Bridges in Mathematics Second Edition Grade 5 Student Book Volumes 1 & 2

The Bridges in Mathematics Grade 5 package consists of:

- Bridges in Mathematics Grade 5 Teachers Guide Units 1–8
- Bridges in Mathematics Grade 5 Assessment Guide
- Bridges in Mathematics Grade 5 Teacher Masters
- Bridges in Mathematics Grade 5 Student Book Volumes 1 & 2
- Bridges in Mathematics Grade 5 Home Connections Volumes 1 & 2
- Bridges in Mathematics Grade 5 Student Book Answer Key
- Bridges in Mathematics Grade 5 Teacher Masters Answer Key
- Bridges in Mathematics Grade 5 Components & Manipulatives
- Bridges Educator Site
- Work Place Games & Activities

Number Corner Grade 5 Teachers Guide Volumes 1–3
- Number Corner Grade 5 Teacher Masters
- Number Corner Grade 5 Student Book
- Number Corner Grade 5 Teacher Masters Answer Key
- Number Corner Grade 5 Student Book Answer Key
- Number Corner Grade 5 Components & Manipulatives
- Word Resource Cards

Digital resources noted in italics.

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Prepared for publication using Mac OS X and Adobe Creative Suite.
Printed in the United States of America.

To reorder this book, refer to number 2B5SB5 (package of 5).

QBB5901 (1 & 2)
Updated 2016-06-27.

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Bridges in Mathematics is a standards-based K–5 curriculum that provides a unique blend of concept development and skills practice in the context of problem solving. It incorporates Number Corner, a collection of daily skill-building activities for students.

The Math Learning Center is a nonprofit organization serving the education community. Our mission is to inspire and enable individuals to discover and develop their mathematical confidence and ability. We offer innovative and standards-based professional development, curriculum, materials, and resources to support learning and teaching. To find out more, visit us at www.mathlearningcenter.org.

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**Work Place Instructions 1A The Product Game**

Each pair of players needs:
- a 1A The Product Game Record Sheet to share
- 2 game markers
- pencils

1. Players decide who is going first. Player 1 is O and Player 2 is X.

2. Player 1 places one of the game markers on any factor.

3. Player 2 places the other game marker on a factor. Then, he multiplies the two factors, draws an X on the product, and writes an equation to match the combination.

   **Player 1** I choose 5.

   **Player 2** I choose 7. Let’s see, 5 × 7 is 35, and I’m X, so I’ll put my X on 35.

4. Player 1 moves one game marker to get a new product. She can move either of the markers.

   **Player 1** I’ll move the factor marker from the 5 to the 3. Since 7 × 3 is 21, I get to put an O on 21.

5. Play continues until a player gets four products in a row across, up and down, or diagonally.
   - Only one factor marker can be moved during a player’s turn.
   - Players can move a game marker so that both are on the same factor. For example, both markers can be on 3. The player would mark the product 9 because 3 × 3 = 9.
   - If the product a player chooses is already covered, the player loses that turn.

**Game Variation**

A. Players play for five in a row.
1 Choose 15 of the problems below to solve.

<table>
<thead>
<tr>
<th>8 × 5</th>
<th>7 × 7</th>
<th>4 × 6</th>
<th>3 × 8</th>
<th>4 × 7</th>
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<tbody>
<tr>
<td>40</td>
<td>49</td>
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<td>24</td>
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<th>6 × 8</th>
<th>8 × 4</th>
<th>3 × 6</th>
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<tbody>
<tr>
<td>36</td>
<td>42</td>
<td>48</td>
<td>32</td>
<td>18</td>
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</table>

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<tr>
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<th>8 × 9</th>
<th>6 × 11</th>
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<td>72</td>
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<td>120</td>
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<th>40 × 6</th>
<th>50 × 8</th>
<th>10 × 9</th>
<th>14 × 9</th>
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<td>240</td>
<td>400</td>
<td>90</td>
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<table>
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<th>6 × 12</th>
<th>12 × 9</th>
<th>7 × 60</th>
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<tbody>
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<td>100</td>
<td>99</td>
<td>72</td>
<td>108</td>
<td>420</td>
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</tbody>
</table>

<table>
<thead>
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<th>30 × 6</th>
<th>13 × 8</th>
<th>11 × 5</th>
<th>25 × 8</th>
<th>12 × 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>104</td>
<td>55</td>
<td>200</td>
<td>96</td>
</tr>
</tbody>
</table>

2 Explain how you decided which problems to solve.

Explanations will vary.
Product Game Problems

Chloe and Ava were playing The Product Game. Their factor markers were on 4 and 5. Ava decided to move the marker from 5 to 7. Write a numerical expression to represent her move.

\[ 4 \times 7 \]

Chris and Katie were playing The Product Game. Their factor markers were on 9 and 2. Chris decided to move the marker from 2 to 6. Write a numerical expression to represent his move.

\[ 9 \times 6 \]

Eric and William were playing The Product Game together. William put an X on 42. One factor marker was on 6. The other factor marker was on ______.

Cindy placed an X on the product 36. What are all the possible locations of the two factor markers?

4 and 9 or both markers on 6

Eli placed an O on the product 24. What are all the possible locations of the two factor markers?

3 and 8
4 and 6

Hannah and Sean were playing The Product Game. Hannah needed to land on the product 18 to win the game. The markers were on 4 and 6.

Which factor marker should Hannah move? She should move the marker that’s on the 4.

Where should she place it? On 3

Solve the following problems.

\[ \begin{array}{cccccc}
8 & \times 10 & \quad & 8 & \times 5 & \quad & 4 & \times 6 & \quad & 8 & \times 6 & \quad & 6 & \times 12 & \quad & 8 & \times 12 & \quad & 4 & \times 8 \\
80 & 40 & 24 & 48 & 72 & 96 & 32 \\
\end{array} \]

Answer Key

9 × 6

4 and 9 or both markers on 6

3 and 8

4 and 6

She should move the marker that’s on the 4.

On 3
More Product Game Problems

1. Jack and Connor are playing The Product Game. They are using light and dark markers instead of X’s and O’s to cover their products on the game board.


   Responses will vary. Example: He should move the marker that's on the 7 to the 2, That way, he'd have $2 \times 6 = 12$, and it would give him 4 in a row.

   b. Connor is using the dark markers. What move should he make next? Tell why.

   Move the marker from the 7 to the 3. Then he'd have $3 \times 6 = 18$, and it would give him 4 in a row.

2. Melanie and Jasmine are also playing The Product Game.


   Responses will vary. Example: Move the marker from 5 to 2. Then she'll have $2 \times 3 = 6$, and it would give her 4 in a row.

   b. Jasmine is using the dark markers. What move should she make next? Tell why.

   Move the marker from the 3 to the 4. Then she'll have $4 \times 5 = 20$, and it would give her 4 in a row.

3. Solve the following.

   \[
   \begin{align*}
   11 & \times 8 \quad & \times 4 & \quad & \times 3 & \quad & \times 6 & \quad & \times 4 & \quad & \times 5 & \quad & \times 9 \\
   88 & & 44 & & 21 & & 42 & & 36 & & 45 & & 81
   \end{align*}
   \]
Facts & Boxes

1. To multiply numbers by 5, Kaylee first multiplies by 10 and then finds half the product.

   a. Write an expression with parentheses to show how Kaylee would solve $9 \times 5$.
      
      $$(9 \times 10) \div 2$$

   b. What is $9 \times 5$?
      
      $45$

   c. Marshall says he would rather use $10 \times 5$ to find $9 \times 5$.
      Write an expression with parentheses that uses $10 \times 5$ to find $9 \times 5$.
      
      $$(10 \times 5) - (1 \times 5)$$

Match each expression with the correct box.

2. 4 layers of 3-by-5 cubes $(3 \times 5) \times 4$

3. 4 layers of 3-by-2 cubes $(3 \times 2) \times 4$

4. 4 layers of 3-by-4 cubes $(3 \times 4) \times 4$

5. Fill in the dimensions of this box: $(\underline{6} \times \underline{2}) \times \underline{2}$

6. Solve the following problems.

   $$\begin{align*}
   8 \times 4 &= 32 \\
   8 \times 8 &= 64 \\
   12 \times 10 &= 120 \\
   12 \times 5 &= 60 \\
   3 \times 7 &= 21 \\
   7 \times 6 &= 42 \\
   \end{align*}$$
Fact Connections

1 Fill in the facts. Look for relationships.

$$\begin{array}{cccccc}
3 & 3 & 3 & 6 & 6 & 6 \\
\times 2 & \times 4 & \times 8 & \times 2 & \times 4 & \times 8 \\
6 & 12 & 24 & 12 & 24 & 48 \\
\end{array}$$

2 Use the above information to help you fill in the blanks.

a  $$3 \times 4 = \underline{2} \times (3 \times 2) = \underline{12}$$

b  $$3 \times 8 = \underline{2} \times (3 \times 4) = \underline{24}$$

c  $$6 \times 2 = (3 \times 2) \times \underline{2} = \underline{12}$$

d  $$6 \times 4 = 2 \times (6 \times \underline{2}) = \underline{24}$$

e  $$2 \times (6 \times 4) = \underline{6} \times 8 = \underline{48}$$

3 Fill in the facts. Look for relationships.

$$\begin{array}{cccccc}
4 & 4 & 4 & 8 & 8 & 8 \\
\times 2 & \times 4 & \times 8 & \times 2 & \times 4 & \times 8 \\
8 & 16 & 32 & 16 & 32 & 64 \\
\end{array}$$

4 Use the above information to help you write an equation that includes parentheses.

ex  $$8 \times 4 = 2 \times (8 \times 2)$$ “To find $8 \times 4$, I can double $8 \times 2$.”

Equations may vary. Examples shown:

a  $$4 \times 6 = 2 \times (2 \times 6)$$

b  $$4 \times 12 = 2 \times (4 \times 6)$$

c  $$8 \times 8 = 2 \times (8 \times 4)$$

5 CHALLENGE Complete the following equations.

a  $$4 \times 67 = \underline{2} \times (2 \times 67)$$

b  $$8 \times 198 = 2 \times (\underline{4} \times 198)$$

c  $$\underline{8} \times 3,794 = 2 \times (4 \times 3,794)$$
Are They Equivalent?

1. Mark each of the following equations true or false and tell how you know. Explanations will vary. Examples shown.
   a. $12 \times 6 = 24 \times 3$  
      - T  
      - Because 24 is double 12, and 3 is half of 6.
   b. $7 \times 4 = 28 \times 2$  
      - F  
      - $7 \times 4 = 28$, not $28 \times 2$.
   c. $48 \times 6 = 24 \times 3$  
      - F  
      - Both factors have been cut in half, so the product of $24 \times 3$ is only $\frac{1}{4}$ as much as $48 \times 6$.
   d. $16 \times 4 = 2 \times 32$  
      - T  
      - 2 is half of 4 and 32 is double 16.
   e. $(22 \times 7) \times 59 = 22 \times (7 \times 59)$  
      - T  
      - When you multiply, the order doesn’t matter.

2. Fill in the blank to make each equation true.
   a. $6 \times 7 = 3 \times 14$
   b. $8 \times 5 = 4 \times 10$
   c. $12 \times 16 = 24 \times 8$
   d. $4 \times 8 = 16 \times 2$
   e. $4 \times 13 = 2 \times 26$

3. **Challenge** Thao says she can find the answer to $8 \times 90$ by halving the 8 and doubling the 90 again and again until she gets down to $1 \times 720$. Is she correct? Prove your answer.
   - Yes; proofs will vary. Example:
   - $8 \times 90 = 4 \times 180 = 2 \times 360 = 1 \times 720 = 720$
Smaller Boxes

Brad needs some boxes to hold his smaller orders of 12 baseballs.

1 List all the possible boxes that could hold 12 balls, if each ball takes up a $1 \times 1 \times 1$ space. You can use numbers, labeled sketches, or words to show, but try to use a system where you can be sure you’ve found all the different boxes.

   $(1 \times 12) \times 1$ or $(1 \times 1) \times 12$ or $(12 \times 1) \times 1$
   $(2 \times 6) \times 1$ or $(6 \times 2) \times 1$ or $(2 \times 1) \times 6$ or $(1 \times 2) \times 6$
   $(3 \times 4) \times 1$ or $(4 \times 3) \times 1$ or $(1 \times 4) \times 3$ or $(1 \times 3) \times 4$
   $(2 \times 3) \times 2$ or $(3 \times 2) \times 2$ or $(2 \times 2) \times 3$

2 Match each expression with the correct box below.

   The numbers in parentheses represent the dimensions of the base and the third number represents the height (number of layers).

   a $2 \times 3 \times 5$
   b $3 \times 5 \times 2$
   c $2 \times 5 \times 3$
# Calculating Cardboard

Below is a list of six possible box designs for Brad’s 24 baseballs. Determine how many units of cardboard are needed to construct each box.

<table>
<thead>
<tr>
<th>Box Design</th>
<th>Brad’s Baseballs</th>
</tr>
</thead>
</table>
| $(1 \times 1) \times 24$ | 98 square units; work will vary. Example: $(2 \times (1 \times 1)) + (4 \times (24 \times 1))$  
$= 2 + 96$  
$= 98$ |
| $(1 \times 2) \times 12$ | 76 square units; work will vary. Example: $(2 \times (1 \times 2)) + (2 \times (1 \times 12)) + (2 \times (2 \times 12))$  
$= 4 + 24 + 48$  
$= 76$ |
| $(1 \times 3) \times 8$ | 70 square units; work will vary. Example: $(2 \times (1 \times 3)) + (2 \times (1 \times 8)) + (2 \times (3 \times 8))$  
$= 6 + 16 + 48$  
$= 70$ |
| $(1 \times 4) \times 6$ | 68 square units; work will vary. Example: $(2 \times (1 \times 4)) + (2 \times (1 \times 6)) + (2 \times (6 \times 4))$  
$= 8 + 12 + 48$  
$= 68$ |
| $(2 \times 2) \times 6$ | 56 square units; work will vary. Example: $(2 \times (2 \times 2)) + (4 \times (2 \times 6))$  
$= 8 + 48$  
$= 56$ |
| $(2 \times 3) \times 4$ | 52 square units; work will vary. Example: $(2 \times (2 \times 3)) + (2 \times (2 \times 4)) + (2 \times (4 \times 3))$  
$= 12 + 16 + 24$  
$= 52$ |
Zack’s Strategies

Zack has been working with a variety of multiplication strategies.

1. Write an expression to describe each of the statements Zack made.
   a. To solve $24 \times 15$, I double and halve.
      $12 \times 30$
   b. To solve $14 \times 8$, I find $14 \times 10$ and remove 2 groups of 14.
      $(14 \times 10) - (14 \times 2)$

2. Evaluate the two expressions above (in other words, find the values).
   a. $360$
   b. $112$

3. Fill in the blanks.
   a. $(2 \times 3) \times \underline{5} = 30$
   b. $4 \times (\underline{3} \times 4) = 48$

4. True or False?
   a. $4 \times 9 = (4 \times 10) - 1$ F
   b. $9 \times 13 = (10 \times 13) - (1 \times 9)$ F

5. Solve the following.
   a. $9 \times 3 = \underline{27}$
      $9 \times 30 = \underline{270}$
   b. $15 \times 4 = \underline{60}$
      $15 \times 40 = \underline{600}$
How Many Boxes?

1 Shane is making boxes to hold baseballs. He wants the dimensions to be $3 \times 5 \times 7$ units. How many balls can one of Shane’s boxes hold?

   105 balls; work will vary.

2 Riley is also making boxes to hold baseballs. She wants the dimensions of her boxes to be $4 \times 6 \times 3$ units. How many balls can one of Riley’s boxes hold?

   72 balls; work will vary.

3 Raquel found two boxes in her storeroom. One box has the dimensions $6 \times 2 \times 3$ and the other is $2 \times 3 \times 6$. Which box holds more balls? Explain your thinking.

   They each hold the same number of balls, 36. Explanations will vary.
Work Place Instructions 1B The Multiple Game

Each pair of players needs:
- 2 colored pencils of different colors
- a 1B Multiple Game Record Sheet to share

1 Player 1 circles a target multiple on the game board.
The number must be a multiple of other numbers (not only itself). In the first turn, any number except 1 can be chosen.

2 Player 2 circles all the factors of the target multiple, not including the target multiple itself.
For example, if Player 1 chooses 12 as a target multiple, Player 2 circles 1, 2, 3, 4, and 6 because 12 is a multiple of each.

3 Then, Player 2 chooses and circles a new target multiple, and Player 1 circles all the available (uncircled) factors of Player 2’s target multiple.
Once a number on the game board has been circled, it may not be used again.

4 Players take turns choosing target multiples and circling factors. When the numbers remaining are not multiples of any uncircled numbers (i.e., when no further plays can be made) the game is over.
If a player chooses a target multiple for which there are no factors that can still be circled, the chosen target multiple must be crossed out and the player loses their turn.

5 Each player finds the sum of the numbers that are circled with their color. The greatest total wins.

Game Variations
A A pair of players may play against another pair.
B Players may create a game board that contains numbers greater than 30.
Milo’s Multiples

1. Help Milo find at least three multiples for each number below.  
   Answers will vary. Examples shown.
   - a. 12: 24, 36, 48
   - b. 16: 32, 64, 128
   - c. 23: 46, 92, 184

2. Help Milo find the factors of each of the numbers below.
   - a. 12: 1, 2, 3, 4, 6, 12
   - b. 16: 1, 2, 4, 8, 16
   - c. 23: 1, 23
   - d. 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

3. What factors do 16 and 24 have in common?
   1, 2, 4, and 8

4. What are two multiples that 8 and 16 have in common?
   Answers will vary. Example:
   32 and 64

5. **CHALLENGE** Farah’s mom told her she's thinking of a number that is a multiple of 12. What else can Farah say with certainty about the number her mom is thinking of?
   Responses will vary. Possibilities include:
   • The number is even.
   • The number is divisible by 12.
   • The number is also a multiple of 2.
   • The number is also a multiple of 3 (or 4 or 6)
Thinking About The Multiple Game

1 List the factors for each number below. Write P next to numbers that are prime and C next to numbers that are composite.

a 29: P
b 25: C
c 24: C
d 23: P

2 Which of the above numbers would you choose if you were going first in The Multiple Game? Why?

Answers and explanations will vary. Example: 29 because it’s prime so my partner will only be able to circle the 1.

3 In The Multiple Game, when would be a good time to choose the number 30?

Responses will vary. Example: Near the end of the game when most of its factors have already been taken.

4 Write an expression for each of the calculations below.

a Multiply 3 by 6, and divide by 9. \((3 \times 6) \div 9\)
b Subtract 10 from 30, and then multiply by 20. \((30 - 10) \times 20\)
c Add 13 and 17 and 21, and multiply the sum by 25. \((13 + 17 + 21) \times 25\)
d Divide 36 by 4, then add 45. \((36 \div 4) + 45\)
1 There is a box of 100 miscellaneous balls that Brad wants to sell.

a What are one or two possible sets of dimensions for the box?

Answers will vary. Possibilities include:

\[(1 \times 1) \times 100 \quad (50 \times 1) \times 2 \quad (25 \times 4) \times 1\]
\[(1 \times 2) \times 50 \quad (25 \times 2) \times 2 \quad (10 \times 10) \times 1\]

b What is the total price for the box of balls if Brad charges $20 per ball?

$2,000; work and expressions or equations will vary.
Example: \(20 \times 100 = 2,000\)

c What if he charges $19 per ball?

$1,900; work and expressions or equations will vary.
Example: \(19 \times 100 = (20 \times 100) − (1 \times 100)\)
\[= 1,900\]

d What if he charges $21 per ball?

$2,100; work and expressions or equations will vary.
Example: \(21 \times 100 = (20 \times 100) − (1 \times 100)\)
\[= 2,100\]

2 Brad noticed there are actually only 99 balls in the box.

a What is the total price if Brad charges $20 per ball?

$1,980; work and expressions or equations will vary.
Example: \(99 \times 20 = (100 \times 20) − (1 \times 20)\)
\[= 1,980\]

b What if he charges $19 per ball?

$1,881; work and expressions or equations will vary.
Example: \(99 \times 19 = (100 \times 19) − (1 \times 19)\)
\[= 1,900 − 19 = 1,881\]

c What if he charges $21 per ball?

$2,079; work and expressions or equations will vary.
Example: \(99 \times 21 = (100 \times 21) − (1 \times 21)\)
\[= 2,100 − 21 = 2,079\]

(continued on next page)
3. Brad keeps championship balls on a rack, as shown in the picture of Brad’s Baseball Storeroom.

a. How many balls are on the championship rack?
   36 balls

b. Write an expression to represent how you could quickly find the number of balls on the championship rack without counting every one.
   Expressions will vary. Example: \((4 \times 5) + (4 \times 4)\)

c. What is the total price if Brad charges $25 per championship ball?
   $900; work will vary. Example: \(36 \times 25 = 18 \times 50 = 9 \times 100 = 900\)

4. Brad has a box of 72 bargain baseballs.

a. What is the total price for all the bargain baseballs if each ball is $10?
   $720; \(72 \times 10 = 720\)

b. What is the total price for the bargain baseballs if each ball is $9?
   $648; \(72 \times 9 = (72 \times 10) - (72 \times 1) = 720 - 72\)

c. If Brad charges $792 for the whole box, how much is each ball?
   $11; work will vary. Example:

   \[
   \frac{720}{720} \quad 720 \quad 792
   \]

   \[
   \frac{72}{72} \quad 10 \quad 11
   \]

d. If Brad charges $648 for the whole box, how much is that for each ball?
   $9; work will vary. Example:

   \[
   \frac{720}{72} \quad 648
   \]

5. CHALLENGE. There is a box of Blue Bombers that contains 72 baseballs. What are the dimensions of all of the possible boxes that contain 72 baseballs? Which one do you think is pictured in the storeroom?

All possible combinations:
- \((1 \times 72) \times 1\) or \((1 \times 1) \times 72\)
- \((1 \times 2) \times 36\) or \((1 \times 36) \times 2\) or \((36 \times 2) \times 1\)
- \((2 \times 2) \times 18\) or \((18 \times 2) \times 2\)
- \((2 \times 3) \times 12\) or \((2 \times 12) \times 3\) or \((3 \times 12) \times 2\)
- \((2 \times 4) \times 9\) or \((2 \times 9) \times 4\) or \((4 \times 2) \times 2\)
- \((2 \times 6) \times 6\) or \((6 \times 6) \times 2\)
- \((3 \times 4) \times 6\) or \((3 \times 6) \times 4\) or \((6 \times 4) \times 3\)
- \((3 \times 3) \times 8\) or \((3 \times 8) \times 3\)

Responses to second question will vary. Example: The front face of the box is square, so it’s either \((2 \times 18) \times 2\) or \((3 \times 8) \times 3\).
Lily’s Lacrosse Team

Lily is the manager of her school’s lacrosse team. Help Lily keep track of the team’s equipment. Show your work using numbers, sketches, or words.

1. Lily brought this box of lacrosse balls to practice on Monday.

   ![Box of lacrosse balls]

   a. How many lacrosse balls does the box hold if one lacrosse ball fits in a 1 unit × 1 unit × 1 unit space?

      728 balls; work will vary.

   b. How much cardboard does it take to make the box?

      502 square units; work will vary. Example:
      \[
      (2 \times (13 \times 8)) + (2 \times (7 \times 8)) + (2 \times (13 \times 7)) \\
      = (2 \times 104) + (2 \times 56) + (2 \times 91) = 208 + 112 + 182 = 502
      \]

2. Lily needs to buy 110 new lacrosse balls for the team. The balls come in sets of 22.

   a. How many sets of lacrosse balls should Lily buy?

      5 sets; work will vary.

   b. If one set of 22 lacrosse balls costs $20, how much will 110 lacrosse balls cost?

      $100; work will vary.

3. Is each equation true or false?

   a. \[98 \times 34 = (100 \times 34) – (1 \times 34)\]  \[F\]

   b. \[46 \times 28 = 23 \times 56\]  \[T\]
Sam’s Sewing Supplies

1. Sam needs more thread for a sewing project. One spool of thread costs 72 cents. Fill out the ratio table below to find out how much 12 spools of thread cost.

<table>
<thead>
<tr>
<th>Spools of thread</th>
<th>1</th>
<th>2</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>72</td>
<td>144</td>
<td>720</td>
<td>864</td>
</tr>
</tbody>
</table>

a. How much do 12 spools of thread cost?

$8.64

b. How would you figure out how much 24 spools of thread cost?

Responses will vary. Example:
Double the cost for 12 spools of thread.

Answers will vary. Example:

$$(12 \times 72) \times 2$$

2. Sam saw a sign advertising thread on sale. The sign said, “Thread Sale! 12 spools for 840 cents, 15 spools for 900 cents, and 18 spools for 990 cents.”

Which is the best deal? Why? Show your thinking.

18 spools for 990 cents; explanations and work will vary.

3. Write an expression for the calculation.

ex To find 18 times 35, I double 35 and halve 18:

$$\left(35 \times 2\right) \times \left(\frac{1}{2}\right) \times \left(18\right) \text{ or } \left(35 \times 2\right) \times \left(18 \div 2\right)$$

a. To find 38 times 14, I multiply 30 times 14 and 8 times 14 and add the two products together.

$$\left(30 \times 14\right) + \left(8 \times 14\right)$$
Charlie’s Chocolates

Charlie is filling boxes with handmade chocolates. She starts with the following boxes.

Expressions and equations will vary somewhat. Examples shown:

1. Charlie fills a box with 5 layers. Each layer has 3 rows of 5 chocolates.
   
   a. Write an expression that shows how many chocolates are in the box.
      
      \((3 \times 5) \times 5\)
   
   b. Write an equation shows how many chocolates are in the box.
      
      \((3 \times 5) \times 5 = 75\)
   
   c. Charlie’s brother dropped his baseball on the box and broke 6 chocolates, which had to be removed. Write an expression that shows how many chocolates are in the box now.
      
      \(((3 \times 5) \times 5) - 6\)
   
   d. Write an equation that shows how many chocolates are in the box now.
      
      \(((3 \times 5) \times 5) - 6 = 69\)

2. Charlie fills 2 more boxes with chocolates. These boxes have 7 layers of chocolates with 12 chocolates in each layer. Then, Charlie puts both boxes together in a larger box.

   a. Write an expression that shows how many chocolates are in the larger box.
      
      \(2 \times (7 \times 12)\)
   
   b. Write an equation that shows how many chocolates are in the larger box.
      
      \(2 \times (7 \times 12) = 168\)
Work Place Instructions 1C Beat the Calculator

Each pair of players needs:

- a set of Beat the Calculator Cards to share
- scratch paper and pencil (optional)
- 1 calculator to share

Some calculators will not work for this game. Check the calculator you want to use by entering \(1 + 3 \times 2 =\). If the answer shown is 7, that calculator will work for this game. If the answer shown is 8, you’ll need to find a different calculator.

1. One player shuffles the cards and places the deck face-down. Players decide which of them will start with the calculator, and they decide on the number of rounds they will play.

2. The player with the calculator turns over a card so both players can see it.

3. The player with the calculator enters the problem exactly as it is written on the card. If the calculator doesn’t have parentheses, the player should enter the parts in parentheses first, then \(=\) to get a result. They must then clear the calculator and enter the rest of the problem using the result in place of the part in parentheses.

4. At the same time, the other player evaluates the expression using an efficient strategy, either mentally or with paper and pencil.

5. The player who gets the correct answer first keeps the card.

6. Players compare answers and share strategies for evaluating the expression.

7. Players switch roles and draw again. (The player who didn’t have the calculator has it now.)

8. The game continues for an agreed-upon number of rounds. The player with the most cards at the end wins.

Game Variations

A. Players write their own problems on cards, mix them up, and then choose from those problems.

B. Instead of racing the calculator, students race each other to find the answer mentally, and check the answer to be sure it’s correct using a calculator if necessary.

C. Players play cooperatively by drawing a card and discussing their preferred mental strategy.

D. Players spread the cards face-down on the table. Each student chooses a different card at the same time and then races to see who gets the correct answer first.
Expressions & Equations

1. Write a numerical expression that includes grouping symbols for each:
   
   a. To find $8 \times 17$, I double and halve.
      
      $$(8 \div 2) \times (17 \times 2)$$
   
   b. To find $36 \times 19$, I find 36 times 20 and remove 1 group of 36.
      
      $$(20 \times 36) - (1 \times 36)$$
   
   c. To find the volume of a box that has a 19 by 22 base and 27 layers, I multiply the area of the base times the height.
      
      $$(19 \times 22) \times 27$$

2. Write an equation for each:
   
   a. To find 7 times 32, I double and halve.
      
      $$(2 \times 7) \times (32 \div 2) = 14 \times 16 = 224$$
   
   b. To find 26 times 13, I multiply 20 times 13 and add it to 6 times 13.
      
      $$(20 \times 13) + (6 \times 13) = 338$$
   
   c. To find 98 times 54, I multiply 100 times 54 and subtract 2 times 54.
      
      $$(100 \times 54) - (2 \times 54) = 5,292$$

3. Show your work for each problem.

   a. Xavier counted 38 balls in one layer of a box. The box has 17 layers. How many balls can the box hold?
      
      **646 balls; work will vary.**

   b. A box holds 448 balls. Each layer has 28 balls. How many layers does the box have?
      
      **16 layers; work will vary.**
Multiplication & Division
Mathematical Thinking

1 Katie says she can multiply $5 \times 68$ by multiplying $10 \times 68$ and dividing the answer in half.

a Do you agree or disagree? Explain your thinking.

Katie is correct; explanations will vary.

b Write an expression that shows Katie’s thinking. Use parentheses.

$\frac{10 \times 68}{2}$

2 Henry says he can multiply $99 \times 57$ by multiplying $100 \times 57$ and then adding one more $57$.

a Do you agree or disagree? Explain your thinking.

Henry is incorrect; explanations will vary.

b Write an expression that shows Henry’s thinking. Use parentheses.

$(100 \times 57) + (1 \times 57)$

3 Paris is packing a crate of flowerpots. She puts 2 rows of 6 flowerpots in each layer of a box. There are 2 layers in the box.

a Write an expression that shows how many flowerpots are in the box.

$(2 \times 6) \times 2$

b Write an equation to show how many flowerpots are in the box.

$(2 \times 6) \times 2 = 24$
Division Story Problems page 1 of 2

For each problem on this page and the next,

- Write an equation, including parentheses if needed, to match the situation.
- Find the answer. You can use base ten pieces to model and solve the part of the problem that requires division.
- Label your answer with the correct units.
- Explain what you did with the remainder and why.

1 Josh and his three friends baked cookies last Saturday. When they were finished, they had 65 cookies. Each of the 4 friends ate 2 cookies right away and divided the rest equally to take home. How many cookies did each of the friends get to take home?

\[(65 - (4 \times 2)) \div 4 = 14 \frac{1}{4} \text{ cookies}\]

Explanations about the remainder will vary somewhat. Example: There was 1 cookie left over, and you can cut up a cookie, so they each got \(\frac{1}{4}\) of the extra cookie.

2 Sara and 5 of her friends did chores all Saturday and earned $75.00. Sara’s dad was so pleased with their work that he gave each of the 6 children a $2.00 tip in addition to the $75.00. They shared the work equally, so they want to share all the money equally. How much money will each person get?

\[(75 + (6 \times 2)) \div 6 = $14.50\]

3 Mrs. O’Donnell is taking 36 fifth graders and 7 parent helpers on a field trip. She wants to give each person, including herself, 2 granola bars for snacks. If there are 5 granola bars in a box, how many boxes will she need to buy?

\[(2 \times (36 + 8)) \div 5 = 17 \text{ R3},\]

So she needs to buy 18 boxes of granola bars. She needs 88 bars. If you divide 88 by 5, it comes out to 17 with 3 left over, so she has to buy an extra box to get the last 3 bars.

