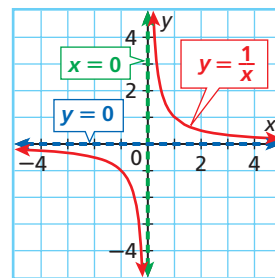
One place that a graph of a rational function may be discontinuous is at an **asymptote**. An **asymptote** is a line that a graph gets closer to as the absolute value of a variable increases. In the graph shown, both the  $x$ - and  $y$ -axes are asymptotes. A graph will get closer and closer to but never touch its asymptotes.



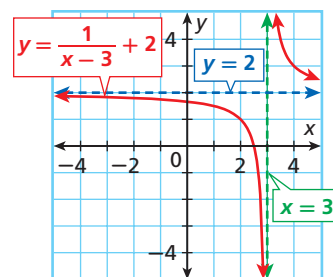
## Writing Math

Vertical lines are written in the form  $x = b$ , and horizontal lines are written in the form  $y = c$ .

Look at the graph of  $y = \frac{1}{x}$ . The denominator is 0 when  $x = 0$ , so **0** is an excluded value. This means there is a vertical asymptote at  $x = 0$ . Notice the horizontal asymptote at  $y = 0$ . This is because there is no value of  $x$  for which  $y = 0$ .



Look at the graph of  $y = \frac{1}{x-3} + 2$ . Notice that there is a vertical asymptote at  $x = 3$  and there is a horizontal asymptote at  $y = 2$ . This is because there is no value of  $x$  for which  $\frac{1}{x-3} = 0$ . If  $\frac{1}{x-3}$  is never 0, then  $y = \frac{1}{x-3} + 2$  can never be equal to 2.



Know it!

Note

## Identifying Asymptotes

| WORDS   | EXAMPLES  |   |
|---|---|---|
| <p>A rational function in the form</p> $y = \frac{a}{x-b} + c$ <p>has a vertical asymptote at the excluded value (<math>x = b</math>) and a horizontal asymptote at <math>y = c</math>.</p> | $y = \frac{2}{x}$ $= \frac{2}{x-0} + 0$ <p>Vertical asymptote:<br/><math>x = 0</math></p> <p>Horizontal asymptote:<br/><math>y = 0</math></p> | $y = \frac{1}{x+2} + 4$ $= \frac{1}{x-(-2)} + 4$ <p>Vertical asymptote:<br/><math>x = -2</math></p> <p>Horizontal asymptote:<br/><math>y = 4</math></p> |

## EXAMPLE 2 Identifying Asymptotes

Identify the asymptotes.

**A**  $y = \frac{1}{x-6}$

Step 1 Write in  $y = \frac{1}{x-b} + c$  form.

$$y = \frac{1}{x-6} + 0$$

Step 2 Identify the asymptotes.

vertical:  $x = 6$

horizontal:  $y = 0$

**B**  $y = \frac{2}{3x-10} - 7$

Step 1 Identify the vertical asymptote.

$$3x - 10 = 0$$

Find the excluded value. Set the denominator equal to 0.

$$\begin{array}{r} +10 \\ \hline 3x = 10 \end{array}$$

Add 10 to both sides.

$$x = \frac{10}{3}$$

Solve for  $x$ .  $\frac{10}{3}$  is an excluded value.

**Step 2** Identify the horizontal asymptote.

$$c = -7 \quad -7 \text{ can be written as } +(-7)$$

$$y = -7 \quad y = c$$

vertical asymptote:  $x = \frac{10}{3}$ ; horizontal asymptote:  $y = -7$



Identify the asymptotes.

2a.  $y = \frac{2}{x-5}$

2b.  $y = \frac{1}{4x+16} + 5$

2c.  $y = \frac{3}{x+77} - 15$

To graph a rational function in the form  $y = \frac{a}{x-b} + c$ , you can use the asymptotes and a table of values.

### EXAMPLE 3 Graphing Rational Functions Using Asymptotes

Graph each function.

**A**  $y = \frac{2}{x+1}$

**Step 1** Identify the asymptotes.

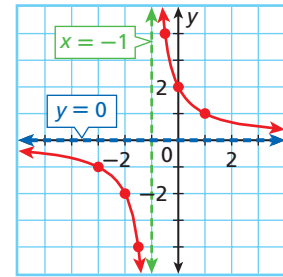
vertical:  $x = -1$  Use  $x = b$ .  $x + 1 = x - (-1)$ , so  $b = -1$ .

horizontal:  $y = 0$  Use  $y = c$ .  $c = 0$

**Step 2** Graph the asymptotes using dashed lines.

**Step 3** Make a table of values. Choose  $x$ -values on both sides of the vertical asymptote.

|     |    |    |                |                |   |   |
|-----|----|----|----------------|----------------|---|---|
| $x$ | -3 | -2 | $-\frac{3}{2}$ | $-\frac{1}{2}$ | 0 | 1 |
| $y$ | -1 | -2 | -4             | 4              | 2 | 1 |



**Step 4** Plot the points and connect them with smooth curves. The curves should not touch the asymptotes.

**B**  $y = \frac{1}{x-2} - 4$

**Step 1** Identify the asymptotes.

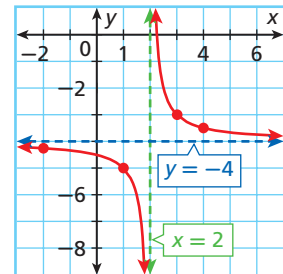
vertical:  $x = 2$  Use  $x = b$ .  $b = 2$

horizontal:  $y = -4$  Use  $y = c$ .  $c = -4$

**Step 2** Graph the asymptotes using dashed lines.

**Step 3** Make a table of values. Choose  $x$ -values on both sides of the vertical asymptote.

|     |       |      |    |    |      |
|-----|-------|------|----|----|------|
| $x$ | -2    | 0    | 1  | 3  | 4    |
| $y$ | -4.25 | -4.5 | -5 | -3 | -3.5 |



**Step 4** Plot the points and connect them with smooth curves. The curves should not touch the asymptotes.



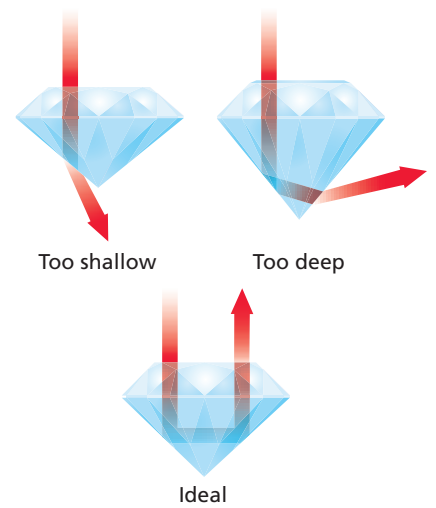
Graph each function.

3a.  $y = \frac{1}{x+7} + 3$

3b.  $y = \frac{2}{x-3} + 2$

**EXAMPLE****4****Gemology Application**

Some diamonds are cut using ratios calculated by the mathematician Marcel Tolkowsky in 1919. The amount of light reflected up through the top of a diamond (brilliance) can be maximized using the ratio between the width of the diamond and the depth of the diamond. A gemologist has a diamond with a width of 9 millimeters. If  $x$  represents the depth of the diamond, then  $y = \frac{9}{x}$  represents the brilliance ratio  $y$ .



- a. Describe a reasonable domain and range.

Both the depth of the diamond and the brilliance ratio will be nonnegative, so nonnegative values are reasonable for the domain and range.

- b. Graph the function.

**Step 1** Identify the vertical and horizontal asymptotes.

vertical:  $x = 0$       Use  $x = b$ .  $b = 0$

horizontal:  $y = 0$       Use  $y = c$ .  $c = 0$

**Step 2** Graph the asymptotes using dashed lines.

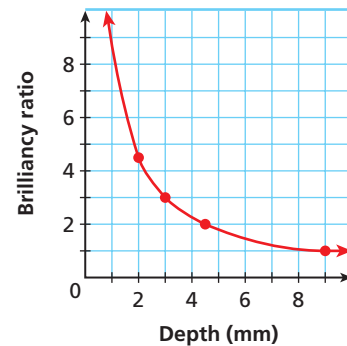
The asymptotes will be the  $x$ - and  $y$ -axes.

**Step 3** Since the domain is restricted to nonnegative values, only choose  $x$ -values on the right side of the vertical asymptote.

|                       |     |   |     |   |
|-----------------------|-----|---|-----|---|
| Depth of Diamond (mm) | 2   | 3 | 4.5 | 9 |
| Brilliance Ratio      | 4.5 | 3 | 2   | 1 |

**Step 4** Plot the points and connect them with smooth curves.

**Brilliance of a Diamond Cut**



4. A librarian has a budget of \$500 to buy copies of a software program. She will receive 10 free copies when she sets up an account with the supplier. The number of copies  $y$  of the program that she can buy is given by  $y = \frac{500}{x} + 10$ , where  $x$  is the price per copy.

- a. Describe a reasonable domain and range.  
b. Graph the function.



The table shows some of the properties of the three types of functions you have studied and their graphs.

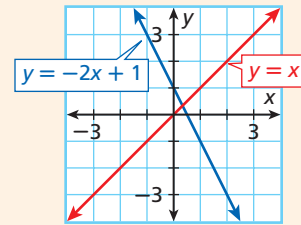


## Types of Functions

### LINEAR FUNCTIONS

$$y = mx + b$$

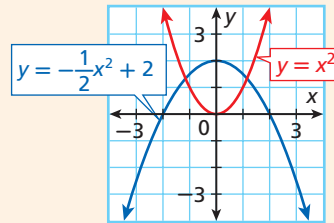
- Graph is a straight line.
- $m$  is the slope. When  $m = 0$ , the graph is a horizontal line.
- When  $m < 0$ , the graph slopes down from left to right.
- When  $m > 0$ , the graph slopes up from left to right.
- $b$  is the  $y$ -intercept.



### QUADRATIC FUNCTIONS

$$y = ax^2 + bx + c, a \neq 0$$

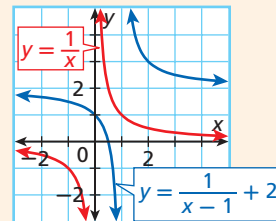
- Graph is a parabola.
- When  $a > 0$ , the parabola opens up.
- When  $a < 0$ , the parabola opens down.
- The axis of symmetry is the vertical line  $x = -\frac{b}{2a}$ .
- The function has a maximum or minimum value at the vertex.



### RATIONAL FUNCTIONS OF THE FORM $y = \frac{a}{x - b} + c$

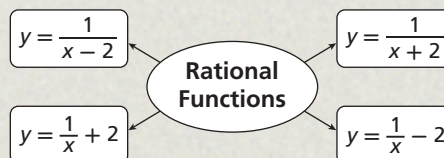
$$y = \frac{1}{x - b} + c$$

- Graph is discontinuous.
- $b$  is an excluded value;  $x = b$  is the vertical asymptote.
- $y = c$  is the horizontal asymptote.



## THINK AND DISCUSS

1. Does  $y = \frac{1}{x-5}$  have any excluded values? Explain.
2. Tell how to find the vertical and horizontal asymptotes of  $y = \frac{1}{x+9} - 5$ .
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, find the asymptotes for the given rational function.





## GUIDED PRACTICE

1. **Vocabulary** An  $x$ -value that makes a function undefined is a(n) \_\_\_\_\_?  
(*asymptote* or *excluded value*)

SEE EXAMPLE 1 Identify any excluded values for each rational function.

p. 634

2.  $y = \frac{4}{x}$

3.  $y = \frac{2}{x^2 + 3}$

4.  $y = -\frac{2}{x}$

5.  $y = \frac{16}{x - 4}$

SEE EXAMPLE 2 Identify the asymptotes.

p. 635

6.  $y = \frac{1}{x - 3}$

7.  $y = \frac{4}{3x + 15}$

8.  $y = \frac{2}{3x - 5} + 2$

9.  $y = \frac{1}{x + 9} - 10$

SEE EXAMPLE 3 Graph each function.

p. 636

10.  $y = \frac{2}{x + 6}$

11.  $y = \frac{1}{x - 2} - 6$

12.  $y = \frac{1}{x} + 2$

13.  $y = \frac{1}{x - 3} - 2$

SEE EXAMPLE 4 14. **Catering** A caterer has \$100 in her budget for fruit. Slicing and delivery of each pound of fruit costs \$5. If  $x$  represents the cost per pound of the fruit itself, then  $y = \frac{100}{x + 5}$  represents the number of pounds  $y$  she can buy.

- Describe a reasonable domain and range.
- Graph the function.

p. 637

## PRACTICE AND PROBLEM SOLVING

## Independent Practice

| For Exercises | See Example |
|---------------|-------------|
| 15–18         | 1           |
| 19–22         | 2           |
| 23–26         | 3           |
| 27            | 4           |

Identify any excluded values for each rational function.

15.  $y = \frac{7}{x}$

16.  $y = \frac{1}{x - 4}$

17.  $y = -\frac{15}{x}$

18.  $y = \frac{12}{x - 5}$

Identify the asymptotes.

19.  $y = \frac{9}{x - 4}$

20.  $y = \frac{2}{x + 4}$

21.  $y = \frac{7}{4x - 12} + 4$

22.  $y = \frac{7}{3x + 5} - 9$

Graph each function.

23.  $y = \frac{5}{x - 5}$

24.  $y = \frac{1}{x + 5} - 6$

25.  $y = \frac{1}{x + 4}$

26.  $y = \frac{1}{x - 4} + 2$

27. **Business** A wholesaler is buying auto parts. He has \$200 to spend. He receives 5 parts free with the order. The number of parts  $y$  he can buy, if the average price of the parts is  $x$  dollars, is  $y = \frac{200}{x} + 5$ .
- Describe a reasonable domain and range.
  - Graph the function.

Find the excluded value for each rational function.

28.  $y = \frac{4}{x}$

29.  $y = \frac{1}{x - 7}$

30.  $y = \frac{2}{x + 4}$

31.  $y = \frac{3}{2x + 1}$

Graph each rational function. Show the asymptotes.

32.  $y = \frac{1}{x - 2}$

33.  $y = \frac{2}{x} + 3$

34.  $y = \frac{3}{x + 1} + 2$

35.  $y = \frac{1}{x - 4} - 1$

36. The function  $y = \frac{60}{x^2}$  relates the luminescence in lumens  $y$  of a 60-watt lightbulb viewed from a distance of  $x$  ft. Graph this function.

## Extra Practice

Skills Practice p. EP20

Application Practice p. EP33

Identify the asymptotes of each rational function.

37.  $y = \frac{7}{x+1}$

38.  $y = \frac{1}{x} - 5$

39.  $y = \frac{12}{x-2} + 5$

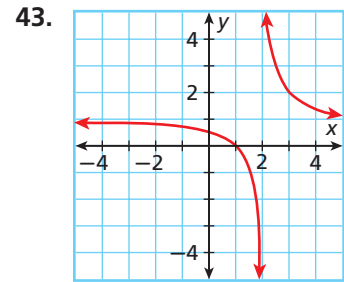
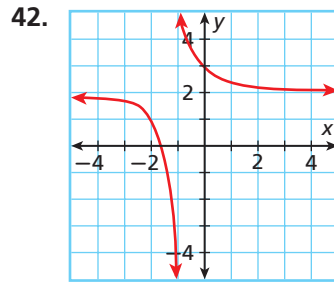
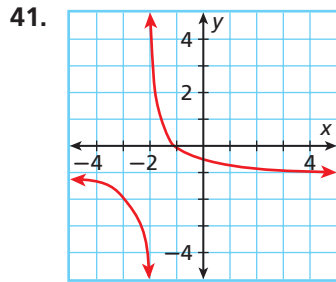
40.  $y = \frac{18}{x+3} - 9$

Match each graph with one of the following functions.

A.  $y = \frac{1}{x+1} + 2$

B.  $y = \frac{1}{x+2} - 1$

C.  $y = \frac{1}{x-2} + 1$



44. **/// ERROR ANALYSIS ///** In finding the horizontal asymptote of  $y = \frac{1}{x+2} - 3$ , student A said the asymptote is at  $y = -3$ , and student B said it is at  $y = -2$ . Who is incorrect? Explain the error.

45. **Finance** The time in months  $y$  that it will take to pay off a bill of \$1200, when  $x$  dollars are paid each month and the finance charge is \$15 per month, is  $y = \frac{1200}{x-15}$ . Describe a reasonable domain and range, and graph the function.



46. The table shows how long it takes different size landscaping teams to complete a project.

- Graph the data.
- Write a rational function to represent the data.
- How many hours would it take 12 landscapers to complete the project?

| Landscapers | Time (h) |
|-------------|----------|
| 2           | 30       |
| 4           | 15       |
| 5           | 12       |
| 10          | 6        |

Graph each function. Compare its graph to the graph of  $y = \frac{1}{x}$ .

47.  $y = \frac{1}{x-6}$

48.  $y = \frac{1}{x+7}$

49.  $y = \frac{1}{x} + 4$

50.  $y = \frac{1}{x-2} - 9$

Find the domain that makes the range positive.

51.  $y = \frac{10}{x-2}$

52.  $y = \frac{10}{x+2}$

53.  $y = \frac{5}{5x+1}$

54.  $y = \frac{4}{3x-7}$

55. **Critical Thinking** In which quadrants would you find the graph of  $y = \frac{a}{x}$  when  $a$  is positive? when  $a$  is negative?

**CONCEPT CONNECTION**



56. This problem will prepare you for the Concept Connection on page 650. It takes a total of 250 workdays to build a house for charity. For example, if 2 workers build the house, it takes them 125 actual construction days. If 10 workers are present, it takes 25 construction days to build the house.

- Write a function that represents the number of construction days to build as a function of the number of workers.
- What is the domain of this function?
- Sketch a graph of the function.



For help with inductive reasoning, see p. 233 and pp. 280–281.

57. **Reasoning** Graph each pair of functions. Then use inductive reasoning to make a conjecture about the relationship between the graphs of the rational functions  $y = \frac{k}{x}$  and  $y = \frac{-k}{x}$ .

a.  $y = \frac{1}{x}; y = \frac{-1}{x}$

b.  $y = \frac{3}{x}; y = \frac{-3}{x}$

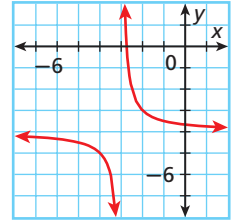
c.  $y = \frac{5}{x}; y = \frac{-5}{x}$

**Multiple Choice** For Exercises 58 and 59, choose the best answer.

58. Which function is graphed?

(A)  $y = \frac{2}{x+3} - 4$  (C)  $y = \frac{2}{x-3} + 4$

(B)  $y = \frac{2}{x+4} - 3$  (D)  $y = \frac{2}{x-4} + 3$



59. Which rational function has a graph with the horizontal asymptote  $y = -1$ ?

(A)  $y = \frac{-1}{x}$  (C)  $y = \frac{1}{x+1}$

(B)  $y = \frac{1}{x-1}$  (D)  $y = \frac{1}{x} - 1$

60. **Short Response** Write a rational function whose graph is the same shape as the graph of  $f(x) = \frac{1}{x}$ , but has a vertical asymptote at  $x = -2$  and a horizontal asymptote at  $y = -3$ . Graph the function.

## CHALLENGE AND EXTEND

61. Graph the equation  $y = \frac{1}{x^2 + 1}$ .

- Does this equation represent a rational function? Explain.
- What is the domain of the function?
- What is the range of the function?
- Is the graph discontinuous?



62. **Graphing Calculator** Are the graphs of  $f(x) = \frac{(x-3)(x-1)}{(x-3)}$  and  $g(x) = x - 1$  identical? Explain. (*Hint:* Are there any excluded values?)

63. **Critical Thinking** Write the equation of a rational function that has a horizontal asymptote at  $y = 3$  and a vertical asymptote at  $x = -2$  and contains the point  $(1, 4)$ .



## SPIRAL STANDARDS REVIEW

2.0, 4.0, 14.0

Solve each inequality. (*Lesson 3-5*)

64.  $4t + 5 < 3(t + 3)$

65.  $2(r + 1) \geq r - 6$

66.  $j + 10 < 4j - 29$

67.  $5(g + 2) \leq 2g - 5$

68.  $c - 5 > 2c + 7$

69.  $6(m - 2) < 2(6 - m)$

Solve each quadratic equation by factoring. (*Lesson 9-5*)

70.  $4 - x^2 = 0$

71.  $3x^2 = x^2 + 2x + 12$

72.  $-x^2 = -6x + 9$

73. Marie has a square piece of cloth. She needs another piece with a length 2 inches shorter than the side of the square piece and a width 2 inches longer. The area for the new piece of cloth is  $780 \text{ in}^2$ . What will be the dimensions of the new piece of cloth? (*Lesson 9-6*)

# 10-3

## Simplifying Rational Expressions



### California Standards

**12.0** Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.

### Vocabulary

rational expression

### Why learn this?

The shapes and sizes of plants and animals are partly determined by the ratio of surface area to volume.

If an animal's body is small and its surface area is large, the rate of heat loss will be high. Hummingbirds must maintain a high metabolism to compensate for the loss of body heat due to having a high surface-area-to-volume ratio. Formulas for surface-area-to-volume ratios are *rational expressions*.

A **rational expression** is an algebraic expression whose numerator and denominator are polynomials. The value of the polynomial expression in the denominator cannot be zero since division by zero is undefined. This means that rational expressions, like rational functions, may have excluded values.

### EXAMPLE 1 Identifying Excluded Values

Find any excluded values of each rational expression.

**A**  $\frac{5}{8r}$

$$8r = 0$$

Set the denominator equal to 0.

$$r = \frac{0}{8} = 0$$

Solve for  $r$  by dividing both sides by 8.

The excluded value is 0.

**B**  $\frac{9d + 1}{d^2 - 2d}$

$$d^2 - 2d = 0$$

Set the denominator equal to 0.

$$d(d - 2) = 0$$

Factor.

$$d = 0 \quad \text{or} \quad d - 2 = 0$$

Use the Zero Product Property.

$$d = 0 \quad \text{or} \quad d = 2$$

Solve for  $d$ .

The excluded values are 0 and 2.