(continued on next page)
**Division Story Problems** page 2 of 2

4 Eighty-nine kids showed up for the first soccer practice. The coaches organized them into groups of 4 for warm-up exercises. How many groups were there?

22 groups. Reasoning will vary. Example: $89 \div 4 = 22 \text{ R1}$, so they’ll need to put 5 kids in one group, and 4 in the other groups. When you divide 89 by 4, there’s 1 left over. It’s easiest to have the leftover kid join one of the other groups.

5 Jamal and three of his friends raked leaves every weekend for a month. By the end of the month, they earned $91.00. They paid $4.00 to Jamal’s little brother for helping out a little on the last day. Then the four friends divided the rest of the money equally. How much did each friend get?

$21.75. Reasoning may vary. Example: \((91.00 - 4.00) \div 4 = 21.75\)

When you divide 87 by 4, you get 3 left over. You can split $3.00 into 4 sets of 75¢ because 

\[ .75 + .75 = 1.50 \text{ and } 1.50 + 1.50 = 3.00 \]

6 The drivers at Pizza Palace were loading up their 3 delivery vans for the evening. There were 65 pizzas. The boss told them to put an equal number of pizzas in each van and said they could split any leftovers equally. How much of a leftover pizza did each of the 3 drivers get?

\[ 65 \div 3 = 21 \text{ R}2 \]

Each driver got \(\frac{2}{3}\) of a pizza, because \(2 \div 3 = \frac{2}{3}\)
What Should You Do with the Remainder?

1. For each problem below:
   - Use numbers, words, or labeled sketches to solve the problem
   - Figure out the best way to handle the remainder for that situation
   - Write an equation to show each problem and the answer

**Story Problem**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td>Four friends made 55 cupcakes and shared them equally. How many cupcakes did each friend get?</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
|   | **13 ¾ cupcakes each**  
|   | **55 ÷ 4 = 13 R3, but you can cut cupcakes, so they can each have another ¾.** |
| Your Work: |  |
| **b** | There are 55 kids in the After-School Club. Tomorrow they are going to the zoo. If each car can carry 4 kids, how many cars will they need to get to the zoo? |
|   |  
|   | **14 cars**  
|   | **55 ÷ 4 = 13 R3, but you can’t leave the last 3 kids behind so they need an extra car.** |
| Your Work: |  |
| **c** | Emma and her 3 friends did chores for Emma’s dad on Saturday. They earned $55.00 and split it evenly. How much money did each of the 4 children get? |
|   |  
|   | **$13.75 each**  
|   | **55 ÷ 4 = 13 R3, but you can divide $3.00 into 12 quarters, and give each friend 3 of them, so they each get $13.75.** |
| Your Work: |  |
Race Car Problems

1. Race cars can drive about 5 miles on 1 gallon of gasoline. If a race car goes 265 miles in one race, about how many gallons of gasoline will it use? Show all your work. Note: You can use the base ten area and linear pieces to help solve this problem.

   53 gallons of gasoline; work will vary.

2. There were 43 cars in the race. They all finished the 265 miles of the race and they each used about 1 gallon of gas to go 5 miles. About how many gallons of gas did the cars use altogether to finish the race? Show all your work.

   2,279 gallons of gasoline; work will vary.
Quotients Win, Game 2

Red Team _________________________  Blue Team _________________________

1. $140 \div 10 = \underline{14}$  
2. $300 \div 20 = \underline{15}$

3. $100 \div 20 = \underline{5}$  
4. $150 \div 15 = \underline{10}$

5. $100 \div 10 = \underline{10}$  
6. $260 \div 10 = \underline{26}$

Red Score _________________________  Blue Score _________________________
Work Place Instructions 1D Quotients Win

Each pair of players needs:
- 2 copies of one of the 1D Quotients Win Record Sheets (there are 6 different sheets; make sure you each get a copy of the same one)
- 2 dice numbered 1–6
- 1 set of base ten area pieces
- 1 set of base ten linear pieces
- 1 red and 1 blue colored pencil or fine-tip felt marker

1. Each player gets 1 die and rolls it at the same time. If they both get the same number, they roll again until they have different numbers.

2. Each player solves the problem that has the same number as the number they rolled. Players solve their problems at the same time.

3. Each player makes a labeled sketch of the problem on the record sheet and fills in the answer, using their colored pencil or marker to sketch the dimensions and a regular pencil for the rest of the work. Players can build a model with base ten pieces first, but they don’t have to.

   Alisha I rolled a 2, so I have to do problem 2 on the game sheet. That’s 130 ÷ 10. First I’ll lay out a linear strip to show 10 and then start fitting in base ten pieces until I get to 130. My rectangle turned out to be 13 along the other side, so that’s the answer. Now I have to make a sketch.

4. Players roll and solve the problems until they have each solved 3. If they roll the number of a problem that has already been solved, they roll again until they get the number of a problem that has not been solved yet. (Players must use the first number that has not been solved). When they are done, they check each other’s work.

5. At the end of the game, players add their quotients and record their score at the bottom of the sheet. The player with the higher score wins.

Game Variations

A. Players take turns instead of both going at once. They sketch and solve their partner’s problems as well as their own on their record sheet. They work together to make sure their answers are correct.

B. Players use Record Sheets 5 and 6, which are much more challenging.
Multiplication & Division Problems

1. Fill in the missing numbers.

\[
\begin{array}{cccc}
6 & \times & 8 & \times 8 \\
\times & 2 & \times 5 & \times 6 \\
\end{array}
\]

\[
\begin{array}{cccc}
6 \times 8 = 48 \\
8 \times 5 = 40 \\
9 \times 6 = 54 \\
4 \times 7 = 28 \\
\end{array}
\]

\[
\begin{array}{cccc}
5 & \times & 9 & \times 7 \\
\times & 6 & \times 4 & \times 7 \\
\end{array}
\]

\[
\begin{array}{cccc}
5 \times 9 = 45 \\
6 \times 7 = 42 \\
7 \times 4 = 28 \\
8 \times 7 = 56 \\
\end{array}
\]

2. Write an equation to answer each question below.

<table>
<thead>
<tr>
<th>Question</th>
<th>Equation</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>a How many cartons of 12 eggs make 36 eggs in all?</td>
<td>(36 \div 12 = 3)</td>
<td>3 cartons</td>
</tr>
<tr>
<td>b There are 6 cans of soda in a pack. How many packs make 42 cans?</td>
<td>(42 \div 6 = 7)</td>
<td>7 packs</td>
</tr>
<tr>
<td>c There are 24 cans of soda in a case. How many cases make 72 cans?</td>
<td>(72 \div 24 = 3)</td>
<td>3 cases</td>
</tr>
<tr>
<td>d There are 3 tennis balls in a can. How many cans make 27 balls?</td>
<td>(27 \div 3 = 9)</td>
<td>9 cans</td>
</tr>
<tr>
<td>e Jim rides his bike at 10 miles per hour. How many hours will it take him to ride 30 miles?</td>
<td>(30 \div 10 = 3)</td>
<td>3 hours</td>
</tr>
</tbody>
</table>
Using Basic Facts to Solve Larger Problems

Knowing the basic multiplication and division facts can help you multiply larger numbers too. Start with the basic facts below and then complete the related fact family of larger numbers. Then make up your own fact family based on other related numbers.

<table>
<thead>
<tr>
<th>Basic Fact Family</th>
<th>Related Fact Family</th>
<th>Your Own Related Fact Family</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ex</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ \underline{4} \times \underline{3} = 12 ]</td>
<td>[ 40 \times 3 = 120 ]</td>
<td>[ \underline{40} \times \underline{30} = 1,200 ]</td>
</tr>
<tr>
<td></td>
<td>[ 3 \times 4 = 12 ]</td>
<td>[ 30 \times 40 = 1,200 ]</td>
</tr>
<tr>
<td></td>
<td>[ \underline{12} \div \underline{4} = 3 ]</td>
<td>[ 120 \div 40 = 3 ]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ \underline{1,200} \div \underline{30} = 40 ]</td>
</tr>
<tr>
<td>1</td>
<td>[ \underline{8} \times \underline{6} = 48 ]</td>
<td>[ 80 \times 6 = 480 ]</td>
</tr>
<tr>
<td></td>
<td>[ 6 \times 8 = 48 ]</td>
<td>[ 6 \times 80 = 480 ]</td>
</tr>
<tr>
<td></td>
<td>[ \underline{48} \div \underline{8} = 6 ]</td>
<td>[ 480 \div 80 = 6 ]</td>
</tr>
<tr>
<td></td>
<td>[ 48 \div 6 = 8 ]</td>
<td>[ 480 \div \underline{6} = 80 ]</td>
</tr>
<tr>
<td>2</td>
<td>[ \underline{4} \times \underline{9} = 36 ]</td>
<td>[ 40 \times 9 = 360 ]</td>
</tr>
<tr>
<td></td>
<td>[ 9 \times 4 = 36 ]</td>
<td>[ \underline{9} \times \underline{40} = 360 ]</td>
</tr>
<tr>
<td></td>
<td>[ \underline{36} \div \underline{4} = 9 ]</td>
<td>[ 360 \div 40 = 9 ]</td>
</tr>
<tr>
<td></td>
<td>[ 36 \div 9 = 4 ]</td>
<td>[ 360 \div \underline{9} = 40 ]</td>
</tr>
<tr>
<td>3</td>
<td>[ \underline{3} \times \underline{7} = 21 ]</td>
<td>[ 30 \times 7 = 210 ]</td>
</tr>
<tr>
<td></td>
<td>[ 7 \times 3 = 21 ]</td>
<td>[ \underline{7} \times \underline{30} = 210 ]</td>
</tr>
<tr>
<td></td>
<td>[ \underline{21} \div \underline{3} = 7 ]</td>
<td>[ 210 \div 30 = 7 ]</td>
</tr>
<tr>
<td></td>
<td>[ 21 \div 7 = 3 ]</td>
<td>[ 210 \div \underline{7} = 307 ]</td>
</tr>
</tbody>
</table>
Money & Fractions

Note: You can use the money value pieces to help solve these problems if you like.

1. Write each amount as a decimal.
   a. 2 quarters = $0.50
   b. 3 dimes and 5 pennies = $0.35

2. Write each amount as a fraction of a dollar.
   a. 3 quarters = \(\frac{3}{4}\) or \(\frac{75}{100}\) of a dollar
   b. 7 dimes = \(\frac{7}{10}\) or \(\frac{70}{100}\) of a dollar

3. Use numbers, labeled sketches, or words to show your work.
   a. Mila has \(\frac{1}{2}\) of a dollar. Claire has \(\frac{1}{4}\) of a dollar. How much money do the girls have together? Record your answer as a fraction and as a decimal.
      \[
      \frac{3}{4} \text{ of a dollar; } 0.75 \\
      \text{Work will vary}
      \]
   b. Henry has \(1\frac{1}{4}\) dollars. Angel has \(1\frac{1}{2}\) dollars. How much money do the boys have together? Record your answer as a fraction and as a decimal.
      \[
      2\frac{3}{4} \text{; } 2.75 \\
      \text{Work will vary}
      \]
   c. \textbf{CHALLENGE} Iris has \(1\frac{3}{5}\) dollars. Violet has \(2\frac{5}{10}\) dollars. How much money do the girls have together? Record your answer as a fraction and as a decimal.
      \[
      4\frac{5}{10} \text{; } 4.10 \\
      \text{Work will vary}
      \]
Fractions & Mixed Numbers

1 Color in the strips to show the fractions named below. Each strip represents 1 whole.

ex \( \frac{1}{4} \) \[ \text{strip colors} \]
\( a \frac{3}{8} \)
\( b \frac{1}{2} \)
\( c \frac{3}{4} \)

2 Color in the strips to show the improper fractions named below. The write the fraction as a mixed number. Each strip represents 1 whole.

ex \( \frac{7}{4} \)
\( a \frac{12}{8} \)
\( b \frac{3}{2} \)
\( c \frac{9}{8} \)

3 Fill in the blanks to show the unit fraction as a fraction of a dollar and as decimal (money) notation.

ex \( \frac{1}{10} = \frac{10}{100} = 0.10 \)
\( a \frac{1}{2} = \frac{50}{100} = 0.50 \)
\( b \frac{1}{4} = \frac{25}{100} = 0.25 \)
\( c \frac{3}{4} = \frac{75}{100} = 0.75 \)
\( d \frac{7}{10} = \frac{70}{100} = 0.70 \)

Write in your math journal using numbers, labeled sketches, or words to explain your answer to the two problems below. (Hint: Use money value pieces to help.)

4 Esther had to solve \( \frac{1}{2} + \frac{1}{4} \). She wrote: $0.05 + $0.75 = $0.80, which is the same as \( \frac{80}{100} \) of a dollar. So \( \frac{1}{2} + \frac{1}{4} = \frac{80}{100} \). Do you agree or disagree with her work?

Disagree; explanations will vary.

5 Thanh had to solve \( \frac{1}{10} + \frac{1}{5} \). He wrote: $0.10 + $0.20 = $0.30, which is the same as \( \frac{3}{10} \) of a dollar, so \( \frac{1}{10} + \frac{1}{5} = \frac{3}{10} \). Do you agree or disagree with his work?

Agree; explanations will vary.
### Fractions on a Clock Face

#### 60 minutes in an hour

- **1/2 = 30 minutes** = 30/60
  - 6 sets of 5 min.
  - 3 sets of 10 min.
  - 2 sets of 15 min.

#### 12 sets of 5 minutes

- **1/4 = 15 minutes** = 15/60
  - 3 sets of 5 min.
  - 1 1/2 sets of 10 min.

- **1/3 = 20 minutes** = 20/60
  - 4 sets of 5 min.
  - 2 sets of 10 min.

#### 12 sets of 5 min.

- **1/6 = 10 minutes** = 10/60
  - 2 sets of 5 min.
  - 1 set of 10 min.

- **1/12 = 5 minutes** = 5/60
  - 1 set of 5 min.
  - 6 sets of 10 min.
Clock Face Fractions

1. Color in the clock to show the fractions below. Each clock represents 1 whole.
   a. $\frac{1}{2}$  
   b. $\frac{1}{4}$  
   c. $\frac{2}{6}$  
   d. $\frac{10}{6}$  
   e. $\frac{5}{3}$

2. Use the pictures above to help complete each comparison below using $<$, $>$, or $=$.
   Ex: $\frac{1}{2} > \frac{5}{12}$  
   a. $\frac{6}{4} = 1\frac{1}{2}$  
   b. $\frac{5}{6} > \frac{5}{12}$  
   c. $\frac{10}{6} > 1\frac{1}{2}$  
   d. $\frac{6}{2} > \frac{6}{4}$  
   e. $\frac{3}{6} < \frac{2}{3}$

3. Subtract these fractions. (Hint: Think about money or clocks to help.)
   a. $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$  
   b. $\frac{3}{4} - \frac{1}{10} = \frac{65}{100}$ or $\frac{26}{40}$ or $\frac{13}{20}$
   c. $1 - \frac{1}{6} = \frac{5}{6}$  
   d. $2 - \frac{1}{4} = \frac{3}{4}$

4. A certain fraction is greater than 2. The denominator is 8. What must be true about the numerator? Explain your answer.
   ?  
   The numerator must be greater than $\frac{16}{8}$ because:
   Explanations will vary.
Clock Fractions Problem String

Problem 1: ________________________

\[
\begin{array}{ccc}
\bullet & + & \bullet \\
& & \\
\bullet & = & \bullet \\
\end{array}
\]

Problem 2: ________________________

\[
\begin{array}{ccc}
\bullet & + & \bullet \\
& & \\
\bullet & = & \bullet \\
\end{array}
\]

Problem 3: ________________________

\[
\begin{array}{ccc}
\bullet & + & \bullet \\
& & \\
\bullet & = & \bullet \\
\end{array}
\]

Problem 4: ________________________

\[
\begin{array}{ccc}
\bullet & + & \bullet \\
& & \\
\bullet & = & \bullet \\
\end{array}
\]

Problem 5: ________________________

\[
\begin{array}{ccc}
\bullet & + & \bullet \\
& & \\
\bullet & = & \bullet \\
\end{array}
\]

Problem 6: ________________________

\[
\begin{array}{ccc}
\bullet & + & \bullet \\
& & \\
\bullet & = & \bullet \\
\end{array}
\]
Work Place Instructions 2A Clock Fractions

Each pair of players needs:

- Two 2A Clock Fractions Record Sheets
- 1 spinner overlay
- colored pencils in several colors
- regular pencils

1. Player 1 spins both spinners and writes the two fractions as an addition expression under the words “Equations for Each Turn.”

2. Then Player 1
   - shades in both fractions on his first clock, using a different color for each fraction.
   - labels each fraction
   - records the sum of the two fractions to finish the equation for that turn.

   Three fourths is the same as nine twelfths, so I’m shading in nine twelfths red. One sixth is the same as two twelfths, so I’m shading in two twelfths green. These two shaded parts add up to eleven twelfths, so \( \frac{3}{4} + \frac{1}{6} = \frac{11}{12} \).

3. Both players check the work to make sure that Player 1 shaded and labeled the fractions and wrote the equation correctly.

4. Player 2 takes her turn and both players check her work.

5. Players do not move on to the next clock until a clock is completely filled. However, if a clock is nearly filled and a player spins a fraction that is too big for it, the player can split the fraction to complete the first clock and put the rest of the fraction in the next clock.

6. When a clock is completely filled, players write an equation that shows the fractions in the clock on the line underneath the clock.

7. The first player to completely fill all three clocks wins the game.

   If a player spins a fraction that is too big for the third clock, she loses that turn. Players must fill the last clock with the exact fraction needed. For example, if a player’s third clock has \( \frac{1}{12} \) to fill in, the player has to spin \( \frac{1}{12} \) to complete the clock.

Game Variations

A. Players can shorten the game by filling only two clocks or lengthen the game by drawing another clock.

B. Players can work together to complete one record sheet, discussing each move and representation.
# Adding Fractions

1. Show the fractions on the strips. Then add them and report the sum.

<table>
<thead>
<tr>
<th></th>
<th>First</th>
<th>Second</th>
<th>Add Them</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>(\frac{3}{4})</td>
<td>(\frac{3}{4})</td>
<td>(\frac{1}{4})</td>
<td>(1 \frac{1}{2}) or (1\frac{1}{2})</td>
</tr>
<tr>
<td>b</td>
<td>(\frac{3}{8})</td>
<td>(\frac{1}{2})</td>
<td>(\frac{7}{8})</td>
<td>(\frac{7}{8})</td>
</tr>
<tr>
<td>c</td>
<td>(\frac{5}{8})</td>
<td>(\frac{3}{4})</td>
<td>(\frac{1}{8})</td>
<td>(1 \frac{3}{8})</td>
</tr>
<tr>
<td>d</td>
<td>(\frac{1}{2})</td>
<td>(\frac{7}{8})</td>
<td>(\frac{1}{8})</td>
<td>(1 \frac{3}{8})</td>
</tr>
</tbody>
</table>

2. Model each problem on a clock to add the fractions. Remember to label your work.

- **a** \(\frac{1}{2} + \frac{1}{6} = \frac{4}{6}\) or \(\frac{2}{3}\)

- **b** \(\frac{2}{3} + \frac{1}{6} = \frac{5}{6}\)

- **c** \(\frac{1}{3} + \frac{2}{6} = \frac{4}{6}\) or \(\frac{2}{3}\)

- **d** \(\frac{2}{3} + \frac{5}{6} = \%\) or \(1\frac{1}{6}\) or \(1\frac{1}{2}\)
**Equivalent Fractions on a Clock**

This clock is broken! The hour hand is stuck at the 12, but the minute hand can still move.

1. Marcus looked at the clock shown above and said, “\(\frac{1}{3}\) of an hour has passed.” Sierra said, “\(\frac{3}{12}\) of an hour has passed.” Ali said, “\(\frac{15}{60}\) of an hour has passed.” Their teacher said they were all correct. Explain how this could be possible.

   **Explanations will vary. (\(\frac{1}{4}\), \(\frac{3}{12}\), and \(\frac{15}{60}\) are equivalent fractions.)**

2. Label each clock with at least 3 equivalent fractions to show what part of an hour has passed.

   ![Clock Images]

   - a: \(\frac{10}{60}\), \(\frac{2}{12}\), \(\frac{1}{6}\)
   - b: \(\frac{15}{60}\), \(\frac{3}{12}\), \(\frac{1}{4}\)
   - c: \(\frac{20}{60}\), \(\frac{4}{12}\), \(\frac{1}{3}\)
   - d: \(\frac{30}{60}\), \(\frac{6}{12}\), \(\frac{2}{4}\), \(\frac{1}{2}\)
   - e: \(\frac{40}{60}\), \(\frac{8}{12}\), \(\frac{2}{3}\)
   - f: \(\frac{45}{60}\), \(\frac{9}{12}\), \(\frac{3}{4}\)
   - g: \(\frac{50}{60}\), \(\frac{10}{12}\), \(\frac{5}{6}\)
   - h: \(\frac{60}{60}\), \(\frac{12}{12}\), \(\frac{4}{4}\), \(\frac{3}{3}\), \(\frac{1}{2}\)
### Adding Fractions

1. Each bar below is divided into 12 equal pieces. Show each fraction on a fraction bar.

   - **ex** \(\frac{1}{3}\)
   - **a** \(\frac{2}{3}\)
   - **b** \(\frac{1}{4}\)
   - **c** \(\frac{3}{4}\)
   - **d** \(\frac{1}{2}\)
   - **e** \(\frac{5}{6}\)

2. Rewrite each pair of fractions so that they have the same denominator. Then use the fraction bar pictures to show their sum. Write an equation to show both fractions and their sum.

<table>
<thead>
<tr>
<th>Fractions to Add</th>
<th>Rewrite with Common Denominator</th>
<th>Picture and Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ex</strong> (\frac{2}{3} + \frac{1}{2})</td>
<td>(\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6})</td>
<td><img src="image1" alt="Equation Picture" /></td>
</tr>
<tr>
<td><strong>a</strong> (\frac{2}{3} + \frac{3}{4})</td>
<td>(\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12})</td>
<td><img src="image2" alt="Equation Picture" /></td>
</tr>
<tr>
<td><strong>b</strong> (\frac{1}{3} + \frac{5}{6})</td>
<td>(\frac{1}{3} + \frac{5}{6} = \frac{2}{6} + \frac{5}{6})</td>
<td><img src="image3" alt="Equation Picture" /></td>
</tr>
<tr>
<td><strong>c</strong> (\frac{7}{12} + \frac{3}{4})</td>
<td>(\frac{7}{12} + \frac{3}{4} = \frac{7}{12} + \frac{9}{12})</td>
<td><img src="image4" alt="Equation Picture" /></td>
</tr>
</tbody>
</table>

Example solutions:
- **ex**: \(\frac{4}{6} + \frac{3}{6} = \frac{7}{6}\) or \(1 \frac{1}{6}\)
- **a**: \(\frac{8}{12} + \frac{9}{12} = \frac{17}{12}\) or \(1 \frac{5}{12}\)
- **b**: \(\frac{2}{6} + \frac{5}{6} = \frac{7}{6}\) or \(1 \frac{1}{6}\)
- **c**: \(\frac{7}{12} + \frac{9}{12} = \frac{16}{12}\) or \(1 \frac{4}{12}\)
Work Place Instructions 2B Racing Fractions

Each pair of players needs:
- 1 Racing Fractions Game Board
- 1 Racing Fractions Record Sheet
- 1 deck of Racing Fractions Cards
- 8 red and 8 blue game markers

1. Each player places 1 game marker at 0 on each of the eight fraction number lines on the game board.
2. Each player chooses a fraction card. The player with the larger fraction goes first.
3. Player 1 takes a new fraction card and then moves one or more game markers the total distance shown on the card.
   
   Student: I got \(\frac{4}{5}\), so I could move \(\frac{4}{5}\) on the fifths track. But I’m really close to 2 on the tenths track, and \(\frac{4}{5}\) is like \(\frac{8}{10}\). I can use \(\frac{2}{10}\) to get to the 2. Then I still have \(\frac{3}{5}\) left to move my marker on the fifths track!

4. The player writes the fraction from the fraction card in the left-hand column of her table and writes the fraction or equation that describes how the game pieces were moved in the right-hand column.
   - If the player selected \(\frac{4}{5}\) and moved \(\frac{4}{5}\), she would write \(\frac{4}{5}\). If the player selected \(\frac{4}{5}\) and moved one marker \(\frac{2}{10}\) and another marker \(\frac{3}{5}\), she would write \(\frac{2}{10} + \frac{3}{5} = \frac{4}{5}\).

5. Player 2 checks Player 1’s work. If Player 2 disagrees, he has to convince Player 1 of his reasoning and Player 1 gets to try again.

6. Then, Player 2 takes a turn and Player 1 checks his work.

7. Players continue to take turns, record moves, and check each other’s work until all of one player’s game markers are on the 2s.
   - If there is no possible move, the player loses the turn.

Game Variation

A. Players play cooperatively and work together to help each other finish the track in a certain time period.
**Fraction Story Problems**

1a  Measure the line below and make a mark along the line to show exactly where each of these fractions belongs. Be sure to label each mark with the name of the fraction.

\[
\frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}
\]

\[0 \quad \frac{1}{6} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{3}{4} \quad 1\]

b  Explain how you figured out where to place each fraction along the line.

*Explanations will vary.*

2  Yesterday Carson recycled \(1\frac{1}{3}\) pounds of paper packaging. He also recycled \(\frac{3}{4}\) of a pound of plastic packaging. Combined, how many pounds of packaging did Carson recycle yesterday? Show all your work.

\(2 \frac{1}{2}\) pounds of packaging. Work will vary.

3  Carmen ran \(1\frac{3}{8}\) miles yesterday. Her sister Lola ran \(2\frac{1}{4}\) miles. How much farther did Lola run than Carmen? Show all your work.

\(\frac{7}{8}\) mile; work will vary.
Double Number Line  page 1 of 2

1  For each of the following:
   - Draw a line from each equation to the matching double number line.
   - Fill in the blanks on the last three double number lines.
   - Record the answer to each equation.

\[
\frac{2}{9} + \frac{1}{5} = \frac{19}{45}
\]

\[
\frac{3}{8} + \frac{2}{6} = \frac{17}{24}
\]

\[
\frac{2}{3} - \frac{1}{7} = \frac{11}{21}
\]

\[
\frac{1}{5} + \frac{1}{7} = \frac{12}{35}
\]

2  Use a double number line to solve these problems.

   a  On Monday, Mr. Miles walked \(\frac{1}{9}\) of the trail to warm up and then he ran \(\frac{7}{8}\) of the trail. What fraction of the trail did Mr. Miles cover on Monday?

\(\frac{7}{8}\) of the trail. Work may vary slightly. Example:

(continued on next page)
**b** On Tuesday, Mr. Miles walked \( \frac{2}{9} \) of the trail to warm up and then he ran \( \frac{4}{6} \) of the trail. What fraction of the trail did Mr. Miles cover on Tuesday?

\( \frac{16}{18} \) or \( \frac{8}{9} \) of the trail; work will vary somewhat. Example:

![Double Number Line](image)

**c** On Wednesday, Mr. Miles walked and ran \( \frac{9}{10} \) of the trail. Then he walked \( \frac{1}{8} \) of the trail back and stopped to rest. What fraction of the course was Mr. Miles from the beginning of the trail when he stopped?

He was \( \frac{31}{40} \) from the beginning of the trail when he stopped.

Work will vary somewhat. Example:

![Double Number Line](image)

**d** On Thursday, Mr. Miles walked and ran \( \frac{19}{20} \) of the trail. Then he walked \( \frac{1}{3} \) of the trail back before he stopped to rest. What fraction of the trail was Mr. Miles from the beginning of the trail when he stopped?

He was \( \frac{37}{60} \) from the beginning of the trail.

Work will vary somewhat. Example:

![Double Number Line](image)
1 Solve the following.
   a  \( \frac{1}{4} \) of 24  = 6  
   b  \( \frac{3}{4} \) of 24  = 18

2 Solve the following.
   a  \( \frac{1}{8} \times 56 \)  = 7  
   b  \( \frac{3}{8} \times 56 \)  = 21

3 Mark both the fraction and the distance traveled on the number line below.
   a  \( \frac{1}{4} \) of 24 km  
   b  \( \frac{3}{4} \times 24 \) km

4 Mark the fraction and the distance traveled on the number line below.
   a  \( \frac{1}{8} \times 56 \) km  
   b  \( \frac{3}{8} \) of 56 km

5 Becky canoed \( \frac{7}{8} \) of the way down a 56 kilometer river. How many kilometers did she canoe?

   49 km; work will vary.
## Add or Subtract Fractions

1. Solve the problems on this page. If your answer is an improper fraction, find its equivalent mixed number.

\[
\frac{2}{3} + \frac{2}{3} = \frac{4}{3} = 1\frac{1}{3}
\]

\(\frac{4}{3}\) is an improper fraction because 4 is greater than 3. \(\frac{3}{3}\) is equal to 1, so \(\frac{4}{3}\) is equal to \(1\frac{1}{3}\).

<table>
<thead>
<tr>
<th>(1\frac{8}{10} - \frac{4}{10})</th>
<th>(\frac{3}{4} - \frac{2}{4})</th>
<th>(\frac{6}{10} + \frac{4}{10})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1\frac{4}{10}) or (1\frac{2}{5})</td>
<td>(\frac{1}{4})</td>
<td>(\frac{10}{10}) or 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\frac{1}{2} + \frac{3}{4})</th>
<th>(\frac{3}{6} - \frac{1}{2})</th>
<th>(\frac{2}{4} + 2\frac{2}{10})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{5}{4}) = (1\frac{1}{4})</td>
<td>0</td>
<td>(2\frac{14}{20}) or (2\frac{7}{10})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\frac{2}{3} - \frac{2}{6})</th>
<th>(1\frac{1}{4} + \frac{2}{6})</th>
<th>(3\frac{3}{4} + 2\frac{1}{8})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{2}{6}) or (\frac{1}{3})</td>
<td>1 (\frac{7}{12})</td>
<td>5 (\frac{7}{8})</td>
</tr>
</tbody>
</table>

2. Find two different ways to show that \(\frac{1}{2} + \frac{1}{4}\) is not equal to \(\frac{2}{6}\). You can use numbers, words, or labeled sketches.

**Explanations will vary. Example:**
1. \(\frac{2}{6} = \frac{1}{3}\), and \(\frac{1}{3}\) is less than \(\frac{1}{2}\), so there’s no way \(\frac{1}{2} + \frac{1}{4}\) could be \(\frac{2}{6}\).

2. [Diagram]

You can see on this sketch that \(\frac{1}{2} + \frac{1}{4} = \frac{3}{4}\), not \(\frac{2}{6}\).
Work Place Instructions 2C Target Practice

Each pair of players needs:
- 1 spinner overlay
- 2 Target Practice Record Sheets
- pencils

1. Players decide who will be Player 1 and who will be Player 2.
2. Player 1 spins Spinner 1 twice and Spinner 2 twice and records all four numbers spun on the Record Sheet.
3. Player 1 chooses two of the numbers spun to create two unit fractions (fractions with 1 as the numerator) that have a sum as close to 1 as possible, and records the fractions created on the Record Sheet.
4. Player 1 finds the sum of the fractions.
5. Then, Player 1 finds her score by figuring out how far from one whole the sum is. Player 2 checks Player 1’s work.
6. Then it is Player 2’s turn.
7. After both players have finished Round 1, they compare scores and determine which is the smaller fraction. The player with the smaller score wins the round and circles his score. If the scores are the same, that round is a tie.
8. For rounds 2–5, players spin each spinner only one time to get two new numbers. They record the newly spun numbers and the two unused numbers from the previous round on their Record Sheet.
9. After five rounds, players compare how many rounds each won to determine the winner of the game.

Game Variations

A. Players spin for four new numbers in each round.
B. Player work together to find the smallest score on one record sheet rather than playing competitively.
C. Players do not determine a winner in each round, instead they add the five scores for each round to determine the winner of the game.
### Better Buys

1. Michael was working on finding the better buy for granola bars: 8 bars for $10 or 20 bars for $23. Fill in each ratio table to find equivalents.

   **a**
<table>
<thead>
<tr>
<th>8 bars</th>
<th>16 bars</th>
<th>24 bars</th>
<th>32 bars</th>
<th>40 bars</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10</td>
<td>$20</td>
<td>$30</td>
<td>$40</td>
<td>$50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>20 bars</th>
<th>40 bars</th>
</tr>
</thead>
<tbody>
<tr>
<td>$23</td>
<td>$46</td>
</tr>
</tbody>
</table>

   **b**
<table>
<thead>
<tr>
<th>20 bars</th>
<th>10 bars</th>
<th>5 bars</th>
<th>1 bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$23</td>
<td>$11.50</td>
<td>$5.75</td>
<td>$1.15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8 bars</th>
<th>4 bars</th>
<th>1 bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10</td>
<td>$5</td>
<td>$1.25</td>
</tr>
</tbody>
</table>

2. Now Michael wants to buy bags of mixed nuts. He can buy 12 bags for 15 dollars or 16 bags for $19.20. Fill in the ratio tables.

   **a**
<table>
<thead>
<tr>
<th>12 bags</th>
<th>6 bags</th>
<th>3 bags</th>
<th>1 bag</th>
</tr>
</thead>
<tbody>
<tr>
<td>$15</td>
<td>$7.50</td>
<td>$3.75</td>
<td>$1.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>16 bags</th>
<th>8 bags</th>
<th>4 bags</th>
<th>1 bag</th>
</tr>
</thead>
<tbody>
<tr>
<td>$19.20</td>
<td>$9.60</td>
<td>$4.80</td>
<td>$1.20</td>
</tr>
</tbody>
</table>

   **b** Explain how you can use the information in the tables to find the better buy of bags of mixed nuts.

   *Explanations will vary. Example: you can see that 16 bags for $19.20 is a better buy because 1 bag at that price is only $1.20 instead of $1.25.*

A previous version of this page included another set of ratio tables as problem 2b (and problem 2b above was listed as problem 2c). The completed ratio tables for the eliminated item are shown here:

<table>
<thead>
<tr>
<th>16 bags</th>
<th>32 bags</th>
<th>48 bags</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10</td>
<td>$20</td>
<td>$30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>20 bags</th>
<th>40 bags</th>
<th>60 bags</th>
<th>80 bags</th>
</tr>
</thead>
<tbody>
<tr>
<td>$15</td>
<td>$30</td>
<td>$45</td>
<td>$60</td>
</tr>
</tbody>
</table>
# Fraction Estimate & Check

Before you solve each problem, look carefully at the fractions and write what you know about the sum or difference. Then find the exact sum or difference. Show all your work. If your answer is greater than 1, write it as a mixed number, not an improper fraction.

<table>
<thead>
<tr>
<th>Problem</th>
<th>What You Know Before You Start</th>
<th>Show your work</th>
<th>Exact Sum or Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{8}{3} + \frac{9}{12}$</td>
<td>$\frac{32}{12} + \frac{9}{12} = \frac{41}{12}$ and $\frac{41}{12} = 3\frac{5}{12}$</td>
<td>$3\frac{5}{12}$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1}{2} - \frac{5}{12}$</td>
<td>Responses will vary. Examples shown. Work will vary. Examples shown.</td>
<td>$\frac{1}{12}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{3}{10} + \frac{4}{5}$</td>
<td>The sum is more than 1 because $\frac{3}{10} &gt; \frac{1}{2}$.</td>
<td>$1\frac{1}{10}$</td>
</tr>
<tr>
<td>4</td>
<td>$\frac{3}{4} - \frac{1}{5}$</td>
<td>The difference is less than $\frac{3}{4}$ but more than $\frac{1}{2}$.</td>
<td>$\frac{11}{20}$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{12}{6} - \frac{8}{12}$</td>
<td>The difference is less than 2, but more than 1.</td>
<td>$1\frac{1}{2}$ or $1\frac{1}{3}$</td>
</tr>
<tr>
<td>6</td>
<td>$\frac{12}{8} + \frac{3}{4}$</td>
<td>The sum is more than 2.</td>
<td>$2\frac{1}{4}$ or 2 $\frac{1}{4}$</td>
</tr>
</tbody>
</table>
Buying Granola page 1 of 2

Granola costs $6 for 5 pounds.

Use the ratio tables to find the prices for different amounts of granola: \(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 2, 3, 8, 13, 15, 28, 31, 50, 60, \text{ and } 80 \text{ pounds.} \) You can find the prices in any order.

<table>
<thead>
<tr>
<th>Price</th>
<th>$6</th>
<th>$1.20</th>
<th>$0.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pounds</td>
<td>5</td>
<td>1</td>
<td>(\frac{1}{4})</td>
</tr>
</tbody>
</table>

The order in which students find the prices, as well as their strategies for doing so will vary. Examples shown.

\(\frac{1}{4}\) pounds cost \$0.30.\n
\(\frac{1}{2}\) pounds cost \$0.60.\n
\(\frac{3}{4}\) pounds cost \$0.90.\n
31 pounds cost \$37.20.\n
2 pounds cost \$2.40.

(continued on next page)
### Buying Granola page 2 of 2

<table>
<thead>
<tr>
<th>Price</th>
<th>$6</th>
<th>$1.20</th>
<th>$3.60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pounds</td>
<td>5</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

3 pounds cost $3.60.

<table>
<thead>
<tr>
<th>Price</th>
<th>$6</th>
<th>$3.60</th>
<th>$9.60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pounds</td>
<td>5</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

8 pounds cost $9.60.

<table>
<thead>
<tr>
<th>Price</th>
<th>$6</th>
<th>$12.00</th>
<th>$3.60</th>
<th>$15.60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pounds</td>
<td>5</td>
<td>10</td>
<td>3</td>
<td>13</td>
</tr>
</tbody>
</table>

13 pounds cost $15.60.

<table>
<thead>
<tr>
<th>Price</th>
<th>$6</th>
<th>$12.00</th>
<th>$18.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pounds</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

15 pounds cost $18.00.

<table>
<thead>
<tr>
<th>Price</th>
<th>$6</th>
<th>$12.00</th>
<th>$24.00</th>
<th>$9.60</th>
<th>$33.60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pounds</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>8</td>
<td>28</td>
</tr>
</tbody>
</table>

28 pounds cost $33.60.

<table>
<thead>
<tr>
<th>Price</th>
<th>$6</th>
<th>$12.00</th>
<th>$18.00</th>
<th>$36.00</th>
<th>$1.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pounds</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>30</td>
<td>1</td>
</tr>
</tbody>
</table>

1 pounds cost $1.20.

<table>
<thead>
<tr>
<th>Price</th>
<th>$6</th>
<th>$60.00</th>
<th>$12.00</th>
<th>$72.00</th>
<th>$24.00</th>
<th>$96.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pounds</td>
<td>5</td>
<td>50</td>
<td>10</td>
<td>60</td>
<td>20</td>
<td>80</td>
</tr>
</tbody>
</table>

50 pounds cost $60.00.

60 pounds cost $72.00

80 pounds cost $96.00
Buying Apples

1  Color in the geoboards to show the fractions below. Each geoboard represents 1 whole.

   a  \( \frac{1}{2} \)

   b  \( \frac{1}{4} \)

   c  \( \frac{3}{4} \)

   d  \( \frac{1}{8} \)

   e  \( \frac{2}{8} \)

   f  \( \frac{5}{8} \)

Add the following fractions. If the sum is greater than 1, write the answer as both an improper fraction and a mixed number.

2  Find equivalent fractions and then add or subtract.

   ex  \( \frac{1}{2} + \frac{5}{8} = \frac{4}{8} + \frac{5}{8} = \frac{9}{8} = 1 \frac{1}{8} \)

   a  \( 1 \frac{1}{2} + \frac{3}{8} = \frac{12}{8} + \frac{3}{8} = \frac{15}{8} = 1 \frac{7}{8} \)

   b  \( 2 \frac{1}{4} + \frac{2}{8} = \frac{18}{8} + \frac{2}{8} = \frac{20}{8} = 2 \frac{4}{8} \text{ or } 2 \frac{1}{2} \)

   c  \( \frac{3}{4} - \frac{2}{8} = \frac{6}{8} - \frac{2}{8} = \frac{4}{8} \text{ or } \frac{1}{2} \)

3  Five pounds of apples cost $8. Use the ratio table to find the cost for 9 pounds and 11 pounds. You do not need to use all of the table cells.

<table>
<thead>
<tr>
<th>Price</th>
<th>$8</th>
<th>$16.00</th>
<th>$1.60</th>
<th>$3.20</th>
<th>$6.40</th>
<th>$14.40</th>
<th>$17.60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pounds</td>
<td>5</td>
<td>10</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>9</td>
<td>11</td>
</tr>
</tbody>
</table>

   a  Cost of 9 pounds of apples: $14.40

   b  Cost of 11 pounds of apples: $17.60
Rewrite & Calculate page 1 of 2

1  Complete the tables to show equivalent fractions of an hour. Use the clocks to show each equivalent fraction. Student work may vary slightly.

a  Equivalent Fractions for \( \frac{1}{6} \)

<table>
<thead>
<tr>
<th>numerator</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>denominator</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

b  Equivalent Fractions for \( \frac{1}{3} \)

<table>
<thead>
<tr>
<th>numerator</th>
<th>1</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>denominator</td>
<td>3</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

c  Equivalent Fractions for \( \frac{3}{4} \)

<table>
<thead>
<tr>
<th>numerator</th>
<th>3</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>denominator</td>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>

2  Use the tables of equivalent fractions above to help you solve these problems.

a  Dylan spent \( \frac{1}{6} \) of an hour sweeping the floor and \( \frac{1}{3} \) of an hour picking weeds to help his parents. What fraction of an hour did Dylan spend helping his parents? \( \frac{3}{6} \) or \( \frac{1}{2} \) hour. Student work may vary.
Rewrite & Calculate  page 2 of 2

b. Lisa spent \( \frac{3}{4} \) of an hour reading her book and \( \frac{1}{3} \) of an hour drawing. What fraction of an hour did Lisa spend reading and drawing?

\[ \frac{13}{12} \text{ or } 1 \frac{1}{12} \text{ hours.} \]

Student work may vary.

c. Rory spent \( 1 \frac{1}{6} \) of an hour swimming and then \( \frac{3}{4} \) of an hour playing basketball. What fraction of an hour did Rory spend swimming and playing basketball?

\[ 1 \frac{11}{12} \text{ hours.} \]

Student work may vary.

d. Lisa spent \( \frac{3}{4} \) of an hour doing chores on Friday, and her sister Eva spent \( \frac{1}{3} \) of an hour doing chores. How much more time (as a fraction of an hour) did Lisa spend on chores than Eva?

Lisa spent \( \frac{5}{12} \) of an hour more time on chores than Eva.

Student work may vary.

3. Rewrite each problem using equivalent fractions with a common denominator. Then find the sum. Students' choice of common denominators may vary.

\[ \text{ex} \quad \frac{1}{3} + \frac{1}{6} \]

\[ \frac{2}{6} + \frac{1}{6} = \frac{3}{6} \]

a. \( \frac{1}{3} + \frac{1}{2} \)

\[ \frac{2}{6} + \frac{3}{6} = \frac{5}{6} \]

b. \( \frac{3}{4} + \frac{1}{6} \)

\[ \frac{9}{12} + \frac{2}{12} = \frac{11}{12} \]
More Equivalent Fractions

1. Fill in the tables to show equivalent fractions. You can use the clocks to help if you want to. Student work may vary slightly. Clocks need not be filled in.

   a. Equivalent Fractions for \(\frac{5}{6}\)
      
      | numerator | 5 | 10 |
      | denominator | 6 | 12 |

   b. Equivalent Fractions for \(\frac{3}{4}\)
      
      | numerator | 3 | 9 |
      | denominator | 4 | 12 |

   c. Equivalent Fractions for \(\frac{1}{3}\)
      
      | numerator | 1 | 2 | 4 |
      | denominator | 3 | 6 | 12 |

2. Rewrite each problem using equivalent fractions with a common denominator. Then find the sum.

   ex. \(\frac{1}{3} + \frac{1}{6}\)
      
      \[\frac{2}{6} + \frac{1}{6} = \frac{3}{6}\]

   a. \(\frac{1}{3} + \frac{1}{4}\)
      
      \[\frac{4}{12} + \frac{3}{12} = \frac{7}{12}\]

   b. \(\frac{3}{4} + \frac{5}{6}\)
      
      \[\frac{9}{12} + \frac{10}{12} = \frac{19}{12} = 1 \frac{7}{12}\]
Fraction Story Problems

Work will vary. Answers shown below.

1. Zack and Noah jogged \(\frac{2}{5}\) of a mile and walked another \(\frac{1}{4}\) of a mile. How far did they go in all? \(\frac{13}{20}\) of a mile

2. Mrs. Brown bought a dozen eggs at the store, but when she got home she discovered that \(\frac{1}{3}\) of the eggs were broken so she threw them away. She used \(\frac{1}{6}\) of the dozen in a recipe. How many eggs were left? What fraction of the dozen was left? 6 eggs; \(\frac{3}{6}\) or \(\frac{1}{2}\) of a dozen

3. Erin had \(3\frac{1}{9}\) boxes of granola bars. She gave \(1\frac{3}{4}\) of the boxes of granola bars to her friends. How much did Erin have left for herself? \(1\frac{13}{36}\) boxes

4. Jada had \(1\frac{1}{2}\) pieces of red licorice. Her cousin gave her \(2\frac{1}{4}\) more pieces of red licorice. Then Jada gave \(1\frac{1}{8}\) of the pieces to her dad. How many pieces did Jada have left? \(2\frac{5}{8}\) pieces

5. Last Saturday, Michael spent \(\frac{2}{6}\) of an hour cleaning the hamster cage, \(\frac{3}{12}\) of an hour walking the dog around the block, and \(\frac{5}{60}\) of an hour feeding the bird. What part of an hour did Michael spend taking care of his pets? \(40\%\) or \(\frac{2}{3}\) of an hour, or 40 minutes

6. Leah was planning to spend \(2\frac{1}{3}\) hours with a friend on Saturday, but her mom said she had to finish her chores first. If it took her \(\frac{3}{4}\) of an hour to do her chores, how much time did Leah have left to spend with her friend? \(1\frac{7}{12}\) hours

7. Josie walked \(\frac{3}{10}\) of a mile to the park. Then she walked \(\frac{2}{5}\) of a mile to Angie’s house. Then she walked \(\frac{1}{20}\) of a mile home. How far did Josie walk in all? \(1\frac{5}{20}\) or \(\frac{3}{4}\) of a mile

8. The class was celebrating reading week with cupcakes. The red and blue table groups ate \(\frac{2}{5}\) of the cupcakes, and the green and purple table groups ate \(\frac{3}{7}\) of the cupcakes. What fraction of the cupcakes was left? How many cupcakes might there have been? \(\frac{6}{35}\) of the cupcakes were left. There might have been 35 cupcakes.

9. **CHALLENGE** Pablo found part of a carton of eggs in the refrigerator. He used \(\frac{1}{3}\) of a dozen for baking a cake and \(\frac{1}{6}\) of a dozen to make brownies. Then he had 2 eggs left over. How many eggs were in the carton when Pablo started? 8 eggs

10. **CHALLENGE** Morgan had \(\frac{1}{2}\) a box of candy left after the movie. She ate \(\frac{1}{2}\) of that on the way home and still had 8 pieces left to give to her little brother. How many pieces of candy were in the box when Morgan bought it? 32 pieces of candy
Ratio Tables to the Rescue!

1. Use the ratio table to multiply $22 \times 45$.

   
<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>20</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>90</td>
<td>900</td>
<td>990</td>
</tr>
</tbody>
</table>

2. Use the ratio tables to find equivalent fractions so that you can add $\frac{2}{5} + \frac{1}{8}$.

   \[
   \frac{2}{5} + \frac{1}{8} = \frac{16}{40} + \frac{5}{40} = \frac{21}{40}
   \]

3. Use a ratio table to simplify a fraction. You might not need all of the columns.

   a. Use the ratio table to simplify $\frac{35}{100}$.

      
      | -5 |
      | 35 | 7 |
      | 100 | 20 |

   b. Use the ratio table to simplify $\frac{6}{24}$.

      
      | 6 | 1 |
      | 24 | 4 |

4. Use the ratio tables to find the better buy. You might not need all of the columns.

   | price in dollars | $30$ | $15$ | $7.50$ | $3.75$ | $1.25$ |
   | number of cans   | 24   | 12   | 6      | 3      | 1      |

   | price in dollars | $36$ | $6.00$ | $1.20$ |
   | number of cans   | 30   | 5      | 1      |

   Work will vary. Example shown.

   Which is the better buy? Explain.

   30 cans for $36; explanations will vary.

5. Regina used a ratio table incorrectly to add $\frac{1}{6} + \frac{1}{3}$. Explain her error.

   \[
   \begin{array}{c|c|c}
   1 & 1 & 2 \\
   \hline
   6 & 3 & q \\
   \end{array}
   \]

   So $\frac{1}{6} + \frac{1}{3} = \frac{2}{q}$.

   Explanations will vary.
   The ratios of the entries are not equivalent.
   $\frac{1}{6} \neq \frac{1}{3} \neq \frac{2}{q}$
Adding Fractions & Mixed Numbers

1 Rewrite each pair of fractions so they have the same denominator. Then find their sum. Simplify the sum if you can.

\[ \frac{5}{8} + \frac{7}{12} = \]
\[ \frac{15}{24} + \frac{14}{24} = \frac{29}{24} \]
\[ \frac{29}{24} = \frac{5}{24} \]

\[ \frac{2}{6} + \frac{8}{12} = \]
\[ \frac{1}{3} + \frac{2}{3} = \frac{3}{3} \]
\[ \frac{3}{3} = 1 \]

Work will vary; examples shown.

\[ a \frac{3}{4} + \frac{2}{8} = \]
\[ \frac{3}{4} + \frac{1}{4} = \frac{4}{4} \]
\[ \frac{4}{4} = 1 \]

\[ b \frac{6}{8} + \frac{9}{12} = \]
\[ \frac{3}{4} + \frac{3}{4} = \frac{6}{4} \]
\[ \frac{6}{4} = 1 \frac{3}{4} \]
\[ \text{or} \quad 1 \frac{1}{2} \]

\[ c 3 \frac{6}{12} + 4 \frac{1}{2} = \]
\[ 3 \frac{1}{2} + 4 \frac{1}{2} = 8 \]

\[ d 1 \frac{5}{8} + 2 \frac{3}{4} = \]
\[ 1 \frac{5}{8} + 2 \frac{6}{8} = 3 \frac{11}{8} = 4 \frac{3}{8} \]

2 Randy solved the following problem: \( \frac{7}{8} + \frac{9}{15} \). He said, “I can add 7 and 9 to get 16 and add 8 and 15 to get 23. The answer is \( \frac{16}{23} \).” Is Randy correct? Explain your answer.

No; explanations will vary.

3 Bobby used a ratio table to find an equivalent fraction to add \( \frac{4}{6} + \frac{3}{5} \). He thought he would simplify \( \frac{4}{6} \) first, and wrote down \( \frac{8}{3} \). Kelly said she didn’t think \( \frac{4}{6} \) was equivalent to \( \frac{8}{3} \). Who is correct? Explain your answer.

Kelly is correct; explanations will vary.
Fraction Addition & Subtraction Review

1. Find the sum or the difference for each pair of fractions.
   a. $\frac{5}{6} - \frac{2}{5} = \frac{13}{30}$
   b. $\frac{1}{3} + \frac{6}{7} = \frac{25}{21} = 1\frac{4}{21}$

2. Annie ran $\frac{5}{8}$ of a mile. Lexi ran $\frac{7}{10}$ of a mile. Who ran farther and by exactly how much? Show all your work.
   
   Lexi ran $\frac{3}{40}$ of a mile farther than Annie; work will vary.

3. Juan and his mother hiked $\frac{3}{8}$ of a mile this morning and $\frac{4}{5}$ of a mile this afternoon. How far did they hike today? Show all your work.
   
   They hiked $\frac{47}{40}$ or $1\frac{7}{40}$ of a mile; work will vary.
More Fraction Problems

1 Fill in the missing fraction or mixed number in each equation.

ex \[ 1 \frac{5}{6} + \underline{\frac{1}{6}} = 2 \]

a \[ 1 = \frac{6}{10} + \underline{\frac{4}{10}} \]

b \[ 2 = 1 \frac{4}{12} + \underline{\frac{8}{12}} \]

c \[ 3 = 1 \frac{1}{8} + 1 \frac{7}{8} \]

d \[ 2 = \frac{10}{12} + 1 \frac{2}{12} or 1 \frac{2}{12} \]

e \[ 2 \frac{6}{8} + 1 \frac{2}{8} = 4 \]

2 Calvin and his family were going on a walk. They wanted to walk to the park, then go to the ice cream parlor, and finally walk home. The map below shows their path and the distances between each stop. How many kilometers will they walk in all? Show all your work.

Ice cream parlor \[ 1 \frac{1}{2} \text{ km} \]

Ice cream parlor \[ \frac{7}{8} \text{ km} \]

Ice cream parlor \[ 1 \frac{3}{4} \text{ km} \]

Ice cream parlor \[ 4 \frac{3}{8} \text{ km; work will vary.} \]

3 Add or subtract to solve each equation. Show all your work.

a \[ 3 \frac{7}{8} + 2 \frac{9}{10} = 6 \frac{31}{40} \]

b \[ \frac{7}{16} + \frac{3}{4} = 1 \frac{19}{16} \text{ or } 1 \frac{3}{16} \]

c \[ 1 \frac{4}{5} - \frac{6}{7} = 3 \frac{33}{35} \]
**Fraction Equivalents** page 1 of 2

1. For each of the following pairs of fractions, draw in lines so they have the same number of pieces. Then write the equivalent fraction name beside both and write an equation under each to match the sketch.

   **ex**
   
   \[
   \frac{1}{2} \quad \frac{3}{6} \\
   \frac{1 \times 3}{2 \times 3} = \frac{3}{6} \\
   \]

   Work will vary somewhat. Examples shown.

   **a**
   
   \[
   \frac{1}{6} \quad \frac{2}{12} \\
   \frac{1 \times 2}{6 \times 2} = \frac{2}{12} \\
   \]

   \[
   \frac{3}{4} \quad \frac{5}{20} \\
   \frac{3 \times 5}{4 \times 5} = \frac{15}{20} \\
   \]

   \[
   \frac{2}{5} \quad \frac{8}{20} \\
   \frac{2 \times 4}{5 \times 4} = \frac{8}{20} \\
   \]

   **c**
   
   \[
   \frac{2}{6} \quad \frac{8}{24} \\
   \frac{2 \times 4}{6 \times 4} = \frac{8}{24} \\
   \]

   (continued on next page)
2 Teri and Jon each got a granola bar from their dad. Teri ate \( \frac{3}{5} \) of hers. Jon ate \( \frac{2}{3} \) of his. Who ate more? Exactly how much more? Use the rectangles below to help solve the problem. Show all of your work.

\[
\begin{align*}
\frac{3 \times 3}{5 \times 3} &= \frac{9}{15} \\
\frac{2 \times 5}{3 \times 5} &= \frac{10}{15}
\end{align*}
\]

\( \frac{9}{15} \) ate exactly \( \frac{1}{15} \) more than \( \frac{10}{15} \)

3 Ryan rode his bike \( \frac{5}{6} \) of a mile. James rode his bike \( \frac{7}{8} \) of a mile. Who rode farther? Exactly how much farther? Use the rectangles below to help solve the problem. Show all of your work.

\[
\begin{align*}
\frac{5 \times 4}{6 \times 4} &= \frac{20}{24} \\
\frac{7 \times 3}{8 \times 3} &= \frac{21}{24}
\end{align*}
\]

\( \frac{20}{24} \) rode exactly \( \frac{1}{24} \) more of a mile than \( \frac{21}{24} \)

4 Find the least common multiple (LCM) of each pair of numbers.

ex 6 and 8

\[
\begin{align*}
6, 12, 18, 24 \\
8, 16, 24 \\
24 \text{ is the LCM of 6 and 8}
\end{align*}
\]

a 3 and 5

\[
\begin{align*}
3, 6, 9, 12, 15 \\
5, 10, 15 \\
15 \text{ is the LCM of 3 and 5}
\end{align*}
\]

b 4 and 5

\[
\begin{align*}
4, 8, 12, 16, 20 \\
5, 10, 15, 20 \\
20 \text{ is the LCM of 4 and 5}
\end{align*}
\]

5 Circle the fraction you think is greater in each pair. Then find out for sure by rewriting the fractions so they have common denominators. (Hint: Use the information from problem 4 to help.) Finally, use the < or > sign to compare the two fractions.

ex \( \frac{3}{8} \) > \( \frac{2}{6} \)

\[
\begin{align*}
\frac{3 \times 3}{8 \times 3} &= \frac{9}{24} \\
\frac{2 \times 4}{6 \times 4} &= \frac{8}{24}
\end{align*}
\]

a \( \frac{2}{3} \) < \( \frac{4}{5} \)

\[
\begin{align*}
\frac{2 \times 5}{3 \times 5} &= \frac{10}{15} \\
\frac{4 \times 3}{5 \times 3} &= \frac{12}{15}
\end{align*}
\]

b \( \frac{1}{4} \) < \( \frac{2}{5} \)

\[
\begin{align*}
\frac{1 \times 5}{4 \times 5} &= \frac{5}{20} \\
\frac{2 \times 4}{5 \times 4} &= \frac{8}{20}
\end{align*}
\]
Which Is Bigger?

1. Compare the fractions using the comparison symbols <, >, and =. Show your work to prove how you know which fraction is greater.

   a. \( \frac{1}{3} \quad \text{_____} \quad \frac{2}{5} \quad \text{Work will vary.} \)

   b. \( \frac{3}{8} \quad \text{_____} \quad \frac{1}{3} \quad \text{Work will vary.} \)

   c. \( \frac{3}{5} \quad \text{_____} \quad \frac{5}{9} \quad \text{Work will vary.} \)

   d. \( \frac{5}{12} \quad \text{_____} \quad \frac{2}{5} \quad \text{Work will vary.} \)

2. Jeff and Eric were painting 2 walls in Jeff’s bedroom. The walls were exactly the same size. Jeff painted \( \frac{2}{3} \) of the first wall. Eric painted \( \frac{4}{7} \) of the other wall. Who painted more? How much more? Use numbers, labeled sketches, or words to solve the problem.

   Jeff painted \( \frac{2}{21} \) of a wall more than Eric. Work will vary.
Using the Greatest Common Factor to Simplify Fractions

1. Simplify each of the fractions. Then fill in the rest of the table.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Factors of the Numerator (Top Number)</th>
<th>Factors of the Denominator (Bottom Number)</th>
<th>Greatest Common Factor</th>
<th>Divide to get the Simplest Form</th>
<th>Picture and Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{4}{12} )</td>
<td>1, 2, 4</td>
<td>1, 2, 3, 6, 12</td>
<td>4</td>
<td>( \frac{4}{12} \div \frac{4}{4} = \frac{1}{3} )</td>
<td>![Diagram of Simplified Fraction]</td>
</tr>
<tr>
<td>( \frac{8}{12} )</td>
<td>1, 2, 4, 8</td>
<td>1, 2, 3, 4, 6, 12</td>
<td>4</td>
<td>( \frac{8}{12} \div \frac{4}{4} = \frac{2}{3} )</td>
<td>![Diagram of Simplified Fraction]</td>
</tr>
<tr>
<td>( \frac{4}{6} )</td>
<td>1, 2, 4</td>
<td>1, 2, 3, 6</td>
<td>2</td>
<td>( \frac{4}{6} \div \frac{2}{2} = \frac{2}{3} )</td>
<td>![Diagram of Simplified Fraction]</td>
</tr>
</tbody>
</table>

2. Find the greatest common factor of each pair of numbers below.

ex 6 and 16
- Factors of 6: 1, 2, 3, 6
- Factors of 16: 1, 2, 4, 8, 16
- Greatest Common Factor: 2

a 6 and 21
- Factors of 6: 1, 2, 3, 6
- Factors of 21: 1, 3, 7, 21
- Greatest Common Factor: 3

b 8 and 24
- Factors of 8: 1, 2, 4, 8
- Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
- Greatest Common Factor: 8

c 18 and 24
- Factors of 18: 1, 2, 3, 6, 9, 18
- Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
- Greatest Common Factor: 6
Using the Greatest Common Factor to Simplify Fractions  page 2 of 2

3  Use your answers from problem 2 to simplify these fractions.

ex \[ \frac{6}{16} \]  \[ \frac{6}{16} \div 2 = \frac{3}{8} \]  \[ \frac{6}{16} = \frac{3}{8} \]

a \[ \frac{21}{6} \]  \[ \frac{21}{6} \div 3 = \frac{7}{2} \]  \[ \frac{21}{6} = \frac{7}{2} \]

b \[ \frac{8}{24} \]  \[ \frac{8}{24} \div 8 = \frac{1}{3} \]  \[ \frac{8}{24} = \frac{1}{3} \]

c \[ \frac{18}{24} \]  \[ \frac{18}{24} \div 6 = \frac{3}{4} \]  \[ \frac{18}{24} = \frac{3}{4} \]

4  A fraction is in its simplest form when its numerator and denominator have no common factor other than 1. Look at the fractions below.

• Circle the fractions that can be simplified.
• Put a line under the fractions that are already in simplest form.

\[ \frac{3}{6} \quad \frac{5}{8} \quad \frac{4}{10} \quad \frac{12}{15} \quad \frac{2}{7} \quad \frac{8}{14} \quad \frac{3}{13} \]

For each problem, show all your work using numbers, words, or labeled sketches. Convert an improper fraction to a mixed number if needed and be sure to write your answer in simplest form.