**C**  $\frac{x + 4}{x^2 + 5x + 6}$

$$x^2 + 5x + 6 = 0$$

Set the denominator equal to 0.

$$(x + 3)(x + 2) = 0$$

Factor.

$$x + 3 = 0 \quad \text{or} \quad x + 2 = 0$$

Use the Zero Product Property.

$$x = -3 \quad \text{or} \quad x = -2$$

Solve each equation for  $x$ .

The excluded values are  $-3$  and  $-2$ .

### Remember!

To review the Zero Product Property, see Lesson 9-5.

To review factoring trinomials, see Chapter 8.



Find any excluded values of each rational expression.

1a.  $\frac{12}{t^2 + 5}$

1b.  $\frac{3b}{b^2 + 5b}$

1c.  $\frac{3k^2}{k^2 + 7k + 12}$

A rational expression is in its simplest form when the numerator and denominator have no common factors except 1. Remember that to simplify fractions you can divide out common factors that appear in both the numerator and the denominator. You can do the same to simplify rational expressions.

### EXAMPLE 2 Simplifying Rational Expressions

Simplify each rational expression, if possible. Identify any excluded values.

**A**  $\frac{3t^3}{12t}$

$$\frac{3t^3}{3 \cdot 4t}$$

Factor 12.

$$\frac{\cancel{3}t^{\cancel{3}^2}}{\cancel{3} \cdot 4t}$$

Divide out common factors. Note that if  $t = 0$ , the expression is undefined.

$$\frac{t^2}{4}; t \neq 0$$

Simplify. The excluded value is 0.

**B**  $\frac{3x^2 - 9x}{x - 3}$

$$\frac{3x(x - 3)}{x - 3}$$

Factor the numerator.

$$\frac{3x(\cancel{x - 3})}{\cancel{x - 3}}$$

Divide out common factors. Note that if  $x = 3$ , the expression is undefined.

$$3x; x \neq 3$$

Simplify. The excluded value is 3.

**C**  $\frac{c}{c + 5}$

$$\frac{c}{c + 5}; c \neq -5$$

The numerator and denominator have no common factors. The excluded value is  $-5$ .

#### Caution!

Be sure to use the original denominator when finding excluded values. The excluded values may not be "seen" in the simplified denominator.



Simplify each rational expression, if possible. Identify any excluded values.

2a.  $\frac{5m^2}{15m}$

2b.  $\frac{6p^3 + 12p}{p^2 + 2}$

2c.  $\frac{3n}{n - 2}$

From this point forward, you do not need to include excluded values in your answers unless they are asked for.

### EXAMPLE 3 Simplifying Rational Expressions with Trinomials

Simplify each rational expression, if possible.

**A**  $\frac{k + 1}{k^2 - 4k - 5}$

$$\frac{k + 1}{(k + 1)(k - 5)}$$

Factor the numerator and the denominator when possible.

$$\frac{\cancel{k + 1}^1}{(\cancel{k + 1})(k - 5)}$$

Divide out common factors.

$$\frac{1}{k - 5}$$

Simplify.

**B**  $\frac{y^2 - 16}{y^2 - 8y + 16}$

$$\frac{(y + 4)(y - 4)}{(y - 4)(y - 4)}$$

$$\frac{(y + 4)(\cancel{y - 4})}{(y - 4)(\cancel{y - 4})}$$

$$\frac{y + 4}{y - 4}$$



Simplify each rational expression, if possible.

3a.  $\frac{r+2}{r^2+7r+10}$

3b.  $\frac{b^2-25}{b^2+10b+25}$

Recall from Chapter 8 that opposite binomials can help you factor polynomials. Recognizing opposite binomials can also help you simplify rational expressions.

Consider  $\frac{x-3}{3-x}$ . The numerator and denominator are opposite binomials. Therefore,

$$\frac{x-3}{3-x} = \frac{x-3}{-x+3} = \frac{\cancel{x}3^1}{-1(\cancel{x-3})} = \frac{1}{-1} = -1.$$

### EXAMPLE 4 Simplifying Rational Expressions Using Opposite Binomials

Simplify each rational expression, if possible.

**A**  $\frac{2x-10}{25-x^2}$

$$\frac{2(x-5)}{(5-x)(5+x)}$$

*Identify opposite binomials.*

$$\frac{2(x-5)}{(5-x)(5+x)}$$

*Rewrite one opposite binomial.*

$$\frac{-1(x-5)(5+x)}{-1(x-5)(5+x)}$$

*Divide out common factors.*

$$-\frac{2}{5+x}$$

*Factor.*

*Identify opposite binomials.*

*Rewrite one opposite binomial.*

*Divide out common factors.*

*Simplify.*

**B**  $\frac{2-2m}{2m^2+2m-4}$

$$\frac{2(1-m)}{2(m+2)(m-1)}$$

*Identify opposite binomials.*

$$\frac{2(1-m)}{2(m+2)(m-1)}$$

*Rewrite one opposite binomial.*

$$\frac{2(1-m)}{2(m+2)(-1)(1-m)}$$

*Divide out common factors.*

$$\frac{2(1-m)^1}{2(m+2)(-1)(1-m)}$$

*Simplify.*

$$-\frac{1}{m+2}$$



Simplify each rational expression, if possible.

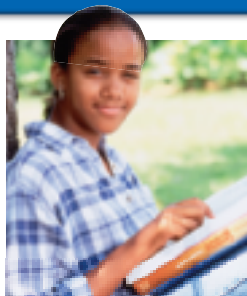
4a.  $\frac{3x-12}{16-x^2}$

4b.  $\frac{6-2x}{2x^2-4x-6}$

4c.  $\frac{3x-33}{x^2-121}$

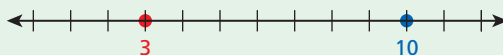
### Student to Student

#### Opposite Binomials



**Tanika Brown,**  
Washington High  
School

I didn't understand why the quotient of opposite binomials simplified to  $-1$ . My teacher showed me an example on a number line:



The distance between 3 and 10 is always the same (7 units). But depending on the order of the subtraction, the difference could be positive or negative.

$$10 - 3 = 7 \quad 3 - 10 = -7$$

So whenever you divide something in the form  $\frac{a-b}{b-a}$ , you get a number divided by its opposite, which is always  $-1$ .

**EXAMPLE 5** *Biology Application*



Water evaporates from a plant's surface. In two plants with different surface areas, the same volume of water will evaporate at a faster rate from the plant with the greater surface area. In the desert, plants must conserve water in order to survive. This means that the greater a plant's surface-area-to-volume ratio, the less likely the plant is to survive in the desert.

- a. What is the surface-area-to-volume ratio of a spherical barrel cactus? (*Hint:* For a sphere,  $S = 4\pi r^2$  and  $V = \frac{4}{3}\pi r^3$ .)

$$\frac{4\pi r^2}{\frac{4}{3}\pi r^3} \quad \text{Write the ratio of surface area to volume.}$$

$$\frac{4\cancel{\pi}r^2}{\frac{4}{3}\cancel{\pi}r^3} \quad \text{Divide out common factors.}$$

$$\frac{4r^2}{\frac{4}{3}r^3} \quad \text{Use properties of exponents.}$$

$$\frac{4}{r} \cdot \frac{3}{4} \quad \text{To divide by } \frac{4}{3}, \text{ multiply by its reciprocal, } \frac{3}{4}.$$

$$\frac{\cancel{4}}{r} \cdot \frac{3}{\cancel{4}} \quad \text{Divide out common factors.}$$

$$\frac{3}{r} \quad \text{Simplify.}$$

- b. Which barrel cactus has a greater chance of survival in the desert, one with a radius of 4 inches or one with a radius of 7 inches? Explain.

$$\frac{3}{r} = \frac{3}{4} \quad \frac{3}{r} = \frac{3}{7} \quad \text{Write the ratio of surface area to volume twice. Substitute 4 and 7 for } r.$$

$$\frac{3}{4} > \frac{3}{7} \quad \text{Compare the ratios.}$$

The cactus with a radius of 7 inches has a greater chance of survival because its surface-area-to-volume ratio is lesser.

**Remember!**

For two fractions with the same numerator, the value of the fraction with a greater denominator is less than the value of the other fraction.

$$9 > 3$$

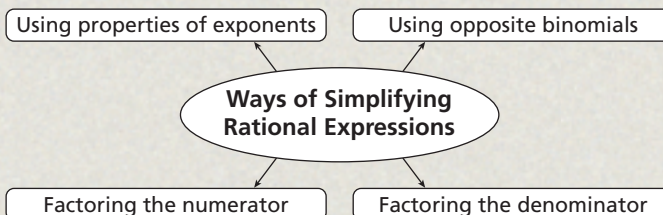
$$\frac{2}{9} < \frac{2}{3}$$



5. Which barrel cactus has less of a chance to survive in the desert, one with a radius of 6 inches or one with a radius of 3 inches? Explain.

**THINK AND DISCUSS**

- Write a rational expression that has an excluded value that cannot be identified when the expression is in its simplified form.
- GET ORGANIZED** Copy and complete the graphic organizer. In each box, write and simplify one of the given rational expressions using the most appropriate method.  $\frac{x-3}{x^2-6x+9}$ ,  $\frac{5x^4}{x^2}$ ,  $\frac{4-x}{x-4}$ ,  $\frac{4x^2-4x}{8x}$







## GUIDED PRACTICE

1. **Vocabulary** What is true about both the numerator and denominator of rational expressions?

**SEE EXAMPLE 1** Find any excluded values of each rational expression.

p. 642

2.  $\frac{5}{m}$

3.  $\frac{x+2}{x^2-8x}$

4.  $\frac{p^2}{p^2-2p-15}$

**SEE EXAMPLE 2** Simplify each rational expression, if possible. Identify any excluded values.

p. 643

5.  $\frac{4a^2}{8a}$

6.  $\frac{2d^2+12d}{d+6}$

7.  $\frac{2}{y+3}$

8.  $\frac{10}{5-y}$

9.  $\frac{2h}{2h+4}$

10.  $\frac{3(x+4)}{6x}$

**SEE EXAMPLE 3** Simplify each rational expression, if possible.

p. 643

11.  $\frac{b+4}{b^2+5b+4}$

12.  $\frac{s^2-4}{s^2+4s+4}$

13.  $\frac{c^2+5c+6}{(c+3)(c-4)}$

14.  $\frac{(x-2)(x+1)}{x^2+4x+3}$

15.  $\frac{j^2-25}{j^2+2j-15}$

16.  $\frac{p+1}{p^2-4p-5}$

**SEE EXAMPLE 4**

p. 644

17.  $\frac{2n-16}{64-n^2}$

18.  $\frac{8-4x}{2x^2-12x+16}$

19.  $\frac{10-5r}{r^2+4r-12}$

20.  $\frac{2x-14}{49-x^2}$

21.  $\frac{5q-50}{100-q^2}$

22.  $\frac{36-12a}{a^2+2a-15}$

**SEE EXAMPLE 5** 23. **Construction** The side of a triangular roof is to have the same height  $h$  and base  $b_2$  as the side of a trapezoidal roof.

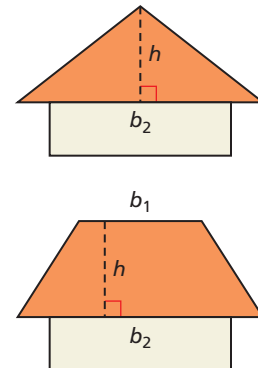
p. 645

- a. What is the ratio of the area of the triangular roof to the area of the trapezoidal roof?

(Hint: For a triangle,  $A = \frac{1}{2}b_2h$ .

For a trapezoid,  $A = \frac{b_1+b_2}{2}h$ .)

- b. Compare the ratio from part a to what the ratio will be if  $b_1$  is doubled for the trapezoidal roof and  $b_2$  is doubled for both roofs.



## PRACTICE AND PROBLEM SOLVING

Find any excluded values of each rational expression.

24.  $\frac{c}{c^2+c}$

25.  $\frac{2}{-3x}$

26.  $\frac{4}{x^2-3x-10}$

27.  $\frac{n^2-1}{2n^2-7n-4}$

Simplify each rational expression, if possible. Identify any excluded values.

28.  $\frac{4d^3+4d^2}{d+1}$

29.  $\frac{3m^2}{m-4}$

30.  $\frac{10y^4}{2y}$

31.  $\frac{2t^2}{16t}$

**Independent Practice**

| For Exercises | See Example |
|---------------|-------------|
| 24–27         | 1           |
| 28–31         | 2           |
| 32–37         | 3           |
| 38–40         | 4           |
| 41            | 5           |

**Extra Practice**

Skills Practice p. EP20

Application Practice p. EP33



**Biology**



As a result of a nationwide policy of protection and reintroduction, the population of bald eagles in the lower 48 states grew from 417 nesting pairs in 1963 to more than 6400 nesting pairs in 2000.

Source: U.S. Fish and Wildlife Service

Simplify each rational expression, if possible.

32.  $\frac{q - 6}{q^2 - 9q + 18}$

33.  $\frac{z^2 - 2z + 1}{z^2 - 1}$

34.  $\frac{t - 3}{t^2 - 5t + 6}$

35.  $\frac{p^2 - 6p - 7}{p^2 - 4p - 5}$

36.  $\frac{x^2 - 1}{x^2 + 4x + 3}$

37.  $\frac{2x - 4}{x^2 - 6x + 8}$

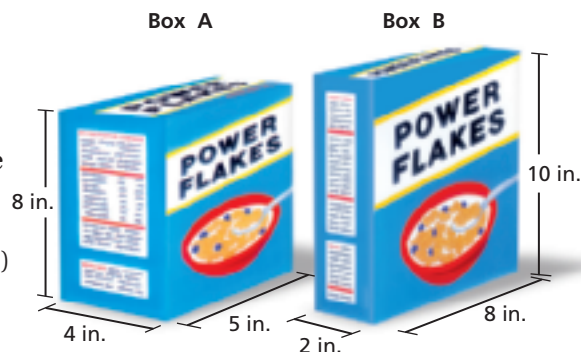
38.  $\frac{20 - 4x}{x^2 - 25}$

39.  $\frac{3 - 3b}{3b^2 + 18b - 21}$

40.  $\frac{3v - 36}{144 - v^2}$

41. **Geometry** When choosing package sizes, a company wants a package that uses the least amount of material to hold the greatest volume of product.

- What is the surface-area-to-volume ratio for a rectangular prism? (Hint: For a rectangular prism,  $S = 2\ell w + 2\ell h + 2wh$  and  $V = \ell wh$ .)
- Which box should the company choose? Explain.



42. **Biology** The table gives information on two populations of animals that were released into the wild. Suppose 16 more predators and 20 more prey are released into the area. Write and simplify a rational expression to show the ratio of predator to prey.

|                          | Predator | Prey |
|--------------------------|----------|------|
| Original Population      | $x$      | $x$  |
| Population 5 Years Later | $4x$     | $5x$ |

Simplify each rational expression, if possible.

43.  $\frac{p^2 + 12p + 36}{12p + 72}$

44.  $\frac{3n^3 + 33n^2 + 15n}{3n^3 + 15n}$

45.  $\frac{a}{2a + a}$

46.  $\frac{j - 5}{j^2 - 25}$

47.  $\frac{6w^2 + 11w - 7}{6w - 3}$

48.  $\frac{n^2 - n - 56}{n^2 - 16n + 64}$

49.  $\frac{(x + 1)^2}{x^2 + 2x + 1}$

50.  $\frac{5}{(x + 5)^2}$

51.  $\frac{25 - x^2}{x^2 - 3x - 10}$

**CONCEPT CONNECTION**



52. This problem will prepare you for the Concept Connection on page 650.

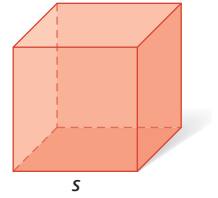
It takes 250 workdays to build a house. The number of construction days is determined by the size of the crew. The crew includes one manager who supervises workers and checks for problems, but does not do any building.

- The table shows the number of construction days as a function of the number of workers. Copy and complete the table.
- Use the table to write a function that represents the number of construction days.
- Identify the excluded values of the function.

| Crew Size ( $x$ ) | Workdays<br>Workers | Construction Days ( $y$ ) |
|-------------------|---------------------|---------------------------|
| 2                 | $\frac{250}{2 - 1}$ | 250                       |
| 3                 | $\frac{250}{3 - 1}$ | ■                         |
| 6                 | ■                   | ■                         |
| 11                | ■                   | 25                        |



53. **Geometry** Let  $s$  represent the length of an edge of a cube.
- Write the ratio of a cube's surface area to volume in simplified form. (*Hint:* For a cube,  $S = 6s^2$ .)
  - What is the ratio of the cube's surface area to volume when  $s = 2$ ?
  - What is the ratio of the cube's surface area to volume when  $s = 6$ ?



54. **Write About It** Explain how to find excluded values for a rational expression.
55. **Critical Thinking** Give an example of a rational expression that has  $x$  in both the numerator and denominator, but cannot be simplified.

**Multiple Choice** For Exercises 56 and 57, choose the best answer.

56. Which expression is undefined for  $x = 4$  and  $x = -1$ ?
- (A)  $\frac{x-1}{x+4}$       (B)  $\frac{x-4}{x+4}$       (C)  $\frac{x}{x^2+3x-4}$       (D)  $\frac{x}{x^2-3x-4}$
57. Which expression is the ratio of the area of a triangle to the area of a rectangle that has the same base and height?
- (A)  $\frac{1}{2}$       (B)  $\frac{bh}{2}$       (C)  $\frac{(bh)^2}{2}$       (D) 2
58. **Gridded Response** What is the excluded value for  $\frac{x-4}{x^2-8x+16}$ ?

## CHALLENGE AND EXTEND



**Reasoning** Tell whether each statement is sometimes, always, or never true. Explain.

- A rational expression has an excluded value.
- A rational expression has a variable inside a square root in the numerator.
- The graph of a rational function has at least one asymptote.

Simplify each rational expression.

62.  $\frac{9v-6v^2}{4v^2-4v-3}$       63.  $\frac{2a^2-7a+3}{2a^2+9a-5}$       64.  $\frac{0.25y-0.10}{0.25y^2-0.04}$

Identify any excluded values of each rational expression.

65.  $\frac{\frac{1}{4}x^2-7x+49}{\frac{1}{4}x^2-49}$       66.  $\frac{-80x+40x^2+40}{-30-30x^2+60x}$       67.  $\frac{6x+12}{12x+6x^2}$



## SPIRAL STANDARDS REVIEW

2.0, 6.0, 17.0

Give the domain and range of each relation. (*Lesson 4-2*)

68. 

|   |   |   |   |   |
|---|---|---|---|---|
| x | 3 | 1 | 2 | 9 |
| y | 4 | 2 | 6 | 5 |

69. 

|   |   |    |    |    |
|---|---|----|----|----|
| x | 4 | 29 | 5  | 25 |
| y | 2 | 2  | -7 | 22 |

Use intercepts to graph the line described by each equation. (*Lesson 5-2*)

70.  $5x - 3y = -15$       71.  $y = 8x - 8$       72.  $\frac{1}{2}x + y = 2$

Simplify. (*Lesson 7-1*)

73.  $5h^{-3}$       74.  $s^{-2}t^6$       75.  $\frac{12}{b^4}$       76.  $\frac{v^{-3}}{w^{-4}}$



# Graph Rational Functions

You can use a graphing calculator to graph rational functions and to compare graphs of rational functions before and after they are simplified.

Use with Lesson 10-3

## Activity

Simplify  $y = \frac{x-1}{x^2-5x+4}$  and give any excluded values. Then graph both the original function and the simplified function, and compare the graphs.

- 1 Simplify the function and find the excluded values.

$$\frac{x-1}{x^2-5x+4} = \frac{x-1}{(x-1)(x-4)} = \frac{1}{x-4}; \text{ excluded values: } 4, 1$$

- 2 Enter  $y = \frac{x-1}{x^2-5x+4}$  and  $y = \frac{1}{x-4}$  into your calculator as shown and press **GRAPH**.

- 3 To compare the graphs, press **TRACE**. At the top of the screen, you can see which graph the cursor is on. To change between graphs, press and .

- 4 The graphs appear to be the same, but check the excluded values, 4 and 1. While on **Y1**, press 4 **ENTER**. Notice that there is no  $y$ -value at  $x = 4$ . The function is undefined.

- 5 Press to switch to **Y2** and press 4 **ENTER**. This function is also undefined at  $x = 4$ . The graphs are the same at this excluded value.

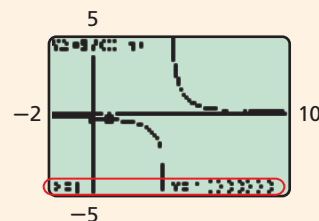
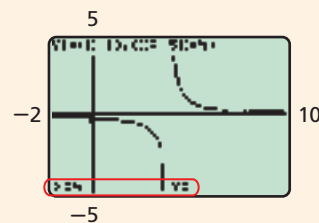
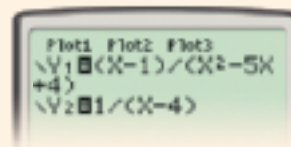
- 6 Return to **Y1** and press 1 **ENTER**. This function is undefined at  $x = 1$ . However, this is not a vertical asymptote. Instead, this graph has a “hole” at  $x = 1$ .

- 7 Switch to **Y2** and press 1 **ENTER**. This function is defined at  $x = 1$ . So the two graphs are the same except at  $x = 1$ .



### California Standards

**13.0** Students add, subtract, multiply, and divide **rational** expressions and **functions**. Students solve both computationally and conceptually challenging problems by using these techniques.



## Try This

1. Why is  $x = 1$  an excluded value for one function but not for the other?
2. Are the functions  $y = \frac{x-1}{x^2-5x+4}$  and  $y = \frac{1}{x-4}$  truly equivalent for all values of  $x$ ? Explain.
3. **Make a Conjecture** Complete each statement.
  - a. If a value of  $x$  is excluded from a function and its simplified form, it appears on the graph as a(n) \_\_\_\_?
  - b. If a value of  $x$  is excluded from a function but not its simplified form, it appears on the graph as a(n) \_\_\_\_?