5  Carlos and Jade are eating mini pizzas for lunch. Jade eats \( \frac{3}{4} \) of her mini pizza. Carlos eats \( \frac{8}{12} \) of his mini pizza. How much pizza do they eat together?

\[ \frac{17}{12} = 1 \frac{5}{12} \text{ pizzas; work will vary. Example:} \]

\[ \frac{3}{4} = \frac{9}{12} \]
\[ \frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1 \frac{5}{12} \]

6  Adam and Sophie are also eating mini pizzas for lunch. Adam ate \( \frac{5}{8} \) of his pizza. Sophie ate \( \frac{2}{3} \) of her pizza.

a  Who ate more?  Sophie ate more.

b  How much more?  \( \frac{1}{24} \) of a pizza more. Work will vary. Example:

\[ \frac{5 \times 3}{8 \times 3} = \frac{15}{24} \]
\[ \frac{2 \times 8}{3 \times 8} = \frac{16}{24} \]
\[ \frac{16}{24} - \frac{15}{24} = \frac{1}{24} \]
Simplifying Fractions

1. List the factors of each number below.
   - a. 12: _________________________________________________________
   - b. 15: _________________________________________________________
   - c. 18: _________________________________________________________

2. Find the simplest form of each fraction below. The factors from item 1 might help.
   - a. \( \frac{12}{15} = \frac{4}{5} \)
   - b. \( \frac{4}{18} = \frac{2}{9} \)
   - c. \( \frac{15}{18} = \frac{5}{6} \)
   - d. \( \frac{18}{12} = 1 \frac{6}{12} = 1 \frac{1}{2} \)

3. Find the simplest form of each fraction. Show your work.
   - a. \( \frac{21}{28} = \frac{3}{4} \)
   - b. \( \frac{36}{45} = \frac{4}{5} \)
   - c. \( \frac{27}{18} = 1 \frac{9}{18} = 1 \frac{1}{2} \)

4. Suzie says that \( \frac{7}{35} \) is a fraction in simplest form. Do you agree or disagree? Explain.
   - No; explanations will vary. \( \frac{1}{5} \) is the simplest form of \( \frac{7}{35} \)

5. Alex says that all unit fractions are in simplest form. Do you agree or disagree? Explain. (A unit fraction has 1 as its numerator, like \( \frac{1}{3} \) or \( \frac{1}{12} \).)
   - Alex is correct; explanations will vary.

6. Find \( \frac{12}{15} + \frac{2}{3} \). Show your work.
   - \( 22 \frac{1}{15} = 1 \frac{7}{15}; \) work will vary.
Problem Solving with the LCM & GCF  page 1 of 2

Show your work as you solve each problem. Make sure your answer is in simplest form.

Work will vary. Examples shown.

1 Julia bought \( \frac{3}{5} \) of a yard of red ribbon and \( \frac{10}{15} \) of a yard of purple ribbon.

a Which piece of ribbon was longer?  the purple ribbon

b Exactly what fraction of a yard longer was it?  \( \frac{1}{15} \) of a yard longer

\[
\frac{3 \times 3}{5 \times 3} - \frac{10}{15} = \frac{9}{15} - \frac{9}{15} = \frac{1}{15}
\]

2 Anthony goes running three times a week. This week, he ran \( \frac{3}{5} \) of a mile on Monday, \( \frac{2}{3} \) of a mile on Wednesday, and \( \frac{3}{4} \) of a mile on Friday. How far did Anthony run this week?

2 1/60 miles. 60 is the LCM of 3, 4, and 5.

\[
\begin{align*}
3 \times 12 &= 36 \\
5 \times 12 &= 60 \\
2 \times 20 &= 40 \\
3 \times 15 &= 45 \\
5 \times 12 &= 60 \\
4 \times 16 &= 60 \\
8 \times 3 &= 24 \\
6 \times 4 &= 24
\end{align*}
\]

\[
\frac{36}{60} + \frac{40}{60} + \frac{45}{60} = \frac{121}{60} = 2 \frac{1}{60}
\]

3 On Monday, Leah spent \( \frac{5}{6} \) of an hour working on her homework, on Tuesday, she spent \( \frac{3}{4} \) of an hour on her homework, and on Wednesday she finished her homework in \( \frac{5}{8} \) of an hour.

a On which day did Leah spent the least amount of time on her homework?  Prove it.  On Wednesday. 24 is the LCM of 4, 6, and 8.

\[
\begin{align*}
\frac{5 \times 4}{6 \times 4} &= \frac{20}{24} \\
\frac{3 \times 6}{4 \times 6} &= \frac{18}{24} \\
\frac{5 \times 3}{8 \times 3} &= \frac{15}{24}
\end{align*}
\]

\[
\frac{5}{8} < \frac{3}{4} < \frac{5}{6}
\]

b How much time did Leah spend doing homework on Monday, Tuesday, and Wednesday in all?

2 5/24 hours

\[
\frac{20}{24} + \frac{18}{24} + \frac{15}{24} = \frac{53}{24} = 2 \frac{5}{24}
\]
On Monday, Kevin spent $\frac{4}{5}$ of an hour working on his homework, on Tuesday he spent $\frac{2}{3}$ of an hour on his homework, and on Wednesday he finished his homework in $\frac{7}{10}$ of an hour. How long did Kevin spend doing homework on Monday, Tuesday, and Wednesday in all?

2 1/6 hours. 30 is the LCM of 3, 5, and 10.

\[
\frac{4 \times 6}{5 \times 6} = \frac{24}{30} \quad \frac{2 \times 10}{3 \times 10} = \frac{20}{30} \quad \frac{7 \times 3}{10 \times 3} = \frac{21}{30}
\]

\[
\frac{24}{30} + \frac{20}{30} + \frac{21}{30} = \frac{65}{30} = 2 \frac{5}{30} = 2 \frac{1}{6}
\]

\[
\frac{5 \div 5}{30 \div 5} = \frac{1}{6}
\]

5 **CHALLENGE** Who spent more time doing homework over Monday, Tuesday, and Wednesday, Leah or Kevin? How much more? How much time did the two of them combined spend doing homework? Express your answers as fractions or mixed numbers, and in hours and minutes as well.

Leah spent $\frac{1}{24}$ of an hour, or 2 1/2 more minutes than Kevin doing homework.

Together, they spent 4 3/8 or 4 hours and 22 1/2 minutes doing homework.

\[
2 \frac{5}{24} + 2 \frac{1}{6} = 2 \frac{5}{24} + 2 \frac{4}{24} = 4 \frac{9}{24} \text{ or } 4 \frac{3}{8} \text{ hours}
\]
Evan’s Turtle

Show your work as you solve each problem. Be sure your answer is in simplest form.

1. One side of the aquarium's base is $\frac{3}{4}$ of a yard long. The other side is $\frac{5}{7}$ of a yard long. What is the perimeter of the base of the aquarium?

   $2 \frac{3}{4}$ yards; work will vary.

2. Evan found two sticks for his turtle’s aquarium. One stick was $\frac{3}{4}$ of a foot long and the other was $\frac{10}{12}$ of a foot long. Which stick was longer? What fraction of a foot longer?

   The $\frac{10}{12}$-foot stick was longer. It was $\frac{1}{12}$ of a foot, or 1 inch, longer. Work will vary.

3. On Friday, Evan’s turtle swam for $\frac{4}{10}$ of an hour. Then, he slept for $\frac{3}{8}$ of an hour.

   a. Did Evan’s turtle swim or sleep longer? How much longer?

      Evan’s turtle swam $\frac{1}{40}$ of an hour longer than he slept.
      Work will vary.

   b. How long did Evan’s turtle swim and sleep?

      $\frac{31}{40}$ of an hour; work will vary.
Fraction Problems

1 Solve the following.
   a \( \frac{2}{5} \) of 60 = 24  
   b \( \frac{2}{3} \) of 60 = 40  
   c \( \frac{3}{4} \) of 60 = 45

2 Find the sum.
   a \( \frac{3}{4} + \frac{2}{3} = 1 \frac{11}{12} \)  
   b \( \frac{5}{6} + \frac{7}{9} = 1 \frac{11}{18} \)  
   c \( \frac{2}{7} + \frac{1}{4} = 1 \frac{5}{28} \)

3 Find the difference.
   a \( \frac{1}{2} - \frac{2}{6} = \frac{1}{6} \)  
   b \( \frac{5}{9} - \frac{1}{7} = \frac{26}{63} \)  
   c \( \frac{8}{14} - \frac{2}{5} = \frac{6}{35} \)

4 Randy jogged in a park by his neighborhood every day after work. On Monday, he jogged 3 \( \frac{2}{9} \) miles, and on Tuesday he jogged 3 \( \frac{3}{8} \) miles.
   a On which day did Randy jog farther? Tuesday
   b How much farther? Show your work.
      \( \frac{11}{72} \) of a mile farther; work will vary.
   c How far did Randy jog on the days combined? Show your work.
      6 \( \frac{43}{72} \) miles; work will vary.

5 Carrie bought 3 watermelons for a school picnic. She used \( \frac{7}{8} \) of a watermelon for one class and \( 1 \frac{1}{5} \) watermelons for another class. How much watermelon does Carrie have left for the last class? Show your work.
   \( \frac{37}{40} \) of a watermelon; work will vary.
Work Place Instructions 3A Beat the Calculator: Fractions

Each pair of players needs:
- 1 set of Beat the Calculator: Fractions Cards
- 2 pencils
- scratch paper
- 1 calculator

1. Shuffle the cards, lay them face down, and decide who will use the calculator first.
2. The player with the calculator turns over a card so both players can see it.
3. The player with the calculator enters the problem shown on the card.
   - Fractions are entered as division expressions.
   - Mixed numbers must be entered with a + between the whole number and the fraction part of the expression.
     (Example: $2 \frac{1}{2} - \frac{3}{4}$ will be entered as $2 + 1 \div 2 - 3 \div 4$.)
4. At the same time, the other player uses the most efficient strategy she can think of for the numbers in the expression. (It’s OK to work mentally or use a piece of scratch paper to do the figuring.)
5. The player who gets the correct answer first keeps the card.
6. Players compare answers and share strategies for evaluating the expression.
   The calculator will give a decimal answer. To see if that decimal is equivalent to the fraction answer, clear the calculator and enter the fraction as a division problem. The calculator will show the decimal equivalent to that fraction.
7. Players switch roles and draw another card to solve.
8. The player with the most cards at the end wins.

Game Variations
A. Players make up their own problems and write them on cards, mix the cards up, and then choose from those problems.
B. Instead of racing the calculator, players race each other.
C. Players play cooperatively by drawing a card and discussing their preferred mental strategy.
D. Players spread the cards face down on the table. Each player chooses a different card at the same time and then players race to see who gets the correct answer first.
**Fraction & Decimal Equivalents**

1. Match each fraction on the left with its decimal equivalent on the right. (Hint: Think about money. Remember that a penny is $\frac{1}{100}$ of a dollar and a dime is $\frac{1}{10}$ of a dollar.)

<table>
<thead>
<tr>
<th>Fractions</th>
<th>Decimals</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{4}{10}$</td>
<td>0.04</td>
</tr>
<tr>
<td>$\frac{33}{100}$</td>
<td>0.25</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>1.00</td>
</tr>
<tr>
<td>$\frac{9}{10}$</td>
<td>0.75</td>
</tr>
<tr>
<td>$\frac{1}{4}$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\frac{65}{100}$</td>
<td>0.12</td>
</tr>
<tr>
<td>$\frac{1}{1}$</td>
<td>0.4</td>
</tr>
<tr>
<td>$\frac{12}{100}$</td>
<td>0.9</td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>0.65</td>
</tr>
<tr>
<td>$\frac{4}{100}$</td>
<td>0.33</td>
</tr>
</tbody>
</table>

2. Susan said, “I multiplied 100 by 47, and then I removed one group of 47.” Write an expression to represent how Susan solved $99 \times 47$. Then evaluate the expression.

$$(100 \times 47) - (1 \times 47)$$

3. Match each expression with the correct rectangular prism below. The numbers in parentheses represent the dimensions of the prism’s base.

- **a** $(4 \times 6) \times 5$
- **b** $(4 \times 5) \times 6$
- **c** $(6 \times 5) \times 4$
Place Value Patterns

Predict the product and record your prediction. Then enter the problem in a calculator and write the results.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Prediction</th>
<th>Calculator Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$184 \times 10$</td>
<td>Predictions will vary.</td>
<td>1,840</td>
</tr>
<tr>
<td>$1.84 \times 10$</td>
<td>Predictions will vary.</td>
<td>18.4</td>
</tr>
<tr>
<td>$1.84 \div 10$</td>
<td>Predictions will vary.</td>
<td>0.184</td>
</tr>
<tr>
<td>$15.9 \times 10$</td>
<td>Predictions will vary.</td>
<td>159</td>
</tr>
<tr>
<td>$6.38 \div 10$</td>
<td>Predictions will vary.</td>
<td>0.638</td>
</tr>
<tr>
<td>$0.86 \times 10$</td>
<td>Predictions will vary.</td>
<td>8.6</td>
</tr>
<tr>
<td>$0.2 \div 10$</td>
<td>Predictions will vary.</td>
<td>0.02</td>
</tr>
<tr>
<td>$398.3 \times 10$</td>
<td>Predictions will vary.</td>
<td>3,983</td>
</tr>
<tr>
<td>$282.5 \div 10$</td>
<td>Predictions will vary.</td>
<td>28.25</td>
</tr>
<tr>
<td>$87.34 \div 10$</td>
<td>Predictions will vary.</td>
<td>8.734</td>
</tr>
<tr>
<td>$9.61 \times 10$</td>
<td>Predictions will vary.</td>
<td>96.1</td>
</tr>
<tr>
<td>$9.61 \times 100$</td>
<td>Predictions will vary.</td>
<td>961</td>
</tr>
<tr>
<td>$87.34 \div 100$</td>
<td>Predictions will vary.</td>
<td>0.8734</td>
</tr>
</tbody>
</table>

1. Take a moment to study the chart above. What patterns or interesting results do you notice? Record your observations.
   
   Observations will vary. Examples:
   - When you multiply a number by 10, the product is 10 times as much as the starting number.
   - If you multiply a decimal number by 10, the decimal point shifts 1 place to the right. If you divide it by 10, the decimal point shifts 1 place to the left.

2. Circle the correct word in each sentence below.
   
   a. When a number is multiplied by 10, the result is (larger, smaller) than the number.
   b. When a number is divided by 10, the result is (larger, smaller) than the number.
Adding & Subtracting Decimals

The Ramirez kids earn money each week for the chores they complete around the house. They made a bar graph showing how much they each earned in the first two weeks of the year. Use the graph to answer the questions below. Show all your work.

1. Look at the information for Sarah’s earnings.
   a. How much did Sarah earn in Week 1? $1.20
   b. How much did Sarah earn in Week 2? $3.60
   c. How much more did Sarah earn in Week 2 than in Week 1? $2.40; work will vary.

2. How much more did Johnny earn in Week 2 than in Week 1? $7.20; work will vary.

3. How much more did Richard earn in Week 2 than in Week 1? $5.40; work will vary.

4. How much more did Mr. and Mrs. Ramirez pay their children in Week 2 than in Week 1? $15.00 more; work will vary.
What’s the Share?

Write and solve an equation to represent each of the problems below.

Equations will vary. Examples shown.

1. Ten friends went out for a special dinner. If each person paid $24, what was the total cost of the dinner?
   \[ \text{Total Cost} = 24 \times 10 = 240 \]

2. After dinner, the friends went out for ice cream. If each of the 10 friends paid $2.40, what was the total cost of the ice cream?
   \[ \text{Total Cost} = 2.40 \times 10 = 24.0 \]

3. Another group of 10 friends bought tickets to a concert, but it was canceled before they could attend. If each of the 10 friends received a refund of $26 for the cost of the tickets, what was the total refund amount?
   \[ \text{Total Refund} = 26 \times 10 = 260 \]

4. The group also received a refund for parking. If each of the ten friends received a $2.60 refund for parking, what was the total parking refund?
   \[ \text{Total Refund} = 2.60 \times 10 = 26.0 \]

5. Jenny had a box with the dimensions \((5 \times 7) \times 2\) and it was filled with baseballs. Each ball took up a \(1 \times 1 \times 1\) space. How many baseballs were in Jenny’s box?
   \[ \text{Number of Baseballs} = (5 \times 7) \times 2 = 70 \]

6. Richard also had a box full of baseballs. The dimensions of his box were \(4 \times (4 \times 5)\). Each ball took up a \(1 \times 1 \times 1\) space. How many baseballs were in Richard’s box?
   \[ \text{Number of Baseballs} = 4 \times (4 \times 5) = 80 \]
1 In each box below, color in the grids to show the number. Then write the number the way you’d read it over the phone to someone. The first one is done for you.

ex

1.2 one and two tenths

a

1.02 one and two hundredths

b

1.12 one and twelve hundredths

c

0.12 zero and twelve hundredths

d

0.21 twenty-one hundredths

e

2 two

2 List the numbers from the boxes above, including the example, on these lines. Write them in order from least to greatest.

0.12 < 0.21 < 1.02 < 1.12 < 1.2 < 2

(continued on next page)
3 Jana says that 0.16 is greater than 0.4 because 16 is greater than 4. Do you agree with her? Use numbers, words, or labeled sketches to explain your answer.

Jana is incorrect. Explanations will vary. Example: She’s wrong because 0.4 is 4 tenths, which is the same as 40 hundredths, but 0.16 is only 16 hundredths.

4 **CHALLENGE** Use the digits 2, 4, and 6 to create six different decimal numbers and write them in the boxes below. When you’re finished, write the numbers in order from least to greatest. **Responses will vary somewhat. Example:**

```
2.46   <   2.64   <   4.26   <   4.62   <   6.24   <   6.42
```
More Decimal Color & Order

1 In each box below, color in the grids to show the number. Then write the number the way you’d read it over the phone to someone. The first one is done for you.

<table>
<thead>
<tr>
<th>ex</th>
<th>a</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="grid1.png" alt="Grid" /></td>
<td><img src="grid2.png" alt="Grid" /></td>
</tr>
<tr>
<td><strong>1.8</strong> one and eight tenths</td>
<td><strong>1.08</strong> one and eight hundredths</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="grid3.png" alt="Grid" /></td>
</tr>
<tr>
<td><strong>1.81</strong> one and eighty-one hundredths</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="grid4.png" alt="Grid" /></td>
</tr>
<tr>
<td><strong>0.18</strong> zero and eighteen hundredths</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="grid5.png" alt="Grid" /></td>
</tr>
<tr>
<td><strong>0.81</strong> eighty-one hundredths</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="grid6.png" alt="Grid" /></td>
</tr>
<tr>
<td><strong>1.18</strong> one and eighteen hundredths</td>
</tr>
</tbody>
</table>

2 List the numbers from the boxes above, including the example, on these lines. Write them in order from least to greatest.

0.18 < 0.81 < 1.08 < 1.18 < 1.8 < 1.81
Decimal Grid  page 1 of 2
# Thinking About Thousandths

1 Label each digit in the numbers below with its place value name. The first one is done for you as an example.

| 3 2 0 3 7 1 | 3 tens | 2 ones | 0 tenths | 3 hundredths | 7 thousandths |
| 3 4 7 5 3 1 | 3 tens | 4 ones | 7 tenths | 5 hundredths | 1 thousandth |
| 1 0 6 1 | 1 one | 0 tenths | 6 hundredths | 1 thousandth |
| 3 4 0 7 5 3 | 3 tens | 4 ones | 7 tenths | 5 hundredths | 3 thousandths |
| 1 4 2 0 0 5 | 1 hundred | 4 tens | 2 ones | 0 tenths | 5 thousandths |

2 Complete the chart below.

<table>
<thead>
<tr>
<th>Number</th>
<th>Number Name Written Out in Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>a 0.540</td>
<td>zero and five hundred forty thousandths</td>
</tr>
<tr>
<td>b 1.503</td>
<td>one and five hundred three thousandths</td>
</tr>
<tr>
<td>c 11.07</td>
<td>eleven and seven hundredths</td>
</tr>
<tr>
<td>d 1.429</td>
<td>one and four hundred twenty-nine thousandths</td>
</tr>
<tr>
<td>e 7.005</td>
<td>seven and five thousandths</td>
</tr>
<tr>
<td>f 0.004</td>
<td>zero and four thousandths</td>
</tr>
</tbody>
</table>

3 Mr. Mugwump is confused. He doesn’t know which is more, 5.200 or 5.002. Draw or write something that will help him understand which number is greater and why.

5.200 is greater than 5.002; explanations will vary.
Work Place Instructions 3B Draw & Compare Decimals

Each pair of players needs:
- 1 deck of Number Cards, 10s and wild cards removed
- 1 spinner overlay
- a 3B Draw & Compare Decimals Record Sheet to share
- 2 pencils

1. Decide who will be Player 1 and who will be Player 2.
2. Remove the 10s and wild cards from the deck of Number Cards, if necessary.
3. One player spins the more or less spinner and circles either more or less for Round 1 on the record sheet.
4. Player 1 draws five Number Cards and records the digits on the “Number Cards I drew” line of the record sheet. A wild card may represent any digit, 0–9.
5. Player 1 uses three of the digits to make either the largest or smallest decimal possible, as determined by the more or less spinner.
   - Then Player 1 reads the decimal aloud to his partner.
   - Player 1 records his decimal on the record sheet and returns the three used cards to the bottom of the deck. The two unused cards will be needed by Player 2.
6. Player 2 draws three new digit cards and records them, along with the two cards remaining after Player 1’s turn, on the “Number Cards I drew” line of the record sheet.
7. Player 2 uses three of the digits to make either the largest or smallest decimal possible, as determined by the more or less spinner.
   - Then Player 2 reads the decimal aloud to her partner.
   - Player 2 records her decimal on the record sheet and returns the three used cards to the bottom of the deck. The two unused cards will be needed by Player 1.
8. The player with the greatest or least 3-digit decimal, as determined at the beginning of the game, wins the round and circles her decimal on the record sheet.
9. Play continues for five rounds. The winner of the game is the player who wins the most rounds.

Game Variations
A. Players each draw three Number Cards instead of five, and use those three to build the largest or smallest decimal possible.
B. Players determine how much larger or smaller their decimal is compared to their partner’s.
Playing Draw & Compare Decimals

1 Carmen is playing Draw & Compare Decimals with her partner. Carmen drew 4, 7, 6, 0, and 2 and has to use three of the cards to make a decimal number less than 1.

0. _____ _____ _____

a If they are playing for “more,” what decimal should Carmen make? 0.764

b If they are playing for “less,” what decimal should Carmen make? 0.024

2 James and Ryan are also playing Draw & Compare Decimals. They are playing for less, and James made the decimal 0.149 with his digit cards.

a Ryan drew the following cards: 5, wild card, 0, 7, and 2. Does Ryan need to use his wild card to win the round? No

b List two decimals that Ryan could create to win the round.

Examples:
0.072
0.057

3 Shawn and Jane are playing Draw & Compare Decimals and are playing for more. Shawn made 0.879 and Jane made 0.987. Both students say they won the round.

a Who is correct? Jane; 0.987 > 0.879

b Explain how you know. Explanations will vary.

4 Find the sums. Show your thinking. Work will vary.

a $57.99 + $14.25 = $72.24

b $23.45 + $19.99 = $43.44

c $1,689 + $145 = $1,834
Work Place Instructions 3C Round & Add Tenths

Each pair of players needs:
- a 3C Round & Add Tenths Record Sheet to share
- colored pencils, 1 red and 1 blue
- 2 regular pencils
- 1 die numbered 0–5
- 1 die numbered 4–9

1. Players take turns rolling one of the dice. The player with the higher number is the Red Player.
   - The Red Player goes first and will record his numbers in red.
   - The other player is the Blue Player and will record her numbers in blue.

2. The Red Player rolls both dice and decides which number to put in the ones place and which to put in the tenths place.
   - The Red Player records the decimal number he made in red under the whole number to which it rounds.

3. The Blue Player rolls both dice and then decides which number to put in the ones place and which to put in the tenths place.
   - The Blue Player records the decimal number she made in blue under the whole number to which it rounds.

4. Players continue taking turns.
   - Each whole number box can only be used once.
   - If a player cannot make a number that rounds to an unclaimed whole number, that turn is lost.

5. Once all the whole numbers are claimed, players predict who will have the largest score.

6. Players add and compare their scores. They circle the highest score on the record sheet to indicate the winner.

Game Variations
A. Each player rolls the dice for herself, but her partner chooses which digit goes in the ones place and which goes in the tenths place.
B. Players roll three dice and make numbers in the hundredths place to play “Roll & Add Hundredths.”
Model, Add & Subtract Decimals

1. Write an expression to match each model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>1.3 + 0.709</td>
</tr>
<tr>
<td>b</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td>1.003 + 0.709</td>
</tr>
<tr>
<td>c</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>2.04 − 1.006</td>
</tr>
<tr>
<td>d</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>2.04 − 1.06</td>
</tr>
</tbody>
</table>

2. Carl has two dogs. They are black Labrador retrievers. The male weighs 31.75 kg and the female weighs 29.48 kg.

a. How much heavier is the male than the female? Show your work.

   \[
   2.27 \text{ kg; work will vary.}
   \]

b. How much do they weigh together? Show your work.

   \[
   61.23 \text{ kg}
   \]
Work Place Instructions 3D Target One

Each pair of players needs:
- a 3D Target One Record Sheet for each player
- 1 deck of Number Cards
- math journals
- 2 pencils

1. Player 1 goes first and Player 2 is the dealer. Player 2 passes out six cards to each player.
2. Player 1 chooses four cards to make two decimal numbers to hundredths.
3. Player 1 adds the two numbers in her math journal, trying to get as close as possible to the target of 1. Each card can only be used once.
4. Player 1 explains how she added the two numbers and then writes an equation with the numbers and their sum on the record sheet.
   Player 2 checks the sum.
5. Player 1 figures her score by finding the difference between the sum and 1. Both players record Player 1’s score on their own record sheets.
   A sum of 0.96 has a score of 0.04. A sum of 1.07 has a score of 0.07. A sum of 1.00 has a score of 0.
6. Then Player 2 takes a turn and Player 1 checks his work.
7. At the end of each turn, players put all the used cards face up in a discard stack and deal out four new cards to each player so that both have six cards again.
8. Players continue to take turns.
9. After five rounds, players add their scores to determine the winner. The lower score wins the game.

Game Variations

A. Players add wild cards to their deck of Number Cards. A wild card can be any numeral 0–9. If a wild card is used, players put a star above the number made from the wild card in the equation on the record sheet.

B. Sums below 1 get a negative score. Sums above 1 get a positive score. Players add those scores together and the final score closest to 0 wins.

C. Play Target One with numbers in the thousandths place instead of the tenths place, using all 6 cards.
Working with Decimals

1. Label each digit in the numbers below with a multiplication expression that shows its place value. The first one is done for you as an example.

\[
\begin{array}{cccccccc}
3 & 2 & . & 5 & 3 & 7 & 2 & . & 1 & 7 & 5 & 6 & 1 & . & 3 & 9 & 4 & 2 & 3 & 6 & . & 9 & 2 & 4 \\
3 \times 10 & 2 \times 1 & 5 \times \left(\frac{1}{10}\right) & 3 \times \left(\frac{1}{100}\right) & 7 \times \left(\frac{1}{1000}\right) & 2 \times 1 & 1 \times \left(\frac{1}{100}\right) & 7 \times \left(\frac{1}{1000}\right) & 5 \times \left(\frac{1}{1000}\right) & 6 \times 10 & 1 \times 1 & 3 \times \left(\frac{1}{100}\right) & 9 \times \left(\frac{1}{1000}\right) & 4 \times \left(\frac{1}{1000}\right) & 2 \times 100 & 3 \times 10 & 6 \times 1 & 9 \times \left(\frac{1}{100}\right) & 2 \times \left(\frac{1}{100}\right) & 4 \times \left(\frac{1}{1000}\right)
\end{array}
\]

2. Round each of the numbers in problem 1 to the nearest tenth and nearest hundredth. The first one is done for you as an example.

<table>
<thead>
<tr>
<th>Number</th>
<th>Rounded to the Nearest Tenth</th>
<th>Rounded to the Nearest Hundredth</th>
</tr>
</thead>
<tbody>
<tr>
<td>32.537</td>
<td>32.5</td>
<td>32.54</td>
</tr>
<tr>
<td>2.175</td>
<td>2.2</td>
<td>2.18</td>
</tr>
<tr>
<td>61.394</td>
<td>61.4</td>
<td>61.39</td>
</tr>
<tr>
<td>236.924</td>
<td>236.9</td>
<td>236.92</td>
</tr>
</tbody>
</table>

3. Complete the chart.

<table>
<thead>
<tr>
<th>Number</th>
<th>Number Name Written in Words</th>
<th>Fraction Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.639</td>
<td>zero and six hundred thirty-nine thousandths</td>
<td>(\frac{639}{1000})</td>
</tr>
<tr>
<td>1.613</td>
<td>one and six hundred thirteen thousandths</td>
<td>(1\ \frac{613}{1000})</td>
</tr>
<tr>
<td>12.067</td>
<td>twelve and sixty-seven thousandths</td>
<td>(12\ \frac{67}{1000})</td>
</tr>
<tr>
<td>2.365</td>
<td>two and three hundred sixty-five thousandths</td>
<td>(2\ \frac{365}{1000})</td>
</tr>
<tr>
<td>9.004</td>
<td>nine and four thousandths</td>
<td>(9\ \frac{4}{1000})</td>
</tr>
<tr>
<td>0.005</td>
<td>zero and five thousandths</td>
<td>(\frac{5}{1000})</td>
</tr>
</tbody>
</table>

4. Compare the pairs of decimals. Fill in each blank with <, >, or =.

\(a\ 25.04 \ < \ 25.4 \quad b\ 67.250 \ > \ 67.205 \quad c\ 11.110 \ > \ 11.011\)
## Fractions & Decimals Chart

*Answers in blank cells, if any, may vary.*

<table>
<thead>
<tr>
<th>Tenths</th>
<th>Fraction</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>(\frac{5}{10})</td>
<td>0.5</td>
</tr>
<tr>
<td>0.2</td>
<td>(\frac{2}{10})</td>
<td>0.2</td>
</tr>
<tr>
<td>0.04</td>
<td>(\frac{4}{10})</td>
<td>0.04</td>
</tr>
<tr>
<td>0.6</td>
<td>(\frac{6}{10})</td>
<td>0.6</td>
</tr>
<tr>
<td>0.8</td>
<td>(\frac{8}{10})</td>
<td>0.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hundredths</th>
<th>Fraction</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>(\frac{25}{100})</td>
<td>0.25</td>
</tr>
<tr>
<td>0.50</td>
<td>(\frac{50}{100})</td>
<td>0.50</td>
</tr>
<tr>
<td>0.20</td>
<td>(\frac{20}{100})</td>
<td>0.20</td>
</tr>
<tr>
<td>0.04</td>
<td>(\frac{4}{100})</td>
<td>0.04</td>
</tr>
<tr>
<td>0.60</td>
<td>(\frac{6}{100})</td>
<td>0.60</td>
</tr>
<tr>
<td>0.80</td>
<td>(\frac{8}{100})</td>
<td>0.80</td>
</tr>
<tr>
<td>0.75</td>
<td>(\frac{75}{100})</td>
<td>0.75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Thousandths</th>
<th>Fraction</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.250</td>
<td>(\frac{250}{1,000})</td>
<td>0.250</td>
</tr>
<tr>
<td>0.500</td>
<td>(\frac{500}{1,000})</td>
<td>0.500</td>
</tr>
<tr>
<td>0.200</td>
<td>(\frac{200}{1,000})</td>
<td>0.200</td>
</tr>
<tr>
<td>0.040</td>
<td>(\frac{400}{1,000})</td>
<td>0.040</td>
</tr>
<tr>
<td>0.600</td>
<td>(\frac{600}{1,000})</td>
<td>0.600</td>
</tr>
<tr>
<td>0.800</td>
<td>(\frac{800}{1,000})</td>
<td>0.800</td>
</tr>
<tr>
<td>0.750</td>
<td>(\frac{750}{1,000})</td>
<td>0.750</td>
</tr>
<tr>
<td>0.125</td>
<td>(\frac{125}{1,000})</td>
<td>0.125</td>
</tr>
<tr>
<td>0.3333…</td>
<td>(\frac{125}{375})</td>
<td>0.3333…</td>
</tr>
</tbody>
</table>
Decimal Grid  page 1 of 2
Unit 3 Module 2  |  Session 5

Decimal Grid  page 2 of 2
Fractions, Decimals & Money

1 Fill in the chart. Use any tools to help except a calculator. The first row has been completed as an example.

<table>
<thead>
<tr>
<th>Fraction of a Dollar</th>
<th>Coin Name</th>
<th>Dollars &amp; Cents Notation</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>half dollar</td>
<td>$0.50$</td>
<td>0.50</td>
</tr>
<tr>
<td>$\frac{1}{4}$</td>
<td>quarter</td>
<td>$0.25$</td>
<td>0.25</td>
</tr>
<tr>
<td>$\frac{1}{10}$</td>
<td>dime</td>
<td>$0.10$</td>
<td>0.10</td>
</tr>
<tr>
<td>$\frac{1}{5}$</td>
<td>2 dimes (or 4 nickels)</td>
<td>$0.20$</td>
<td>0.20</td>
</tr>
<tr>
<td>$\frac{1}{20}$</td>
<td>nickel</td>
<td>$0.05$</td>
<td>0.05</td>
</tr>
<tr>
<td>$\frac{1}{100}$</td>
<td>penny</td>
<td>$0.01$</td>
<td>0.01</td>
</tr>
</tbody>
</table>

2 How would you write 0.35 as:
   a a fraction? $\frac{35}{100}$
   b in dollars and cents notation? $0.35$

3 How would you write $0.60:
   a as a fraction? $\frac{60}{100}$
   b as a decimal? 0.60
Decimal Practice

1 Practice adding decimals by playing this game. Please don’t use a calculator. If you can get the answers in your head, that’s fine. If you need to do some paper and pencil work, show your work next to the game board.

a Choose 2 numbers from the box at the right and add them.

b Circle the sum of the numbers on the game board.

c Try to find four sums in a row, column, or diagonal.

d There is one number on the board that is a mistake. As you play, see if you can tell which number is the mistake and circle it. The sooner you find it, the easier it will be to get four in a row! Correct combination noted below each sum on the board.

<table>
<thead>
<tr>
<th>3.26</th>
<th>5.16</th>
<th>7.12</th>
<th>8.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 + 2.76</td>
<td>2.76 + 2.4</td>
<td>3.12 + 4</td>
<td>4.05 + 4</td>
</tr>
<tr>
<td>6.4</td>
<td>4.55</td>
<td>3.62</td>
<td>4.5</td>
</tr>
<tr>
<td>2.4 + 4</td>
<td>0.5 + 4.05</td>
<td>0.5 + 3.12</td>
<td>0.5 + 4</td>
</tr>
<tr>
<td>6.81</td>
<td>1.27</td>
<td>2.9</td>
<td>6.45</td>
</tr>
<tr>
<td>2.76 + 4.05</td>
<td>0.5 + 2.4</td>
<td>2.4 + 4.05</td>
<td></td>
</tr>
<tr>
<td>5.88</td>
<td>7.17</td>
<td>5.52</td>
<td>6.76</td>
</tr>
<tr>
<td>2.76 + 3.12</td>
<td>3.12 + 4.05</td>
<td>3.12 + 2.4</td>
<td>2.76 + 4</td>
</tr>
</tbody>
</table>

2 Write four decimal numbers that have an even digit in the tenths place, an odd digit in the hundredths place, and an even number in the thousandths place.

Responses will vary. Example: 0.234, 1.876, 5.418, 26.636

3 Put the decimals you wrote for problem 2 in order from least to greatest.

Responses will vary, depending on the answers given for problem 2.

_________ < _________ < _________ < _________

4 Write the four decimals using number names (words).

Responses will vary, depending on the answers given for problem 2.
Decimals on a Number Line

1. Use a base ten linear piece to locate and mark these decimals on the number line. Write the numbers above the line.

0.1  0.4  0.8  1.2  1.5  1.8

2. Mark and label the approximate locations of these decimals on the number line. Write the numbers below the line.

0.25  0.75  0.62  1.55  0.04  1.91  1.08  1.69

3. Continue to use a base ten linear piece to help you determine which numbers on the number line are:

a. between $\frac{1}{2}$ and $\frac{9}{10}$: 0.62, 0.75, 0.8

b. closest to but not equal to 0.7: 0.75

c. between 0.9 and 1.2: 1.08

d. less than $\frac{1}{2}$: 0.04, 0.1, 0.25, 0.4

e. less than $1\frac{3}{4}$ but greater than $1\frac{1}{5}$: 1.5, 1.55, 1.69
Round, Add & Subtract Decimals

1. Round each decimal number to the nearest whole number.
   - a. 2.6 → 3
   - b. 3.35 → 3
   - c. 17.8 → 18

2. Round each decimal number to the nearest tenth.
   - a. 0.15 → 0.2
   - b. 0.72 → 0.7
   - c. 2.03 → 2.0

3. CHALLENGE Round each decimal number to the nearest hundredth.
   - a. 0.678 → 0.68
   - b. 3.196 → 3.20
   - c. 0.997 → 1.00

4. Solve.
   - \[ 1.43 + 2.58 = 4.01 \]
   - \[ 5.99 - 3.26 = 2.73 \]
   - \[ 3.09 + 2.67 = 5.76 \]

5. Solve.
   - \[ 16.03 - 12.42 = 3.61 \]
   - \[ 10.18 + 15.07 = 25.25 \]
   - \[ 99.99 - 3.79 = 96.20 \]
Memory Bytes page 1 of 2

Use the information below to help you with the problems that follow.

- 1 gigabyte (GB) is equal to 1,000 megabytes (MB).
- 1 megabyte (MB) is equal to 1,000 kilobytes (KB).
- 1 kilobyte (KB) is equal to 1,000 bytes.

Paula downloaded some music to her new MP3 player.

1. She downloaded one song that was 3.82 MB and another song that was 2.69 MB.
   a. How many megabytes of memory do the two songs use?
   b. How many kilobytes of memory do Paula’s two songs use?
   c. How many bytes do Paula’s two songs use?

2. The next day, Paula downloaded more songs. Once she was finished, her MP3 player said it had 23.15 MB of memory used.
   a. How many megabytes did the new songs use?
   b. How many kilobytes did the new songs use?

3. A few days later, Paula deleted 6.51 MB of songs from her MP3 player.
   a. Now how much memory is being used?
   b. How many bytes did Paula delete?

(continued on next page)
Memory Bytes page 2 of 2

Tyler’s media player plays both music and video (such as television shows and movies). It holds 7.92 GB of songs and videos.

4. The memory display on Tyler’s media player says “7.59 GB used.”

   a. How many gigabytes of memory are still available in the media player?

      \[
      \begin{align*}
      &0.33 \text{ GB; work will vary.} \\
      &\text{Example:} \\
      &7.92 - 7.59 = 0.33
      \end{align*}
      \]

   b. How many megabytes is that?

      \[
      \begin{align*}
      &330 \text{ MB; work will vary.} \\
      &\text{Example:} \\
      &0.33 \times 1,000 = 330
      \end{align*}
      \]

   c. How many kilobytes is that?

      \[
      \begin{align*}
      &330,000 \text{ KB; work will vary.} \\
      &\text{Example:} \\
      &330 \times 1,000 = 330,000
      \end{align*}
      \]

   d. How many bytes is that?

      \[
      \begin{align*}
      &330,000,000 \text{ bytes.} \\
      &\text{Work will vary. Example:} \\
      &330,000 \times 1,000 = 330,000,000
      \end{align*}
      \]

5. Tyler deleted 2.75 GB of TV shows from his media player. Now how much memory is being used?

   \[
   \begin{align*}
   &4.84 \text{ GB} \\
   &\text{Work will vary.} \\
   &\text{Example:} \\
   &2.75 + 0.25 + 4.59 = 7.59
   \end{align*}
   \]

   \[
   \begin{align*}
   &7.59 - 2.75 = 4.84
   \end{align*}
   \]

6. After deleting the TV shows, Tyler added two movies to his media player. The memory display now says “7.61 GB used.” How many gigabytes of memory do his new movies use?

   \[
   \begin{align*}
   &2.77 \text{ GB; work will vary.} \\
   &\text{Example:} \\
   &6.15 - 4.84 = 1.31 \text{ GB}
   \end{align*}
   \]

7. **Challenge** Tyler added some songs to his media player, and now 7.69 GB of the player’s storage is full. He wants to download some episodes of a TV show that together take up 2,250 MB. Does he have enough room?

   \[
   \begin{align*}
   &\text{No, he doesn’t have enough room. He only has } \frac{8.92}{2.77} \text{ GB available.} \\
   &230 \text{ MB left. Work will vary. Example:} \\
   &0.23 \times 1,000 = 230 \text{ MB}
   \end{align*}
   \]
1. In the 2012 London Summer Olympics, Jamaican sprinter Usain Bolt ran the 200-meter sprint in 19.32 seconds, coming in first place. The sprinter who came in second, Yohan Blake, finished the race in 19.44 seconds. By how much did Bolt win the race? Show all your work.

   Bolt won the race by 0.12 seconds; work will vary.

2. The sprinter who came in third, Warren Weir, finished in 19.84 seconds. Did Bolt run the race more or less than a half-second faster than the third place finisher? Show all your work and explain how you can tell.

   More than a half-second faster; work will vary.

   \[
   \begin{array}{c}
   19.84 \\
   - 19.32 \\
   \hline
   0.52 > 0.5 \text{ second}
   \end{array}
   \]

2. In the 2012 London Summer Olympics, Usain Bolt set a new Olympic record when he ran the 100-meter sprint in 9.63 seconds. Is that less than half, exactly half, or more than half as long as it took him to run the 200-meter sprint? Show all your work.

   9.63 is less than half the time it took Bolt to run the 200-meter sprint.

   \[
   \begin{array}{c}
   19.26 \text{ < } 19.32
   \end{array}
   \]
Vertical Problems page 1 of 2

Give and Take Strategy for Addition

1 Fill in the answer.

\[
\begin{align*}
999 + 1 &= 1000 \\
+ 457 - 1 &= 456 \\
\hline
1456
\end{align*}
\]

2 Fill in the answers. As you work, think about how the give and take strategy is used.

\[
\begin{align*}
4.78 - 0.01 &= 4.77 \\
+ 2.39 + 0.01 &= 2.40 \\
\hline
7.17
\end{align*}
\]

\[
\begin{align*}
4.78 + 0.22 &= 5.00 \\
+ 2.39 - 0.22 &= 2.17 \\
\hline
7.17
\end{align*}
\]

Constant Difference Strategy for Subtraction

3 Fill in the answer.

\[
\begin{align*}
921 + 4 &= 925 \\
- 496 + 4 &= -500 \\
\hline
425
\end{align*}
\]

4 Fill in the answers. As you work, think about the two different ways in which the problem has been changed, and compare them. Which seems easier? Why?

\[
\begin{align*}
7.78 + 0.11 &= 7.89 \\
- 2.89 + 0.11 &= -3.00 \\
\hline
4.89
\end{align*}
\]

Most students will likely choose the first way as easier, explaining that it’s easier to subtract if the subtrahend has zeros in the tenth and hundredth places.

(continued on next page)
Adding & Subtracting Decimals

5 Use the give and take strategy for addition to solve these problems. What can you take from one addend and give to the other to make each problem easier?

\[
\begin{align*}
75.6 & - 0.1 & 75.5 & & 4.76 & - 0.02 & 4.74
+ 29.9 & + 0.1 & + 30.0 & & + 4.38 & + 0.02 & + 4.40 \\
& & & & & & & & & \\
& & & & & & & & & 105.5
\end{align*}
\]

\[
\begin{align*}
1.93 & + 0.7 & 2.00 & & 0.68 & - 0.03 & 0.65
+ 7.38 & - 0.7 & + 7.31 & & + 0.97 & + 0.03 & + 1.00 \\
& & & & & & & & & 9.31
\end{align*}
\]

\[
\begin{align*}
57.80 & + 0.20 & 58.00 & & 0.88 & + 0.12 & 1.00
+ 7.38 & - 0.20 & + 7.18 & & + 20.37 & - 0.12 & + 20.25 \\
& & & & & & & & & 65.18
\end{align*}
\]

6 Use the constant difference strategy for subtraction to solve these problems. What can you add or subtract to or from both numbers to make each problem easier?

\[
\begin{align*}
7.78 & + 0.11 & 7.89 & & 13.02 & + 0.01 & 13.03
- 2.89 & + 0.11 & + 3.00 & & - 1.99 & + 0.01 & - 2.00 \\
& & & & & & & & & 4.89
\end{align*}
\]

\[
\begin{align*}
5.30 & + 0.11 & 5.41 & & 14.32 & + 0.05 & 14.37
- 2.89 & + 0.11 & - 3.00 & & - 3.95 & + 0.05 & - 4.00 \\
& & & & & & & & & 2.41
\end{align*}
\]

\[
\begin{align*}
6.10 & + 0.07 & 6.17 & & 25.35 & + 0.20 & 25.55
- 0.93 & + 0.07 & - 1.00 & & - 2.80 & + 0.20 & - 3.00 \\
& & & & & & & & & 5.17
\end{align*}
\]
More Memory Bytes

Use the information below to help you solve the following problems:

- 1 gigabyte (GB) is equal to 1,000 megabytes (MB).
- 1 megabyte (MB) is equal to 1,000 kilobytes (KB).
- 1 kilobyte (KB) is equal to 1,000 bytes.

1 Write and solve an equation for each problem.

   a How many bytes are in 6 KB?  \( 6 \times 1,000 = 6,000 \) bytes
   b How many bytes are in 84 KB?  \( 84 \times 1,000 = 84,000 \) bytes
   c How many kilobytes are in 4 MB?  \( 4 \times 1,000 = 4,000 \) KB
   d How many kilobytes are in 39 MB?  \( 39 \times 1,000 = 39,000 \) KB
   e How many megabytes are in 8 GB?  \( 8 \times 1,000 = 8,000 \) MB
   f How many megabytes are in 92 GB?  \( 92 \times 1,000 = 92,000 \) MB
   g How many bytes are in 7 MB?  \( 7 \times 1,000 \times 1,000 = 7,000,000 \) bytes
   h How many bytes are in 15 MB?  \( 15 \times 1,000 \times 1,000 = 15,000,000 \) bytes
   i How many kilobytes are in 2 GB?  \( 2 \times 1,000 \times 1,000 = 2,000,000 \) KB
   j How many bytes are in 3 GB?  \( 3 \times 1,000 \times 1,000 \times 1,000 = 3,000,000,000 \) bytes

3 Madeline has a song that uses 2.35 MB of memory.

   a How many kilobytes is that?  2,350 KB
   b How many bytes is that?  2,350,000 bytes

4 Madeline buys three songs. One uses 1.73 MB of memory, another uses 2.08 MB, and the third uses 3.99 MB. How many megabytes does Madeline need to store her new songs? Show your work.

   7.8 MB; work will vary.
Equivalent Measures

1 kilometer (km) = 1,000 meters (m)
1 meter (m) = 100 centimeters (cm)
1 centimeter (1 cm) = 10 millimeters (mm)

1 kilogram (kg) = 1,000 grams (g)
1 gram (g) = 1,000 milligrams (mg)

1 liter (l) = 1,000 milliliters (ml)
Different Measures

1. Solve each of the following.
   a. \(64 \text{ cm} = \underline{640} \text{ mm}\)
   b. \(125 \text{ km} = \underline{125,000} \text{ m}\)
   c. \(3,500 \text{ mg} = \underline{3.5} \text{ g}\)
   d. \(4.3 \text{ l} = \underline{4,300} \text{ ml}\)
   e. \(300 \text{ mg} = \underline{0.3} \text{ g}\)

2. Carlton ran 1.3 kilometers on Monday and 2.4 kilometers on Tuesday. How many meters did he run on both days? Show your work.
   
   \(3,700 \text{ meters}; \text{ work will vary.}\)

3. Maria Jose weighed both of her pets. Her parakeet weighs 30 grams, and her turtle weighs 600 grams. How many kilograms do her pets weigh together? Show your work.
   
   \(0.63 \text{ kg}; \text{ work will vary.}\)

4. Walt drinks an average of 10.5 liters of water every Monday through Friday, and only 3 liters of water on the weekend. How many more milliliters of water does Walt usually drink during the weekdays than during the weekends? Show your work.
   
   \(7,500 \text{ ml}; \text{ work will vary.}\)

5. **CHALLENGE** Lindy was making an obstacle course for her friends to follow. She marked a 0.8 kilometer run, a 100 meter jump rope path, and a 50 meter belly crawl path. After her friends complete the course, how many kilometers will they have gone? How many meters? Centimeters? Show your work.
   
   \(0.95 \text{ km}\)
   \(950 \text{ m}\)
   \(95,000 \text{ cm}\)
   \(\text{Work will vary.}\)
Meters & Meters

1 Use the information below to help you with the following problems:
   • 1 kilometer (km) = 1,000 meters (m)
   • 1 meter (m) = 100 centimeters (cm)
   • 1 centimeter (cm) = 10 millimeters (mm)

   a  How many millimeters are in 5 cm?  50 mm
   b  How many millimeters are in 48 cm?  480 mm
   c  How many centimeters are in 9 m?  900 cm
   d  How many centimeters are in 37 m?  3,700 cm
   e  How many meters are in 6 km?  6,000 m
   f  How many meters are in 79 km?  79,000 m
   g  How many meters are in 7 km?  7,000 m
   h  How many millimeters are in 8 km?  8,000,000 mm

2 Tyler is training for a running race. On Monday, he ran 8.67 km. On Tuesday, he ran 9.54 km. On Wednesday, he ran 7.99 km.

   a  How far did Tyler run on Monday, Tuesday, and Wednesday? Show your work.  
      26.2 km; work will vary.

   b  How much farther did Tyler run on Tuesday than on Wednesday? Show your work.  
      1.55 km; work will vary.
Measurements

1. Round the following measurements to the nearest whole number.
   a. 4.32 cm **4 cm**
   b. 10.09 ml **10 ml**
   c. 287.5 km **288 km**

2. Round the following measurements to the nearest tenth.
   a. 3.01 g **3.0 g**
   b. 67.54 m **67.5 m**
   c. 599.93 l **599.9 l**

3. Round the following measurements to the nearest hundredth.
   a. 15.175 kg **15.18 kg**
   b. 25.105 mm **25.11 mm**
   c. 1.006 MB **1.01 MB**

4. There are 1000 meters in a kilometer. How many meters are in 8.59 kilometers?
   **8,590 m**

5. There are 100 grams in a hectogram. How many grams are in 17.84 hectograms?
   **1,784 grams**

6. Fill in the blanks.
   a. 0.68 + **0.39** = 0.7 + 0.37
   b. 1.26 – 0.74 = 1.25 – **0.73**
Round to the nearest 10 and then divide to estimate each quotient.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Rounded</th>
<th>Estimated Quotient</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ex</strong> 123 ÷ 2</td>
<td>120 ÷ 2 = 60</td>
<td>123 ÷ 2 is about equal to 60.</td>
</tr>
<tr>
<td>a 177 ÷ 3</td>
<td>180 ÷ 3</td>
<td>177 ÷ 3 is about equal to 60.</td>
</tr>
<tr>
<td>b 237 ÷ 6</td>
<td>240 ÷ 6</td>
<td>237 ÷ 6 is about equal to 40.</td>
</tr>
<tr>
<td>c 452 ÷ 5</td>
<td>450 ÷ 5</td>
<td>452 ÷ 5 is about equal to 90.</td>
</tr>
</tbody>
</table>
**Division Practice**

1. Choose one division problem below and circle it. Pick the one that seems best for you—not too hard and not too easy.

   180 ÷ 12  220 ÷ 20  440 ÷ 22  520 ÷ 26
   
   15    11    20    20

   a. Write a story problem to match the division problem you just circled.

   **Students’ choice of division problems, and the story problems they write, will vary. See above for answers.**

   b. Make a labeled sketch on the grid below to show the problem you chose. **Sketches and work will vary, depending on the problem selected.**

   c. Find the answer to the problem you chose using your sketch. Show all your work.

   Example:
   
   \[
   520 \div 26 \\
   10 \times 26 = 260 \\
   260 + 260 = 520 \\
   \text{so } 520 \div 26 = 20 \]
Session 3

Work Place Instructions 3E Division Showdown  page 1 of 2

Each pair of players needs

- a 3E Division Showdown Starter Sheet (Regular or Challenge Version) to share
- a 3E Division Showdown Continuation Sheet A to share
- a 3E Division Showdown Continuation Sheet B to share
- 1 spinner overlay
- 1 red and 1 blue colored pencil or markers

1 Players record their names at the top of the starter sheet, then decide which player will go first and what color each player will use.

2 Player 1 spins the spinners and records a division problem using the two numbers. Then he marks and labels the known dimension (the divisor) on the grid with his or her color.

3 Next, Player 1 loops and labels 10 times the divisor, and records the results in the figuring box to the right of the grid, working in his or her own color.

For example, if the divisor spun was 12, loop and label $12 \times 10$ on the grid, then mark it by drawing the dimension across the top. Work in the figuring box to subtract 120 from the dividend to see how much is still left.

Lindsey OK, I'm first. I spun 376 and 12, so now I have to write $376 \div 12$ in the division problem box. Then I have to mark 12 on the side of the grid, and loop and label $10 \times 12$, which is 120. The last thing I have to do is subtract 120 from 376 so we can see how much we still have. It's 256, and it's your turn now, Maya.

4 Player 2 takes a turn to loop and label 10 times the divisor using his or her own color, and record the results in the figuring box.

(continued on next page)
Work Place Instructions 3E Division Showdown page 2 of 2

5 Players take turns looping and labeling 10 times the divisor and recording the results in the figuring box until there isn’t enough left of the dividend to subtract 10 times the divisor again.

6 When there isn’t enough left to subtract 10 times the divisor anymore, the player whose turn it is gets to subtract as many groups of the divisor as he or she wants.

The player should try to remove as many groups as possible, since the player who makes the last removal wins the round.

7 The player who makes the last move must do the following:
   - Loop and label the final groups on the grid.
   - Subtract the final amount in the figuring box.
   - Show the remainder, if there is one, with Xs on the grid.
   - Write the answer to the division problem.
   - Multiply the quotient times the divisor, and add the remainder if there is one, to double-check the answer.

8 The player who makes the last move scores points equal to the remainder, if there is one. If there is no remainder he scores 1 point for being the last to make a move.

Before the winner of the round can take the points, the other player gets to add up the pieces, including the remainder, on the grid to be sure they total the dividend. If they don’t, both players have to find the mistake and fix it.

9 Players play four more rounds (using 3E Division Showdown Continuation Sheets A & B to record the results), then find the sum of their points from all 5 rounds to see who wins the game. (High score wins.)

Players should take turns starting first. If the red player started Round 1, the blue player should start Round 2, and they should continue to alternate starting each new round.

Game Variations

A Use the Challenge Version of the Starter Sheet, which has higher dividends and divisors.
**Metric Conversions**

Knowing how to multiply and divide by 10, 100, and 1,000 can help you make conversions between units in the metric system.

1. **Metric Units of Length/Distance**
   a. Complete the following sentences.
      - There are **1,000** millimeters in 1 meter.
      - There are **100** centimeters in 1 meter.
      - There are **1,000** meters in 1 kilometer.

   b. Use the information in part a to complete the equivalencies below.
      - **10** millimeters = 1 centimeter
      - **100,000** centimeters = 1 kilometer
      - **1,000,000** millimeters = 1 kilometer

2. **Metric Units of Volume/Capacity**
   a. Complete the following sentences.
      - There are **1,000** milliliters in 1 liter.
      - There are **100** centiliters in 1 liter.
      - There are **1,000** liters in 1 kiloliter.

   b. Use the information in part a to complete the equivalencies below.
      - **3,000** milliliters = 3 liters
      - **400** centiliters = 4 liters
      - **7,000** liters = 7 kiloliters

3. **Metric Units of Mass**
   a. Complete the following sentences.
      - There are **1,000** milligrams in 1 gram.
      - There are **100** centigrams in 1 gram.
      - There are **1,000** grams in 1 kilogram.

   b. Use the information in part a to complete the equivalencies below.
      - **2,500** milligrams = 2.5 grams
      - **450** centigrams = 4.5 grams
      - **3,500** grams = 3.5 kilograms

4. **CHALLENGE** Complete the following conversions.
   a. **10,000,000** millimeters = 10 kilometer
   b. **4,500,000** millimeters = 4.5 kilometers
More Rounding & Estimation Practice

1. Complete the following multiplication and division fact families.

<table>
<thead>
<tr>
<th>ex</th>
<th>40 × 3 = 120</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>30 × 5 = 150</td>
</tr>
<tr>
<td>b</td>
<td>20 × 6 = 120</td>
</tr>
<tr>
<td>c</td>
<td>40 × 7 = 280</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3 × 40 = 120</th>
<th>5 × 30 = 150</th>
</tr>
</thead>
<tbody>
<tr>
<td>120 ÷ 40 = 3</td>
<td>150 ÷ 30 = 5</td>
</tr>
<tr>
<td>120 ÷ 3 = 40</td>
<td>150 ÷ 5 = 30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6 × 20 = 120</th>
<th>7 × 40 = 280</th>
</tr>
</thead>
<tbody>
<tr>
<td>120 ÷ 20 = 6</td>
<td>280 ÷ 40 = 7</td>
</tr>
<tr>
<td>120 ÷ 6 = 20</td>
<td>280 ÷ 7 = 40</td>
</tr>
</tbody>
</table>

2. Use rounding and estimation to answer each question yes or no without doing all of the calculations.

a. Mrs. Jackson has 3 grandchildren who go to Park Heights Elementary School. At Back to School Night, she wanted to buy each of them 2 T-shirts with the school mascot on them. The T-shirts cost $18 each, and she has $150 to spend. Can she buy 2 T-shirts for each grandchild?

* Yes   ○ No

b. It costs $27 per person to go to an amusement park. Mr. Lee is taking his two children to the amusement park, and he has $120 to spend. Can he afford for each of his children to bring a friend?

○ Yes   ● No  Assuming it also costs $27 for Mr. Lee.

c. Rachel is reading a book that is 293 pages long. If she reads 38 pages per night, will she be able to finish the book in a week?

○ Yes   ● No

d. Dante’s cousin Carl was bragging that he biked 82 miles last week. If Dante bikes 18 miles a day for 5 days, will he ride more miles than Carl did?

● Yes   ○ No

3. CHALLENGE  Bakery A sells boxes of 6 muffins for $5.85. Bakery B sells boxes of 8 muffins for $8.25. Which bakery offers the better deal on muffins? How can you tell?

Bakery A offers the better deal. Explanations will vary, but even without doing the calculations, you can tell that at $5.85 for 6 muffins, each muffin costs less than $1.00, while at $8.25 for 8 muffins, each muffin costs more than $1.00.
**Fraction & Decimal Review**

1. Find the sum or difference. Show your work.

   a. \( \frac{1}{3} + \frac{3}{8} = \frac{17}{24} \)

   b. \( \frac{6}{7} + \frac{2}{5} = \frac{44}{35} \text{ or } 1 \frac{9}{35} \)

   c. \( \frac{6}{9} - \frac{1}{4} = \frac{15}{36} \text{ or } \frac{5}{12} \)

   d. \( \frac{5}{12} - \frac{1}{8} = \frac{7}{24} \)

2. Isabel and Jared each made a pan of brownies. Their pans of brownies were exactly the same size. After the first day, there was \( \frac{1}{3} \) of one pan and \( \frac{2}{12} \) of the other pan left. What fraction of the brownies were eaten? Show your work.

   1 \( \frac{7}{12} \) of the brownies were eaten; work will vary.

3. Which of the following describes the value of the number 6.21? (Mark all that are true.)
   - six hundred twenty-one hundredths
   - six and twenty-one hundredths
   - sixty-two tenths and one hundredth
   - six hundred twenty-one tenths

4. Round 156.789 to the nearest:
   - one \( 157 \)
   - tenth \( 156.8 \)
   - hundredth \( 156.79 \)
Work Place Instructions 4A The Product Game, Version 2

Each pair of players needs:

- 1 Product Game, Version 2 Record Sheet
- 2 game markers
- pencils

1. Players decide who is going first. Player 1 is O and Player 2 is X (or players can use two different colors).
2. Player 1 places one of the game markers on any factor.
3. Player 2 places the other game marker on a factor. Then she multiplies the two factors, draws an X on the product, and writes an equation to match the combination.

   Tabitha: I choose 6.
   Ambrose: I choose 9. I am drawing my X on 54 because $6 \times 9$ is 54.

4. Player 1 moves one game marker to get a new product. He can move either of the markers.

   Tabitha: I moved the factor marker from the 6 to the 10. Since $9 \times 10$ is 90, I’ll put my O on 90.

5. Play continues until a player gets four products in a row across, up and down, or diagonally.
   - Only one factor marker can be moved during a player’s turn.
   - Players can move a marker so that both are on the same factor. For example, both markers can be on 9. The player would mark the product 81 because $9 \times 9$ = 81.
   - If the product a player chooses is already covered, the player loses that turn.

Game Variations

A. Players can try for three in a row (easier) or five in a row (harder).

B. Players can introduce wild numbers by adding a blank circle to the list of factors. Players can also cover some of the products to make them blank. Then players can make the blank spaces any factor or product they want or need as they play.

C. Players can make their own game board in which they rearrange the numbers. They can also make a game board with different factors and products, but they need to make sure they include all of the products of the factors they choose.
**Product Problems**

1. Find the product.
   
   **a** \((18 \times 4) \times 5 = 360\)
   
   **b** \(22 \times (6 \times 10) = 1,320\)
   
   **c** \(15 \times (4 \times 20) = 1,200\)

2. Find the quotient.
   
   **a** \(1,300 \div 100 = 13\)
   
   **b** \(1,300 \div 10 = 130\)
   
   **c** \(1,300 \div 5 = 260\)

3. Solve the problems in this string. Use the answers from the first few combinations to help solve the rest.
   
   **a** \(48 \times 10 = 480\)
   
   **b** \(48 \times 5 = 240\)
   
   **c** \(48 \times 15 = 720\)
   
   **d** \(48 \times 100 = 4,800\)
   
   **e** \(48 \times 50 = 2,400\)
   
   **f** \(2,448 \div 48 = 51\)
Callie’s Cake Pops

Callie is trying to earn money to purchase a new pair of soccer cleats. She has decided to make her famous cake pops and sell them to her friends.

1. Callie’s mom is willing to loan her the money to get the fundraiser started. Callie knows that her cake pops cost $1.25 to make, and she’d like to make 36 of them. How much money does Callie need to borrow from her mom?

   $45.00; work will vary. Example:
   $1.25 = $1.00 + $0.25
   $1.00 × 36 = $36.00
   $0.25 × 36 is the same as 36 quarters.
   Since there are 4 quarters in a dollar, this is $9.00.
   $36.00 + $9.00 = $45.00

2. Callie priced her cake pops at $1.75 each and sold 32 of them.

   a. How much money did Callie collect for her 32 cake pops?

   $56.00; work will vary. Example:

<table>
<thead>
<tr>
<th>cake pops</th>
<th>1</th>
<th>2</th>
<th>10</th>
<th>20</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>cost per pop</td>
<td>$1.75</td>
<td>$3.50</td>
<td>$17.50</td>
<td>$35.00</td>
<td>$56.00</td>
</tr>
</tbody>
</table>

   b. How much did Callie earn to put toward her soccer cleats? (What was her profit?)

   $11.00; work will vary. Example:

   $56.00
   − $45.00
   $11.00
Multiplication Strategy

Here is a completed box challenge puzzle. If you look at it closely, you’ll see that the number at the top is the product of the two numbers in the middle, and the number at the bottom is the sum of the two numbers in the middle.

4 × 3 = 12
4 + 3 = 7

1 Fill in the blanks to complete each of the box challenge puzzles below. Remember that the number at the top is the product of the two numbers in the middle, and the number at the bottom is the sum of the two numbers in the middle.