## SECTION 10A



## Rational Functions and Expressions

## Construction Daze

Robert is part of a volunteer crew constructing houses for low-income families. The table shows how many construction days it takes to complete a house for work crews of various sizes.

| Crew Size | Construction Days | Workdays |
|-----------|-------------------|----------|
| 2         | 100               | 200      |
| 4         | 50                | 200      |
| 8         | 25                | 200      |
| 10        | 20                | 200      |
| 20        | 10                | 200      |

- Working at the same rate, how many construction days should it take a crew of 40 people to build the house?
- Express the number of construction days as a function of the crew size. Define the variables. What type of relationship is formed in the situation?
- Explain how the crew size affects the number of construction days.
- About how many construction days would it take a crew of 32 to complete a house?
- If a crew can complete a house in 12.5 days, how many people are in the crew?
- What are a reasonable domain and range of the function?
- Suppose there are two managers that do not contribute to the work of building the house, yet are counted as part of the crew. Express the number of construction days as a function of the crew size. What are the asymptotes of this function? Graph the function.



## Quiz for Lessons 10-1 Through 10-3

### 10-1 Inverse Variation

Tell whether each relationship represents an inverse variation. Explain.

1. 

|     |    |    |    |
|-----|----|----|----|
| $x$ | -5 | -4 | -3 |
| $y$ | 10 | -8 | 6  |

2. 

|     |    |   |   |
|-----|----|---|---|
| $x$ | 18 | 9 | 6 |
| $y$ | 2  | 4 | 6 |

3.  $y = \frac{3}{x}$

4.  $y + x = \frac{3}{4}$

5.  $xy = -2$

6.  $y = \frac{x}{5}$

7. Write and graph the inverse variation in which  $y = 3$  when  $x = 2$ .
8. Write and graph the inverse variation in which  $y = 4$  when  $x = -1$ .
9. The number of calculators Mrs. Hopkins can buy for the classroom varies inversely as the cost of each calculator. She can buy 24 calculators that cost \$60 each. How many calculators can she buy if they cost \$80 each?

### 10-2 Rational Functions

Identify any excluded values and the asymptotes for each rational function. Then graph each function.

10.  $y = \frac{12}{x}$

11.  $y = \frac{6}{x+2}$

12.  $y = \frac{4}{x-1}$

13.  $y = \frac{2}{x+1} - 3$

14. Jeff builds model train layouts. He has \$75 to spend on packages of miniature landscape items. He receives 6 free packages with each order. The number of packages  $y$  that Jeff can buy is given by  $y = \frac{75}{x} + 6$ , where  $x$  represents the cost of each package in dollars. Describe the reasonable domain and range values and graph the function.

### 10-3 Simplifying Rational Expressions

Find any excluded values of each rational expression.

15.  $\frac{15}{n}$

16.  $\frac{p}{p-8}$

17.  $\frac{x+2}{x^2+6x+8}$

18.  $\frac{t-1}{t^2+t}$

Simplify each rational expression, if possible. Identify any excluded values.

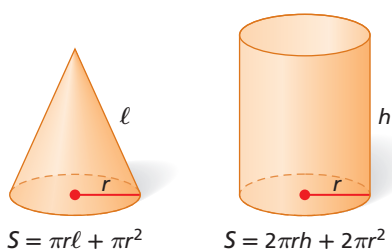
19.  $\frac{3x^2}{6x^3}$

20.  $\frac{2n}{n^2-3n}$

21.  $\frac{s+1}{s^2-4s-5}$

22.  $\frac{12-3x}{x^2-8x+16}$

23. Suppose a cone and a cylinder have the same radius and that the slant height  $\ell$  of the cone is the same as the height  $h$  of the cylinder. Find the ratio of the cone's surface area to the cylinder's surface area.



# 10-4

## Multiplying and Dividing Rational Expressions



### California Standards

**13.0** Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.

### Why learn this?

You can multiply rational expressions to determine the probabilities of winning prizes at carnivals. (See Example 5.)

The rules for multiplying rational expressions are the same as the rules for multiplying fractions. You multiply the numerators, and you multiply the denominators.



### Multiplying Rational Expressions

If  $a$ ,  $b$ ,  $c$ , and  $d$  are nonzero polynomials, then  $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ .

### EXAMPLE 1 Multiplying Rational Expressions

Multiply. Simplify your answer.

**A**  $\frac{a+3}{2} \cdot \frac{6}{3a+9}$

$$\frac{6(a+3)}{2(3a+9)}$$

Multiply the numerators and denominators.

$$\frac{6(a+3)}{2 \cdot 3(a+3)}$$

Factor.

$$\frac{\cancel{6}(a+3)^1}{\cancel{6}(a+3)}$$

Divide out the common factors.

$$1$$

Simplify.

**B**  $\frac{12b^3c^2}{5ac} \cdot \frac{15a^2b}{3b^2c}$

$$\frac{(12)(15)a^2(b^3 \cdot b)c^2}{(5)(3)ab^2(c \cdot c)}$$

Multiply the numerators and the denominators. Arrange the expression so like variables are together.

$$\frac{180a^2b^4c^2}{15ab^2c^2}$$

Simplify.

$$12a^1b^2c^0$$

Divide out common factors. Use properties of exponents.

$$12ab^2$$

Simplify. Remember that  $c^0 = 1$ .

**C**  $\frac{5x^2}{2y^3} \cdot \frac{3x}{2y^2}$

$$\frac{15x^3}{4y^5}$$

Multiply. There are no common factors, so the product cannot be simplified.

### Remember!

Review the Quotient of Powers Property in Lesson 7-4.

$$\frac{a^m}{a^n} = a^{m-n}$$



Multiply. Simplify your answer.

1a.  $\frac{(c-4)}{5} \cdot \frac{45}{(-4c+16)}$

1b.  $\frac{5y^5z}{3xy^2z} \cdot \frac{2x^4y^2}{4xy}$

## EXAMPLE 2 Multiplying a Rational Expression by a Polynomial

### Remember!

Just as you can write an integer as a fraction, you can write any expression as a rational expression by writing it with a denominator of 1.

Multiply  $(x^2 + 8x + 15) \frac{4}{2x+6}$ . Simplify your answer.

$$\frac{x^2 + 8x + 15}{1} \cdot \frac{4}{2x + 6}$$

*Write the polynomial over 1.*

$$\frac{(x+3)(x+5)}{1} \cdot \frac{4}{2(x+3)}$$

*Factor the numerator and denominator.*

$$\frac{\cancel{(x+3)}(x+5)\cancel{4}^2}{\cancel{2}(x+3)}$$

*Divide out common factors.*

$$2x + 10$$

*Multiply remaining factors.*



2. Multiply  $\frac{m-5}{m^2-4m-12} \cdot 3m+6$ . Simplify your answer.

There are two methods for simplifying rational expressions. You can **simplify first** by dividing out common factors and **then multiply** the remaining factors. You can also **multiply first** and **then simplify**. Using either method will result in the same answer.

## EXAMPLE 3 Multiplying Rational Expressions Containing Polynomials

Multiply  $\frac{4d^3+4d}{16f} \cdot \frac{2f}{7d^2f+7f}$ . Simplify your answer.

**Method 1** Simplify first.

$$\frac{4d^3 + 4d}{16f} \cdot \frac{2f}{7d^2f + 7f}$$

$$\frac{4d(d^2 + 1)}{16f} \cdot \frac{2f}{7f(d^2 + 1)}$$

*Factor.*

$$\frac{\cancel{4}d\cancel{(d^2+1)}}{\cancel{16}^2f} \cdot \frac{\cancel{2}f}{7f\cancel{(d^2+1)}}$$

*Divide out common factors.*

Then multiply.

$$\frac{d}{14f}$$

*Simplify.*

**Method 2** Multiply first.

$$\frac{4d^3 + 4d}{16f} \cdot \frac{2f}{7d^2f + 7f}$$

$$\frac{(4d^3 + 4d)2f}{16f(7d^2f + 7f)}$$

*Multiply.*

$$\frac{8d^3f + 8df}{112d^2f^2 + 112f^2}$$

*Distribute.*

Then simplify.

$$\frac{8df(d^2 + 1)}{112f^2(d^2 + 1)}$$

*Factor.*

$$\frac{\cancel{8}df\cancel{(d^2+1)}}{\cancel{112}^{14}f^2(d^2+1)}$$

*Divide out common factors.*

$$\frac{d}{14f}$$

*Simplify.*



Multiply. Simplify your answer.

3a.  $\frac{n-5}{n^2+4n} \cdot \frac{n^2+8n+16}{n^2-3n-10}$

3b.  $\frac{p+4}{p^2+2p} \cdot \frac{p^2-3p-10}{p^2+16}$



The rules for dividing rational expressions are the same as the rules for dividing fractions. To divide by a rational expression, multiply by its reciprocal.



### Dividing Rational Expressions

If  $a$ ,  $b$ ,  $c$ , and  $d$  are nonzero polynomials, then  $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$ .

#### EXAMPLE 4 Dividing by Rational Expressions and Polynomials

Divide. Simplify your answer.

**A**  $\frac{1}{x} \div \frac{x-2}{2x}$

$$\frac{1}{x} \cdot \frac{2x}{x-2}$$

$$\frac{1(2x)}{x(x-2)}$$

$$\frac{2\cancel{x}}{\cancel{x}(x-2)}$$

$$\frac{2}{x-2}$$

Write as multiplication by the reciprocal.

Multiply the numerators and the denominators.

Divide out common factors.

Simplify.

**B**  $\frac{x^2-2x}{x} \div \frac{2-x}{x^2+2x+1}$

$$\frac{x^2-2x}{x} \cdot \frac{x^2+2x+1}{2-x}$$

$$\frac{x(x-2)}{x} \cdot \frac{(x+1)(x+1)}{2-x}$$

$$\frac{x(x-2)}{x} \cdot \frac{(x+1)(x+1)}{-1(x-2)}$$

$$\frac{\cancel{x}(\cancel{x-2})^1}{\cancel{x}} \cdot \frac{(x+1)(x+1)}{-1(\cancel{x-2})}$$

$$-(x+1)^2$$

Write as multiplication by the reciprocal.

Factor.

Rewrite one opposite binomial.

Divide out common factors.

Multiply.

**C**  $\frac{3a^2b}{b} \div (3a^2+6a)$

$$\frac{3a^2b}{b} \div \frac{3a^2+6a}{1}$$

$$\frac{3a^2b}{b} \cdot \frac{1}{3a^2+6a}$$

$$\frac{3a^2b}{b(3a^2+6a)}$$

$$\frac{\cancel{3}a^2\cancel{b}}{\cancel{b}[3\cancel{a}(a+2)]}$$

$$\frac{a}{(a+2)}$$

Write the binomial over 1.

Write as multiplication by the reciprocal.

Multiply the numerators and the denominators.

Factor. Divide out common factors.

Simplify.



Divide. Simplify your answer.

4a.  $\frac{3}{x^2} \div \frac{x^3}{(x-5)}$

4b.  $\frac{18vw^2}{6v} \div \frac{3v^2x^4}{2w^4x}$

4c.  $\frac{x^2-x}{x+2} \div (x^2+2x-3)$

## EXAMPLE 5 Probability Application

Marty is playing a carnival game. He needs to pick two items out of a bag without looking. The bag has red and blue items. There are three more red items than blue items.

- a. Write and simplify an expression that represents the probability that Marty will pick two blue items without replacing the first item.

Let  $x$  = the number of blue items.

$$\text{Blue} + \text{Red} = \text{Total}$$

$$x + x + 3 = 2x + 3$$

Write expressions for the number of each color item and for the total number of items.

1st pick: blue items  $\rightarrow$   $x$       2nd pick: blue items  $\rightarrow$   $x-1$

$$P(\text{blue, blue}) = \frac{x}{2x+3} \cdot \frac{x-1}{2x+2}$$

1st pick: total items  $\rightarrow$   $2x+3$       2nd pick: total items  $\rightarrow$   $2x+2$

$$= \frac{x}{(2x+3)} \cdot \frac{x-1}{2(x+1)}$$
$$= \frac{x(x-1)}{2(2x+3)(x+1)}$$

### Remember!

For a review of probability topics, see Skills Bank pp. SB29–SB32.

- b. What is the probability that Marty picks two blue items if there are 10 blue items in the bag before his first pick? Round your answer to the nearest hundredth.

Since  $x$  represents the number of blue items, substitute 10 for  $x$ .

$$P(\text{blue, blue}) = \frac{10(10-1)}{2(2 \cdot 10 + 3)(10 + 1)}$$

Substitute.

$$= \frac{10(9)}{2(23)(11)} = \frac{90}{506} \approx 0.18$$

Use the order of operations to simplify.

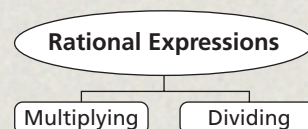
The probability is approximately 0.18.



5. **What if...?** There are 50 blue items in the bag before Marty's first pick. What is the probability that Marty picks two blue items? Round your answer to the nearest hundredth.

## THINK AND DISCUSS

1. Explain how to divide by a polynomial.
2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe how to perform the operation with rational expressions.





## GUIDED PRACTICE

SEE EXAMPLE 1 Multiply. Simplify your answer.

p. 652

1.  $\frac{4hj^2}{10j^3} \cdot \frac{3h^3k}{h^3k^3}$

2.  $\frac{4y}{x^5} \cdot \frac{2yz^2}{9x^2}$

3.  $\frac{x-2}{x+3} \cdot \frac{4x+12}{6}$

4.  $\frac{ab}{c} \cdot \frac{2a^2}{3c}$

5.  $\frac{7c^4d}{10c} \cdot \frac{5a}{21c^3d}$

6.  $\frac{12p^2q}{5p} \cdot \frac{15p^4q^3}{12q}$

SEE EXAMPLE 2 7.  $\frac{12}{4y+8}(y^2-4)$ 

p. 653

8.  $\frac{x+2}{6x^2}(5x+10)$

9.  $\frac{3m}{6m+18}(m^2-7m-30)$

10.  $\frac{4p}{8p+16}(p^2-5p-14)$

11.  $\frac{a^2}{a}(a^2+10a+25)$

12.  $\frac{-c}{4c+4}(c^2-c-2)$

SEE EXAMPLE 3 13.  $\frac{a^2+6ab}{b} \cdot \frac{5+3a}{3a^2b+5ab}$ 

p. 653

14.  $\frac{x^2+5x+4}{x-4} \cdot \frac{x^2-2x-8}{x^2+6x+8}$

15.  $\frac{j-1}{j^2-4j+3} \cdot \frac{j^2-5j+6}{2j-4}$

16.  $\frac{p^3+4pq}{p} \cdot \frac{6q^3-8}{2q}$

17.  $\frac{r^2+15r+14}{r^2-16} \cdot \frac{2r+8}{r+1}$

18.  $\frac{y-8}{y^2-1} \cdot \frac{y+2}{y^2-49}$

SEE EXAMPLE 4 Divide. Simplify your answer.

p. 654

19.  $\frac{3a^4b}{2a^2c^3} \div \frac{12a^2c}{8c^4}$

20.  $\frac{2m^3+2m}{m^2-2m} \div \frac{4m^2+4}{m-1}$

21.  $\frac{x^2+4x-5}{3x-3} \div (x^2-25)$

SEE EXAMPLE 5 22. **Probability** While playing a game, Rachel pulls two tiles out of a bag without looking and without replacing the first tile. The bag has two colors of tiles—black and white. There are 10 more white tiles than black tiles.

p. 655

a. Write and simplify an expression that represents the probability that Rachel will pick a black tile, then a white tile.

b. What is the probability that Rachel pulls a black tile and then a white tile if there are 5 black tiles in the bag before her first pick? Round your answer to the nearest hundredth.

## PRACTICE AND PROBLEM SOLVING

## Independent Practice

Multiply. Simplify your answer.

23.  $\frac{p^6q^2}{7r^3} \cdot \frac{-3p^2}{r}$

24.  $\frac{3r^2t}{6st^3} \cdot \frac{2r^2s^3t^2}{8r^4s^2}$

25.  $\frac{10}{y+5} \cdot \frac{y+2}{3}$

26.  $\frac{3}{2a+6}(a^2+4a+3)$

27.  $\frac{4m^2-8m}{m^2+6m-16}(m^2+7m-8)$

28.  $\frac{x}{2x^2-12x+18}(2x^2-4x-6)$

29.  $\frac{6n^2+18n}{n^2+9n+8} \cdot \frac{n^2-1}{2n+6}$

30.  $\frac{3a^2b}{5a^3+10a^2b} \cdot \frac{2a+4b}{6a^3b+6a^2b^2}$

31.  $\frac{t^2-100}{5t+50} \cdot \frac{5}{t-10}$

## Extra Practice

Skills Practice p. EP21

Application Practice p. EP33

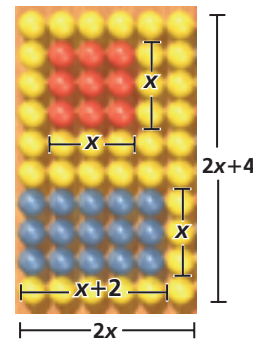
Divide. Simplify your answer.

32.  $\frac{6j^2k^5}{5j} \div \frac{4j^3k^3}{3j}$

33.  $\frac{a-4}{a^2} \div (8a-2a^2)$

34.  $\frac{x^2-9}{x^2+6x+9} \div \frac{4x^2-12x}{16x}$

35. **Entertainment** A carnival game board is covered completely in small balloons. You throw darts at the board and try to pop the balloons.
- Write and simplify an expression describing the probability that the next two balloons popped are red and then blue. (*Hint:* Write the probabilities as ratios of the areas of rectangles.)
  - What is the probability that the next two balloons popped are red and then blue if  $x = 3$ ?
36. **ERROR ANALYSIS** Which is incorrect? Explain the error.



**A**

$$\frac{4a^2 - b^2}{a^2} \cdot \frac{a}{2a - b}$$

$$\frac{2\cancel{4}a^2 - b^2}{\cancel{a}^2} \cdot \frac{\cancel{a}}{2\cancel{a} - b} = \frac{2 - b^2}{b}$$

**B**

$$\frac{4a^2 - b^2}{a^2} \cdot \frac{a}{2a - b}$$

$$\frac{(2a - b)(2a + b)}{a^2} \cdot \frac{a}{2a - b} = \frac{2a + b}{a}$$

37. **Critical Thinking** Which of the following expressions is NOT equivalent to the other three? Explain why.
- $\frac{4x^2}{x^2 - 3x} \cdot \frac{2x - 6}{8y^2}$
  - $\frac{6xy^2}{x^2} \div \frac{3y^4}{2x^2}$
  - $\frac{10x^4y}{5xy^2} \div 2x^2y$
  - $\frac{4x}{xy^2 + 2y^2} \cdot \frac{x^2 - 4}{4x - 8}$

Multiply or divide. Simplify your answer.

- $\frac{5p^3}{p^2q} \cdot \frac{2q^3}{p^2}$
  - $\frac{2x^2}{4x - 8} \cdot \frac{x^2 - 5x + 6}{x^5}$
  - $\frac{33m - 3m^2}{-2m - 4} \div \frac{6m - 66}{m^2 - 4m}$
  - $\frac{6m^2 - 18m}{12m^3 + 12m^2} \div \frac{m^2 - 9}{m^2 + 4m + 3}$
  - $\frac{x^2 - 9}{4x} \div (4x^2 - 36)$
  - $\frac{12w^4x^7}{3w^3} \cdot \frac{w^{-1}x^{-7}}{4}$
44. **Write About It** Explain how to divide  $\frac{1}{m} \div \frac{3}{4m}$ .

**CONCEPT CONNECTION**



45. This problem will prepare you for the Concept Connection on page 684.
- The size of an image projected on a screen depends on how far the object is from the lens, the magnification of the lens, and the distance between the image and the lens. Magnification of a lens is  $M = \frac{I}{O} = \frac{y}{x}$  where  $I$  is the height of the image,  $O$  is the height of the object,  $x$  is the distance of the object from the lens, and  $y$  is the distance of the image from the lens.
- If an object 16 cm high is placed 15 cm from the lens, it forms an image 60 cm from the lens. What is the height of the image?
  - Marie moves the same object to a distance of 20 cm from the lens. If the image is the same size as part a, what is the distance between the image and the lens?
  - What is the magnification of the lens?

**Multiple Choice** For Exercises 46–48, choose the best answer.

46. Which expression is equivalent to  $\frac{t+4}{9} \cdot \frac{t+4}{3}$ ?
- (A)  $\frac{(t+4)^2}{27}$       (B)  $\frac{t^2+16}{27}$       (C)  $\frac{1}{3}$       (D)  $\frac{1}{27}$
47. Identify the product  $-\frac{20b^2}{a^2} \cdot \frac{3ab}{15b}$ .
- (A)  $-\frac{a}{4b^2}$       (B)  $-4b^2$       (C)  $-\frac{4b^2}{a}$       (D)  $-\frac{b^2}{4a}$
48. Which of the following is equivalent to  $\frac{2x}{x+5}$ ?
- (A)  $\frac{x-2}{8x} \cdot \frac{4}{x^2+3x-10}$       (C)  $\frac{x-2}{4} \div \frac{x^2+3x-10}{8x}$
- (B)  $\frac{x^2-3x-10}{8x} \cdot \frac{4}{x-2}$       (D)  $\frac{x^2-3x-10}{4} \div \frac{x-2}{8x}$
49. **Short Response** Simplify  $\frac{x^2-10x+24}{3x^2-12x} \div (x^2-3x-18)$ . Show your work.

## CHALLENGE AND EXTEND

Simplify.

50.  $\frac{x-3}{3x-6} \cdot \frac{3x+12}{x+1} \cdot \frac{2x-4}{x^2+x-12}$       51.  $\frac{x^2-1}{x+2} \div \frac{3x+3}{x+2} \div (x-1)$

A *complex fraction* is a fraction that contains one or more fractions in the numerator or the denominator. Simplify each complex fraction.

(Hint: Use the rule  $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d}$ .)