24
10
36
15

2 The craft store sells boxes of modeling clay. Each box holds 14 sticks of clay. Complete the ratio table to find out how many sticks there are in different numbers of boxes.

<table>
<thead>
<tr>
<th>Boxes</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>10</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sticks of Clay</td>
<td>14</td>
<td>28</td>
<td>42</td>
<td>84</td>
<td>140</td>
<td>126</td>
</tr>
</tbody>
</table>

3 You can also buy individual sticks of modeling clay for $0.35 each. Find out how much it would cost to buy different numbers of individual sticks of clay.

<table>
<thead>
<tr>
<th>Clay Sticks</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>20</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$0.35</td>
<td>$0.70</td>
<td>$1.40</td>
<td>$2.80</td>
<td>$7.00</td>
<td>$6.65</td>
</tr>
</tbody>
</table>

4 Miranda was asked to solve the problem 1,300 ÷ 26. How can she use multiplication to solve this problem? Find the answer and describe the strategy you used.

50; strategies will vary. Example: I can use a ratio table to find that 26 × 50 = 1,300. That means 1,300 ÷ 26 = 50.

<table>
<thead>
<tr>
<th>26</th>
<th>260</th>
<th>130</th>
<th>1,300</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>5</td>
<td>50</td>
</tr>
</tbody>
</table>
Box Puzzle Challenges

Complete the box puzzle challenges. Remember that the top box shows the *product* of the two middle numbers, and the bottom box shows the *sum* of the two middle numbers.

1. \[
\begin{array}{c}
100 \\
4 \\
29 \\
4 \\
\end{array}
\]

2. \[
\begin{array}{c}
225 \\
9 \\
34 \\
25 \\
\end{array}
\]

3. \[
\begin{array}{c}
400 \\
16 \\
41 \\
25 \\
\end{array}
\]

4. \[
\begin{array}{c}
350 \\
14 \\
39 \\
25 \\
\end{array}
\]

5. \[
\begin{array}{c}
150 \\
2 \\
77 \\
75 \\
\end{array}
\]

6. \[
\begin{array}{c}
600 \\
8 \\
83 \\
75 \\
\end{array}
\]

Challenge

7. \[
\begin{array}{c}
375 \\
75 \\
80 \\
5 \\
\end{array}
\]

8. \[
\begin{array}{c}
375 \\
15 \\
40 \\
25 \\
\end{array}
\]
Work Place Instructions 4B Multiplication Battle page 1 of 2

Each pair of players needs:

- 1 4B Multiplication Battle Record Sheet
- 1 deck of Number Cards (remove the 0s, 1s, and wild cards)
- 1 more/less die
- pencils

1. Players work together to remove the 0s, 1s, and wild cards from the deck and set them aside. Then they shuffle the remaining cards and place them in a stack, face-down, between them.

2. Each player draws a card from the stack. The player with the higher number goes first.

3. Players should place the cards they just drew at the bottom of the stack for use during the game.

4. Player 1 rolls the more/less die to see whether more or less is the goal, and then circles the word on the record sheet.

5. Player 1 draws three cards from the top of the stack, records the numbers on the record sheet, and thinks about the best order for multiplying these three numbers. It may help to move the cards around.

6. Player 1 writes an expression to show the order to multiply the numbers. The two numbers that will be multiplied first are written in parentheses, with the third number outside the parentheses.

7. Player 1 multiplies the first two numbers inside the parentheses and writes the product, along with the third number, on the next line.

8. Player 1 finds and records the product, using the work space to do any figuring if necessary. If Player 2 does not agree with the answer, Player 1 must record his thinking in the work space to prove that he is correct (or make corrections if he is not).

Pablo: I can just do this one in my head. That's why I multiplied the 6 × 4 first, because any number times 10 is really easy.

9. The Last Draw Option: If a player is not happy with his total, he can choose to draw one more card from the top of the stack and then multiply or divide the total by that number. He can do the work in his head if he likes, but if Player 2 does not agree with the answer, Player 1 must record his thinking in the work space to prove that he is correct (or make corrections if he is not).

(continued on next page)
Work Place Instructions 4B Multiplication Battle page 2 of 2

10 Player 2 takes a turn drawing three cards, finding the product, and exercising the Last Draw Option if she chooses to do so.

11 Players compare their totals and circle the winner.
   The lower total wins if players rolled “less” at the start of the round. The higher total wins if they rolled “more” at the start of the round.

12 Players begin a new round. Best out of three rounds wins the game.

Game Variations

A Start with a whole deck. Remove the 0s, 7s, 8s, 9s, and wild cards so you’re playing the game with 1s, 2s, 3s, 4s, 5s, 6s, and 10s.

B Eliminate the Last Draw Option for fewer steps and less computation.

C Leave the deck set up as instructed in the first step on the previous page, but put the wild cards back in and shuffle the deck thoroughly. If a player draws a wild card, she can assign it any value between 11 and 20, even if she draws it for the Last Draw Option. She must write the assigned value on one of the lines, just as if she had drawn that number from the stack of cards.
1. Complete the box challenges.
   a) \[
   \begin{array}{c}
   8 \\
   0.25 \\
   \hline
   2 \\
   \hline
   \end{array}
   \quad \begin{array}{c}
   2 \\
   \hline
   8.25 \\
   \hline
   \end{array}
   \]
   b) \[
   \begin{array}{c}
   0.25 \\
   24 \\
   \hline
   6 \\
   \hline
   \end{array}
   \quad \begin{array}{c}
   24.25 \\
   \hline
   \end{array}
   \]
   c) \[
   \begin{array}{c}
   0.75 \\
   2 \\
   \hline
   1.50 \\
   \hline
   \end{array}
   \quad \begin{array}{c}
   2 \\
   \hline
   2.75 \\
   \hline
   \end{array}
   \]
   d) \[
   \begin{array}{c}
   0.75 \\
   4 \\
   \hline
   3 \\
   \hline
   \end{array}
   \quad \begin{array}{c}
   4.75 \\
   \hline
   \end{array}
   \]

2. Find the product.
   a) \(\frac{1}{5}\) of 20 = \(4\)  
   b) \(\frac{1}{3}\) of 18 = \(6\)  
   c) \(\frac{4}{5}\) of 20 = \(16\)  
   d) \(\frac{2}{3}\) of 18 = \(12\)  
   e) \(\frac{1}{6}\) of 30 = \(5\)  
   f) \(\frac{1}{15}\) of 60 = \(4\)  
   g) \(\frac{5}{6}\) of 30 = \(25\)  
   h) \(\frac{2}{15}\) of 60 = \(8\)

3. Brooke and Kaden each sold 15 boxes of cookies for $2.25 per box. How much money did they collect together? Show your work.

   \$33.75; work will vary.
Callie’s Soccer Cleats  page 1 of 2

Callie is still trying to make money to buy some soccer cleats. She decided to make some charts to help her choose what to do.

1  She could make beaded bracelets. They cost $2.25 each to make. Fill in the table to show how much it would cost to make different numbers of beaded bracelets.

<table>
<thead>
<tr>
<th>Number of Beaded Bracelets</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$2.25</td>
<td>$4.50</td>
<td>$9.00</td>
<td>$11.25</td>
<td>$18.00</td>
<td>$20.25</td>
<td>$22.50</td>
<td>$42.75</td>
<td>$45.00</td>
</tr>
</tbody>
</table>

2  Callie can sell the beaded bracelets for $3.50 each. Fill in the table to show how much money Callie could make by selling different numbers of beaded bracelets.

<table>
<thead>
<tr>
<th>Number of Beaded Bracelets</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue (money from sale)</td>
<td>$3.50</td>
<td>$7.00</td>
<td>$14.00</td>
<td>$17.50</td>
<td>$28.00</td>
<td>$31.50</td>
<td>$35.00</td>
<td>$66.50</td>
<td>$70.00</td>
</tr>
</tbody>
</table>

3  She could also make woven bracelets. They cost $1.20 each to make. Fill in the table to show how much it would cost to make different numbers of woven bracelets.

<table>
<thead>
<tr>
<th>Number of Woven Bracelets</th>
<th>1</th>
<th>5</th>
<th>9</th>
<th>10</th>
<th>15</th>
<th>49</th>
<th>50</th>
<th>99</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$1.20</td>
<td>$6.00</td>
<td>$10.80</td>
<td>$12.00</td>
<td>$18.00</td>
<td>$58.80</td>
<td>$60.00</td>
<td>$118.80</td>
<td>$120.00</td>
</tr>
</tbody>
</table>

(continued on next page)
The woven bracelets sell for $1.50. Fill in the table to show how much money Callie could make by selling different numbers of woven bracelets.

<table>
<thead>
<tr>
<th>Number of Woven Bracelets</th>
<th>1</th>
<th>5</th>
<th>9</th>
<th>10</th>
<th>15</th>
<th>49</th>
<th>50</th>
<th>99</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue (money from sale)</td>
<td>$1.50</td>
<td>$7.50</td>
<td>$13.50</td>
<td>$15.00</td>
<td>$22.50</td>
<td>$73.50</td>
<td>$75.00</td>
<td>$148.50</td>
<td>$150.00</td>
</tr>
</tbody>
</table>

Which kind of bracelet do you think Callie should make? Why?

*Responses and explanations will vary.*
**Multiplication Battle**

When you play Multiplication Battle, you get to draw 3 cards and multiply the numbers on those cards in any order you want. Part of the idea is to put them in an order that makes it easy to get the answer. For example, if you got the cards 7, 8, and 5, you might decide to multiply them in this order: \((7 \times 8) \times 5\) because \(7 \times 8 = 56\), and you can multiply 56 by 10 and then cut the product in half because 5 is half of 10.

1. Dana and Tyler were playing Multiplication Battle. Dana went first, and got the cards 5, 8, and 6.
   
   a. Dana said she solved the problem by finding the product of 8 and 5, and then multiplying that by 6. Write an expression to show her thinking.
      
      \((8 \times 5) \times 6\)
      
   b. What is the product of 6, 8, and 5? Show your work. (You can put the numbers in a different order if you want; you don’t have to use Dana’s idea.)

      240; work will vary.

2. Tyler went next and got 4, 7, and 5.
   
   a. Tyler said he solved the problem by finding the product of 7 and 5, and then doubling it twice. Write an expression to show his thinking.
      
      \(((7 \times 5) \times 2) \times 2\)
      
   b. What is the product of 4, 7, and 5? (You can put the numbers in a different order if you want; you don’t have to use Tyler’s idea.)

      140; work will vary.

3. If you were playing Multiplication Battle and got the cards 6, 7, and 9, what order would you use to make the problem easy to solve? Write an expression to show, and then solve the problem.  

   378; expressions and work will vary. Example:

   \[
   (6 \times 7) \times 9 \quad = (42 \times 10) - (42 \times 1) \\
   \quad = 420 - 42 \\
   \quad = 378
   \]

4. Round 13,674.947 to the nearest:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ten</td>
<td>13,670</td>
</tr>
<tr>
<td>one</td>
<td>13,675</td>
</tr>
<tr>
<td>tenth</td>
<td>13,674.9</td>
</tr>
<tr>
<td>hundredth</td>
<td>13,674.95</td>
</tr>
</tbody>
</table>
More Planning for Callie page 1 of 3

As Callie was planning how to raise money, she wondered about making other numbers of bracelets and cake pops.

1 Beaded bracelets cost $2.25 each to make. How much would it cost to make 61 beaded bracelets? Use the ratio table below to record your strategy.

   **Note** You don’t need to use all the boxes on this table or any of the others below; just use what you need and leave the rest.

<table>
<thead>
<tr>
<th>Number of Beaded Bracelets</th>
<th>1</th>
<th>10</th>
<th>5</th>
<th>6</th>
<th>60</th>
<th>61</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$2.25</td>
<td>$22.50</td>
<td>$11.25</td>
<td>$13.50</td>
<td>$135.00</td>
<td>$137.25</td>
</tr>
</tbody>
</table>

It would cost **$137.25** to make 61 beaded bracelets.

2 Beaded bracelets sell for $3.50 each. How much money would Callie bring in if she sold 42 beaded bracelets? Use the ratio table below to record your strategy.

<table>
<thead>
<tr>
<th>Number of Beaded Bracelets</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>40</th>
<th>42</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue (money from sale)</td>
<td>$3.50</td>
<td>$7.00</td>
<td>$14.00</td>
<td>$140.00</td>
<td>$147.00</td>
</tr>
</tbody>
</table>

42 beaded bracelets would sell for a total of **$147.00**.

3 Woven bracelets cost $1.20 each to make. How much would it cost to make 22 woven bracelets? Use the ratio table below to record your strategy.

<table>
<thead>
<tr>
<th>Number of Woven Bracelets</th>
<th>1</th>
<th>2</th>
<th>10</th>
<th>20</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$1.20</td>
<td>$2.40</td>
<td>$12.00</td>
<td>$24.00</td>
<td>$26.40</td>
</tr>
</tbody>
</table>

It would cost **$26.40** to make 22 woven bracelets.

(continued on next page)
More Planning for Callie  page 2 of 3

4 Woven bracelets sell for $1.50 each. How much money would Callie bring in if she sold 55 woven bracelets? Use the information on the ratio table below, and write in your own to solve the problem.

<table>
<thead>
<tr>
<th>Number of Woven Bracelets</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>10</th>
<th>5</th>
<th>45</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue (money from sale)</td>
<td>$1.50</td>
<td>$3.00</td>
<td>$6.00</td>
<td>$15.00</td>
<td>$7.50</td>
<td>$67.50</td>
<td>$82.50</td>
</tr>
</tbody>
</table>

55 woven bracelets would sell for a total of $82.50.

5 Callie spilled frosting on several of her tables. Fill in all the spots that are covered. Use the information to answer the questions.

a Callie was figuring the cost to make cake pops at $1.25 each and found how many cake pops she could make if she had $67.50.

<table>
<thead>
<tr>
<th>Number of Cake Pops</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>40</th>
<th>10</th>
<th>54</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$1.25</td>
<td>$2.50</td>
<td>$5.00</td>
<td>$50.00</td>
<td>$12.50</td>
<td>$67.50</td>
</tr>
</tbody>
</table>

How many cake pops can she make for $67.50? 54

b Callie made a different table to figure out how many cake pops she could make if she had $95.00.

<table>
<thead>
<tr>
<th>Number of Cake Pops</th>
<th>1</th>
<th>2</th>
<th>20</th>
<th>10</th>
<th>5</th>
<th>50</th>
<th>25</th>
<th>75</th>
<th>76</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$1.25</td>
<td>$2.50</td>
<td>$25.00</td>
<td>$12.50</td>
<td>$6.25</td>
<td>$62.50</td>
<td>$31.25</td>
<td>$93.75</td>
<td>$95.00</td>
</tr>
</tbody>
</table>

How many cake pops can she make for $95.00? 76

(continued on next page)
More Planning for Callie  page 3 of 3

C One of Callie’s tables was for beaded bracelets.

<table>
<thead>
<tr>
<th>Number of Beaded Bracelets</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>40</th>
<th>10</th>
<th>5</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$2.25</td>
<td>$4.50</td>
<td>$9.00</td>
<td>$90.00</td>
<td>$22.50</td>
<td>$11.25</td>
<td>$101.25</td>
</tr>
</tbody>
</table>

How many beaded bracelets can she make for $101.25? 45

D The last table showed woven bracelets.

<table>
<thead>
<tr>
<th>Number of Woven Bracelets</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>10</th>
<th>5</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue (money from sale)</td>
<td>$1.50</td>
<td>$3.00</td>
<td>$6.00</td>
<td>$15.00</td>
<td>$7.50</td>
<td>$67.50</td>
</tr>
</tbody>
</table>

How many woven bracelets can she make for $67.50? 45

6 CHALLENGE If Callie wants to make a profit of $100 or more selling bracelets, how many of each type do you think she should make? Explain your thinking.

Answers and explanations will vary. Example:
She should make 80 beaded bracelets, and sell them all.
It costs $1.20 to make woven bracelets, and they sell for $1.50, so she only makes 30¢ profit on each.
It costs $2.25 to make beaded bracelets, and they sell for $3.50, so she makes $1.25 on each.

<table>
<thead>
<tr>
<th>beaded bracelets</th>
<th>1</th>
<th>10</th>
<th>2</th>
<th>8</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>profit</td>
<td>$1.25</td>
<td>$12.50</td>
<td>$2.50</td>
<td>$10.00</td>
<td>$100.00</td>
</tr>
</tbody>
</table>
### With or Without

Callie’s friend Vanessa also wants to raise money and decided to sell homemade frozen yogurt to her friends and neighbors.

Students’ use of ratio tables will vary. Examples shown.

1. Vanessa will sell vanilla yogurt for $2.50 a cup. Fill in the table to show how much money Vanessa will make if she sells 19 cups of vanilla yogurt.

   **Note** You don’t need to use all the boxes in this table or those below. Just use as many as you need and leave the rest.

<table>
<thead>
<tr>
<th>Number of vanilla yogurt cups sold</th>
<th>1</th>
<th>10</th>
<th>20</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue (money from sale)</td>
<td>$2.50</td>
<td>$25.00</td>
<td>$50.00</td>
<td>$47.50</td>
</tr>
</tbody>
</table>

   Vanessa will make **$47.50** if she sells 19 cups of vanilla yogurt.

2. Vanessa will sell vanilla yogurt in waffle cones for $2.75 each. Fill in the table to show how much money Vanessa will make if she sells 21 cones of vanilla yogurt.

<table>
<thead>
<tr>
<th>Number of vanilla yogurt cones sold</th>
<th>1</th>
<th>10</th>
<th>20</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue (money from sale)</td>
<td>$2.75</td>
<td>$27.50</td>
<td>$55.00</td>
<td>$57.75</td>
</tr>
</tbody>
</table>

   Vanessa will make **$57.75** if she sells 21 cones of vanilla yogurt.

3. Vanessa will also offer a variety of toppings for her yogurt for $0.35 a scoop. Fill in the table to show how much money Vanessa will make if she sells 49 scoops of toppings.

<table>
<thead>
<tr>
<th>Number of scoops of toppings</th>
<th>1</th>
<th>10</th>
<th>5</th>
<th>50</th>
<th>49</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue (money from sale)</td>
<td>$0.35</td>
<td>$3.50</td>
<td>$1.75</td>
<td>$17.50</td>
<td>$17.15</td>
</tr>
</tbody>
</table>

   Vanessa will make **$17.15** if she sells 49 scoops of toppings.
Over & Under

1  Use relationships among the problems to help you solve them.

\[
\begin{align*}
\frac{1}{4} \text{ of 44 is } & \underline{11} & 0.25 \times 44 = \underline{11} & 25 \times 44 = \underline{1,100} \\
26 \times 44 = \underline{1,144} & 24 \times 44 = \underline{1,056} & 0.24 \times 44 = \underline{10.56} \\
\frac{1}{4} \text{ of 45 is } & \underline{11.25} & 0.25 \times 45 = \underline{11.25} & 25 \times 45 = \underline{1,125} \\
26 \times 45 = \underline{1,170} & 24 \times 45 = \underline{1,080} & 0.24 \times 45 = \underline{10.80} 
\end{align*}
\]

2  Use relationships among the problems to help you solve them.

\[
\begin{align*}
\frac{3}{4} \text{ of 32 is } & \underline{24} & 0.75 \times 32 = \underline{24} & 75 \times 32 = \underline{2,400} \\
74 \times 32 = \underline{2,368} & 76 \times 32 = \underline{2,432} & 0.76 \times 32 = \underline{24.32} \\
\frac{1}{4} \text{ of 33 is } & \underline{8.25} & 0.75 \times 33 = \underline{24.75} & 75 \times 33 = \underline{2,475} \\
74 \times 33 = \underline{2,442} & 76 \times 33 = \underline{2,508} & 0.76 \times 33 = \underline{25.08} 
\end{align*}
\]

3  CHALLENGE  Use the answers to the problems above to help solve some of those below.

\[
\begin{align*}
76 \times 48 = \underline{3,648} & 24 \times 96 = \underline{2,304} & 0.74 \times 32 = \underline{23.68} \\
25 \times 9 = \underline{225} & 75 \times 37 = \underline{2,775} & 76 \times 112 = \underline{8,512}
\end{align*}
\]
Making Cupcakes

1. A fancy cupcake costs $0.85 to make. Fill in the table to show how much it would cost to make 21 fancy cupcakes.

   Students' use of ratio tables will vary. Examples shown.

   Note: You don’t need to use all the boxes in this table or the other one below. Just use as many as you need and leave the rest.

<table>
<thead>
<tr>
<th>Number of Fancy Cupcakes</th>
<th>1</th>
<th>10</th>
<th>20</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$0.85</td>
<td>$8.50</td>
<td>$17.00</td>
<td>$17.85</td>
</tr>
</tbody>
</table>

   It would cost **$17.85** to make 21 fancy cupcakes.

2. How many fancy cupcakes could you make for $85.85? Fill in the table to solve the problem.

<table>
<thead>
<tr>
<th>Number of Fancy Cupcakes</th>
<th>1</th>
<th>10</th>
<th>100</th>
<th>101</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$0.85</td>
<td>$8.50</td>
<td>$85.00</td>
<td>$85.85</td>
</tr>
</tbody>
</table>

   You could make **101** fancy cupcakes for $85.85.

3. Find $47 \times 98$, using the most efficient strategy you can. Show your work.

   4,606; work will vary. Example

   \[
   47 \times 98 = (47 \times 100) - (47 \times 2) \\
   = 4700 - 94 \\
   = 4606
   \]

4. Find $987 \div 47$, using the most efficient strategy you can. Show your work.

   21; work will vary. Example

   \[
   \begin{array}{c|c|c|c|c}
   & 1 & 10 & 20 & 21 \\
   \hline
   47 & 470 & 940 & 987 \\
   \end{array}
   \]
Double-Digit Multiplication Sketches page 1 of 2

1 Mrs. Hill’s preschool classroom is 16 feet wide and 28 feet long. She is planning to divide it into 4 sections. Here is her plan. Use a multiplication equation to label the area of each section (in square feet).

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rug</td>
<td>Tables</td>
</tr>
<tr>
<td>16’</td>
<td>28’</td>
</tr>
<tr>
<td>10 × 20 = 200</td>
<td>10 × 8 = 80</td>
</tr>
<tr>
<td>6 × 20 = 120</td>
<td>6 × 8 = 48</td>
</tr>
</tbody>
</table>

a What is the total area of the 16-by-28-foot classroom? Show your work.

448 sq. feet; work may vary. Example:

\[
\begin{array}{c}
200 \\
80 \\
120 \\
\hline
448
\end{array}
\]

\[
\begin{array}{c}
28 \\
\times 16 \\
\hline
448 \text{ sq. ft.}
\end{array}
\]

b Write the answers.

\[
\begin{array}{cccccccc}
20 & 30 & 50 & 40 & 10 & 60 & 30 \\
\times 30 & \times 40 & \times 30 & \times 20 & \times 90 & \times 30 & \times 30 \\
600 & 1,200 & 1,500 & 800 & 900 & 1,800 & 900
\end{array}
\]

(continued on next page)
3 Sketch an array for each of the frames below. Label each part with a multiplication equation to show its area. Then find the total area of the array. **Work will vary. Examples shown.**

**a**

```
  26
---
 13
---
```

```
10 \times 20 = 200 \\
10 \times 6 = 60 \\
3 \times 20 = 60 \\
3 \times 6 = 18 \\
```

**total area = 338**

**b**

```
  15
---
 14
---
```

```
10 \times 10 = 100 \\
10 \times 5 = 50 \\
4 \times 10 = 40 \\
4 \times 5 = 20 \\
```

**total area = 210**

**c**

```
  23
---
 17
---
```

```
10 \times 20 = 200 \\
10 \times 3 = 30 \\
7 \times 20 = 140 \\
7 \times 3 = 21 \\
```

**total area = 391**

4 Write the answers.

```
20 \times 9 = 180 \\
40 \times 8 = 320 \\
50 \times 7 = 350 \\
70 \times 4 = 280 \\
30 \times 8 = 240 \\
60 \times 5 = 300 \\
80 \times 8 = 640 \\
```

**Work will vary. Examples shown.**
Work Place Instructions 4C Beat the Calculator: Multiplication

Each pair of players needs:
- 1 set of Beat the Calculator: Multiplication Cards
- scratch paper and pencils (optional)
- 1 calculator that follows order of operations

Some calculators will not work for this game. You can check a calculator by entering $1 + 3 \times 2 = $.
If the answer shown is 7, the calculator will work for this game.

1. Shuffle the cards, lay them face-down, and decide who will use the calculator first.
2. Player 1 turns over a card so both players can see it.
3. Player 1 enters the problem exactly as it is written on the card.
   Remember, $\frac{1}{4}$ of 60 means $\frac{1}{4} \times 60$ and can be entered on a calculator as $1 \div 4 \times 60$.
   $\frac{3}{5}$ of 50 means $\frac{3}{5} \times 50$ and can be entered on a calculator as $3 \div 5 \times 50$.
4. At the same time, Player 2 evaluates the expression using the most efficient strategy she can think of, either mentally or on paper.
5. The player who gets the correct answer first keeps the card.
6. Players compare answers and share strategies for evaluating the expression.

7. Players switch roles and draw again.
   - This time, Player 2 has the calculator.
8. The player with the most cards at the end wins.

Game Variations
A. Players make up their own problems, mix them up, and then choose from those problems.
B. Instead of racing the calculator, players race each other.
C. Players play cooperatively by drawing a card and discussing their preferred mental strategy.
D. Players spread the cards face-down on the table. Each player chooses a different card at the same time and then they race to see who gets the correct answer to their problem first.
More About Quarters

1 Fill in the blanks to complete each of the box challenge puzzles below. Remember that the number at the top is the product of the two numbers in the middle, and the number at the bottom is the sum of the two numbers in the middle.

\[
\begin{array}{ccc}
400 & 25 & 16 \\
41 & 10 & 35 \\
500 & 25 & 45 \\
1.0 & 0.25 & 4.0 \\
4.25 & 0.25 & 36.25 \\
60 & 0.25 & 60.25 \\
\end{array}
\]

2 Jami is completing the following box challenge and says that the missing number on the right is 100 and the missing number on the bottom is 100 \( \frac{1}{4} \). Do you agree or disagree? Explain why.

Jami is incorrect; explanations will vary. (The missing number on the right is 1, and the missing number on the bottom is 1 \( \frac{1}{4} \).)

3 Find the product or quotient.

\[
\begin{align*}
a & \quad 30 \times 25 = 750 \\
b & \quad 750 \div 25 = 30 \\
c & \quad 7500 \div 25 = 300 \\
d & \quad 7550 \div 25 = 302 \\
\end{align*}
\]

4 Tell how you used one of the combinations in problem 3 to help solve another one of the combinations in that problem.

Explanations will vary
Double-Digit Multiplication page 1 of 2

1. Find the product of each pair of numbers below. Make a labeled sketch to help, or just use numbers. Show all of your work.

```
ex 35
   × 23
   805
```

Some students will continue to use sketches, while others will use numbers only. Both sketches and numbers shown below.

```
a  43
   × 27
   1,161
```
```
b  38
   × 28
   1,064
```
```
c  36
   × 18
   648
```
```
d  46
   × 36
   1,656
```

Some students will continue to use sketches, while others will use numbers only. Both sketches and numbers shown below.
Double-Digit Multiplication page 2 of 2

2 Solve the story problems below. Make a labeled sketch to help, or just use numbers. Show all of your work.

a Jon works at T-Shirts R Us. Yesterday, he unpacked 36 boxes of new shirts. Each box had 24 shirts in it. How many shirts did he unpack?

\[
864 \text{ shirts} \quad \begin{array}{c}
36 \\
\times 24
\end{array}
\]

\[
\begin{array}{c}
20 \times 30 = 600 \\
20 \times 6 = 120 \\
4 \times 30 = 120 \\
4 \times 6 = 24
\end{array}
\]

864 shirts

b Jon made 27 stacks of long-sleeved T-shirts. He put 18 shirts in each stack. How many shirts did he stack in all?

\[
486 \text{ shirts} \quad \begin{array}{c}
27 \\
\times 18
\end{array}
\]

\[
\begin{array}{c}
10 \times 20 = 200 \\
10 \times 7 = 70 \\
8 \times 20 = 160 \\
8 \times 7 = 56
\end{array}
\]

486 shirts

C CHALLENGE Then Jon made 28 stacks of short-sleeved T-shirts. He put 25 shirts in each stack. The store he works for had to pay $3.99 for each shirt. How much did they have to pay for all the shirts Jon stacked?

\[\text{\$2,793.00; work will vary. Example:}\]

\[\frac{1}{4} \text{ or 0.25 of 28 is 7, so } 25 \times 28 = 700\]

\[700 \times \$4.00 = 2,800.00, \text{ but it's only } \$3.99 \text{ for each shirt, so you have to take away a penny for each shirt, and } 0.01 \times 700 = \$7.00\]

\[2,800.00 - 7.00 = \text{\$2,793.00}\]
Reasonable Estimates & Partial Products

1 Circle the most reasonable estimate for each multiplication problem.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>23 × 21</td>
<td>400</td>
<td>600</td>
<td>4,000</td>
</tr>
<tr>
<td>b</td>
<td>31 × 19</td>
<td>600</td>
<td>700</td>
<td>6,000</td>
</tr>
<tr>
<td>c</td>
<td>312 × 18</td>
<td>600</td>
<td>800</td>
<td>6,000</td>
</tr>
<tr>
<td>d</td>
<td>96 × 33</td>
<td>270</td>
<td>1,000</td>
<td>3,000</td>
</tr>
</tbody>
</table>

2 Use partial products to solve each problem below. Draw lines between the digits to show which numbers you multiplied.

ex 63

\[ \times 21 \]

\[ \begin{align*}
20 \times 60 &= 1,200 \\
20 \times 3 &= 60 \\
1 \times 60 &= 60 \\
1 \times 3 &= 3 \\
\hline
1,323 \\
\end{align*} \]

a 27

\[ \times 46 \]

\[ \begin{align*}
40 \times 20 &= 800 \\
40 \times 7 &= 280 \\
6 \times 20 &= 120 \\
6 \times 7 &= 42 \\
\hline
1,242 \\
\end{align*} \]

b 36

\[ \times 43 \]

\[ \begin{align*}
40 \times 30 &= 1,200 \\
40 \times 6 &= 240 \\
3 \times 30 &= 90 \\
3 \times 6 &= 18 \\
\hline
1,548 \\
\end{align*} \]

c 29

\[ \times 67 \]

\[ \begin{align*}
60 \times 20 &= 1,200 \\
60 \times 9 &= 540 \\
7 \times 20 &= 140 \\
7 \times 9 &= 63 \\
\hline
1,943 \\
\end{align*} \]

d 37

\[ \times 59 \]

\[ \begin{align*}
50 \times 30 &= 1,500 \\
50 \times 7 &= 350 \\
9 \times 30 &= 270 \\
9 \times 7 &= 63 \\
\hline
2,183 \\
\end{align*} \]

e 47

\[ \times 56 \]

\[ \begin{align*}
50 \times 40 &= 2,000 \\
50 \times 7 &= 350 \\
6 \times 40 &= 240 \\
6 \times 7 &= 42 \\
\hline
2,632 \\
\end{align*} \]
Moving Toward the Standard Algorithm page 1 of 2

How does the standard multiplication algorithm work?

- It goes from bottom to top.
- It goes from right to left.
- It starts with the 1s.