52.  $\frac{\frac{c+5}{c^2-4}}{\frac{c^2+6c+5}{c+2}}$       53.  $\frac{\frac{x^2y}{xz^3}}{\frac{x^2y}{x^2z}}$       54.  $\frac{\frac{x^2}{2} \cdot \frac{x}{3}}{\frac{x}{6}}$       55.  $\frac{\frac{a+1}{a^2+6a+5}}{\frac{2a+2}{a+5}}$



## SPIRAL STANDARDS REVIEW

4.0, 8.0, 12.0

56. Danny runs laps around the track each week. This week, he wants to run at least 12 laps. He ran 3 laps earlier in the week, and today he ran 2 laps. Write and solve an inequality to determine how many more laps Danny has to run to meet his goal. (Lesson 3-4)
57. Pierce has \$30 to spend on a night out. He already spent \$12 on dinner and \$9 on a movie ticket. He will spend some money  $m$  on movie-theatre snacks. Write and solve an inequality that will show all the values of  $m$  that Pierce can spend on snacks. (Lesson 3-4)

Identify which lines are parallel. (Lesson 5-7)

58.  $y = 3x + 4$ ;  $y = 7$ ;  $y = -3x + 4$ ,  $y = 2$
59.  $y = 5x - 7$ ;  $y = -x + 5$ ;  $y = -x - 7$ ,  $y = 5x + 5$
60.  $y = -7x$ ;  $y = 12$ ;  $y = -x + 4$ ,  $y = -7x + 5$

Simplify each rational expression, if possible. Identify any excluded values. (Lesson 10-3)

61.  $\frac{x+2}{x^2-4}$       62.  $\frac{2(x-3)}{3(x+4)}$       63.  $\frac{4x-10}{2x^2-8}$

# 10-5

## Adding and Subtracting Rational Expressions



California Standards

**13.0** Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.

Also covered: **15.0**

### Who uses this?

Kayakers can use rational expressions to figure out travel time for different river trips. (See Example 5.)



The rules for adding rational expressions are the same as the rules for adding fractions. If the denominators are the same, you add the numerators and keep the common denominator.

$$\frac{3}{8} + \frac{2}{8} = \frac{3+2}{8} = \frac{5}{8}$$



### Adding Rational Expressions with Like Denominators

If  $a$ ,  $b$ , and  $c$  represent polynomials and  $c \neq 0$ , then  $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ .

#### EXAMPLE 1 Adding Rational Expressions with Like Denominators

Add. Simplify your answer.

$$\begin{aligned} \text{A} \quad \frac{3b}{b^2} + \frac{5b}{b^2} \\ \frac{3b + 5b}{b^2} &= \frac{8b}{b^2} \\ &= \frac{8}{b} \end{aligned}$$

Combine like terms in the numerator. Divide out common factors. Simplify.

$$\begin{aligned} \text{B} \quad \frac{x^2 - 8x}{x - 4} + \frac{2x + 8}{x - 4} \\ \frac{x^2 - 8x + 2x + 8}{x - 4} &= \frac{x^2 - 6x + 8}{x - 4} \\ &= \frac{(x - 2)(\cancel{x - 4})}{\cancel{x - 4}} \\ &= x - 2 \end{aligned}$$

Combine like terms in the numerator. Factor. Divide out common factors. Simplify.

$$\begin{aligned} \text{C} \quad \frac{2m + 4}{m^2 - 9} + \frac{2}{m^2 - 9} \\ \frac{2m + 4 + 2}{m^2 - 9} &= \frac{2m + 6}{m^2 - 9} \\ &= \frac{2(m + 3)}{(m - 3)(m + 3)} \\ &= \frac{2}{m - 3} \end{aligned}$$

Combine like terms in the numerator. Factor. Divide out common factors. Simplify.



Add. Simplify your answer.

1a.  $\frac{n}{2n} + \frac{3n}{2n}$

1b.  $\frac{3y^2}{y+1} + \frac{3y}{y+1}$

## EXAMPLE 2 Subtracting Rational Expressions with Like Denominators

Subtract. Simplify your answer.

$$\frac{3m-6}{m^2+m-6} - \frac{-m+2}{m^2+m-6}$$

$$\frac{3m-6 - (-m+2)}{m^2+m-6} = \frac{3m-6+m-2}{m^2+m-6} \quad \text{Subtract numerators.}$$

$$= \frac{4m-8}{m^2+m-6} \quad \text{Combine like terms.}$$

$$= \frac{4(m-2)}{(m+3)(m-2)} \quad \text{Factor. Divide out common factors.}$$

$$= \frac{4}{m+3} \quad \text{Simplify.}$$

### Caution!

Make sure you add the opposite of each term in the numerator of the second expression when subtracting rational expressions.



Subtract. Simplify your answer.

2a.  $\frac{5a+2}{a^2-4} - \frac{2a-4}{a^2-4}$       2b.  $\frac{2b+14}{b^2+3b-4} - \frac{-2b+2}{b^2+3b-4}$

As with fractions, rational expressions must have a common denominator before they can be added or subtracted. If they do not have a common denominator, you can use any common multiple of the denominators to find one. You can also use the least common multiple (LCM) of the denominators.

To find the LCM of two expressions, write the prime factorization of both expressions. Line up the factors as shown. To find the LCM, multiply one number from each column.

$$\begin{array}{l} 6x^2 = 2 \cdot 3 \cdot x \cdot x \\ 8x = 2 \cdot 2 \cdot 2 \cdot x \\ \text{LCM} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x = 24x^2 \end{array} \quad \begin{array}{l} 5x + 15 = 5(x + 3) \\ x^2 - 9 = (x + 3)(x - 3) \\ \text{LCM} = 5(x + 3)(x - 3) \end{array}$$

## EXAMPLE 3 Identifying the Least Common Multiple

Find the LCM of the given expressions.

**A**  $24a^3, 4a$  *Write the prime factorization of each expression. Align common factors.*

$$24a^3 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot a$$

$$4a = 2 \cdot 2 \cdot a$$

$$\text{LCM} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot a = 24a^3$$

**B**  $2d^2 + 10d + 12, d^2 + 7d + 12$

$$2d^2 + 10d + 12 = 2(d^2 + 5d + 6) \quad \text{Factor each expression.}$$

$$= 2(d+3)(d+2) \quad \text{Align common factors.}$$

$$d^2 + 7d + 12 = (d+3)(d+4)$$

$$\text{LCM} = 2(d+3)(d+2)(d+4)$$



Find the LCM of the given expressions.

3a.  $5f^2h, 15fh^2$       3b.  $x^2 - 4x - 12, (x-6)(x+5)$

The LCM of the denominators of rational expressions is also called the least common denominator, or LCD, of the rational expressions. You can use the LCD to add or subtract rational expressions.

| Adding or Subtracting Rational Expressions |   |
|--|---|
| <b>Step 1</b>                              | Identify a common denominator.  |
| <b>Step 2</b>                              | Multiply each expression by an appropriate form of 1 so that each term has the common denominator as its denominator. |
| <b>Step 3</b>                              | Write each expression using the common denominator.   |
| <b>Step 4</b>                              | Add or subtract the numerators, combining like terms as needed.   |
| <b>Step 5</b>                              | Factor as needed.   |
| <b>Step 6</b>                              | Simplify as needed.   |

#### EXAMPLE 4 Adding and Subtracting with Unlike Denominators

Add or subtract. Simplify your answer.

**A**  $\frac{3x}{6x^2} + \frac{2x}{4x}$

$$6x^2 = 2 \cdot 3 \cdot x \cdot x$$

**Step 1**  $4x = 2 \cdot 2 \cdot x$  Identify the LCD.

$$\text{LCD} = 2 \cdot 2 \cdot 3 \cdot x \cdot x = 12x^2$$

**Step 2**  $\frac{3x}{6x^2} \left(\frac{2}{2}\right) + \frac{2x}{4x} \left(\frac{3x}{3x}\right)$  Multiply each expression by an appropriate form of 1.

**Step 3**  $\frac{6x}{12x^2} + \frac{6x^2}{12x^2}$  Write each expression using the LCD.

**Step 4**  $\frac{6x + 6x^2}{12x^2}$  Add the numerators.

**Step 5**  $\frac{\cancel{6}x(1+x)}{\cancel{6} \cdot 2x^{\cancel{2}x}}$  Factor and divide out common factors.

**Step 6**  $\frac{1+x}{2x}$  Simplify.

**B**  $\frac{1}{m-3} - \frac{5}{3-m}$

**Step 1** The denominators are **opposite binomials**.

The LCD can be either  $m-3$  or  $3-m$ . Identify the LCD.

**Step 2**  $\frac{1}{m-3} - \frac{5}{3-m} \left(\frac{-1}{-1}\right)$  Multiply the second expression by  $\frac{-1}{-1}$  to get an LCD of  $m-3$ .

**Step 3**  $\frac{1}{m-3} - \frac{-5}{m-3}$  Write each expression using the LCD.

**Step 4**  $\frac{1 - (-5)}{m-3}$  Subtract the numerators.

**Steps 5, 6**  $\frac{6}{m-3}$  No factoring is needed, so just simplify.

#### Remember!

Expressions like  $m-3$  and  $3-m$  are opposite binomials.  
 $3-m = -1(m-3)$   
 and  
 $m-3 = -1(3-m)$



Add or subtract. Simplify your answer.

4a.  $\frac{4}{3d} - \frac{2d}{2d^3}$

4b.  $\frac{a^2 + 4a}{a^2 + 2a - 8} + \frac{8}{a - 2}$



## EXAMPLE 5 Recreation Application

Katy wants to find out how long it will take to kayak 1 mile up a river and return to her starting point. Katy's average paddling rate is 4 times the speed of the river's current.

- a. Write and simplify an expression for the time it will take Katy to kayak the round-trip in terms of the rate of the river's current.

**Step 1** Write expressions for the distances and rates in the problem.

The **distance** in both directions is **1 mile**.

Let  $x$  represent the rate of the current, and let  $4x$  represent Katy's paddling rate.

Katy's **rate against the current** is  $4x - x$ , or  $3x$ .

Katy's **rate with the current** is  $4x + x$ , or  $5x$ .

**Step 2** Use a table to write expressions for time.

| Direction                  | Distance (mi) | Rate (mi/h) | Time (h) = $\frac{\text{distance}}{\text{rate}}$ |
|----------------------------|---------------|-------------|--|
| Upstream (against current) | 1             | $3x$        | $\frac{1}{3x}$                                   |
| Downstream (with current)  | 1             | $5x$        | $\frac{1}{5x}$                                   |

**Step 3** Write and simplify an expression for the total time.

total time = time upstream + time downstream

$$\begin{aligned} \text{total time} &= \frac{1}{3x} + \frac{1}{5x} \\ &= \frac{1}{3x} \left( \frac{5}{5} \right) + \frac{1}{5x} \left( \frac{3}{3} \right) \\ &= \frac{5}{15x} + \frac{3}{15x} \\ &= \frac{8}{15x} \end{aligned}$$

*Substitute known values.*

*Multiply each fraction by an appropriate form of 1.*

*Write each expression using the LCD,  $15x$ .*

*Add the numerators.*

- b. The rate of the river is 2 miles per hour. How long will it take Katy to kayak round trip?

$$\frac{8}{15(2)} = \frac{4}{15}$$

*Substitute 2 for  $x$ . Simplify.*

It will take Katy  $\frac{4}{15}$  of an hour, or 16 minutes, to kayak the round-trip.

### Helpful Hint

To write expressions for time in terms of distance and rate, solve  $d = rt$  for  $t$ .

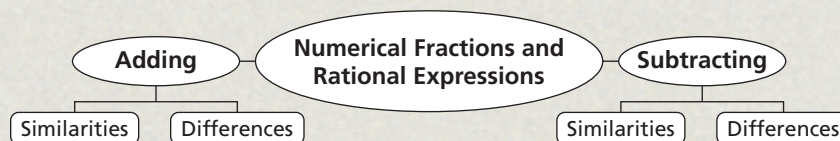
$$t = \frac{d}{r}$$



5. **What if?...** Katy's average paddling rate increases to 5 times the speed of the current. Now how long will it take Katy to kayak the round trip?

## THINK AND DISCUSS

- Explain how to find the least common denominator of rational expressions.
- GET ORGANIZED** Copy and complete the graphic organizer. In each box, compare and contrast operations with fractions and rational expressions.





## GUIDED PRACTICE

SEE EXAMPLE 1 Add. Simplify your answer.

p. 659

1.  $\frac{y}{3y^2} + \frac{5y}{3y^2}$

2.  $\frac{4m+30}{m+5} + \frac{m^2+8m+5}{m+5}$

3.  $\frac{x}{x^2-16} + \frac{4}{x^2-16}$

SEE EXAMPLE 2 Subtract. Simplify your answer.

p. 660

4.  $\frac{7}{2x^3} - \frac{3}{2x^3}$

5.  $\frac{7a-2}{a^2+3a+2} - \frac{5a-6}{a^2+3a+2}$

6.  $\frac{3x^2+1}{2x+2} - \frac{2x^2-2x}{2x+2}$

SEE EXAMPLE 3 Find the LCM of the given expressions.

p. 660

7.  $3xy^2, 6x^3yz$

8.  $x^2+9x+20, (x+5)(x-4)$

9.  $y^2-16, (y+9)(y-4)$

SEE EXAMPLE 4 Add or subtract. Simplify your answer.

p. 661

10.  $\frac{3}{c} - \frac{4}{3c}$

11.  $\frac{x^2+x}{x^2+3x+2} + \frac{3}{x+2}$

12.  $\frac{2x}{x-5} + \frac{x}{5-x}$

SEE EXAMPLE 5 13. **Travel** The Escobar family went on a car trip. They drove 100 miles on country roads and 240 miles on the highway. They drove 50% faster on the highway than on the country roads. Let  $r$  represent their rate on country roads in miles per hour.

p. 662

- Write and simplify an expression that represents the number of hours it took the Escobar family to complete their trip in terms of  $r$ . (*Hint*: 50% faster means 150% of the original rate.)
- Find their total travel time if they drove the posted speed limit.



## PRACTICE AND PROBLEM SOLVING

## Independent Practice

| For Exercises | See Example |
|---------------|-------------|
| 14–16         | 1           |
| 17–19         | 2           |
| 20–25         | 3           |
| 26–31         | 4           |
| 32            | 5           |

Add. Simplify your answer.

14.  $\frac{4y}{y^3} + \frac{4y}{y^3}$

15.  $\frac{a^2-3}{a+3} + \frac{2a}{a+3}$

16.  $\frac{4x-13}{x^2-5x+6} + \frac{1}{x^2-5x+6}$

Subtract. Simplify your answer.

17.  $\frac{m^2}{m-6} - \frac{6m}{m-6}$

18.  $\frac{c+3}{4c^2-25} - \frac{-c+8}{4c^2-25}$

19.  $\frac{-2a^2-9a}{a-2} - \frac{-5a^2-4a+2}{a-2}$

Find the LCM of the given expressions.

20.  $4jk^4m, 25jm$

21.  $12a^2+4a, 27a+9$

22.  $p^2-3p, pqr^2$

23.  $5xy^2z, 10y^3$

24.  $5x^2, 7x-14$

25.  $y^2+7y+10, y^2+9y+20$

Add or subtract. Simplify your answer.

26.  $\frac{2x}{5x} + \frac{10x}{3x^2}$

27.  $\frac{y^2-y}{y^2-4y+3} - \frac{2y-2}{3y-9}$

28.  $\frac{-3t}{t-4} - \frac{2t+4}{4-t}$

29.  $\frac{z}{3z^2} + \frac{4}{7z}$

30.  $\frac{5x}{2x-6} + \frac{x+2}{3-x}$

31.  $\frac{3m}{4m-8} - \frac{m^2}{m^2-4m+4}$

## Extra Practice

Skills Practice p. EP21

Application Practice p. EP33



## Travel



The first transcontinental railroad was completed in Utah on May 10, 1869. The occasion was commemorated with a golden spike that connected the eastern and western tracks.

32. **Fitness** Ira walks one mile from his house to the recreation center. After playing basketball, he walks home at only 85% of his normal walking speed. Let  $w$  be Ira's normal rate of walking.
- Write an expression to represent Ira's round-trip walking time.
  - If Ira's normal rate of walking is 3 miles per hour, how long did it take for him to complete his walking?
33. **Travel** A train travels 500 miles across the Midwest—50 miles through cities and 450 miles through open country. As it passes through cities, it slows to one-fifth the speed it travels through open territory. Let  $r$  represent the rate in open territory in miles per hour.
- Write and simplify an expression that represents the number of hours it takes the train to travel 500 miles in terms of  $r$ .
  - Find the total travel time if the train's rate through open territory is 50 miles per hour.
  - Critical Thinking** If you knew the time it took the train to make the round-trip, how could you find its average rate?

Add or subtract. Simplify your answer.

34.  $\frac{10}{5+y} + \frac{2y}{5+y}$

35.  $\frac{7}{49-c^2} - \frac{c}{49-c^2}$

36.  $\frac{6a}{a-12} + \frac{4}{12-a}$

37.  $\frac{b}{2b^3} + \frac{3}{3b^2}$

38.  $\frac{r^2+2r}{r+3} - \frac{2r+9}{r+3}$

39.  $\frac{x^2-2x}{3x-15} - \frac{8x-25}{3x-15}$

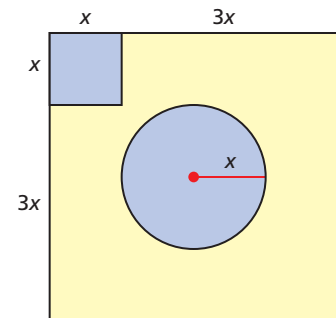
40.  $\frac{2y}{8y^2} + \frac{9}{4y^3}$

41.  $\frac{2}{x+2} + \frac{6}{x+4}$

42.  $\frac{2y}{3y-9} - \frac{y+1}{y^2-9}$

43. **/// ERROR ANALYSIS ///** Two students were asked to find the excluded values of the expression  $\frac{p}{p^2-p-12} - \frac{4}{p^2-p-12}$ . Student A identified the excluded value as  $p = -3$ . Student B identified the excluded values as  $p = -3$  and  $p = 4$ . Who is incorrect? What is the error?

44. **Multi-Step** At the spring fair there is a square Velcro target as shown. A player tosses a ball, which will stick to the target in some random spot. If the ball sticks to a spot in either the small square or the circle, the player wins a prize. What is the probability that a player will win a prize, assuming the ball sticks somewhere on the target? Round your answer to the nearest hundredth.



45. **Critical Thinking** Write two expressions whose sum is  $\frac{x}{x+1}$ .



46. This problem will prepare you for the Concept Connection on page 684. Jonathan is studying light in his science class. He finds that a magnifying glass can be used to project upside-down images on a piece of paper. The equation  $\frac{1}{f} = \frac{1}{x} + \frac{1}{y}$  relates the focal length of the lens  $f$ , the distance of the object from the lens  $x$ , and the distance of the image from the lens  $y$ . The focal length of Jonathan's lens is 12 cm.
- Jonathan wants to write  $y$ , the distance of the image from the lens, as a function of  $x$ , the distance of the object from the lens. To begin, he rewrote the equation as  $\frac{1}{y} = \frac{1}{12} - \frac{1}{x}$ . Explain how he did this.
  - Explain how Jonathan simplified the equation in part a to  $\frac{1}{y} = \frac{x-12}{12x}$ .

47. **Critical Thinking** Identify three common denominators that could be used to add  $\frac{3}{2x^2}$  to  $\frac{3}{4x}$ .



48. **Write About It** Explain how to find the least common denominator of two rational expressions when the denominators are opposite binomials.

**Multiple Choice** For Exercises 49–51, choose the best answer.

49. What is the LCD of  $\frac{6}{3p+3}$  and  $\frac{4}{p+1}$ ?  
 (A)  $p+1$       (B) 12      (C)  $3p+1$       (D)  $3p+3$
50. Simplify  $\frac{4}{2x} - \frac{1}{x}$ .  
 (A)  $\frac{1}{x}$       (B)  $\frac{3}{x}$       (C)  $\frac{5}{x}$       (D)  $\frac{3}{2x}$
51. Which of the following is equivalent to  $\frac{2x}{x-2}$ ?  
 (A)  $\frac{x}{x+2} + \frac{x}{x-2}$       (C)  $\frac{x^2+4x}{x^2-4} + \frac{x}{x+2}$   
 (B)  $\frac{2x}{x^2-4} + \frac{4}{x-2}$       (D)  $\frac{x}{x+2} + \frac{x^2+6x}{x^2-4}$
52. **Extended Response** Andrea biked 3 miles to the post office and 5 miles to the library. The rate at which she biked to the library was three times faster than her rate to the post office  $r$ .
- Write an expression that represents Andrea's total biking time in hours. Explain what each part of your expression means in the situation.
  - Simplify the expression.
  - How long did it take Andrea to bike the 8 miles if her biking rate to the post office was 3 miles per hour?

## CHALLENGE AND EXTEND

Add or subtract and simplify. Find the excluded values.

53.  $\frac{3}{x+y} - \frac{2x+y}{x^2-y^2}$       54.  $\frac{3}{2m} + \frac{4}{m^2} + \frac{2}{5m}$       55.  $\frac{a}{xy} + \frac{b}{xz} + \frac{c}{yz}$
56. Simplify the complex fraction  $\frac{\frac{1}{x} - \frac{1}{y}}{\frac{x}{xy} - \frac{1}{x}}$ . (*Hint: Simplify the numerator and denominator of the complex fraction first.*)



## SPIRAL STANDARDS REVIEW

11.0, 12.0, 14.0

Factor each polynomial. Check your answer. (*Lesson 8-2*)

57.  $2x^2 - 5 - 13x + 4x^2$       58.  $6a + 2a^3 - 2 - 10a^2$       59.  $15h + 5h^2 - 20h - 60$   
 60.  $12s^3 + 8s^2$       61.  $56t^3 - 14t^2 - 42t$       62.  $-10 + 2m^2 + m^3 - 5m$

Solve each quadratic equation by factoring. Check your answer. (*Lesson 9-5*)

63.  $d^2 - 4d - 12 = 0$       64.  $2g^2 - 9g = -4$       65.  $9x^2 + 6x + 1 = 0$

Simplify each rational expression, if possible. Identify any excluded values. (*Lesson 10-3*)

66.  $\frac{2t^2 - 8}{t^2 - 4}$       67.  $\frac{n^2 + 5n}{n^2 + 3n - 10}$       68.  $\frac{4 - x}{x^2 - 16}$



# Model Polynomial Division

Some polynomial divisions can be modeled by algebra tiles. If a polynomial can be modeled by a rectangle, then its factors are represented by the length and width of the rectangle. If one factor is a divisor, then the other factor is a quotient.

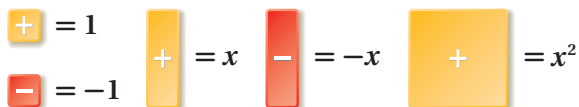


## California Standards

**10.0 Students** add, subtract, multiply, and **divide** monomials and **polynomials**. Students solve multistep problems, including word problems, by using these techniques.

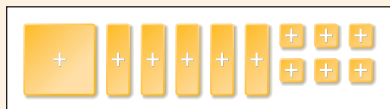
Use with Lesson 10-6

### KEY

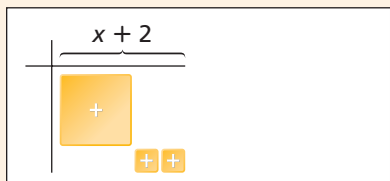


### Activity 1

Use algebra tiles to find the quotient  $(x^2 + 5x + 6) \div (x + 2)$ .

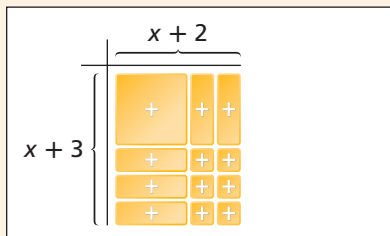


Model  $x^2 + 5x + 6$ .



Try to form a rectangle with a length of  $x + 2$ .

Place the  $x^2$ -tile in the upper-left corner. Then place two unit tiles in a row at the lower-right corner.



Try to use all the remaining tiles to complete a rectangle.

If you can complete a rectangle, then the width of the rectangle is the quotient.

The rectangle has length  $x + 2$  and width  $x + 3$ . So,  $(x^2 + 5x + 6) \div (x + 2) = x + 3$ .

You can check your answer by multiplying.

$$\begin{aligned} &(x + 3)(x + 2) \\ &x^2 + 2x + 3x + 6 \\ &x^2 + 5x + 6 \quad \checkmark \end{aligned}$$

Use the FOIL method.

### Try This

Use algebra tiles to find each quotient.

- $(x^2 + 5x + 4) \div (x + 1)$
- $(x^2 + 7x + 10) \div (x + 5)$
- $(x^2 + 4x - 5) \div (x - 1)$
- $(2x^2 + 5x + 2) \div (x + 2)$
- $(x^2 - 6x + 8) \div (x - 2)$
- $(2x^2 - x - 3) \div (x + 1)$
- Describe what happens when you try to model  $(x^2 - 4x + 3) \div (x + 1)$ .

# 10-6

## Dividing Polynomials



### California Standards

**10.0** Students add, subtract, multiply, and **divide** monomials and **polynomials**. Students solve multistep problems, including word problems, by using these techniques.

**12.0** Students **simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.**

### Why learn this?

Division of polynomials can be used to compare the energy produced by solar panels.

The electrical power (in watts) produced by a solar panel is directly proportional to the surface area of the solar panel. Division of polynomials can be used to compare energy production by solar panels of different sizes.

To divide a polynomial by a monomial, you can first write the division as a rational expression. Then divide each term in the polynomial by the monomial.



### EXAMPLE 1 Dividing a Polynomial by a Monomial

Divide  $(6x^3 + 8x^2 - 4x) \div 2x$ .

$$\frac{6x^3 + 8x^2 - 4x}{2x}$$

Write as a rational expression.

$$\frac{6x^3}{2x} + \frac{8x^2}{2x} - \frac{4x}{2x}$$

Divide each term in the polynomial by the monomial  $2x$ .

$$\frac{6^3 x^{3-1}}{2x} + \frac{8^4 x^{2-1}}{2x} - \frac{4^2 x}{2x}$$

Divide out common factors in each term.

$$3x^2 + 4x - 2$$

Simplify.



Divide. Check your answer.

1a.  $(8p^3 - 4p^2 + 12p) \div (-4p^2)$

1b.  $(6x^3 + 2x - 15) \div 6x$

Division of a polynomial by a binomial is similar to division of whole numbers.



### Dividing Polynomials

|        | WORDS  | NUMBERS                                  | POLYNOMIALS   |
|--------|--|--|---|
| Step 1 | Factor the numerator and/or denominator if possible. | $\frac{168}{3} = \frac{56 \cdot 3}{3}$   | $\frac{r^2 + 3r + 2}{r + 2} = \frac{(r + 2)(r + 1)}{(r + 2)}$ |
| Step 2 | Divide out any common factors.                       | $\frac{56 \cdot \cancel{3}}{\cancel{3}}$ | $\frac{(r + 2)(r + 1)}{(r + 2)}$                              |
| Step 3 | Simplify.  | 56                                       | $r + 1$   |

## EXAMPLE 2 Dividing a Polynomial by a Binomial

### Helpful Hint

Put each term of the numerator over the denominator only when the denominator is a monomial. If the denominator is a polynomial, try to factor first.

Divide.

$$\text{A } \frac{c^2 + 4c - 5}{c - 1} = \frac{(c + 5)(c - 1)}{c - 1}$$

*Factor the numerator.*

$$\frac{(c + 5)\cancel{(c - 1)}}{\cancel{(c - 1)}}$$

*Divide out common factors.*

$$\frac{c + 5}{1} = c + 5$$

*Simplify.*

$$\text{B } \frac{3x^2 - 10x - 8}{4 - x} = \frac{(3x + 2)(x - 4)}{4 - x}$$

*Factor the numerator.*

$$\frac{(3x + 2)(x - 4)}{-1(x - 4)}$$

*Factor one opposite binomial.*

$$\frac{(3x + 2)\cancel{(x - 4)}}{-1\cancel{(x - 4)}}$$

*Divide out common factors.*

$$-3x - 2$$

*Simplify.*



Divide. Check your answer.

2a.  $\frac{10 + 7k + k^2}{k + 2}$

2b.  $\frac{b^2 - 49}{b + 7}$

2c.  $\frac{s^2 + 12s + 36}{s + 6}$

Recall how you used long division to divide whole numbers as shown at right. You can also use long division to divide polynomials. An example is shown below.

$$\begin{array}{r} 15 \\ 23 \overline{)345} \\ \underline{-23} \phantom{0} \\ 115 \\ \underline{-115} \\ 0 \end{array}$$

$$(x^2 + 3x + 2) \div (x + 2)$$

$$\begin{array}{r} \phantom{x} + 1 \\ x + 2 \overline{)x^2 + 3x + 2} \\ \underline{x^2 + 2x} \phantom{0} \\ \phantom{x^2} + x + 2 \\ \underline{\phantom{x^2} + x + 2} \\ \phantom{x^2} \phantom{x} + 0 \end{array}$$

Divisor points to  $x + 2$  in the divisor.  
Quotient points to  $x + 1$ .  
Dividend points to  $x^2 + 3x + 2$ .

### Using Long Division to Divide a Polynomial by a Binomial

- |   |
|---|
| <b>Step 1</b> Write the binomial and polynomial in standard form.   |
| <b>Step 2</b> Divide the first term of the dividend by the first term of the divisor. This is the first term of the quotient.                 |
| <b>Step 3</b> Multiply this first term of the quotient by the binomial divisor and place the product under the dividend, aligning like terms. |
| <b>Step 4</b> Subtract the product from the dividend.   |
| <b>Step 5</b> Bring down the next term in the dividend.   |
| <b>Step 6</b> Repeat Steps 2–5 as necessary until you get 0 or until the degree of the remainder is less than the degree of the binomial.     |

### EXAMPLE 3 Polynomial Long Division

Divide using long division. Check your answer.

**A**  $(x^2 + 2 + 3x) \div (x + 2)$

**Step 1**  $x + 2 \overline{)x^2 + 3x + 2}$

Write in long division form with expressions in standard form.

**Step 2**  $x + 2 \overline{)x^2 + 3x + 2}$   
 $\phantom{x + 2} \phantom{)}x^2 + 2x$

Divide the first term of the dividend by the first term of the divisor to get the first term of the quotient.

**Step 3**  $x + 2 \overline{)x^2 + 3x + 2}$   
 $\phantom{x + 2} \phantom{)}x^2 + 2x$   
 $\phantom{x + 2} \phantom{)}\phantom{x^2} + x$

Multiply the first term of the quotient by the binomial divisor. Place the product under the dividend, aligning like terms.

**Step 4**  $x + 2 \overline{)x^2 + 3x + 2}$   
 $\phantom{x + 2} \phantom{)}x^2 + 2x$   
 $\phantom{x + 2} \phantom{)}\phantom{x^2} + x$   
 $\phantom{x + 2} \phantom{)}\phantom{x^2} + 0$

Subtract the product from the dividend.

**Step 5**  $x + 2 \overline{)x^2 + 3x + 2}$   
 $\phantom{x + 2} \phantom{)}x^2 + 2x$   
 $\phantom{x + 2} \phantom{)}\phantom{x^2} + x + 2$

Bring down the next term in the dividend.

**Step 6**  $x + 2 \overline{)x^2 + 3x + 2}$   
 $\phantom{x + 2} \phantom{)}x^2 + 2x$   
 $\phantom{x + 2} \phantom{)}\phantom{x^2} + x + 2$   
 $\phantom{x + 2} \phantom{)}\phantom{x^2} + x + 2$   
 $\phantom{x + 2} \phantom{)}\phantom{x^2} + 0$

Repeat Steps 2–5 as necessary. The remainder is 0.

**Check** Multiply the answer and the divisor.  $(x + 2)(x + 1)$   
 $x^2 + x + 2x + 2$   
 $x^2 + 3x + 2 \checkmark$

**B**  $\frac{x^2 + 4x + 3}{x + 1}$

$x + 1 \overline{)x^2 + 4x + 3}$

Write in long division form.

$x + 1 \overline{)x^2 + 4x + 3}$   
 $\phantom{x + 1} \phantom{)}x^2 + x$   
 $\phantom{x + 1} \phantom{)}\phantom{x^2} + 3x + 3$   
 $\phantom{x + 1} \phantom{)}\phantom{x^2} + 3x + 3$   
 $\phantom{x + 1} \phantom{)}\phantom{x^2} + 0$

$x^2 \div x = x$   
 Multiply  $x \cdot (x + 1)$ . Subtract.  
 Bring down the 3.  $3x \div x = 3$   
 Multiply  $3(x + 1)$ . Subtract.  
 The remainder is 0.

**Check** Multiply the answer and the divisor.  $(x + 1)(x + 3)$   
 $x^2 + 3x + 1x + 3$   
 $x^2 + 4x + 3 \checkmark$

#### Helpful Hint

When the remainder is 0, you can check your simplified answer by multiplying it by the divisor. You should get the numerator.



Divide using long division. Check your answer.

3a.  $(2y^2 - 5y - 3) \div (y - 3)$       3b.  $(a^2 - 8a + 12) \div (a - 6)$



Sometimes the divisor is not a factor of the dividend, so the remainder is not 0. Then the remainder can be written as a rational expression.

### EXAMPLE 4 Long Division with a Remainder

Divide  $(2x^2 + 3x - 6) \div (x - 2)$ .

$$\begin{array}{r}
 x - 2 \overline{)2x^2 + 3x - 6} \\
 \underline{2x - 4} \phantom{- 6} \\
 7x - 6 \\
 \underline{7x - 14} \\
 8
 \end{array}$$

*Write in long division form.*

$$\begin{array}{r}
 2x + 7 \\
 x - 2 \overline{)2x^2 + 3x - 6} \\
 \underline{-(2x^2 - 4x)} \phantom{- 6} \\
 7x - 6 \\
 \underline{-(7x - 14)} \\
 8
 \end{array}$$

$2x^2 \div x = 2x$

*Multiply  $2x(x - 2)$ . Subtract.*

*Bring down the  $-6$ .  $7x \div x = 7$*

*Multiply  $7(x - 2)$ . Subtract.*

*The remainder is 8.*

$$\frac{8}{x - 2}$$

*Write the remainder as a rational expression using the divisor as the denominator.*

$$2x + 7 + \frac{8}{x - 2}$$

*Write the quotient with the remainder.*



Divide.

4a.  $(3m^2 + 4m - 2) \div (m + 3)$     4b.  $(y^2 + 3y + 2) \div (y - 3)$

Sometimes you need to write a placeholder for a term using a zero coefficient. This is best seen if you write the polynomials in standard form.

### EXAMPLE 5 Dividing Polynomials That Have a Zero Coefficient

Divide  $(3x - 4x^3 - 15) \div (2x + 3)$ .

$$\begin{array}{r}
 (-4x^3 + 3x - 15) \div (2x + 3) \\
 2x + 3 \overline{)-4x^3 + 0x^2 + 3x - 15} \\
 \underline{-2x^2 + 3x - 3} \\
 2x + 3 \overline{)-4x^3 + 0x^2 + 3x - 15} \\
 \underline{-(-4x^3 - 6x^2)} \phantom{- 15} \\
 6x^2 + 3x \\
 \underline{-(6x^2 + 9x)} \phantom{- 15} \\
 -6x - 15 \\
 \underline{-(-6x - 9)} \\
 -6
 \end{array}$$

*Write the polynomials in standard form.*

*Write in long division form. Use  $0x^2$  as a placeholder for the  $x^2$  term.*

$-4x^3 \div 2x = -2x^2$

*Multiply  $-2x^2(2x + 3)$ . Subtract.*

*Bring down  $3x$ .  $6x^2 \div 2x = 3x$*

*Multiply  $3x(2x + 3)$ . Subtract.*

*Bring down  $-15$ .  $-6x \div 2x = -3$*

*Multiply  $-3(2x + 3)$ . Subtract.*

*The remainder is  $-6$ .*

$$(3x - 4x^3 - 15) \div (2x + 3) = -2x^2 + 3x - 3 + \frac{-6}{2x + 3}$$

#### Remember!

Recall from Chapter 7 that a polynomial in one variable is written in standard form when the degrees of the terms go from greatest to least.



Divide.

5a.  $(1 - 4x^2 + x^3) \div (x - 2)$

5b.  $(4p - 1 + 2p^3) \div (p + 1)$

## THINK AND DISCUSS

- When dividing a polynomial by a binomial, what does it mean when the remainder is 0?
- Suppose that the final answer to a polynomial division problem is  $x - 5 + \frac{3}{x+2}$ . Find an excluded value. Justify your answer.
- GET ORGANIZED** Copy and complete the graphic organizer. In each box, show an example.



## 10-6

## Exercises



California Standards Practice

10.0, 12.0, 25.2

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KEYWORD: MA8CA 10-6

Parent Resources Online

KEYWORD: MA8CA Parent

### GUIDED PRACTICE

SEE EXAMPLE 1

p. 667

Divide. Check your answer.

1.  $(4x^2 - x) \div 2x$

2.  $(16a^4 - 4a^3) \div 4a$

3.  $(21b^2 - 14b + 24) \div 3b$

4.  $(18r^2 - 12r + 6) \div -6r$

5.  $(6x^3 + 12x^2 + 9x) \div 3x^2$

6.  $(5m^4 + 15m^2 - 10) \div 5m^3$

SEE EXAMPLE 2

p. 668

7.  $\frac{2x^2 - x - 3}{x + 1}$

8.  $\frac{a^2 - a - 12}{a - 4}$

9.  $\frac{6y^2 + 11y - 10}{3y - 2}$

10.  $\frac{t^2 - 6t + 8}{t - 4}$

11.  $\frac{x^2 + 16x + 15}{x + 15}$

12.  $\frac{p^2 - p - 20}{p + 4}$

SEE EXAMPLE 3

p. 669

Divide using long division. Check your answer.

13.  $(c^2 + 7c + 12) \div (c + 4)$

14.  $(3s^2 - 12s - 15) \div (s - 5)$

15.  $\frac{x^2 + 5x - 14}{x + 7}$

16.  $\frac{x^2 + 4x - 12}{x - 2}$

SEE EXAMPLE 4

p. 670

17.  $(a^2 + 4a + 3) \div (a + 2)$

18.  $(2r^2 + 11r + 5) \div (r - 3)$

19.  $(n^2 + 8n + 15) \div (n + 4)$

20.  $(2t^2 - t + 4) \div (t - 1)$

21.  $(8n^2 - 6n - 7) \div (2n + 1)$

22.  $(b^2 - b + 1) \div (b + 2)$

SEE EXAMPLE 5

p. 670

23.  $(3x - 2x^3 - 10) \div (3 + x)$

24.  $(3p^3 - 2p^2 - 4) \div (p - 2)$

25.  $(m^2 + 2) \div (m - 1)$

26.  $(3x^2 + 4x^3 - 5) \div (5 + x)$

27.  $(4k^3 - 2k - 8) \div (k + 1)$

28.  $(j^3 + 6j + 2) \div (j + 4)$

### PRACTICE AND PROBLEM SOLVING

Divide. Check your answer.

29.  $(9t^3 + 12t^2 - 6t) \div 3t^2$

30.  $(5n^3 - 10n + 15) \div (-5n)$

31.  $(-16p^4 + 4p^3 + 8) \div 4p^3$

32.  $\frac{4r^2 - 9r + 2}{r - 2}$

33.  $\frac{8t^2 + 2t - 3}{2t - 1}$

34.  $\frac{3g^2 + 7g - 6}{g + 3}$

**Independent Practice**

| For Exercises | See Example |
|---------------|-------------|
| 29–31         | 1           |
| 32–34         | 2           |
| 35–38         | 3           |
| 39–41         | 4           |
| 42–44         | 5           |

**Extra Practice**

Skills Practice p. EP21

Application Practice p. EP33

Divide using long division. Check your answer.

35.  $(x^2 - 5x + 6) \div (x - 2)$       36.  $(2m^2 + 8m + 8) \div (m + 2)$   
 37.  $(6a^2 + 7a - 3) \div (2a + 3)$       38.  $(3x^2 - 10x - 8) \div (x - 4)$   
 39.  $(3x^2 - 2x + 6) \div (x - 2)$       40.  $(2m^2 + 5m + 8) \div (m + 1)$   
 41.  $(6x^2 - x - 3) \div (2x - 1)$       42.  $(2m^3 - 4m - 30) \div (2m - 10)$   
 43.  $(6t^3 + 21t + 9) \div (3t + 9)$       44.  $(p^4 - 7p^2 + p + 1) \div (p - 3)$

45. **Multi-Step** Find the value of  $n$  so that  $x - 4$  is a factor of  $x^2 + x + n$ .

**Geometry** The area of each of three rectangles is  $(2x^2 - 3x - 2)$  cm<sup>2</sup>. Below are the different widths of the rectangles. Find each corresponding length.

46.  $x - 2$       47.  $x + 1$       48.  $2x + 1$



49. **Graphing Calculator** Use the table of values for  $f(x) = \frac{(x^2 + 3x + 4)}{x - 5}$  to answer the following.

- a. Describe what is happening to the values of  $y$  as  $x$  increases from 2 to 4.  
 b. Describe what is happening to the values of  $y$  as  $x$  increases from 6 to 8.  
 c. Explain why there is no value in the  $y$  column when  $x$  is 5.

| X | Y1    |
|---|-------|
| 2 | 4.667 |
| 3 | 3.5   |
| 4 | 2.667 |
| 5 | ERR   |
| 6 | 2     |
| 7 | 1.429 |
| 8 | 1     |

50. **Estimation** Estimate the value of  $\frac{x^2 + 10x + 25}{x^2 - 25} \div \frac{x^4 - 4x^3 - 45x^2}{x^2 - 14x + 45}$  for  $x = 2.88$ .

51. **Solar Energy** The greater the area of a solar panel, the greater the number of watts of energy produced. The area of two solar panels  $A$  and  $B$ , in square meters, can be represented by  $A = m^2 + 3m + 2$  and  $B = 2m + 2$ . Divide the polynomials to find an expression that represents the ratio of the area of  $A$  to the area of  $B$ .

52. **ERROR ANALYSIS** Two students attempted to divide  $\frac{4x^2 - 6x + 12}{-2x}$ . Which is incorrect? Explain the error.

**A**

$$\begin{array}{r} 4x^2 - 6x + 12 \\ -2x \\ \hline 2 \cancel{4x^2}^1 - \cancel{6x}^3 + \frac{6 \cancel{12}}{-2x} \\ \hline -2x + 3 - \frac{6}{x} \end{array}$$

**B**

$$\begin{array}{r} 4x^2 - 6x + 12 \\ -2x \\ \hline 2 \cancel{4x^2}^1 - \cancel{6x}^3 + \frac{6 \cancel{12}}{-2x} \\ \hline -2x - 3 - \frac{6}{x} \end{array}$$

**California LINK**

**Solar Energy**



High school students can compete in the Dell-Winston School Solar Car Challenge, in which teams of participants design, build, and race cars powered by solar energy. The team above is from Walnut High School in Walnut, CA. The name of their car is "QuikSolar."

**CONCEPT CONNECTION**



53. This problem will prepare you for the Concept Connection on page 684. Jonathan continues to study lenses and uses the equation  $\frac{1}{y} = \frac{x - 12}{12x}$ .
- a. Jonathan wants to write  $y$ , the distance of the image from the lens, as a function of  $x$ , the distance of the object from the lens. What is the equation solved for  $y$ ?
- b. Use a graphing calculator to create a table of values for the function  $y(x)$ . For which value of  $x$  is the function undefined?



54. **Write About It** When dividing a polynomial by a binomial, what does it mean when there is a remainder?

55. **Critical Thinking** Divide  $2x + 3 \overline{)2x^2 + 7x + 6}$ . Find a value for each expression by substituting 10 for  $x$  in the original problem. Repeat the division. Compare the results of each division.



56. **Write About It** Is  $3x + 2$  a factor of  $3x^2 + 14x + 8$ ? Explain.

**Multiple Choice** For Exercises 57–60, choose the best answer.

57. Which expression has an excluded value of  $-\frac{1}{2}$ ?

- (A)  $\frac{4x^2 - 2x - 2}{4x - 2}$  (B)  $\frac{4x^2 - 2x - 2}{2x - 4}$  (C)  $\frac{4x^2 - 2x - 2}{4x + 2}$  (D)  $\frac{4x^2 - 2x - 2}{2x + 4}$

58. Find  $(x^2 - 1) \div (x + 2)$ .

- (A)  $x - 2 + \frac{-5}{x + 2}$  (B)  $x - 2 + \frac{3}{x + 2}$  (C)  $x + 2 + \frac{-5}{x - 2}$  (D)  $x + 2 + \frac{3}{x - 2}$

59. Which expression is a factor of  $x^2 - 4x - 5$ ?

- (A)  $x - 1$  (B)  $x + 1$  (C)  $x - 4$  (D)  $x + 5$

60. Which of the following expressions is equivalent to  $(x^3 + 2x^2 + 3x + 1) \div (x - 1)$ ?

- (A)  $x^2 + 3x + 6 + \frac{7}{x - 1}$  (C)  $x^2 + 3x + 6 + \frac{-5}{x - 1}$   
 (B)  $x^2 + x + 2 + \frac{-1}{x - 1}$  (D)  $x^2 + x + 2 + \frac{3}{x - 1}$

## CHALLENGE AND EXTEND

Divide. Simplify your answer.