**ex**

<table>
<thead>
<tr>
<th>Estimate:</th>
<th>Fill in the area model, but start in the bottom right corner.</th>
<th>List the partial products, and add.</th>
</tr>
</thead>
<tbody>
<tr>
<td>36 × 24</td>
<td></td>
<td>36 × 24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 × 6 = 24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 × 30 = 120</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20 × 6 = 120</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20 × 30 = 600</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total = 864</td>
</tr>
</tbody>
</table>

1 Practice working from bottom to top, right to left.

<table>
<thead>
<tr>
<th>Estimate:</th>
<th>Fill in the area model, but start in the bottom right corner.</th>
<th>List the partial products, and add.</th>
</tr>
</thead>
<tbody>
<tr>
<td>28 × 26</td>
<td></td>
<td>28 × 26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 8 = 48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 20 = 120</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20 × 8 = 160</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20 × 20 = 400</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total = 728</td>
</tr>
</tbody>
</table>
2. Practice again. This time, list the partial products in the third column. Remember to go in order from region A to region D.

<table>
<thead>
<tr>
<th>Estimate: 29 × 18</th>
<th>Fill in the area model, but start in the bottom right corner.</th>
<th>List the partial products, and add.</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>9</td>
<td>29 × 18</td>
</tr>
<tr>
<td>200</td>
<td>90</td>
<td>A 8 × 9 = 72</td>
</tr>
<tr>
<td>160</td>
<td>72</td>
<td>B 8 × 20 = 160</td>
</tr>
<tr>
<td>200</td>
<td>90</td>
<td>C 10 × 9 = 90</td>
</tr>
<tr>
<td>100</td>
<td>35</td>
<td>D 10 × 20 = 200</td>
</tr>
<tr>
<td>Total = 522</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Practice again. This time, you use the frame to help draw in the area model. Then label it, list the partial products, and add. Go in order from region A to region D.

<table>
<thead>
<tr>
<th>Estimate: 35 × 27</th>
<th>Use the frame to help draw the area model. Then fill it in to find the partial products.</th>
<th>List the partial products, and add.</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>5</td>
<td>35 × 27</td>
</tr>
<tr>
<td>600</td>
<td>100</td>
<td>A 7 × 5 = 35</td>
</tr>
<tr>
<td>210</td>
<td>35</td>
<td>B 7 × 30 = 210</td>
</tr>
<tr>
<td>100</td>
<td>35</td>
<td>C 20 × 5 = 100</td>
</tr>
<tr>
<td>600</td>
<td>35</td>
<td>D 20 × 30 = 600</td>
</tr>
<tr>
<td>Total = 945</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


\[
\begin{array}{ccccccc}
30 & 40 & 20 & 50 & 80 & 60 \\
\times 40 & \times 50 & \times 70 & \times 50 & \times 30 & \times 70 \\
1,200 & 2,000 & 1,400 & 2,500 & 2,400 & 4,200 \\
\end{array}
\]
**Bottom to Top, Right to Left**

1. For each problem below, sketch and label a 4-part area model. Then list the partial products in order from bottom right corner to top left corner, and add them to get the total.

   **ex** \(23 \times 27\)
   
   \[
   \begin{array}{c}
   20 \\
   60 \\
   20 \\
   \end{array}
   \begin{array}{c}
   400 \\
   140 \\
   \end{array}
   \]
   
   Total = \(621\)

   \[
   \begin{array}{c}
   27 \\
   \times 23 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   3 \times 7 = 21 \\
   3 \times 20 = 60 \\
   20 \times 7 = 140 \\
   20 \times 20 = 400 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   \text{Total} = \end{array}
   \]

   **a** \(24 \times 35\)
   
   \[
   \begin{array}{c}
   30 \\
   4 \end{array}
   \begin{array}{c}
   20 \\
   4 \end{array}
   \]
   
   Total = \(840\)

   \[
   \begin{array}{c}
   24 \\
   \times 35 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   4 \times 5 = 20 \\
   4 \times 30 = 120 \\
   20 \times 5 = 100 \\
   20 \times 30 = 600 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   \text{Total} = \end{array}
   \]

   **b** \(26 \times 43\)
   
   \[
   \begin{array}{c}
   40 \\
   6 \end{array}
   \begin{array}{c}
   20 \\
   \end{array}
   \]
   
   Total = \(1,118\)

   \[
   \begin{array}{c}
   43 \\
   \times 26 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   6 \times 3 = 18 \\
   6 \times 40 = 240 \\
   20 \times 3 = 60 \\
   20 \times 40 = 800 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   \text{Total} = \end{array}
   \]

2. Practice listing and adding the partial products in the same order as you did above, without the labeled sketches.

   \[
   \begin{array}{c}
   38 \\
   \times 43 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   3 \times 8 = 24 \\
   3 \times 30 = 90 \\
   40 \times 8 = 320 \\
   40 \times 30 = 1200 \\
   \text{Total} = 1634 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   29 \\
   \times 29 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   9 \times 9 = 81 \\
   9 \times 20 = 180 \\
   20 \times 9 = 180 \\
   20 \times 20 = 400 \\
   \text{Total} = 841 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   65 \\
   \times 54 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   4 \times 5 = 20 \\
   4 \times 60 = 240 \\
   50 \times 5 = 250 \\
   50 \times 60 = 3,000 \\
   \text{Total} = 3,510 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   48 \\
   \times 37 \\
   \end{array}
   \]
   
   \[
   \begin{array}{c}
   7 \times 8 = 56 \\
   7 \times 40 = 280 \\
   30 \times 8 = 240 \\
   30 \times 40 = 1,200 \\
   \text{Total} = 1,776 \\
   \end{array}
   \]
Example
Let’s use the standard algorithm first.

```
   46
× 36
---
 276
+1380
---
1656
```

Now let’s use a labeled area model and the 4 partial products method to check our work. Remember to work from bottom to top, right to left, starting with region A.

```
   40
×  6
---
 240
+1200
---
1656
```

1. Can you find the numbers we got by using the sketch and the four partial products method in the standard algorithm? Where are they? Responses will vary. Example: If you add the numbers in regions A & B, you get the product in the first row of the standard algorithm. The numbers in Regions C & D added match the product in the second row.

2. What are some things you need to pay attention to when you use the standard multiplication algorithm?

Responses will vary from one class to another. Possibilities include:
- Place value of the numbers being multiplied
- Accuracy in multiplying each pair of digits
- Lining up the products correctly and adding them correctly.
- Keeping the numbers you carry straight.
3 Practice on your own. For each problem below:

- Use the standard algorithm to get the answer.
- Then complete the area model for the problem by labeling each region.
- Finally, write out the four partial products and add them to double-check your work with the standard algorithm.

<table>
<thead>
<tr>
<th>Standard Algorithm</th>
<th>Area Model</th>
<th>Four Partial Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 × 38</td>
<td>1200</td>
<td>43 × 38</td>
</tr>
<tr>
<td>40 × 3</td>
<td>90</td>
<td>A (8 \times 3) = 24</td>
</tr>
<tr>
<td>30 × 4</td>
<td>320</td>
<td>B (8 \times 40) = 320</td>
</tr>
<tr>
<td>8 × 43</td>
<td>1200</td>
<td>C (30 \times 3) = 90</td>
</tr>
<tr>
<td>Total = 1634</td>
<td></td>
<td>D (30 \times 40) = 1200</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Standard Algorithm</th>
<th>Area Model</th>
<th>Four Partial Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 × 29</td>
<td>600</td>
<td>35 × 29</td>
</tr>
<tr>
<td>30 × 5</td>
<td>100</td>
<td>A (9 \times 5) = 45</td>
</tr>
<tr>
<td>20 × 9</td>
<td>270</td>
<td>B (9 \times 30) = 270</td>
</tr>
<tr>
<td>9 × 315</td>
<td>1015</td>
<td>C (20 \times 5) = 100</td>
</tr>
<tr>
<td>Total = 1015</td>
<td></td>
<td>D (20 \times 30) = 600</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Standard Algorithm</th>
<th>Area Model</th>
<th>Four Partial Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 × 37</td>
<td>1200</td>
<td>42 × 37</td>
</tr>
<tr>
<td>40 × 2</td>
<td>60</td>
<td>A (7 \times 2) = 14</td>
</tr>
<tr>
<td>30 × 7</td>
<td>280</td>
<td>B (7 \times 40) = 280</td>
</tr>
<tr>
<td>7 × 42</td>
<td>1554</td>
<td>C (30 \times 2) = 60</td>
</tr>
<tr>
<td>Total = 1554</td>
<td></td>
<td>D (30 \times 40) = 1200</td>
</tr>
</tbody>
</table>
Al’s Practice Sheet

1. For each problem below:
   - Use the standard algorithm to get the answer.
   - Then complete the area model for the problem by labeling each region.
   - Finally, write out the four partial products and add them to double-check your work with the standard algorithm.

<table>
<thead>
<tr>
<th>Standard Algorithm</th>
<th>Area Model</th>
<th>Four Partial Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 × 9</td>
<td>30 9</td>
<td>39 × 9</td>
</tr>
<tr>
<td>× 3 9</td>
<td></td>
<td>A 9 × 9 = 81</td>
</tr>
<tr>
<td>3 5 1</td>
<td>900 270</td>
<td>B 9 × 30 = 270</td>
</tr>
<tr>
<td>+ 1 1 7 0</td>
<td></td>
<td>C 30 × 9 = 270</td>
</tr>
<tr>
<td>1 5 2 1</td>
<td></td>
<td>D 30 × 30 = 900</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total = 1,521</td>
</tr>
</tbody>
</table>

   | 2 × 8              | 20 8      | 28 × 28               |
   | × 2 8              |            | A 8 × 8 = 64          |
   | 2 2 4              | 400 160   | B 8 × 20 = 160        |
   | + 5 6 0            |            | C 20 × 8 = 160        |
   | 7 8 4              |            | D 20 × 20 = 400       |
   |                    |            | Total = 784           |

2. Al is using the standard multiplication algorithm, but he hasn’t filled in all the numbers. Help him complete each problem by filling in the gray boxes correctly.
Fill in the Boxes

For each problem below, fill in the boxes using the standard multiplication algorithm. Then double-check your work by listing and adding the partial products.

**a**

\[
\begin{array}{c}
\phantom{\times} \ 56 \\
\times \ 27 \\
\hline \\
\end{array}
\]

List and add the four partial products for problem a to double-check your work.

\[
\begin{align*}
7 \times 6 &= 42 \\
7 \times 50 &= 350 \\
20 \times 6 &= 120 \\
20 \times 50 &= 1,000 \\
\text{Total} &= 1,512
\end{align*}
\]

**b**

\[
\begin{array}{c}
\phantom{\times} \ 308 \\
\times \ 7 \\
\hline \\
\end{array}
\]

List and add the three partial products for problem b to double-check your work.

\[
\begin{align*}
7 \times 8 &= 56 \\
7 \times 0 &= 0 \\
7 \times 300 &= 2,100 \\
\text{Total} &= 2,156
\end{align*}
\]

**c**

\[
\begin{array}{c}
\phantom{\times} \ 445 \\
\times \ 34 \\
\hline \\
\end{array}
\]

List and add all six partial products for problem c to double-check your work.

\[
\begin{align*}
4 \times 5 &= 20 \\
4 \times 40 &= 160 \\
4 \times 400 &= 1,600 \\
30 \times 5 &= 150 \\
30 \times 40 &= 1,200 \\
30 \times 400 &= 12,000 \\
\text{Total} &= 15,130
\end{align*}
\]
Solving Problems with the Standard Algorithm page 1 of 2

1 For each problem below:
- Use the standard algorithm to solve the problem or fill in the boxes that have been left blank.
- List and add the partial products to double-check your answer.

**Note** If you want, you can use the partial product method first, and then use the standard algorithm to solve the problem again.

- \(37 \times 86\)
  - \(6 \times 7 = 42\)
  - \(6 \times 30 = 180\)
  - \(80 \times 7 = 560\)
  - \(80 \times 30 = 2400\)
  - Total = 3,182

- \(54 \times 25\)
  - \(5 \times 4 = 20\)
  - \(5 \times 50 = 250\)
  - \(20 \times 4 = 80\)
  - \(20 \times 50 = 1000\)
  - Total = 1,350
Choose one combination from problem 1 that you could solve more efficiently with a strategy other than the standard algorithm or listing and adding the partial products. How would you solve it? Show your work on a separate sheet.

Responses will vary.
Alex & the Algorithm

1. Alex is practicing solving problems using the standard algorithm for multiplication. He knows the first step, but then he gets stuck. Finish these problems Alex started.

$$
\begin{array}{c}
5 \\
28 \\
\times 67 \\
\hline
196 \\
+ 1,680 \\
\hline
1,876
\end{array}
\quad
\begin{array}{c}
2 \\
93 \\
\times 87 \\
\hline
651 \\
+ 7,440 \\
\hline
8,091
\end{array}
\quad
\begin{array}{c}
4 \\
56 \\
\times 48 \\
\hline
448 \\
+ 2,240 \\
\hline
2,688
\end{array}
$$

2. When using the algorithm, Alex doesn’t understand why he needs to write a zero in the ones place of the second partial product.

a. Explain to Alex why he needs to do this.

b. What would happen if Alex did not place a zero there?

$$
\begin{array}{c}
\frac{1}{5} \\
37 \\
\times 26 \\
\hline
222 \\
+ 740 \\
\hline
962
\end{array}
$$

3. Fill in the boxes to complete the problems.

$$
\begin{array}{c}
1 2 3 \\
\times 5 6 \\
\hline
7 3 8 \\
+ 6,1 5 0 \\
\hline
6,8 8 8
\end{array}
\quad
\begin{array}{c}
7 8 9 \\
\times 1 2 \\
\hline
1,5 7 8 \\
+ 7,8 9 0 \\
\hline
9,4 6 8
\end{array}
$$
25 \times 64

Use each of the strategies below to solve 25 \times 64.

1. Area Model & Four Partial Products

\[
\begin{array}{c|c|c}
|  & 60 & 4 \\
\hline
20 & | & |
\midrule
5 & 1,200 & A 20 \\
\hline
B 300 & | & |
\end{array}
\]

\[
\begin{array}{c|c|c}
|  & 80 &  \\
\hline
20 & | & |
\midrule
C  &  & |
\end{array}
\]

Total = 1,600

2. Doubling & Halving

25 \times 64 = 50 \times 32 = 100 \times 16 = 1,600

3. Ratio Table

<table>
<thead>
<tr>
<th>64</th>
<th>640</th>
<th>1280</th>
<th>320</th>
<th>1,600</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>20</td>
<td>5</td>
<td>25</td>
</tr>
</tbody>
</table>

Use of ratio table will vary. Example:

4. Using Quarters

a. 64 \times \frac{1}{4} = 16

b. 64 \times 0.25 = 16

c. How can you use these results to find 25 \times 64?

Answers will vary. Example: Multiply the answer to 64 \times 0.25 by 100 because 25 is 100 times more than 0.25

5. The Standard Multiplication Algorithm

a. Solve the problem.

b. Which strategy do you think is best for this combination? Why?

Responses will vary.
Story Problems page 1 of 3

Solve each problem. Use the standard multiplication algorithm for two problems and any strategy you choose for the rest. Show your work. Explain your choice of strategy.

Strategies selected, work and explanations will vary. Examples shown:

1. Connor is trying to drive his car less frequently. He started by figuring out how much he drives in a typical year. If Connor drives about 98 miles each week, how much does he drive in one year (52 weeks)?

Solve the problem: 5,096 miles

\[
98 \times 52 = (100 \times 52) - (2 \times 52) \\
= 5,200 - 104 \\
= 5,096 \text{ miles}
\]

What strategy did you use? Why?

Over strategy because 98 is very close to 100. It’s easy to multiply 52 by 100 and then subtract 2 sets of 52.

2. Taylor has a cupcake business. She packages cupcakes in cartons that hold 25 cupcakes. The Wildwood School ordered 184 cartons of Taylor’s cupcakes. How many cupcakes did the Wildwood School order?

Solve the problem:

4,600 cupcakes

\[
184 \times 25 \\
\frac{1}{4} \text{ of } 184 \text{ is } 46, \text{ so } 0.25 \times 184 = 46 \\
\text{Then multiply the product, } 46, \text{ by } 100 \text{ because } 25 \text{ is 100 times more than } 0.25. \\
46 \times 100 = 4,600 \text{ cupcakes}
\]

What strategy did you use? Why?

Quarters because you’re multiplying by 25, and it’s easy to figure out \(\frac{1}{4}\) of 184 by cutting it in half once and then again.

(continued on next page)
3 Victoria signed up for a two-year cell phone plan. She will pay $37.50 a month for 24 months. How much will Victoria have paid at the end of her two-year plan?

Solve the problem: \( $900.00 \)

<table>
<thead>
<tr>
<th>Number of Months</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>10</th>
<th>20</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money Owed</td>
<td>$37.50</td>
<td>$75.00</td>
<td>$150.00</td>
<td>$375.00</td>
<td>$750.00</td>
<td>$900.00</td>
</tr>
</tbody>
</table>

What strategy did you use? Why?

**Ratio table because it’s easy to double $37.50 and multiply it by 10, and then you can put the parts together.**

4 Aaron wants to visit Australia. He found a plane ticket for $2,150. If Aaron saves $86 a week, how many weeks will it take him to save enough money to go to Australia?

Solve the problem: **25 weeks**

<table>
<thead>
<tr>
<th>Weeks</th>
<th>1</th>
<th>10</th>
<th>20</th>
<th>5</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money Saved</td>
<td>$86</td>
<td>$860</td>
<td>$1,720</td>
<td>$430</td>
<td>$2,150</td>
</tr>
</tbody>
</table>

What strategy did you use? Why?

**Ratio table, because it’s easy to divide by multiplying, and the ratio table makes it easy to build up to the total.**

5 Tina’s family drinks about 128 ounces of milk in one week. How many ounces of milk do they drink in 36 weeks?

Solve the problem: **4,608 ounces of milk**

\[
\begin{array}{c}
128 \\
\times \ 36 \\
768 \\
+ 3,840 \\
\hline 4,608 \\
\end{array}
\]

What strategy did you use? Why?

**Standard algorithm because the numbers were big, so it seemed like the easiest way.**

(continued on next page)
6  Max is building a cage for his ducks. The base of the cage is 208 square feet. If one side is 13 feet, how long is the other side? The cage is a rectangular prism.

Solve the problem: 16 feet

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>10</th>
<th>5</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>13</td>
<td>130</td>
<td>65</td>
<td>208</td>
</tr>
</tbody>
</table>

What strategy did you use? Why?

**Ratio table because it’s an easy way to do division.**

7  Zoe is saving money to go on a trip to Mexico. She earns $16.75 for mowing the lawn. If Zoe mows the lawn 28 times, how much money will she earn?

Solve the problem: $469.00

\[
\begin{align*}
16.75 &= 16 + 0.75 \\
16 \times 28 &= 8 \times 56 = 4 \times 112 = 2 \times 224 = 1 \times 448 = 448 \\
0.75 \times 28 &= \text{as} \quad \frac{3}{4} \times 28. \quad \frac{1}{4} \text{of} \ 28 \text{is} \ 7, \text{so} \quad \frac{3}{4} \text{of} \ 28 \text{is} \ 21 \\
448 + 21 &= 469
\end{align*}
\]

What strategy did you use? Why?

**I split up $16.75 into $16.00 + $0.75. Then I used double and half for 16 \times 28 because it’s easy to keep cutting 16 in half. I used quarters for 0.75 \times 28 because 0.75 is the same as 3 quarters, and you can divide 28 by 4.**

8  Briana is making a box for her art supplies. The box has a base of 176 square inches. The height of the box is 26 inches. What is the volume of the box?

Solve the problem: 4,576 cubic inches

\[
\begin{align*}
176 &\times 26 \\
1056 &+ 3520 \\
4576
\end{align*}
\]

What strategy did you use? Why?

**Standard algorithm because it seemed easiest with these numbers.**
Leah’s Problems

1. Leah needs to solve the three problems below. She has to use the standard algorithm for multiplication at least once. For each problem, decide which strategy Leah should use and then solve the problem.

   \[
   \begin{align*}
   541 \times 32 & = 17,312 \\
   58 \times 25 & = 1,450 \\
   199 \times 65 & = 12,935
   \end{align*}
   \]

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Strategy</th>
<th>Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   Strategies selected will vary.

2. Leah solved \(302 \times 67\) by multiplying \(300\) by \(60\) and \(2\) by \(7\) and adding those products together. Did she get the right answer? Why or why not?

   No; explanations will vary.

Review

3. What is \(\frac{3}{4}\) of 96? 72

4. What is \(\frac{4}{5}\) of 80? 64

5. What is \(\frac{2}{3}\) of 45? 30
Which Estimate Makes the Most Sense?

1. For problems a–d, circle the estimate that makes the most sense. Explain why you chose the estimate you did.

   a. \(29 \div 4\)  
      
      Why?  
      
      Because \(7 \times 4 = 28\), and that’s as close as you can get to 29.

   b. \(57 \div 9\)  
      
      Why?  
      
      Because \(9 \times 6 = 54\), and \(9 \times 7 = 63\). 54 is closer to 57 than 63 is.

   c. \(108 \div 10\)  
      
      Why?  
      
      \(10 \times 10 = 100\), and \(10 \times 11 = 110\), so 11 is the closest estimate.

   d. \(147 \div 12\)  
      
      Why?  
      
      Because \(12 \times 12 = 144\), so you can’t get any closer.

2. **Challenge** In the two boxes below, make up your own division estimation problems to share with a classmate. **Problems will vary.**
Work Place Instructions 4D Estimate & Check

Each pair of players needs:

- a 4D Estimate & Check Record Sheet to share
- 1 deck of Estimate & Check Cards
- 1 die marked 1–6
- pencils

1. Players shuffle the deck of cards and place them in a stack, face-down, between them. Then they roll the die to determine which of them gets to start.

2. Player 1 takes the first card from the top of the stack and writes the division problem in the first box on his part of the record sheet. The player chooses the best estimate from the six numbers at the top of the sheet and explains his thinking to the other player.

3. Player 2 takes the next card from the top of the stack and follows the instructions in step 2.

4. Both players use a calculator to find the exact answers to their problems and record those answers on their part of the record sheet. Then they record the differences between their estimates and the answers. They put their cards they just used at the bottom of the stack.

5. Players repeat steps 2–4 until each has taken 5 turns. Then they each add up all the differences between their estimates and the actual answers. The player with the lower score wins.

Game Variations

A. Develop a new set of cards that can be used with the two record sheets.
1. Solve each problem below using the traditional (standard) multiplication algorithm.

\[
\begin{align*}
785 \times 39 & = 7065 + 23,550 = 30,615 \\
804 \times 26 & = 4824 + 16,080 = 20,904 \\
653 \times 98 & = 5224 + 58,770 = 63,994
\end{align*}
\]

2. Choose one problem above that you could solve easily with a different strategy. Explain which strategy you would use and why.

Responses and explanations will vary.

3. Fill in the boxes.

\[
\begin{align*}
67 \times 76 & = 4020 + 4690 = 5092 \\
49 \times 27 & = 3430 + 980 = 1323
\end{align*}
\]

Review

4. Claudia says that \(17 \times 80\) is the same as \(17 \times 8 \times 10\). Do you agree or disagree? Explain.

Claudia is correct. Explanations will vary.

5. Andre says that \(4 \times 27\) is the same as \(4 \times 3 \times 9\). Do you agree or disagree? Explain.

Andre is correct. Explanations will vary.
### Story Problem Paper

**Division Combination:** ______

**Story Problem to Match:**

<table>
<thead>
<tr>
<th>Answer: ______</th>
</tr>
</thead>
</table>

### Ratio Table

**Figuring Box**
Division on a Base Ten Grid

1. Complete the following multiplication problems.

\[
\begin{array}{ccccccc}
14 & 14 & 14 & 14 & 14 & 14 & 14 \\
\times 2 & \times 3 & \times 10 & \times 5 & \times 20 & \times 30 & \\
\hline
28 & 42 & 140 & 70 & 280 & 420 & \\
\end{array}
\]

2. Solve the following division problems. Use the multiplication problems above and the grids to help. **Work will vary. Examples shown.**

\[322 \div 14 = 23\]

\[
\begin{array}{c}
322 \\
-140 \\
\hline
182 \\
-140 \\
\hline
42 \\
-42 \\
\hline
0 \\
\end{array}
\]

\[10 + 10 + 3 = 23\]

\[238 \div 14 = 17\]

\[
\begin{array}{c}
238 \\
-140 \\
\hline
98 \\
-70 \\
\hline
28 \\
-28 \\
\hline
0 \\
\end{array}
\]

\[10 + 5 + 2 = 17\]
Water Conservation page 1 of 2

Do you want to help conserve water? Here are some water-saving tips. Be sure to show all of your work for each of these problems.

1. If you leave the faucet running while you take a 5-minute shower, you use about 400 cups of water. How many gallons is that?

   - **Ratio Table**
     | Gallons | 1 | 10 | 20 | 5 | 2 | 4 | 8 |
     | Cups    | 16 | 160 | 320 | 80 | 32 | 64 | 128 |
   - **Solution**
     \[
     \begin{align*}
     \text{Gallons} & : 1, 10, 20, 5, 2, 4, 8 \\
     \text{Cups} & : 16, 160, 320, 80, 32, 64, 128 \\
     \frac{400}{16} & = 25 \text{ gallons}
     \end{align*}
     \]

   a. If you get wet, turn off the water to soap up, and turn the water back on to rinse off, you only use about 64 cups of water. How many gallons is that?

     \[
     \begin{align*}
     \frac{64}{16} & = 4 \text{ gallons} \\
     \end{align*}
     \]

   b. If you take a shower every day and use the method described in part a above, how many gallons of water can you save in a day? How many gallons of water can you save in a week?

     \[
     25 - 4 = 21 \text{ gallons in a day}
     \]

     \[
     21 \times 7 = (20 \times 7) + (1 \times 7) \\
     = 140 + 7 \\
     = 147
     \]

     You can save 21 gallons in a day.

     You can save 147 gallons in a week.
If you fill the bathtub all the way, it takes about 576 cups of water. How many gallons is that? \[ \text{36 gallons} \]

If you fill the bathtub just enough to wash yourself, it takes about 144 cups of water. How many gallons is that?

\[ \text{9 gallons} \]

If you take a bath 3 times a week and use the second method described above, how many gallons of water can you save in a week? How many gallons of water can you save in a month?

\[ \frac{36}{27} = \text{gallons in a day} \]

\[ 27 \times 3 = (20 \times 3) + (7 \times 3) = 60 + 21 = 81 \text{ gallons in a week} \]

\[ 81 \times 4 = 324 \text{ gallons in a month} \]
Water Conservation Challenge

1 If you leave the hose running the whole time you wash a car, it takes about 4,800 cups of water. If you fill a bucket, wash the car, and then rinse it with the hose, it takes about 240 cups of water. How many gallons of water can you save by using a bucket and hose instead of leaving the water running?

285 gallons; work will vary. Example:

\[
\begin{align*}
1600 & \quad 300 \text{ gallons} \\
16 \div 4800 & \quad 15 \text{ gallons} \\
- 3200 & \quad - 160 \\
1600 & \quad 80 \\
- 1600 & \quad - 80 \\
0 & \quad 0
\end{align*}
\]

\[300 - 15 = 285\]

2 Mr. Mugwamp has a leaky faucet. It leaks 2 drops of water every second. If there are 3,840 drops of water in a cup, how many gallons of water will be wasted in a single day (24 hours)?

2 \(\frac{13}{16}\) gallons or 2.8125 gallon or almost 3 gallons

Work will vary. Example:

\[
\begin{align*}
24 \times 60 \times 60 & = 86,400 \text{ seconds in 24 hours} \\
86,400 \times 2 & = 172,800 \text{ drops in 24 hours.} \\
172,800 \div 3,840 & = 45 \text{ cups} \\
16 \div 45 & = 32 \\
13
\end{align*}
\]
Division with Tables & Sketches

1 Fill in the ratio table for 19.

<table>
<thead>
<tr>
<th>Number of Groups</th>
<th>1</th>
<th>2</th>
<th>10</th>
<th>5</th>
<th>20</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>19</td>
<td>38</td>
<td>190</td>
<td>95</td>
<td>380</td>
<td>285</td>
</tr>
</tbody>
</table>

2 Solve the two division problems using the ratio table above and sketches to help. You can add to the ratio table if you want to.

ex 304 ÷ 19 = \_6\_  
\[\begin{array}{c|cc|c|c|c|c|c} 
& & & \underline{304} & & \underline{5} & \underline{10} & \underline{16} \\
19 & -190 & & & & & & \\
114 & & & & & & & \\
-95 & & & & & & & \\
-19 & & & & & & & \\
0 & & & & & & & \\
\end{array}\]

\[\begin{array}{c|cc|c|c|c|c|c} 
& & & \underline{304} & & \underline{5} & \underline{10} & \underline{16} \\
19 & -190 & & & & & & \\
114 & & & & & & & \\
-95 & & & & & & & \\
-19 & & & & & & & \\
0 & & & & & & & \\
\end{array}\]

Work will vary

Work will vary

3 Use the standard multiplication algorithm to solve the problems below. Show your work.

\[\begin{array}{c|c|c|c|c|c|c|c} 
& \underline{84} & \underline{79} & \underline{86} & \underline{92} \\
\times 36 & & & & \\
\underline{504} & \underline{474} & \underline{172} & \underline{644} \\
+2520 & +1580 & +2580 & +2760 \\
\underline{3024} & \underline{2054} & \underline{2752} & \underline{3404} \\
\end{array}\]

Answer Key: Work will vary
Work Place Instructions 4E Lowest Remainder Wins

Each pair of players needs:
- a 4E Lowest Remainder Wins Record Sheet for each player
- a clear spinner overlay
- 1 die marked 0–5
- 2 dice marked 1–6

1. Players roll one of the dice to decide who goes first. Player 1 then spins the spinner to get the first divisor for both players.

2. Both players start a ratio table for that divisor in the Round 1 box on their record sheet and fill in the table for 1 and 10 groups of the divisor.
   Players can add more entries to their ratio tables if they need them while they are playing.

3. Each player takes a turn to roll the 3 dice one time and then arranges the digits any way he or she likes to make a dividend.
   Players each try to make a 3-digit number that won’t leave a remainder when it’s divided by the divisor. If they can’t do that, they try to make a number that will leave a very small remainder. Players can add entries to their ratio tables to help them decide how to arrange the digits if they like.

4. Both players record their division problem on their own record sheet and do the division.
   Players can continue to add any useful entries to their ratio tables to help as they go along.

5. When both players have finished their division problems, they explain their work to each other. When they both agree that the other’s work is correct, they enter their score and that of the other player’s at the bottom of their record sheet.
   A player gets 0 points if she had no remainder. Otherwise, the player gets the number of points that matches her remainder.

6. Players play two more rounds of the game and then add up their scores at the bottom of the sheet to find their total. The player with the lower score wins.

Game Variations

A. Use 2 dice numbered 4–9 instead of the 3 dice on the materials list.

B. Use the challenge record sheets instead of the regular record sheets for this game. The challenge sheets have a spinner with higher divisors.

C. Use 2 dice marked 4–9 and one die marked 1–6 to get higher dividends.

D. Use 2 dice marked 4–9 and 2 dice marked 1–6 to get 4-digit dividends.
Divisibility Rules

It’s easy to tell if a small number like 12 is divisible by another number. With bigger numbers, like 435, it can be harder to tell. You already know how to tell if a number is divisible by 2, 5, or 10. There are also rules that can help you tell if any number is divisible by 3, 6, or 9.