61.  $(6x^3y - x^2 + 4xy^2) \div (2x^2y)$

62.  $(x^3 - 1) \div (x - 1)$

63.  $(x^3 + 2x^2 - x - 2) \div (x^2 - 1)$

64.  $(x^3 + 8) \div (x + 2)$



65. **Geometry** The base of a triangle is  $(2x + 4)$  m and the area is  $(2x^2 + 5x + 2)$  m<sup>2</sup>. How much longer is the base than the height?



66. **Geometry** The formula for finding the volume of a cylinder is  $V = BH$ , where  $B$  is the area of the base of the cylinder and  $H$  is the height.

a. Find the height of the cylinder given that  $V = \pi(x^3 + 4x^2 + 5x + 2)$  and  $B = \pi(x^2 + 2x + 1)$ .

b. Find an expression for the radius of the base.



## SPIRAL STANDARDS REVIEW

3.0, 5.0, 13.0

67. Billy and James are collecting guitar picks. James has 38 picks and adds 2 to his collection every week. Billy starts with 2 picks and collects 14 picks a week. Write and solve an equation to find the number of weeks it will take Billy to accumulate the same number of picks as James. (Lesson 2-4)

68. Jane is shopping for a new phone. She wants to spend \$130 on the phone, but she is willing to pay within \$25 of her ideal price. Write and solve an absolute-value equation to find the maximum and minimum prices Jane is willing to pay. (Lesson 2-7)

Multiply. Simplify your answer. (Lesson 10-4)

69.  $(x^2 + 4x + 3) \cdot \frac{8}{2(x + 3)}$

70.  $\frac{9xy^2}{2x^3} \cdot \frac{8y}{3x^4}$

71.  $\frac{2k^2 + 4k^3}{k + 1} \cdot \frac{k^2 + 3k + 2}{2k^2}$

# 10-7

# Solving Rational Equations



California Standards

**Preparation for 15.0**  
Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

### Who uses this?

Athletes can use rational equations to determine how to improve their statistics. (See Exercise 44.)

A **rational equation** is an equation that contains one or more rational expressions. If a rational equation is a proportion, it can be solved using the Cross Product Property.

### EXAMPLE 1 Solving Rational Equations by Using Cross Products

#### Vocabulary

rational equation  
extraneous solution

Solve  $\frac{3}{t-3} = \frac{2}{t}$ . Check your answer.

$$\frac{3}{t-3} = \frac{2}{t} \quad \text{Use cross products.}$$

$$3t = (t-3)(2)$$

$$3t = 2t - 6 \quad \text{Distribute 2 on the right side.}$$

$$t = -6 \quad \text{Subtract } 2t \text{ from both sides.}$$

Check  $\frac{3}{t-3} = \frac{2}{t}$  *Substitute -6 for t in the original equation.*

|                  |                  |
|------------------|------------------|
| $\frac{3}{-6-3}$ | $\frac{2}{-6}$   |
| $\frac{3}{-9}$   | $\frac{2}{-6}$   |
| $-\frac{1}{3}$   | $-\frac{1}{3}$ ✓ |



Solve. Check your answer.

1a.  $\frac{1}{n} = \frac{3}{n+4}$

1b.  $\frac{4}{h+1} = \frac{2}{h}$

1c.  $\frac{21}{x-7} = \frac{3}{x}$

Some rational equations contain sums or differences of rational expressions. To solve these, you must find a common denominator for all the rational expressions in the equation.

### EXAMPLE 2 Solving Rational Equations by Using the LCD

Solve  $\frac{1}{c} + \frac{3}{2c} = \frac{2}{c+1}$ . Check your answer.

Step 1 Find the LCD.

$$2c(c+1) \quad \text{Include every factor of the denominators.}$$

Step 2 Multiply both sides by the LCD.

$$2c(c+1)\left(\frac{1}{c} + \frac{3}{2c}\right) = 2c(c+1)\left(\frac{2}{c+1}\right)$$

$$2c(c+1)\left(\frac{1}{c}\right) + 2c(c+1)\left(\frac{3}{2c}\right) = 2c(c+1)\left(\frac{2}{c+1}\right)$$

*Distribute on the left side.*

**Step 3** Simplify and solve.

$$2\cancel{c}(c+1)\left(\frac{1}{\cancel{c}}\right) + 2\cancel{c}(c+1)\left(\frac{3}{2\cancel{c}}\right) = 2c\cancel{(c+1)}\left(\frac{2}{\cancel{c+1}}\right) \quad \text{Divide out common factors in each term.}$$

$$2(c+1) + (c+1)3 = (2c)2 \quad \text{Simplify.}$$

$$2c + 2 + 3c + 3 = 4c \quad \text{Distribute and multiply.}$$

$$5c + 5 = 4c \quad \text{Combine like terms.}$$

$$c + 5 = 0 \quad \text{Subtract } 4c \text{ from both sides.}$$

$$c = -5 \quad \text{Subtract 5 from both sides.}$$

**Check**

$$\frac{1}{c} + \frac{3}{2c} = \frac{2}{c+1}$$

|  |                           |
|--|---------------------------|
| $\frac{1}{-5} + \frac{3}{2(-5)}$         | $\frac{2}{-5+1}$          |
| $\frac{2}{-10} + \frac{3}{-10}$          | $\frac{2}{-4}$            |
| $-\frac{5}{10} \text{ or } -\frac{1}{2}$ | $-\frac{1}{2} \checkmark$ |



Solve each equation. Check your answer.

2a.  $\frac{2}{a+1} + \frac{1}{a+1} = \frac{4}{a}$

2b.  $\frac{6}{j+2} - \frac{10}{j} = \frac{4}{2j}$

When you multiply each side of an equation by the LCD, you may get an *extraneous solution*. An **extraneous solution** is a solution to a resulting equation that is not a solution to the original equation. Because of extraneous solutions, it is especially important to check your answers.

### EXAMPLE 3 Extraneous Solutions

Solve  $\frac{x-9}{x^2-9} = \frac{-3}{x-3}$ . Check your answer.

$$(x-9)(x-3) = -3(x^2-9) \quad \text{Use cross products.}$$

$$x^2 - 12x + 27 = -3x^2 + 27 \quad \text{Multiply the left side. Distribute } -3 \text{ on the right side.}$$

$$4x^2 - 12x + 27 = 27 \quad \text{Add } 3x^2 \text{ to both sides.}$$

$$4x^2 - 12x = 0 \quad \text{Subtract 27 from both sides.}$$

$$4x(x-3) = 0 \quad \text{Factor the quadratic expression.}$$

$$4x = 0 \text{ or } x-3 = 0 \quad \text{Use the Zero Product Property.}$$

$$x = 0 \text{ or } x = 3 \quad \text{Solve for } x.$$

**Check**

$$\frac{x-9}{x^2-9} = \frac{-3}{x-3} \quad \frac{x-9}{x^2-9} = \frac{-3}{x-3}$$

|                     |                  |
|---------------------|------------------|
| $\frac{0-9}{0^2-9}$ | $\frac{-3}{0-3}$ |
| $\frac{-9}{-9}$     | $\frac{-3}{-3}$  |
| 1                   | 1 $\checkmark$   |

|                     |                  |
|---------------------|------------------|
| $\frac{3-9}{3^2-9}$ | $\frac{-3}{3-3}$ |
| $\frac{-6}{0}$      | $\frac{-3}{0}$   |

Because both  $\frac{-6}{0}$  and  $\frac{-3}{0}$  are undefined, 3 is not a solution.

3 is an extraneous solution. The only solution is 0.



Solve. Check your answer.

3a.  $\frac{3}{x-7} = \frac{x-2}{x-7}$

3b.  $\frac{x+1}{x-2} = \frac{4}{x-3}$

3c.  $\frac{9}{x^2+2x} = \frac{6}{x^2}$

## THINK AND DISCUSS

- Why is it important to check your answers to rational equations?
- For what values of  $x$  are the rational expressions in the equation  $\frac{x}{x-3} = \frac{2}{x+3}$  undefined?
- Explain why some rational equations, such as  $\frac{x}{x-4} = \frac{4}{x-4}$ , have no solutions.
- GET ORGANIZED** Copy and complete the graphic organizer. In each box, write the solution and check.



### Solving Rational Equations

Solve by using cross products.

$$\frac{3}{x} = \frac{2}{x+1}$$

Solve by using the LCD.

$$\frac{7}{x-1} - \frac{4}{x-1} = \frac{6}{x}$$

## 10-7

## Exercises



California Standards Practice

Preparation for **15.0;**  
**25.1**



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## GUIDED PRACTICE

**Vocabulary** Apply the vocabulary from this lesson to answer each question.

- A(n) \_\_\_\_\_? \_\_\_\_\_ contains one or more rational expressions. (*extraneous solution* or *rational equation*)
- A(n) \_\_\_\_\_? \_\_\_\_\_ is a solution to a resulting equation that is not a solution to the original equation. (*extraneous solution* or *rational equation*)

Solve. Check your answer.

SEE EXAMPLE 1

p. 674

3.  $\frac{3}{x+4} = \frac{2}{x}$

4.  $\frac{5}{s-6} = \frac{4}{s}$

5.  $\frac{20}{p+100} = \frac{-10}{2p}$

6.  $\frac{4}{j} = \frac{1}{j+2}$

7.  $\frac{3}{x-4} = \frac{9}{x-2}$

8.  $\frac{6}{2x-1} = 3$

SEE EXAMPLE 2

p. 674

9.  $\frac{6}{x} - \frac{5}{x} = \frac{1}{3}$

10.  $\frac{a}{9} + \frac{1}{3} = \frac{2}{5}$

11.  $\frac{3}{x+1} = \frac{2}{x} + \frac{3}{x}$

12.  $\frac{8}{d} = \frac{1}{d+2} - \frac{3}{d}$

13.  $\frac{3}{s-6} = \frac{4}{s} + \frac{1}{2s}$

14.  $\frac{7}{r} + \frac{2}{r-1} = \frac{-1}{2r}$

15.  $\frac{3}{a-4} = \frac{a}{a-2}$

16.  $\frac{r}{2} - \frac{2}{r} = \frac{5}{6}$

17.  $\frac{6}{n} = \frac{7}{n^2} - 1$

18.  $\frac{4}{x+1} = x - 2$

19.  $\frac{5}{a^2} = \frac{-4}{a} + 1$

20.  $\frac{1}{p} = \frac{-3}{p^2} + 2$

SEE EXAMPLE 3

p. 675

Solve. Check your answer.

21.  $\frac{3}{c-4} = \frac{c-1}{c-4}$

22.  $\frac{w+3}{w^2-1} - \frac{2w}{w-1} = 1$

23.  $\frac{3x-7}{x-5} + \frac{x}{2} = \frac{8}{x-5}$

## PRACTICE AND PROBLEM SOLVING

### Independent Practice

| For Exercises | See Example |
|---------------|-------------|
| 24–27         | 1           |
| 28–35         | 2           |
| 36–43         | 3           |

### Extra Practice

Skills Practice p. EP21

Application Practice p. EP33

Solve. Check your answer.

24.  $\frac{8}{x-2} = \frac{2}{x+1}$       25.  $\frac{12}{3n-1} = \frac{3}{n}$       26.  $\frac{x}{x+4} = \frac{x}{x-1}$       27.  $\frac{9}{x+5} = \frac{4}{x}$
28.  $\frac{6}{s} - \frac{2}{s} = 5$       29.  $\frac{1}{2x} + \frac{1}{4x} = \frac{7}{8x}$       30.  $\frac{7}{c} - \frac{2}{c} = \frac{4}{c-1}$       31.  $\frac{9}{m} - \frac{3}{2m} = \frac{15}{m}$
32.  $\frac{3}{x^2} = \frac{2}{x}$       33.  $\frac{r}{3} - 3 = -\frac{6}{r}$       34.  $\frac{6}{x^2} = \frac{1}{2} + \frac{1}{2x}$       35.  $\frac{8}{3x^2} = \frac{2}{x} - \frac{1}{3}$

Solve. Check your answer.

36.  $\frac{x+4}{x+1} = \frac{3}{x-3}$       37.  $\frac{x+1}{x-4} = \frac{3(x+1)}{x-4}$
38.  $\frac{5x}{x-3} = 8 + \frac{15}{x-3}$       39.  $\frac{3t}{t-3} = \frac{t+4}{t-3}$
40.  $\frac{2}{x} = \frac{x+1}{x^2-1}$       41.  $\frac{1}{x} = \frac{x-4}{x^2-16}$
42.  $\frac{x+2}{3} = \frac{x+7}{x+3}$       43.  $\frac{x+4}{2x-2} = \frac{x+3}{x-1}$

44. **Multi-Step** Clancy has been keeping his free throw statistics. Use his data to write the ratio of the number of free throws Clancy has made to the number of attempts.

| Clancy's Free Throws |      |
|----------------------|------|
| Attempts             | Made |
| 45                   | 39   |

- a. What percentage has he made?
- b. Write and solve an equation to find how many free throws  $f$  Clancy would have to make in a row to improve his free-throw percentage to 90%. (*Hint:* Clancy needs to make  $f$  more free throws in  $f$  more attempts.)
45. Karla and Andrew are sorting their book collections. Karla has 12 books and divides them evenly into stacks. Andrew has 18 books and evenly divides his books into 2 fewer stacks than Karla. He also has 6 more books in each stack than Karla. Copy and complete the table. Then find the number of stacks of books Karla has.

|                 | Karla | Andrew           |
|-----------------|-------|------------------|
| Books           |       | 18               |
| Stacks          | $x$   |                  |
| Books per Stack |       | $\frac{18}{x-2}$ |

### CONCEPT CONNECTION



46. This problem will prepare you for the Concept Connection on page 684. Blanca sets up a lens with a focal length  $f$  of 15 cm and places a candle 24 cm from the lens. She knows that  $\frac{1}{f} = \frac{1}{x} + \frac{1}{y}$  where  $x$  is the distance of the object from the lens and  $y$  is the distance of the image from the lens.
- a. Write the equation using the given values.
- b. For the values of  $f$  and  $x$  given above, how far will the image appear from the lens?
- c. How will distance between the image and the lens be affected if Blanca uses a lens with a focal length of 18 cm?



47. **Critical Thinking** Can you cross multiply to solve all rational equations? If so, explain. If not, how do you identify which ones can be solved using cross products?



48. **Write About It** Solve  $\frac{1}{x} + \frac{3}{x} = \frac{3}{x-1}$ . Explain each step and why you chose the method you used.

**Multiple Choice** For Exercises 49–51, choose the best answer.

49. Which value is an extraneous solution to  $\frac{x}{x+4} - \frac{4}{x-4} = \frac{x^2+16}{x^2-16}$ ?

- (A) -16      (B) -4      (C) 4      (D) 16

50. Which is a solution to  $\frac{x+2}{x-3} - \frac{1}{x} = \frac{3}{x^2-3x}$ ?

- (A) -1      (B) 0      (C) 1      (D) 3

51. What are the solutions of  $\frac{5}{x^2} = \frac{1}{3} + \frac{2}{3x}$ ?

- (A) -3 and 5      (B) -3 and 2      (C) 2 and 3      (D) 3 and -5

## CHALLENGE AND EXTEND



52. **Reasoning** Below is a solution to the rational equation  $\frac{3}{x} = \frac{6}{x+4}$ . Use an algebraic property to justify each step.

| Statements        | Reasons |
|-------------------|---------|
| a. $3(x+4) = 6x$  | _____   |
| b. $3x + 12 = 6x$ | _____   |
| c. $12 = 3x$      | _____   |
| d. $4 = x$        | _____   |

53. For what value of  $a$  will the equation  $\frac{x+4}{x-a} = \frac{7}{x-a}$  have no solution?

54. Jill has a 10-year-old sister and a sister who will be 12 next year. The equation  $\frac{10}{j} + \frac{12}{j+1} = 4$ , where  $j$  is Jill's age, describes the relationship between the ages of Jill and her sisters. Angela has one sister who is 16 this year and a sister who will be 18 next year. The equation  $\frac{16}{a} + \frac{18}{a+1} = 4$ , where  $a$  is Angela's age, describes the relationship between the ages of Angela and her sisters. What is the difference between Jill's and Angela's ages?



## SPIRAL STANDARDS REVIEW

8.0, 9.0

Identify which lines are parallel and which lines are perpendicular. (Lesson 5-8)

55.  $y = \frac{1}{3}x$ ;  $y = 3x + 1$ ;  $y = 3x - 1$

56.  $y = -2x$ ;  $y = 2x - 2$ ;  $y = \frac{1}{2}x + 4$

57.  $y = -x - 3$ ;  $y = x - 2$ ;  $y = x + 3$

58.  $y = -\frac{2}{3}x + 2$ ;  $y = \frac{3}{2}x + 3$ ;  $y = -\frac{3}{2}x - 1$

Solve each system by elimination. (Lesson 6-3)

59.  $\begin{cases} x - y = -3 \\ 2x + 2y = 22 \end{cases}$

60.  $\begin{cases} x - 2y = 18 \\ 2x + 3y = 15 \end{cases}$

61.  $\begin{cases} -3x - 2y = 27 \\ 4x - 3y = -2 \end{cases}$

62. Find the number of solutions of  $7x^2 + 5x - 13 = 0$  using the discriminant. (Lesson 9-9)

# 10-8

# Applying Rational Equations



California Standards

**15.0** Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

### Why learn this?

You can use rational equations to find out how long it takes two people working together to complete a job.



When two people team up to complete a job, each person completes a fraction of the whole job. You can use this idea to write and solve a rational equation to find how long it will take to complete a job.

### EXAMPLE 1 Gardening Application

Danielle can weed a garden in 2 hours. It takes Omar 3 hours to weed the same garden. How long will it take them to weed the garden if they work together?

Let  $h$  be the number of hours Danielle and Omar need to weed the garden. Danielle weeds the garden in 2 hours, so she weeds  $\frac{1}{2}$  of the garden per hour. Omar weeds the garden in 3 hours, so he weeds  $\frac{1}{3}$  of the garden per hour. The table shows the part of the garden that each person weeds in  $h$  hours.

| Danielle's Part | + | Omar's Part    | = | Whole Garden |
|-----------------|---|----------------|---|--------------|
| $\frac{1}{2}h$  | + | $\frac{1}{3}h$ | = | 1            |

$$\frac{1}{2}h + \frac{1}{3}h = 1$$

*Solve this equation for  $h$ .*

$$6\left(\frac{1}{2}h + \frac{1}{3}h\right) = 6(1)$$

*Multiply both sides by the LCD, 6.*

$$3h + 2h = 6$$

*Distribute 6 on the left side.*

$$5h = 6$$

*Combine like terms.*

$$h = \frac{6}{5} = 1\frac{1}{5}$$

*Divide by 5 on both sides.*

Working together, Danielle and Omar can weed the garden in  $1\frac{1}{5}$  hours, or 1 hour 12 minutes.

**Check** Danielle weeds  $\frac{1}{2}$  of the garden per hour, so in  $1\frac{1}{5}$  hours, she weeds  $\frac{1}{2} \cdot \frac{6}{5} = \frac{3}{5}$  of the garden. Omar weeds  $\frac{1}{3}$  of the garden per hour, so in  $1\frac{1}{5}$  hours, he weeds  $\frac{1}{3} \cdot \frac{6}{5} = \frac{2}{5}$  of the garden. Together, they weed  $\frac{3}{5} + \frac{2}{5} = 1$  garden.



1. Cindy mows a lawn in 50 minutes. It takes Sara 40 minutes to mow the same lawn. How long will it take them to mow the lawn if they work together?

**EXAMPLE 2** *Chemistry Application*

A chemist has 500 milliliters of a solution that is half alcohol. He needs a solution that is 60% alcohol. How many milliliters of alcohol should he add?

Let  $a$  be the number of milliliters of alcohol that the chemist should add.

The table shows the amount of alcohol and the total amount of the solution.

|          | Alcohol (mL) | Total (mL) |
|----------|--------------|------------|
| Original | 250          | 500        |
| New      | $250 + a$    | $500 + a$  |

The new solution is 60% alcohol, so  $\frac{250 + a}{500 + a} = 0.6$ . Solve for  $a$ .

$$250 + a = 0.6(500 + a) \quad \text{Multiply both sides by } 500 + a.$$

$$250 + a = 300 + 0.6a \quad \text{Distribute } 0.6 \text{ on the right side.}$$

$$0.4a = 50 \quad \text{Subtract } 250 \text{ from both sides and } 0.6a \text{ from both sides.}$$

$$a = 125 \quad \text{Divide both sides by } 0.4.$$

The chemist should add 125 mL of alcohol to the solution.



2. Suppose the chemist wants a solution that is 80% alcohol. How many milliliters of alcohol should he add?

**EXAMPLE 3** *Transportation Application*

A passenger train travels 20 mi/h faster than a freight train, and it takes the passenger train 2 hours less time to travel 240 miles. How long does the freight train take to make the trip?

Let  $t$  be the time it takes the freight train to travel 240 miles.

|                 | Distance (mi) | Time (h) | Rate (mi/h)         |
|-----------------|---------------|----------|---------------------|
| Freight Train   | 240           | $t$      | $\frac{240}{t}$     |
| Passenger Train | 240           | $t - 2$  | $\frac{240}{t - 2}$ |

The passenger train is 20 mi/h faster, so  $\frac{240}{t - 2} = \frac{240}{t} + 20$ .

$$t(t - 2) \frac{240}{t - 2} = t(t - 2) \left( \frac{240}{t} + 20 \right) \quad \text{Multiply both sides by the LCD.}$$

$$240t = (t - 2)240 + (t^2 - 2t)(20) \quad \text{Distribute } t(t - 2) \text{ on the right side. Simplify.}$$

$$240t = 240t - 480 + 20t^2 - 40t \quad \text{Distribute and multiply.}$$

$$0 = 20t^2 - 40t - 480 \quad \text{Subtract } 240t \text{ from both sides.}$$

$$0 = t^2 - 2t - 24 \quad \text{Divide both sides by } 20.$$

$$0 = (t - 6)(t + 4), \text{ so } t = -4, 6 \quad \text{Factor and solve. Time is nonnegative, so } -4 \text{ is extraneous.}$$

The freight train makes the trip in 6 hours.



3. Ryan drives 10 mi/h slower than Maya, and it takes Ryan 1 hour longer to travel 300 miles. How long does it take Maya to make the trip?

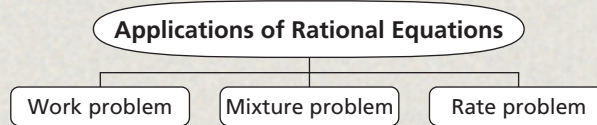
**Remember!**

distance = rate • time,  
so rate =  $\frac{\text{distance}}{\text{time}}$ .

## THINK AND DISCUSS



1. Explain why it makes sense that the answer to Example 1 is less than the time it takes Danielle or Omar to weed the garden when working alone.
2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of each type of application and its solution.



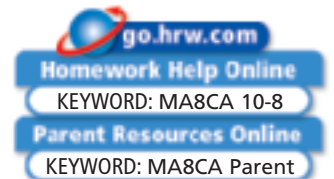
## 10-8

## Exercises



California Standards Practice

15.0, 25.2



### GUIDED PRACTICE

SEE EXAMPLE 1  
p. 679

1. Summer can paint a room in 3 hours. Louise can paint the room in 5 hours. How many hours will it take them to paint the room if they work together?

SEE EXAMPLE 2  
p. 680

2. **Cooking** A chef has 4 quarts of a soup that consists of equal parts chicken stock and vegetable stock. She wants to make a soup that is 75% chicken stock. How many quarts of chicken stock should she add to the mixture?

SEE EXAMPLE 3  
p. 680

3. **Fitness** Connor and Matt walk a 12-mile course as part of a fitness program. Matt walks 1 mi/h faster than Connor, and it takes him 1 hour less than Connor to complete the course. How long does it take Connor to complete the course?

### PRACTICE AND PROBLEM SOLVING

#### Independent Practice

| For Exercises | See Example |
|---------------|-------------|
| 4–5           | 1           |
| 6             | 2           |
| 7             | 3           |

#### Extra Practice

Skills Practice p. EP21

Application Practice p. EP33

4. **Technology** Lawrence's old robotic vacuum can clean his apartment in  $1\frac{1}{2}$  hours. His new robotic vacuum can clean his apartment in 45 minutes. How long will it take both vacuums working together to clean his apartment?

5. The table shows the time it takes for pipes of different sizes to fill a reservoir with water. How long would it take for both pipes working at the same time to fill the reservoir?

| Pipe Output   |                            |
|---------------|----------------------------|
| Diameter (ft) | Time to Fill Reservoir (h) |
| 2             | 12                         |
| 3             | 6                          |

6. Maria has 8 cups of a fruit punch that consists of 25% orange juice and 75% apple juice. She wants to make a drink that is 40% orange juice. How many cups of orange juice should Maria add to the mixture?
7. **Transportation** The average speed of an express train is 15 mi/h faster than the average speed of a local train. It takes the local train 2 hours longer than the express train to cover 360 miles. What is the average speed of the express train?

8. **Sports** A race consists of a 20-mile bike ride and a 12-mile run. Jen's rate while biking is 4 mi/h faster than while running. She completes the race in 4 hours.

- a. Let  $r$  be Jen's rate while biking. Copy and complete the table.
- b. Use the information in the Time column to write a rational equation.
- c. Solve the equation to find Jen's rate while biking.

|         | Distance (mi) | Rate (mi/h) | Time (h)       |
|---------|---------------|-------------|----------------|
| Biking  | ■             | $r$         | $\frac{20}{r}$ |
| Running | 12            | ■           | ■              |



9. **Geometry** Both rectangle A and rectangle B have areas of  $96 \text{ m}^2$ . The length of B is twice the length of A. The width of A is 4 m greater than the width of B.

- a. Let  $\ell$  be the length of rectangle A. Copy and complete the table.

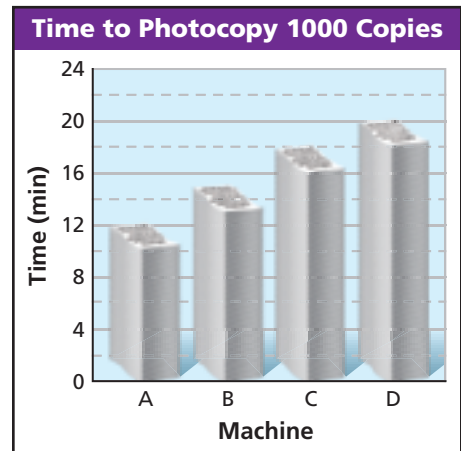
|             | Area ( $\text{m}^2$ ) | Length (m) | Width (m)         |
|-------------|-----------------------|------------|-------------------|
| Rectangle A | 96                    | $\ell$     | $\frac{96}{\ell}$ |
| Rectangle B | ■                     | ■          | ■                 |

- b. Use the information in the Width column to write a rational equation.
- c. Solve the equation to find the length and width of rectangle A.

10. **Multi-Step** The manager of a copy center can use two of the machines in the graph to make 1000 photocopies of a leaflet. Will the job be completed more quickly with machines A and D or with machines B and C? Explain.

11. A number plus the reciprocal of the number is  $\frac{5}{4}$  of the number. What is the number?

12. **ERROR ANALYSIS** Jordan can wash the windows in his house in 4 hours. His brother can do the job in 6 hours. They want to know how long it will take if they work together. Which solution to the problem is incorrect? Explain the error.



**A**

$$\frac{h}{4} + \frac{h}{6} = 1 \rightarrow 12\left(\frac{h}{4} + \frac{h}{6}\right) = 12(1)$$

$$3h + 2h = 12$$

$$5h = 12$$

$$h = 2.4$$

**B**

$$\frac{4}{h} + \frac{6}{h} = 1 \rightarrow h\left(\frac{4}{h} + \frac{6}{h}\right) = h(1)$$

$$4 + 6 = h$$

$$10 = h$$


**CONCEPT CONNECTION**



13. This problem will prepare you for the Concept Connection on page 684.

Melinda has a lens with a focal length  $f$  of 12 cm. She wants to place a candle so that the distance from the candle to the lens is 10 cm more than the distance from the candle's image to the lens. She knows that  $\frac{1}{f} = \frac{1}{x} + \frac{1}{y}$ , where  $x$  is the distance from the object to the lens and  $y$  is the distance from the image to the lens.

- a. Write and solve a rational equation to find the distance from the candle to the lens.
- b. How far will the candle's image be from the lens?

14. **Critical Thinking** Working individually, Tyrone and Cheryl can each mow the lawn in 48 minutes. How long will it take them to mow the lawn if they work together? What do you notice about your answer? Explain why this makes sense.
-  15. **Write About It** Write an original problem that can be solved using a rational equation. Include the solution to the problem.

**Multiple Choice** For Exercises 16 and 17, choose the best answer.

16. A chemist has 80 milliliters of a solution that consists of equal amounts of water and ethanol. The chemist wants to make a solution that is 70% water. Which equation can be used to find the additional amount of water  $w$  that should be added to the solution?
- (A)  $\frac{40 + w}{80 + w} = 0.7$     (B)  $\frac{w}{80} + \frac{w}{40} = 0.7$     (C)  $\frac{80 + w}{40 + w} = 0.7$     (D)  $\frac{80}{w} + \frac{40}{w} = 0.7$
17. Ms. Yamashiro can grade her students' homework in 30 minutes. Her teaching assistant can grade the same papers in 45 minutes. If they work together, how long will it take them to grade the papers?
- (A) 9 minutes    (B) 18 minutes    (C) 37.5 minutes    (D) 75 minutes

### CHALLENGE AND EXTEND

18. A water tank contains three drains. The table shows the time it takes for each drain to empty the tank. How long does it take to empty the tank when all three drains are used?
19. Luke, Eddie, and Ryan can complete a job in 1 hour and 20 minutes if they work together. Working alone, it takes Ryan 1 hour more to complete the job than it takes Luke, and Luke works twice as fast as Eddie. How much time would it take each to complete the job working alone?
20. A bowl contains fruit punch that is  $\frac{1}{2}$  cranberry juice. When 6 cups of cranberry juice are added to the bowl, the resulting punch is  $\frac{2}{3}$  cranberry juice. How many cups of punch were in the bowl to start with?

| Time Needed to Empty Tank |         |
|---------------------------|---------|
| Drain A                   | 3 hours |
| Drain B                   | 6 hours |
| Drain C                   | 4 hours |



### SPIRAL STANDARDS REVIEW

7.0, 11.0, 12.0

Without graphing, tell whether each point is on the graph of  $4x + 2y = 14$ . (Lesson 5-1)

21. (1, 5)                      22. (0, 8)                      23. (-2, 11)  
 24. (5, -2)                    25. (9, -11)                    26. (4, -2)

Determine whether each trinomial is a perfect square. If so, factor. If not, explain. (Lesson 8-5)

27.  $4x^2 + 12x + 9$                       28.  $9x^2 - 6x + 1$                       29.  $16x^2 - 24x - 9$   
 30.  $49x^2 - 4x + 4$                       31.  $25x^2 + 8x - 16$                       32.  $36x^2 + 8x + 16$

Divide. Check your answer. (Lesson 10-6)

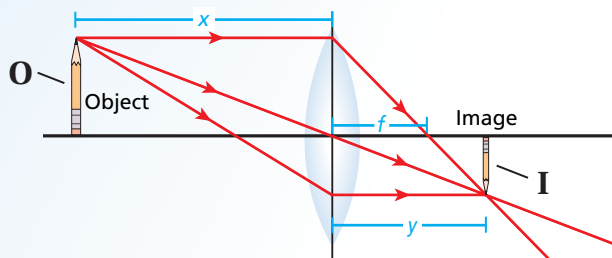
33.  $\frac{x^2 + 3x + 2}{x + 2}$                       34.  $\frac{x^2 - 6x - 16}{x - 8}$                       35.  $\frac{x^2 + 7x + 12}{x + 3}$   
 36.  $\frac{x^2 - 4x + 21}{x - 7}$                       37.  $\frac{x^2 - 9x + 18}{x - 3}$                       38.  $\frac{x^2 - 3x - 40}{x + 5}$

## SECTION 10B



## Operations with Rational Expressions and Equations

**An Upside-Down World** Jamal is studying lenses and their images for a science project. He finds in a science book that a magnifying glass can be used to project upside-down images on a screen. The equation  $\frac{1}{f} = \frac{1}{x} + \frac{1}{y}$  relates the focal length of the lens  $f$ , the distance of the object from the lens  $x$ , and the distance of the image from the lens  $y$ . The focal length of Jamal's lens is 10 cm.



1. Solve the given equation for  $y$  using the given value of  $f$ .
2. Jamal experiments with a candle, the lens, and a screen. Given that the focal length remains constant, use a table for the  $x$ -values 0, 2, 4, 6, 8, 10, 12, 14, and 16 cm. For which  $x$ -values are the  $y$ -values positive?
3. Graph the function  $y(x)$ . Label the axes.

Magnification for images is the ratio of the height of the image to the height of the object. This is also equal to the ratio of the distance between the image and the lens and the distance between the object and the lens:  $M = \frac{I}{O} = \frac{y}{x}$ .  $I$  is the height of the image,  $O$  is the height of the object,  $y$  is the distance of the image from the lens, and  $x$  is the distance of the object from the lens.

4. If the height of a candle is 15 cm and the projected image of that candle is 37.5 cm, what is the magnification of the lens?
5. As Jamal moves the candle further from the lens (increases  $x$ ), and the distance between the lens and the screen decreases ( $y$  decreases), does the magnification  $M$  stay the same, increase, or decrease?



## Quiz for Lessons 10-4 Through 10-8

### 10-4 Multiplying and Dividing Rational Expressions

Multiply. Simplify your answer.

$$1. \frac{n+3}{n-5} \cdot (n^2 - 5n)$$

$$2. \frac{6xy^2}{2x^2y^6} \cdot \frac{6x^4y^4}{9x^3}$$

$$3. \frac{3h^3 - 6h}{10g^2} \cdot \frac{4g}{gh^2 - 2g}$$

$$4. \frac{m^2 + m - 2}{m^2 - 2m - 8} \cdot \frac{m^2 - 8m + 16}{3m - 3}$$

Divide. Simplify your answer.

$$5. \frac{2}{n^3} \div \frac{n-6}{n^5}$$

$$6. \frac{2x^2 + 8x + 6}{x} \div \frac{2x^2 + 2x}{x^3 - x^2}$$

$$7. \frac{8b^3c}{b^2c} \div (4b^2 + 4b)$$

### 10-5 Adding and Subtracting Rational Expressions

Add or subtract. Simplify your answer.

$$8. \frac{15}{2p} - \frac{13}{2p}$$

$$9. \frac{3m^2}{4m^5} + \frac{5m^2}{4m^5}$$

$$10. \frac{x^2 + 8x}{x-2} - \frac{3x + 14}{x-2}$$

$$11. \frac{2t}{4t^2} + \frac{2}{t}$$

$$12. \frac{m^2 - m - 2}{m^2 + 6m + 5} - \frac{2}{m + 5}$$

$$13. \frac{4x}{x-2} + \frac{3x}{2-x}$$

### 10-6 Dividing Polynomials

Divide. Check your answer.

$$14. (6d^2 + 4d) \div 2d$$

$$15. (15x^4 + 3x^3 - x) \div (-3x^2)$$

$$16. (2x^2 - 7x - 4) \div (2x + 1)$$

Divide using long division. Check your answer.

$$17. (a^2 + 3a - 10) \div (a - 2)$$

$$18. (4y^2 - 9) \div (2y - 3)$$

$$19. (2x^2 + 5x - 8) \div (x + 2)$$

### 10-7 Solving Rational Equations

Solve. Check your answer.

$$20. \frac{3}{x} = \frac{4}{x-1}$$

$$21. \frac{1}{x} = \frac{2}{x^2}$$

$$22. \frac{2}{t} + \frac{4}{3t} = \frac{4}{t+2}$$

$$23. \frac{4}{n^2} = \frac{7}{n} + 2$$

$$24. \frac{d+2}{d+8} = \frac{-6}{d+8}$$

$$25. \frac{x-6}{x^2-6} = \frac{-4}{x-4}$$

### 10-8 Applying Rational Equations

26. It takes Dustin 2 hours to shovel the snow from his driveway and sidewalk. It takes his sister 3 hours to shovel the same area. How long will it take them to shovel the walk if they work together?
27. A chemistry student needs to make a solution that is 70% water and 30% hydrochloric acid. The student's current mixture of 300 mL is 60% water and 40% hydrochloric acid. How much water must the student add to achieve his desired solution?



**Vocabulary**

|                                  |     |                               |     |
|----------------------------------|-----|-------------------------------|-----|
| asymptote . . . . .              | 634 | inverse variation . . . . .   | 627 |
| discontinuous function . . . . . | 634 | rational equation . . . . .   | 674 |
| excluded value . . . . .         | 634 | rational expression . . . . . | 642 |
| extraneous solution . . . . .    | 675 | rational function . . . . .   | 634 |

Complete the sentences below with vocabulary words from the list above.

- A(n)     ?      is an algebraic expression whose numerator and denominator are polynomials.
- A function whose rule is a quotient of polynomials in which the denominator has a degree of at least 1 is a(n)     ?     .
- A(n)     ?      is an equation that contains one or more rational expressions.
- A(n)     ?      is a relationship that can be written in the form  $y = \frac{k}{x}$ , where  $k$  is a nonzero constant.
- A function is a(n)     ?      if its graph contains one or more jumps, breaks, or holes.

**10-1 Inverse Variation (pp. 627–633)**



Prep for **13.0**

**EXAMPLE**

- Write and graph the inverse variation in which  $y = 2$  when  $x = 3$ .

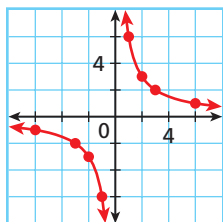
$y = \frac{k}{x}$     Use the form  $y = \frac{k}{x}$ .

$2 = \frac{k}{3}$     Substitute known values.

$6 = k$     Multiply by 3 to find the value of  $k$ .

$y = \frac{6}{x}$     Substitute 6 for  $k$  in  $y = \frac{k}{x}$ .

|          |    |    |    |    |      |   |   |   |   |
|----------|----|----|----|----|------|---|---|---|---|
| <b>x</b> | -6 | -3 | -2 | -1 | 0    | 1 | 2 | 3 | 6 |
| <b>y</b> | -1 | -2 | -3 | -6 | und. | 6 | 3 | 2 | 1 |



Make a table of values and plot the points.

**EXERCISES**

Tell whether each relationship represents an inverse variation. Explain.

6. 

| x   | y  |
|-----|----|
| 4   | -3 |
| -12 | 1  |
| 6   | -2 |

7. 

| x  | y  |
|----|----|
| 2  | 4  |
| 6  | 8  |
| 10 | 12 |

- Write and graph the inverse variation in which  $y = 4$  when  $x = -1$ .
- Write and graph the inverse variation in which  $y = \frac{1}{2}$  when  $x = 2$ .
- Let  $x_1 = 5$ ,  $y_1 = -6$ , and  $x_2 = 2$ . Let  $y$  vary inversely as  $x$ . Find  $y_2$ .
- The number of fleet vehicles a town can afford to buy varies inversely as the price of each car. If the town can afford 3 cars priced at \$22,000 each, what must the price of a car be in order for the town to purchase 5 of them?

## 10-2 Rational Functions (pp. 634–641)

### EXAMPLE

- Graph the function  $y = \frac{1}{x+1} + 3$ .

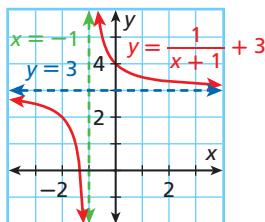
Find the asymptotes.

$$x = -1 \quad b = -1$$

$$y = 3 \quad c = 3$$

Graph the asymptotes using dashed lines.

Make a table of values. Choose  $x$ -values on both sides of the vertical asymptote.



|     |               |    |   |               |
|-----|---------------|----|---|---------------|
| $x$ | -3            | -2 | 0 | 1             |
| $y$ | $\frac{5}{2}$ | 2  | 4 | $\frac{7}{2}$ |

Plot the points and connect them with smooth curves.

### EXERCISES

Identify any excluded values and the asymptotes for each rational function.

12.  $y = \frac{1}{x+4}$                       13.  $y = \frac{1}{x+1} + 3$

14.  $y = \frac{-5}{2x+6} - 4$                       15.  $y = \frac{2}{4x-7} + 5$

Graph each function.

16.  $y = \frac{3}{x}$                                       17.  $y = \frac{4}{x+5}$

18.  $y = \frac{1}{x+4} - 2$                       19.  $y = \frac{1}{x-6} + 2$

20. A rectangle has an area of  $24 \text{ cm}^2$ . If  $x$  represents the width, then  $y = \frac{24}{x}$  represents the length  $y$ . Describe the reasonable domain and range values and graph the function.

## 10-3 Simplifying Rational Expressions (pp. 642–648)

### EXAMPLE

Simplify the rational expression, if possible. Identify any excluded values.

■  $\frac{x-1}{x^2+2x-3}$

$$\frac{x-1}{(x+3)(x-1)}$$

*Factor the denominator.*

$$\frac{\cancel{x-1}^1}{(x+3)\cancel{(x-1)}}$$

*Divide out common factors.*

$$\frac{1}{x+3}$$

*Simplify.*

Identify the excluded values.

$$x^2 + 2x - 3 = 0 \quad \text{Set the denominator equal to 0.}$$

$$(x+3)(x-1) = 0 \quad \text{Factor.}$$

$$x+3 = 0 \text{ or } x-1 = 0 \quad \text{Use the Zero Product Property.}$$

$$x = -3 \text{ or } x = 1 \quad \text{Solve each equation.}$$

The excluded values are  $-3$  and  $1$ .

### EXERCISES

Identify any excluded values of each rational expression.

21.  $\frac{3}{5p}$                                       22.  $\frac{-2}{r-7}$

23.  $\frac{t}{t^2-t}$                                       24.  $\frac{-4}{x^2-4x-5}$

25.  $\frac{x-1}{x^2-25}$                                       26.  $\frac{x+4}{x^2-11x+28}$

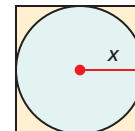
Simplify each rational expression, if possible. Identify any excluded values.

27.  $\frac{7r^2}{21r^3}$                                       28.  $\frac{3k^2}{6k^3-9k^2}$

29.  $\frac{x+6}{x^2+4x-12}$                                       30.  $\frac{2x-6}{9-x^2}$

31.  $\frac{3x+15}{x^2+4x-5}$                                       32.  $\frac{x^2+9x+18}{x^2+x-30}$

33. What is the ratio of the area of the square to the area of the circle?



## 10-4 Multiplying and Dividing Rational Expressions (pp. 652–658)



### EXAMPLE

- Divide. Simplify your answer.

$$\frac{32x^2y^2}{7z} \div \frac{2xy^2}{28xz^3}$$

$$\frac{32x^2y^2}{7z} \cdot \frac{28xz^3}{2xy^2} \quad \text{Multiply by the reciprocal.}$$

$$\frac{\cancel{3}2^{16}x^2y^{\cancel{2}}}{\cancel{7}z} \cdot \frac{\cancel{2}8^4x^{\cancel{1}}z^{\cancel{3}}z^2}{\cancel{2}xy^{\cancel{2}}} \quad \text{Divide out common factors.}$$

$$64x^2z^2 \quad \text{Simplify.}$$

### EXERCISES

Multiply or divide. Simplify your answer.

34.  $\frac{2b}{3b-6} \cdot (b^2 - b - 2)$

35.  $\frac{4x}{3x+9} \cdot (x^2 - 9)$

36.  $\frac{5ab^2}{2ab} \cdot \frac{3a^2b^2}{a^2b}$

37.  $16n^3 \div \frac{4m^2n}{3mn}$

38.  $\frac{b+2}{2b^2+12b} \cdot \frac{b^2+2b-24}{b^2-16}$

39.  $\frac{x^2+2x-3}{4x} \div \frac{x^2-4}{x}$

## 10-5 Adding and Subtracting Rational Expressions (pp. 659–665)



### EXAMPLES

Add or subtract. Simplify your answer.

■  $\frac{7x}{3xy} - \frac{x^2-3x}{3xy}$

$$\frac{7x - (x^2 - 3x)}{3xy} \quad \text{Subtract numerators.}$$

$$\frac{7x - x^2 + 3x}{3xy} \quad \text{Distribute.}$$

$$\frac{10x - x^2}{3xy} \quad \text{Simplify.}$$

■  $\frac{3w}{w-5} + \frac{4}{w^2-2w-15}$

$$\frac{3w}{w-5} + \frac{4}{(w-5)(w+3)} \quad \text{Factor to find the LCD.}$$

$$\frac{3w(w+3)}{(w-5)(w+3)} + \frac{4}{(w-5)(w+3)} \quad \text{Write each expression using the LCD.}$$

$$\frac{3w^2+9w}{(w-5)(w+3)} + \frac{4}{(w-5)(w+3)} \quad \text{LCD.}$$

$$\frac{3w^2+9w+4}{(w-5)(w+3)} \quad \text{Add and simplify.}$$

### EXERCISES

Find the LCM of the given expressions.

40.  $5a^2b, 10ab^2$

41.  $2x^2 - 6x, 5x - 15$

Add or subtract. Simplify your answer.

42.  $\frac{b^2}{2b} + \frac{8}{2b}$

43.  $\frac{3x^2-4}{x^2-2} + \frac{2x}{x^2-2}$

44.  $\frac{8p}{p^2-4p+2} - \frac{2}{p^2-4p+2}$

45.  $\frac{3b+4}{7-b} - \frac{5-2b}{7-b}$

46.  $\frac{n-5}{n^2-1} - \frac{n+5}{n^2-1}$

47.  $\frac{3}{5m} + \frac{m+2}{10m^2}$

48.  $\frac{h^2+2h}{h-5} - \frac{3h-1}{5-h}$

49. A scout troop hikes 10 miles to the top of a mountain. Because the return trip is downhill, the scouts are able to hike 3 times faster on their way down. Let  $r$  represent the troop's rate to the mountaintop. Write and simplify an expression for the round-trip hiking time in terms of  $r$ .

## 10-6 Dividing Polynomials (pp. 667–673)



10.0, 12.0

### EXAMPLE

- Divide  $(4x^3 - 2x^2 + 5x - 1) \div (x - 2)$ .

$$\begin{array}{r}
 4x^2 + 6x + 17 \\
 x - 2 \overline{) 4x^3 - 2x^2 + 5x - 1} \\
 \underline{-(4x^3 - 8x^2)} \phantom{- 1} \\
 6x^2 + 5x \phantom{- 1} \\
 \underline{-(6x^2 - 12x)} \phantom{- 1} \\
 17x - 1 \\
 \underline{-(17x - 34)} \\
 33 \\
 4x^2 + 6x + 17 + \frac{33}{x - 2}
 \end{array}$$

### EXERCISES

Divide. Check your answer.

50.  $(-5x^3 + 10x - 25) \div (-5x^2)$

51.  $\frac{x^2 - 8x - 20}{x - 10}$

52.  $\frac{6n^2 - 13n - 5}{2n - 5}$

Divide using long division. Check your answer.

53.  $(x^2 + 5x + 6) \div (x + 3)$

54.  $(x^2 + x - 30) \div (x - 5)$

55.  $(3b^3 - 4b + 2) \div (b - 2)$

## 10-7 Solving Rational Equations (pp. 674–678)



Prep for 15.0

### EXAMPLE

- Solve  $\frac{3}{x+3} = \frac{2}{x}$ . Check your answer.

$3x = 2(x + 3)$  *Use cross products.*

$3x = 2x + 6$  *Distribute.*

$\frac{-2x}{x} = \frac{-2x}{6}$  *Subtract 2x from both sides.*

Check

$$\begin{array}{r|l}
 \frac{3}{x+3} = \frac{2}{x} & \\
 \frac{3}{6+3} & \frac{2}{6} \\
 \frac{3}{9} & \frac{1}{3} \\
 \frac{1}{3} & \frac{1}{3} \quad \checkmark
 \end{array}$$

### EXERCISES

Solve. Check your answer.

56.  $-4 = \frac{3}{r}$

57.  $\frac{6}{7} = \frac{x}{2}$

58.  $\frac{6}{b} = \frac{-5}{3+b}$

59.  $\frac{7}{3y^2} = \frac{-2}{y}$

60.  $\frac{2}{x-1} = \frac{3x}{1-x}$

61.  $\frac{2x}{x^2} + \frac{1}{x^2} = 3$

62.  $\frac{2}{3} + \frac{4}{x} = \frac{6}{3x}$

63.  $-\frac{1}{3x} + \frac{x}{4} = -\frac{1}{12x}$

64.  $\frac{2}{3b} + 4 = \frac{1}{3b}$

65.  $\frac{4}{x-4} = \frac{8}{x^2-16}$

66.  $\frac{5x-10}{x+1} = \frac{x}{2}$

67.  $\frac{2x}{x+3} + \frac{x}{4} = \frac{3}{x+3}$

68.  $\frac{9m}{m-5} = 7 - \frac{3}{m-5}$

69.  $\frac{x-4}{x^2-4} = \frac{-2}{x-2}$

## 10-8 Applying Rational Equations (pp. 679–683)



15.0

### EXAMPLE

- Armin can clean a house in 5 hours. It takes Greg 7 hours to clean the same house. How long will it take them if they work together?

$35\left(\frac{1}{5}h + \frac{1}{7}h\right) = 35(1)$  *Multiply both sides by the LCD, 35*

$7h + 5h = 35$  *Distribute 35 on the left side.*

$12h = 35$  *Combine like terms.*

$h = \frac{35}{12} = 2\frac{11}{12}$  *Divide both sides by 12.*

Together they can clean the house in  $2\frac{11}{12}$  h, or 2 h 55 min.

### EXERCISES

70. A liquid poured into pipe A fills a storage tank in 12 hours. The same liquid poured into pipe B fills the tank in 18 hours. How long would it take to fill the tank if the liquid is poured into both pipes at the same time?
71. A chemist has 400 milliliters of a solution that is 40% water and 60% chlorine. She wants a solution that is 50% water and 50% chlorine. How many milliliters of water should she add to the solution?

- Write and graph the inverse variation in which  $y = -4$  when  $x = 2$ .
- The number of posters the Spanish Club can buy varies inversely as the cost of each poster. The club can buy 15 posters that cost \$2.60 each. How many posters can the club buy if they cost \$3.25 each?

Identify any excluded values and the asymptotes for each rational function.

3.  $y = \frac{3}{x+1}$

4.  $y = \frac{1}{2x-1} + 5$

5.  $y = \frac{1}{x+3} - 3$

Simplify each rational expression, if possible. Identify any excluded values.

6.  $\frac{2b}{4b^2}$

7.  $\frac{x^2 - 16}{x^2 + 3x - 4}$

8.  $\frac{b^2 - 2b - 15}{5 - b}$

9.  $\frac{x^2 + 4x - 5}{x^2 - 25}$

Multiply. Simplify your answer.

10.  $\frac{-4}{x^2 - 9} \cdot (x - 3)$

11.  $\frac{2a^2b^2}{5b^3} \cdot \frac{15a^2b}{8a^4}$

12.  $\frac{x^2 - x - 12}{x^2 - 16} \cdot \frac{x^2 + x - 12}{x^2 + 3x + 2}$

Divide. Simplify your answer.

13.  $\frac{4x^2y^4}{3xy^2} \div \frac{12xy}{15x^3y^2}$

14.  $\frac{3b^2 - 6b}{2b^3 + 3b^2} \div \frac{2b - 4}{8b + 12}$

15.  $\frac{x^2 + 2x - 15}{x^2 - 9} \div \frac{x^2 - 25}{x^2 + 3x + 2}$

Add or subtract. Simplify your answer.

16.  $\frac{b^2 + 3}{5b} + \frac{4}{5b}$

17.  $\frac{5x - 2}{x^2 + 2} - \frac{2x}{x^2 + 2}$

18.  $\frac{2}{3x^2} - \frac{5 - 2x}{3x^2}$

19.  $\frac{3m}{2m^2} + \frac{1}{2m}$

20.  $\frac{3x}{2x + 4} - \frac{1}{x + 2}$

21.  $\frac{y^2 + 4}{y - 3} + \frac{y^2}{3 - y}$

Divide.

22.  $(8t^2 - 2t) \div 2t$

23.  $\frac{3x^2 + 2x - 8}{x + 2}$

24.  $\frac{k^2 - 2k - 35}{k + 5}$

Divide using long division.

25.  $(2w^2 + 5w - 12) \div (w + 4)$

26.  $(x^2 - 4x + 9) \div (x + 2)$

27. The area of rectangle can be modeled by  $A(x) = x^3 - 1$ . The length is  $x - 1$ .

- Find a polynomial to represent the width of the rectangle.
- Find the width when  $x$  is 6 cm.

Solve. Check your answer.

28.  $\frac{2}{x-1} = \frac{9}{2x-3}$

29.  $\frac{3}{n-1} = \frac{n}{n+4}$

30.  $\frac{2}{n+2} = \frac{n-4}{n^2-4}$

31. Julio can wash and wax the family car in 2 hours. It takes Leo 3 hours to wash and wax the same car. How long will it take them to wash and wax the car if they work together?

# COLLEGE ENTRANCE EXAM PRACTICE



CHAPTER  
**10**

## FOCUS ON SAT MATHEMATICS SUBJECT TESTS

The topics covered on each SAT Mathematics Subject Test vary only slightly each time the test is given. Find out the general distribution of test items across topics, and then identify the areas you need to concentrate on while studying.



To prepare for the SAT Math Subject Tests, start reviewing material several months before your test date. Take sample tests to find the areas you need to focus on. You are not expected to have studied all topics on the test.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. Which set of ordered pairs satisfies an inverse variation?

- (A) (6, 3) and (8, 4)
- (B) (2, -3) and (4, 5)
- (C) (4, -2) and (-5, 10)
- (D) (2, 6) and (-3, -4)
- (E)  $(4, \frac{1}{4})$  and  $(-4, \frac{1}{4})$

2. If  $\frac{3}{x+3} = \frac{7x}{x^2-9}$ , what is  $x$ ?

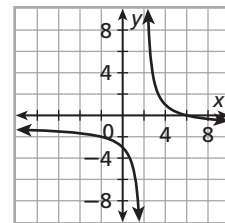
- (A) -12
- (B) -3
- (C)  $-\frac{9}{4}$
- (D)  $\frac{9}{4}$
- (E) 3

3. What is  $h$  if  $(x^3 + 2x^2 - 4x + h) \div (x + 1)$  has a remainder of 15?

- (A) -10
- (B) -5
- (C) 5
- (D) 10
- (E) 20

4. The graph of which function is shown?

- (A)  $f(x) = \frac{2}{x+4} + 1$
- (B)  $f(x) = \frac{4}{x+2} - 1$
- (C)  $f(x) = \frac{4}{x-2} + 1$
- (D)  $f(x) = \frac{4}{x-2} - 1$
- (E)  $f(x) = \frac{2}{x-4} + 1$



5. Which function has the same graph as  $f(x) = \frac{x^2 - 4x - 5}{x^2 - 3x - 10}$  except at  $x = 5$ ?

- (A)  $g(x) = \frac{x-1}{x-2}$
- (B)  $g(x) = \frac{x+1}{x+2}$
- (C)  $g(x) = \frac{x+1}{(x-5)(x+2)}$
- (D)  $g(x) = \frac{(x+5)(x-1)}{x+2}$
- (E)  $g(x) = \frac{(x-5)(x+1)}{x-2}$

## Multiple Choice: Choose Combinations of Answers

Some multiple-choice test items require selecting a combination of correct answers. The correct response is the most complete option available. To solve this type of test item, determine if each statement is true or false. Then choose the option that includes each correct statement.

### EXAMPLE 1

Which of the following has an excluded value of  $-5$ ?

- I.  $\frac{5}{x-5}$       III.  $\frac{x^2-10}{5x+25} \cdot \frac{5}{x-10}$
- II.  $\frac{8x^2+36x-20}{2(x+5)}$       IV.  $\frac{2(x+2)}{2x^2+12x+10}$
- (A) I only      (C) II, III, and IV
- (B) II and III      (D) III and IV

Look at each statement separately and determine whether it is true. You can keep track of which statements are true in a table.

#### Statement I

The denominator,  $x - 5$ , equals 0 when  $x = 5$ .

Statement I does not answer the question, so it is false.

#### Statement II

The denominator,  $2(x + 5)$ , equals 0 when  $x = -5$ .

Statement II does answer the question, so it is true.

#### Statement III

The denominator,  $(5x + 25)(x + 10)$ , equals 0 when  $x = -5$  or  $x = -10$ .

Statement III does answer the question, so it is true.

#### Statement IV

The denominator,  $2x^2 + 12x + 10$ , can be factored as  $2(x + 5)(x + 1)$ . This expression equals 0 when  $x = -5$  or  $x = -1$ .

Statement IV does answer the question, so it is true.

| Statement | True/False |
|-----------|------------|
| I         | False      |
| II        | True       |
| III       | True       |
| IV        | True       |

Statements II, III, and IV are all true. Option C is the correct response because it includes all the true statements.

Options B and D contain some of the true statements, but option C is the most complete answer.



Evaluate all of the statements before deciding on an answer choice. Make a table to keep track of whether each statement is true or false.

Read each test item and answer the questions that follow.

#### Item A

Which dimensions represent a rectangle that has an area equivalent to the expression  $2x^2 + 18x + 16$ ?

I.  $\ell = x + 8$

$$w = 2(x + 1)$$

II.  $\ell = 2x + 2$

$$w = \frac{x^2 + 3x - 40}{x - 5}$$

III.  $\ell = x + 2$

$$w = \frac{(2x + 2)(x + 4)}{1} \cdot \frac{(3x - 1)}{(3x^2 - 11x - 4)}$$

(A) I only

(C) I and II

(B) III only

(D) I, II, and III

- How do you determine the area of a rectangle?
- Daisy realized that the area of the rectangle described in I was equivalent to the given area and selected option A as her response. Do you agree? Explain your reasoning.
- Write a simplified expression for the width of the rectangle described in II.
- Explain each step for determining the area of the rectangle described in III.
- If the rectangle described in II has an area equivalent to the given expression, then which options can you eliminate?

#### Item B

Which expression is undefined for  $x = 3$  or  $x = -2$ ?

I.  $2x + 12 \cdot \frac{4}{2(x - 3)(x + 6)}$

III.  $\frac{(x - 2)}{(x + 2)(x - 1)}$

II.  $\frac{9x - 1}{x^2 + 3}$

IV.  $\frac{14}{x^2 - x - 6}$

(A) I, III, and IV

(C) III and IV

(B) I and II

(D) I and IV

- When is an expression undefined?
- Henry determined that expression I is undefined when  $x = 3$ . He decides it is an incorrect answer because the expression is defined when  $x = -2$ . Should he select option C by process of elimination? Explain your reasoning.
- Make a table to determine the correct response.

#### Item C

Which rational function has a graph with a horizontal asymptote of  $y = 4$ ?

I.  $y = \frac{-4}{x}$

III.  $y = \frac{1}{x - 4}$

II.  $y = \frac{1}{x} + 4$

IV.  $y = -\frac{1}{x} + 4$

(A) I and III

(C) I and II

(B) II only

(D) II and IV

- Where does the horizontal asymptote of the function in statement I occur?
- Using your answer from Problem 9, which option(s) can you eliminate? Explain your reasoning.
- Look at the options remaining. Which statement would be best to check next? Explain your reasoning.

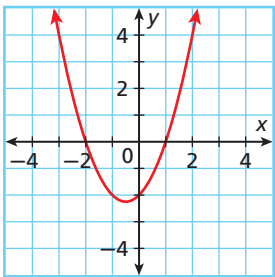


## CUMULATIVE ASSESSMENT, CHAPTERS 1-10

### Multiple Choice

- At how many points does the graph of  $y = (x - 6)^2$  intersect the  $x$ -axis?
 

(A) none                      (C) two  
(B) one                        (D) three
- Which point lies on the line described by  $2x - 3y - 9 = 0$ ?
 

(A)  $(-2, 2)$                 (C)  $(0, 3)$   
(B)  $(4, -5)$                 (D)  $(3, -1)$
- Which function is shown in the graph?
 

(A)  $y = x^2 + x - 2$   
(B)  $y = x^2 - x - 2$   
(C)  $y = -x^2 - x + 2$   
(D)  $y = x^2 - 2$
- A city map is laid out on a coordinate plane. Elm Street is described by the line  $x + 2y = -6$ . Oak Street intersects Elm Street at a right angle. Which of the following could be the equation for Oak Street?
 

(A)  $2x + y = 5$   
(B)  $-2x + y = 3$   
(C)  $x + 2y = 4$   
(D)  $-x - 2y = 8$

- Which expression is NOT equivalent to  $\frac{3}{x-1}$ ?
 

(A)  $\frac{3x+6}{x^2+x-2}$                 (C)  $\frac{3x+3}{x^2-1}$   
(B)  $\frac{3x-3}{x^2-2x+1}$                 (D)  $\frac{3x-3}{x-1}$

- What are the  $x$ - and  $y$ -intercepts of the line described by  $6x - 2y = 4$ ?
 

(A)  $x$ -intercept:  $\frac{2}{3}$   
 $y$ -intercept: 2  
(B)  $x$ -intercept:  $-\frac{2}{3}$   
 $y$ -intercept: 2  
(C)  $x$ -intercept:  $\frac{2}{3}$   
 $y$ -intercept: -2  
(D)  $x$ -intercept: -2  
 $y$ -intercept:  $\frac{2}{3}$

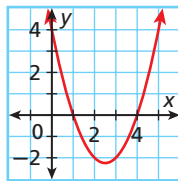
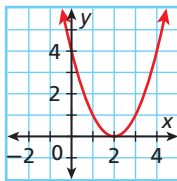
- Which expression is equivalent to  $\frac{3m^2n}{5m} \cdot \frac{20mn}{n^6}$ ?
 

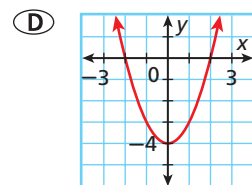
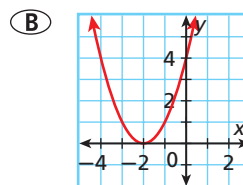
(A)  $\frac{12m^2}{n^4}$                         (C)  $12m^2n^3$   
(B)  $\frac{12m^3}{n^3}$                         (D)  $\frac{12m}{n}$

- Simplify  $\frac{3}{x} + \frac{3}{5x}$ .
 

(A)  $\frac{1}{x}$                             (C)  $\frac{18}{5x}$   
(B)  $\frac{18}{10x}$                         (D)  $\frac{1}{2x}$

- Which of these is the graph of  $y = x^2 - 4x + 4$ ?
 

(A)                 (C) 



10. What is  $(-12x^6 + x) \div (-4x^2)$ ?

- (A)  $3x^3 - \frac{1}{4x}$       (C)  $3x^5$   
(B)  $3x^4 - \frac{1}{4x}$       (D)  $3x^4 + x$

11. What is the greatest common factor of the terms of the polynomial  $6x^3 - 18x^2 + 12x + 3$ ?

- (A) 3      (C)  $6x$   
(B)  $3x$       (D)  $6x^2$

12. Which is a solution to  $\frac{n}{n+2} = \frac{-8}{n}$ ?

- (A) -4      (C) 2  
(B) -2      (D) 4

13. Which of the following is equivalent to  $\left(\frac{2x^5y^2}{8x}\right)^{-2}$ ?

- (A)  $\frac{16}{x^8y^4}$       (C)  $\frac{4}{x^4y^2}$   
(B)  $\frac{x^8}{16y^4}$       (D)  $\frac{x^5}{16}$

14. Which of the following is a true statement about the function  $y = \frac{1}{x}$ ?

- (A) The domain is all real numbers.  
(B) The range is all real numbers.  
(C) The domain is all real numbers except 0.  
(D) The range is all real numbers except 1.



If you are allowed to write in your test booklet, you may want to add additional information to a given diagram. Be sure to mark your answer on the answer sheet because you will not receive any credit for marks in the test booklet.

### Gridded Response

15. What is the excluded value for the rational expression  $\frac{x^2 - 4}{3x - 6}$ ?

16. The trinomial  $x^2 - 6x + c$  is a perfect square trinomial. What is the value of  $c$ ?

17. Find the positive solution of the equation  $x^2 + 5x - 4 = 0$ . Round your answer to the nearest tenth.

18. What is the value of  $4^0 - (2^{-3})$ ?

19. Identify the excluded value for  $y = \frac{x - 4}{x - 2}$ .

### Short Response

20. Mr. Lui wrote  $\frac{15 - 5x}{x^2 - 9x + 18}$  on the board.

- a. Explain what kind of expression it is.  
b. Simplify the expression. Show your work.  
c. Identify any excluded values.

21. Lynne can paint a wall in 40 minutes. Jeff can paint the same wall in 60 minutes. How long will it take Lynne and Jeff to paint the wall if they work together? Show your work.

22. What are 2 values of  $b$  that will make  $2x^2 - bx - 20$  factorable? Explain your answer.

23. The sum of the digits of a two-digit number is 11. If 45 is added to the number, the digits will be reversed.

- a. Write a system of equations that you can use to find the digits of the number. Tell what each variable represents.  
b. Solve the system of equations to find the two-digit number. Show your work.

### Extended Response

24. Principal Farley has \$200 to pay for some teachers to attend a technology conference. The company hosting the conference is allowing 2 teachers to attend for free. The number of teachers  $y$  that can be sent to the conference is given by the function  $y = \frac{200}{x} + 2$ , where  $x$  is the cost per teacher.

- a. Describe the reasonable domain and range values for this function.  
b. Identify the vertical and horizontal asymptotes.  
c. Graph the function.  
d. Give two points on the graph whose coordinates are whole numbers and describe what they mean in the context of this situation.