<table>
<thead>
<tr>
<th>Rule</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>A number is divisible by 3 if the sum of its digits is divisible by 3.</td>
<td>957 is divisible by 3 because $9 + 5 + 7 = 21$, and 21 is divisible by 3. $(21 \div 3 = 7)$</td>
</tr>
<tr>
<td>A number is divisible by 6 if it is divisible by 3 (see above) and it is divisible by 2 (has a 0, 2, 4, 6, or 8 in the ones place).</td>
<td>786 is divisible by 6 because $7 + 8 + 6 = 21$, and 21 is divisible by 3. $(21 \div 3 = 7)$ 786 also ends in 6, which means it is even (divisible by 2).</td>
</tr>
<tr>
<td>A number is divisible by 9 if the sum of its digits is divisible by 9.</td>
<td>837 is divisible by 9 because $8 + 3 + 7 = 18$, and 18 is divisible by 9.</td>
</tr>
</tbody>
</table>

1. Use the chart below to help you figure out if the numbers are divisible by 3, 6, or 9. In the last column, you don’t have to list all the factors of the number. Just list any other numbers you know for sure that the number is divisible by.

<table>
<thead>
<tr>
<th>Number</th>
<th>Sum of the Digits</th>
<th>Divisible by 3?</th>
<th>Divisible by 6?</th>
<th>Divisible by 9?</th>
<th>Also Divisible by:</th>
</tr>
</thead>
<tbody>
<tr>
<td>ex 495</td>
<td>$4 + 9 + 5 = 18$</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>5</td>
</tr>
<tr>
<td>a 987</td>
<td>$9 + 8 + 7 = 24$</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>1</td>
</tr>
<tr>
<td>b 540</td>
<td>$5 + 4 + 0 = 9$</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>10, 2, 5</td>
</tr>
<tr>
<td>c 762</td>
<td>$7 + 6 + 2 = 15$</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>2</td>
</tr>
<tr>
<td>d 747</td>
<td>$7 + 4 + 7 = 18$</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>1</td>
</tr>
<tr>
<td>e 570</td>
<td>$5 + 7 + 0 = 12$</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>2, 5, 10</td>
</tr>
<tr>
<td>f 645</td>
<td>$6 + 4 + 5 = 15$</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>5</td>
</tr>
<tr>
<td>g 792</td>
<td>$7 + 9 + 2 = 18$</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>2</td>
</tr>
</tbody>
</table>

Answers will vary.
Multiplication Problems & Mazes

1. Complete the multiplication problems below. Use problems you have already solved to help solve other ones.
   a. $18 \times 2 = \underline{36}$
   b. $23 \times 2 = \underline{46}$
   c. $34 \times 2 = \underline{68}$
   a. $18 \times 3 = \underline{54}$
   b. $23 \times 3 = \underline{69}$
   c. $34 \times 3 = \underline{102}$
   a. $18 \times 10 = \underline{180}$
   b. $23 \times 10 = \underline{230}$
   c. $34 \times 10 = \underline{340}$
   a. $18 \times 5 = \underline{90}$
   b. $23 \times 5 = \underline{115}$
   c. $34 \times 5 = \underline{170}$

2. Use the problems above to write three more combinations for each number. Show as much work as you need to find each product.
   a. $18 \times \underline{_____} = \underline{_____}$
   b. $23 \times \underline{_____} = \underline{_____}$
   c. $34 \times \underline{_____} = \underline{_____}$
   a. $18 \times \underline{_____} = \underline{_____}$
   b. $23 \times \underline{_____} = \underline{_____}$
   c. $34 \times \underline{_____} = \underline{_____}$
   a. $18 \times \underline{_____} = \underline{_____}$
   b. $23 \times \underline{_____} = \underline{_____}$
   c. $34 \times \underline{_____} = \underline{_____}$

3. Use multiplication and division to find the secret path through each maze. The starting and ending points are marked for you. You can only move one space up, down, over, or diagonally each time. Write four equations to explain the path through the maze.

**Example:**

- $36 \div 4 = 9$
- $9 \times 20 = 180$
- $180 \div 3 = 60$
- $60 \div 20 = 3$

- $240 \div 60 = 4$
- $4 \times 30 = 120$
- $120 \div 6 = 20$
- $20 \div 4 = 5$

- $420 \div 70 = 6$
- $6 \times 40 = 240$
- $240 \div 8 = 30$
- $30 \div 6 = 5$
Mike’s Measurements

Mike is moving to a new house. He is measuring his furniture to see which items will fit in his new room. Help Mike determine the measurements of his furniture.

1. Mike’s bed has a base of 192 cm by 96 cm. What is the perimeter of the base of Mike’s bed in meters? Show your work.

   5.76 meters; work will vary.

2. Mike’s wooden storage box is 25 inches by 36 inches by 39 inches. What is the volume of Mike’s box? Show your work.

   35,100 cubic inches; work will vary.

3. Mike’s room is rectangular. One wall of Mike’s new room is 3.96 meters long.
   a. How long is this wall in centimeters?
      396 cm
   b. How long is this wall in millimeters?
      3,960 mm

4. Another wall in Mike’s room is 2.51 meters long.
   a. How long is this wall in centimeters?
      251 cm
   b. How long is this wall in millimeters?
      2,510 mm

5. What is the area of Mike’s room in square centimeters? Show your work.

   99,396 sq cm.; work will vary.
Work Place Instructions 5A Target One Fractions

Each pair of partners needs:
- 2 Target One Fractions Record Sheets
- 1 deck of Number Cards, with the 0s, 7s, 9s, and wild cards removed
- math journals

1. Players shuffle the deck of cards and decide who will be the dealer. The dealer gives 5 cards to each player.
2. Player 1 chooses three cards to make a whole number and a fraction that will result in a product that is as close to 1 as possible. Each card is only used once.
   For the fraction, one card is the numerator and the other is the denominator. For example, if the cards 2, 5, and 8 are used, players could choose 5 × \(\frac{2}{8}\), 5 × \(\frac{8}{2}\), 2 × \(\frac{5}{8}\), etc.

```
5 8 6 2 5
```

Player 1 OK, I got two 5s, a 2, a 6, and an 8. Hmm... I think I'm going to use one of the 5s as my whole number, and the 2 and the 8 for the fraction, like this.

3. Player 1 multiplies the two numbers and shows her work in her math journal. Then players discuss how to multiply the two numbers.
   Player 1 I think that 5 × \(\frac{2}{8}\) is \(\frac{10}{8}\), which is \(\frac{5}{4}\), and that's the same as \(\frac{1}{4}\).
   Player 2 I thought of it as 5 × \(\frac{2}{8}\) = 5 × \(\frac{1}{4}\), so it's \(\frac{5}{4}\). That's also \(\frac{1}{4}\).

4. Player 1 writes an equation to represent her work on her record sheet.

5. Player 1 figures her score by finding the difference between the product and 1. Both Player 1 and Player 2 both record Player 1's score on their record sheets.
   A product of \(\frac{1}{4}\) has a score of \(\frac{1}{4}\). A product of \(\frac{5}{4}\) has a score of \(\frac{1}{4}\). A product of 1 has a score of 0.
   At the end of each turn, players keep the 2 cards they did not use. The dealer passes out 3 new cards to each player, so each player has a total of 5 cards to begin the next round.

6. Player 2 takes a turn. Players continue to take turns until they have played 5 rounds of the game.

7. Players add the scores of all 5 rounds. The lower total score wins.

Game Variations

A Include the wild cards in the deck. A wild card can be any numeral 1–6. Players put a star above the number made from a wild card in the equation on their record sheets.

B Products below 1 get a positive score. Products above 1 get a negative score. Players add those scores together and the final score closest to 0 wins.
   For example, a product of \(\frac{7}{8}\) would be scored as +\(\frac{3}{8}\), and a product of \(1\frac{1}{4}\) would be scored as –\(\frac{1}{4}\).

C Players play Target Two Fractions, trying to get as close as possible to 2 instead of 1.
Target One Fractions

In Target One Fractions, players choose 3 numbers to create a whole number and a fraction that have a product close to 1. Their score is the difference between their product and 1.

1. Claudia is playing Target One Fractions. She has these cards: 2, 3, 5, 5, 8. Help Claudia by choosing 3 cards and writing an equation she will solve.

   Responses will vary.
   The numbers that will yield a result closest to 1 are 2, 3, and 5 used in one of the following equations:

   \( 2 \times \frac{3}{5} = \frac{6}{5} \) or \( 3 \times \frac{2}{5} = \frac{6}{5} \)

2. Ernesto is playing Target One Fractions. He chose these cards: 2, 3, 8. He made the problem \( 8 \times \frac{2}{3} \).

   a. What is \( 8 \times \frac{2}{3} \)? Show your work.

      \( 16 \frac{2}{3} = 5 \frac{1}{3}; \) work will vary.

   b. What is Ernesto’s score? _________

   c. How else could you arrange the numbers? Responses will vary.

      The arrangement that will yield a product closest to 1 is \( 2 \times \frac{3}{8} \).

   d. What would your product be? Show your work.

      Answers to 2d, e, and f will vary depending on how each student chooses to arrange the numbers.

   e. What is your score? ________

   f. What is the difference between your score and Ernesto’s? Show your work.
A Fraction of a Whole  page 1 of 2

Use numbers and labeled sketches to solve the problems on this page. Show your work.

1 Nathan participated in a 5K (5 kilometer) race. He walked $\frac{1}{4}$ of the way, jogged $\frac{3}{5}$ of the way, and ran $\frac{3}{20}$ of the way.

a How many kilometers did Nathan walk?

$$1 \frac{1}{4} \text{ km}$$

$$\frac{1}{4} \times 5 = (\frac{1}{4} \times 4) + (\frac{1}{4} \times 1) = 1 + \frac{1}{4} = 1 \frac{1}{4}$$

b How many meters did Nathan walk?

$$1,250 \text{ m}$$

$$1 \times 1,000 = 1,000$$

$$\frac{1}{4} \times 1,000 = 250 \Rightarrow 1,250$$

c How many kilometers did Nathan jog?

$$3 \text{ km}$$

$$\frac{1}{2} \times 5 = 1, \text{ so } \frac{3}{5} \times 5 = 1 + 1 + 1 = 3$$

d How many meters did Nathan jog?

$$3,000 \text{ m}$$

$$3 \times 1,000 = 3,000$$

e How far in kilometers did Nathan run?

$$\frac{3}{4} \text{ km}$$

$$\frac{1}{20} \times 5 = \frac{1}{20} + \frac{1}{20} + \frac{1}{20} + \frac{1}{20} + \frac{1}{20} = \frac{5}{20} = \frac{1}{4}, \text{ so } \frac{3}{20} \times 5 = \frac{3}{4}$$

f How many meters did Nathan run?

$$750 \text{ m}$$

$$\frac{1}{4} \times 1,000 = 250, \text{ so } \frac{3}{4} \times 1,000 = 3 \times 250 = 750$$

2 Deja’s bedroom is 9 square meters. She just got a new rug that covers $\frac{3}{5}$ of her bedroom floor. How big is Deja’s new rug?

$$5 \frac{2}{5}$$

$$\frac{1}{5} \times 9 = (\frac{1}{5} \times 5) + (\frac{1}{5} \times 4) = 1 \frac{4}{5}, \text{ so } \frac{3}{5} \times 9 = 1 \frac{4}{5} + 1 \frac{4}{5} + 1 \frac{4}{5} = 3 \frac{12}{5} = 5 \frac{2}{5} \text{ sq. m}$$

(continued on next page)
3 Write a story problem for \( \frac{2}{3} \times 4 \). Then solve your own problem.

\[ \frac{2}{3} \times 4 = \frac{8}{3} \text{ or } 2 \frac{2}{3} \]

Story problems and work will vary.

4 Solve the following combinations. Show your work for each. **Work will vary. Examples shown.**

*Hint:* Use one or more of the strategies on the chart you made with your classmates.

\[ 3 \times \frac{3}{4} = \frac{9}{4} \text{ or } 2 \frac{1}{4} \quad 6 \times \frac{2}{5} = 2 \frac{1}{5} \quad 8 \times \frac{2}{3} = 4 \frac{2}{3} \text{ or } 5 \frac{1}{3} \]

\[ \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{9}{4} \quad 6 \times \frac{2}{5} = \left(5 \times \frac{2}{5}\right) + \left(1 \times \frac{2}{5}\right) = 2 + \frac{2}{5} \quad 8 \times \frac{2}{3} = \left(6 \times \frac{2}{3}\right) + \left(2 \times \frac{2}{3}\right) = 4 + \frac{2}{3} + \frac{2}{3} = 4 \frac{2}{3} \]

5 Fill in the blanks.

\[ \frac{1}{4} \times \underline{36} = 9 \quad \frac{2}{4} \times \underline{18} = 9 \quad \frac{3}{4} \times \underline{12} = 9 \quad \frac{4}{4} \times \underline{9} = 9 \]

Find and describe at least one pattern in the 4 combinations above.

**Observations will vary. Examples:**
- The greater the fraction, the smaller the whole number you multiply it by to get 9.
- If you multiply the whole number by the numerator in each fraction, you get 36: \(1 \times 36 = 36\), \(2 \times 18 = 36\), \(3 \times 12 = 36\), and \(4 \times 9 = 36\).

6 Fill in the blanks.

\[ \frac{1}{5} \times \underline{75} = 15 \quad \frac{1}{5} \times \underline{150} = 30 \quad \frac{1}{5} \times \underline{300} = 60 \quad \frac{1}{5} \times \underline{600} = 120 \]

Find and describe at least one pattern in the 4 combinations above.

**Observations will vary. Examples:**
- The whole number by which you multiply \( \frac{1}{5} \) doubles each time, which makes sense because the product also doubles each time.

7 **CHALLENGE** Fill in the blanks.

\[ \frac{5}{4} \times \underline{8} = 10 \quad \frac{8}{4} \times \underline{4 \frac{1}{2}} = 9 \quad \frac{3}{5} \times \underline{50} = 30 \quad \frac{4}{5} \times \underline{75} = 60 \]
Fractions of Wholes

1. Find the product.
   a. \( \frac{1}{4} \) of 7 = \( \frac{7}{4} \) or 1 \( \frac{3}{4} \)
   b. \( \frac{1}{5} \times 25 = \) __5__
   c. \( \frac{1}{3} \) of 36 = __12__
   d. \( \frac{3}{4} \) of 7 = \( \frac{5}{4} \)
   e. \( \frac{4}{3} \times 25 = \) __20__
   f. \( \frac{2}{3} \times 36 = \) __24__

2. True or False?
   a. \( \frac{3}{4} \times 11 = 8 \frac{1}{4} \) T F
   b. \( \frac{3}{5} \) of 20 = 15 T F
   c. \( \frac{2}{5} \) of 30 = 18 T F
   d. \( 16 \times \frac{1}{5} = \frac{16}{5} \) T F
   e. \( \frac{2}{6} \times 21 = 7 \) T F
   f. \( 24 \times \frac{2}{3} = \frac{48}{3} \) T F

3. Madeline read \( \frac{2}{3} \) of her favorite book on the car ride to her grandparents’ house. If the book had 225 pages, how many pages of the book has she read?
   150 pages; work will vary.

4. Theo entered a race that required him to ride his bike 54 kilometers and run \( \frac{1}{6} \) as far as he bikes.
   a. How many kilometers will Theo run?
      9 km
      Work will vary.
   b. How many meters will he run?
      9,000 m
Thinking About Strategy

1  Anna is playing Target One Fractions. She has these cards: 4, 3, 6, 4, 2.
   a  What three cards should she choose to make a whole number and fraction
      whose product is close to 1?  Answers will vary. The 3 numbers that will
      result in the product closest to 1 are 2, 3, and 6 used in one of two
      equations: 2 × 3/6 or 3 × 2/6.
   b  Write an equation for the problem Anna will solve.
      Answers to problems 1b, c, and d will vary depending on the 3
      numbers selected.
   c  Solve the problem.
   d  What is Anna’s score for this round?

2  Multiply.
   Hint: Use one of the strategies on the class poster from today’s session.
   \[ \frac{3}{5} \times 4 = \frac{12}{5} \text{ or } 2 \frac{2}{5} \quad \quad \frac{4}{5} \times 3 = \frac{12}{5} \text{ or } 2 \frac{2}{5} \quad \quad \frac{4}{5} \times 16 = \frac{64}{5} \text{ or } 12 \frac{4}{5} \]
   Work will vary.

3  CHALLENGE  Morgan thought she would be able to sell 75 plants for the school
               fundraiser. So far, she has sold \( \frac{2}{3} \) of her goal. Morgan’s friend Billy has sold \( \frac{4}{5} \) of his
               goal of 60 plants.
   a  Make a prediction: which of the two students has sold more plants?
      Predictions will vary.
   b  How many plants has each student sold? Show your work.
      Morgan has sold 50 plants (\( \frac{2}{3} \times 75 = 50 \)).
      Billy has sold 48 plants (\( \frac{4}{5} \times 60 = 48 \)).
Ryan’s Baseball Cards

1 Ryan has 48 baseball cards. He gives some of them away to his friends. Help Ryan figure out how many cards each of his friends will get. Show your work.

a If Ryan gives \( \frac{1}{4} \) of his 48 cards to Anna, how many cards does he give Anna?

12 cards; work will vary.

b If Ryan gives \( \frac{3}{8} \) of his 48 cards to Josiah, how many cards does he give Josiah?

18 cards; work will vary.

c If Ryan gives \( \frac{2}{6} \) of his 48 cards to Max, how many cards does he give Max?

16 cards; work will vary.

d How many cards does Ryan have left? What fraction of his original 48 cards is this?

2 cards; \( \frac{3}{48} \) or \( \frac{1}{24} \) of the original 48 cards.

2 Solve the problems below.

\[
\frac{2}{3} + \frac{5}{6} = \underline{\frac{9}{6} \text{ or } 1 \frac{3}{6}} \text{ or } \frac{1}{12} \quad 1 \frac{1}{3} - \frac{7}{8} = \underline{\frac{11}{24}} \quad 1 \frac{4}{5} + 1 \frac{3}{10} = \underline{2 \frac{11}{10} \text{ or } 3 \frac{1}{10}}
\]

\[
16 \times \frac{7}{8} = \underline{14} \quad \frac{4}{5} \times 24 = \underline{\frac{96}{5} \text{ or } 19 \frac{1}{5}} \quad 27 \times \frac{4}{3} = \underline{12}
\]

\[
16 \frac{1}{8} - 15 \frac{3}{5} = \underline{21\frac{1}{40}} \quad 208 \frac{4}{7} + 201 \frac{3}{4} = \underline{410 \frac{9}{28}} \quad 20 \frac{1}{6} - 15 \frac{3}{5} = \underline{4 \frac{17}{30}}
\]
More Geoboard Perimeters

P = ____________ linear units  
P = ____________ linear units

P = ____________ linear units  
P = ____________ linear units

P = ____________ linear units  
P = ____________ linear units
More Geoboard Perimeters, Challenge

P = ____________ linear units

P = ____________ linear units

P = ____________ linear units

P = ____________ linear units

P = ____________ linear units

P = ____________ linear units
Reviewing the Standard Algorithm for Multiplication

1. Solve these multiplication problems.

<table>
<thead>
<tr>
<th></th>
<th>80</th>
<th>80</th>
<th>80</th>
<th>600</th>
<th>600</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>×60</td>
<td>4,800</td>
<td>×70</td>
<td>5,600</td>
<td>×80</td>
<td>6,400</td>
<td>×10</td>
</tr>
</tbody>
</table>

2. Solve these multiplication problems using the *standard algorithm*. Use the answers above to make sure your answers are reasonable.

**Example (ex):**

\[
\begin{align*}
21 & \times 36 \\
72 & \\
184 & \\
\end{align*}
\]

\[
\begin{align*}
\underline{1,104} \\
\underline{+ 5,520} \\
\underline{6,624} \\
\end{align*}
\]

**Exercise (a):**

\[
\begin{align*}
78 & \times 76 \\
5,460 & + 468 \\
\underline{5,928} \\
\end{align*}
\]

**Exercise (b):**

\[
\begin{align*}
80 & \times 72 \\
& \underline{160} \\
& \underline{+ 5,600} \\
& \underline{5,760} \\
\end{align*}
\]

**Exercise (c):**

\[
\begin{align*}
78 & \times 59 \\
& \underline{702} \\
& \underline{+ 3,900} \\
& \underline{4,602} \\
\end{align*}
\]

**Exercise (d):**

\[
\begin{align*}
587 & \times 13 \\
& \underline{1,761} \\
& \underline{+ 5,870} \\
& \underline{7,631} \\
\end{align*}
\]

**Exercise (e):**

\[
\begin{align*}
602 & \times 26 \\
& \underline{3,612} \\
& \underline{+ 12,040} \\
& \underline{15,652} \\
\end{align*}
\]
Fraction Multiplication Grids
Simplifying Fractions

1. Write all the factors of each number below. Try to think of the factors in pairs.

<table>
<thead>
<tr>
<th></th>
<th>Factors of 2</th>
<th>Factors of 4</th>
<th>Factors of 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1, 2</td>
<td>1, 2, 4</td>
<td>1, 2, 4, 8</td>
</tr>
<tr>
<td>3</td>
<td>1, 3</td>
<td>1, 2, 3, 6</td>
<td>1, 2, 3, 4, 6, 12</td>
</tr>
</tbody>
</table>

2. You can simplify a fraction by dividing the numerator and the denominator by the same number. If you divide the numerator and denominator by the largest factor they have in common (the greatest common factor), you can show the fraction in its simplest form. Look carefully at the example below. Then fill in the rest of the table.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Factors of the Numerator (top number)</th>
<th>Factors of the Denominator (bottom number)</th>
<th>Greatest Common Factor</th>
<th>Divide to get the simplest form.</th>
<th>Picture and Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ex (\frac{4}{12})</td>
<td>1, 2, 4</td>
<td>1, 2, 3, 4, 6, 12</td>
<td>4</td>
<td>(\frac{4}{12} ÷ 4 = \frac{1}{3})</td>
<td>![Picture]</td>
</tr>
<tr>
<td>a (\frac{4}{6})</td>
<td>1, 2, 4</td>
<td>1, 2, 3, 6</td>
<td>2</td>
<td>(\frac{4}{6} ÷ 2 = \frac{2}{3})</td>
<td>![Picture]</td>
</tr>
<tr>
<td>b (\frac{3}{12})</td>
<td>1, 3</td>
<td>1, 2, 3, 4, 6, 12</td>
<td>3</td>
<td>(\frac{3}{12} ÷ 3 = \frac{1}{4})</td>
<td>![Picture]</td>
</tr>
</tbody>
</table>
Area of the Addition Key

1. Julissa measured the + key on her calculator. It was $\frac{1}{2}$ inch tall and $\frac{1}{4}$ inch wide. What is the area of the + key?

   a. Use your geoboard to make a model of the key. In the model, the total area of the geoboard represents 1 square inch, so each side would be exactly 1 inch long.

   b. Draw a sketch of the geoboard model of the key here. Label the dimensions and area of the key.

   c. Write an equation to show the dimensions and area of the + key on Julissa’s calculator.

      \[
      \frac{1}{2} \text{ in.} \times \frac{1}{4} \text{ in.} = \frac{1}{8} \text{ sq. in.}
      \]
      
      or \[
      \frac{1}{4} \text{ in.} \times \frac{1}{2} \text{ in.} = \frac{1}{8} \text{ sq. in.}
      \]
Multiplying Fractions with the Area Model  page 1 of 2

1  For each problem, make a sketch, label the dimensions and area, and write an equation.  
Work may vary somewhat. Examples shown.

a  There was a price tag on Isabel’s new book that was \(\frac{1}{4}\) inch wide and \(\frac{3}{4}\) inch long. What was the area of the price tag?

\[
\begin{array}{c|c|c|c|c\}
\hline
& \frac{3}{4}'' & & & \\
\hline
\frac{3}{4}'' & & & & \\
\hline
\end{array}
\]

\[
\frac{3}{4} \times \frac{1}{4} = \frac{3}{16} \text{ sq. in.}
\]

b  Tomas’s teacher has little stickers she likes to give out when her students have had a good day. Each sticker is \(\frac{1}{2}\) inch wide and \(\frac{1}{2}\) inch tall. What is the area of each sticker?

\[
\begin{array}{c|c|c|c|c\}
\hline
& \frac{3}{4}'' & & & \\
\hline
\frac{1}{2}'' & & & & \\
\hline
\end{array}
\]

\[
\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \text{ sq. in. (or } \frac{2}{8} \text{ or } \frac{4}{16} \text{ sq. in.)}
\]

c  There is a special holiday stamp that measures \(\frac{3}{4}\) inch by \(\frac{1}{2}\) inch. What is the area of the stamp?

\[
\begin{array}{c|c|c|c|c\}
\hline
& \frac{3}{4}'' & & & \\
\hline
\frac{1}{2}'' & & & & \\
\hline
\end{array}
\]

\[
\frac{3}{4} \times \frac{1}{2} = \frac{3}{8} \text{ sq. in. (or } \frac{6}{16} \text{ sq. in.)}
\]

(continued on next page)
For each multiplication problem, draw an array, label the dimensions and area, and write an equation.

2  For each multiplication problem, draw an array, label the dimensions and area, and write an equation.

a  \( \frac{1}{2} \times \frac{1}{8} \)

\( \frac{1}{2}'' \)  \( \frac{1}{8}'' \)

\( \frac{1}{16} = \frac{1}{16} \)

\( \frac{1}{2} \times \frac{1}{8} = \frac{1}{16} \) (or \( \frac{4}{64} \))

b  \( \frac{5}{8} \times \frac{1}{4} \)

\( \frac{5}{8}'' \)  \( \frac{1}{4}'' \)

\( \frac{5}{32} = \frac{5}{32} \)

\( \frac{5}{8} \times \frac{1}{4} = \frac{5}{32} \) (or \( \frac{10}{64} \))

3  Choose two fractions and show how you could multiply them on this grid. Label the dimensions and area, and write an equation to show the fractions and their product.
### Missing Fractions

1. Fill in the missing fraction or mixed number in each equation.

\[ 1 \frac{3}{4} + \frac{1}{4} = 2 \]
\[ 1 = \frac{2}{5} + \frac{3}{5} \]
\[ 2 = \frac{7}{8} + 1 \frac{1}{8} \]

\[ 3 = 1 \frac{1}{5} + 1 \frac{4}{5} \]
\[ 4 = 1 \frac{2}{6} + 2 \frac{2}{6} \]
\[ 2 \frac{3}{5} + 1 \frac{3}{5} = 4 \frac{2}{10} \]

2. Calvin and his family went on another walk. This time, they went to the park, and then to the ice cream parlor. They stopped at the new fountain on their way home. In all, they walked 4 \( \frac{3}{8} \) kilometers. How far was it from the fountain to their home? Use the map to help solve the problem. Show all your work.

\[ \frac{3}{8} \text{ km; work will vary.} \]
Brian’s Boxes & Kevin’s Pictures

1. Brian is filling in puzzle boxes like those below. He’s done the first one already. Look at how he filled it in, then complete the others.

![Example box]

\[
\begin{array}{c}
\text{a} \\
6 \quad \frac{3}{8}
\end{array}
\]

\[
\begin{array}{c}
\text{b} \\
5 \quad \frac{3}{4}
\end{array}
\]

2. Make up a box challenge for Brian to solve. Fill in the entire first box to show the completed box. Then, fill in just two parts of the second box so it is ready for Brian to solve. Think carefully about which parts of the box you will leave blank.

Responses will vary.

3. Kevin has 140 pictures. He wants to sell them for $4.50 each. If Kevin sells all of his pictures, how much money will be he make? Think about the fastest way to solve this problem. Show your work.

\[\$630.00; \text{ work will vary.}\]

4. Kevin has sold pictures twice before. He made $172.95 at his first sale and $398.26 at his second sale. Show your work for the following problems.

\[\text{a} \quad \text{How much did Kevin make from his first two sales?} \]

\[\$571.21; \text{ work will vary.}\]

\[\text{b} \quad \text{How much more did Kevin make at his second sale than his first sale?} \]

\[\$225.31; \text{ work will vary.}\]
Find the Product

1. Compete the puzzle boxes.
   - **Example**
   - a
   - b

2. Fill in the blanks.
   - a \(13 \times \frac{7}{7} = 13\)
   - b \(\frac{4}{4} \times 12 = 12\)
   - c \(5 \times \frac{2}{3} = 3\frac{1}{3}\)

3. Thanh ate \(\frac{7}{12}\) of his candy bar on Monday and another \(\frac{1}{3}\) of his candy bar on Tuesday. What fraction of the candy bar does Thanh have left? Show your work.

   He has \(\frac{1}{12}\) of the candy bar remaining; work will vary.

4. Charlotte and Riley walked \(\frac{6}{10}\) of a mile to school one day, and then after school they walked \(\frac{3}{8}\) of a mile to the store. From the store back home was another \(\frac{2}{5}\) of a mile. How far did the girls walk in all? Show your work.

   1 \(\frac{1}{6}\) miles; work will vary.
Picturing Fraction Multiplication

1. Each of the pictures below shows the results of multiplying one fraction by another. Label each of the shaded regions with its dimensions and area. Then write a multiplication equation to match.

   **ex**
   \[
   \frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}
   \]

   **a**
   \[
   \frac{4}{5} \times \frac{4}{5} = \frac{16}{25}
   \]

   **b**
   \[
   \frac{3}{4} \times \frac{6}{8} = \frac{18}{32} = \frac{9}{16}
   \]

   **c**
   \[
   \frac{3}{5} \times \frac{8}{10} = \frac{24}{50} = \frac{12}{25}
   \]

2. Pedro is using paper rectangles that are all the same size to make a collage. Each piece is \(\frac{3}{4}\) inch by \(\frac{1}{2}\) inch. What is the area of each piece? Use numbers, words, or pictures to solve the problem. Show your work.

   **Work will vary.**

   Each piece had an area of \(\frac{3}{8}\) square inch.
Modeling Fraction Multiplication

1. Circle the picture that best represents each problem.

   a. \( \frac{4}{7} \times \frac{3}{4} = \frac{12}{28} = \frac{3}{7} \)

   - A
   - B
   - C
   - D

   b. \( \frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6} \)

   - A
   - B
   - C
   - D

2. Use the squares to model each combination and find the products. You will need to divide the sides of each square in order to represent each fraction as a dimension.

   a. \( \frac{4}{5} \times \frac{5}{6} = \frac{20}{30} = \frac{2}{3} \)

   - \( \frac{4}{5} \)
   - \( \frac{5}{6} \)

   b. \( \frac{7}{8} \times \frac{2}{3} = \frac{14}{40} = \frac{7}{20} \)

   - \( \frac{7}{8} \)
   - \( \frac{2}{3} \)

   c. \( \frac{1}{4} \times \frac{2}{6} = \frac{2}{24} = \frac{1}{12} \)

   - \( \frac{1}{4} \)
   - \( \frac{2}{6} \)
### Reasoning About Multiplying with Fractions

Write =, >, or < to make each statement true.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3 \times 45 = \text{B}$</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>B &gt; 3</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>B &gt; 45</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$3 \times 1 = \text{C}$</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>C = 3</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>C &gt; 1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$\frac{3}{4} \times 1 = \text{A}$</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>A &lt; 1</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>A = $\frac{3}{4}$</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$\frac{3}{7} \times \frac{4}{15} = \text{P}$</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>P &lt; $\frac{3}{7}$</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>P &lt; $\frac{4}{15}$</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>P &lt; 1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$1\frac{7}{9} \times \frac{5}{6} = \text{Q}$</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>Q &lt; $1\frac{7}{9}$</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>Q &gt; $\frac{5}{6}$</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>Q &gt; 1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$\frac{6}{17} \times 7 = \text{S}$</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>S &gt; $\frac{6}{17}$</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>S &lt; 7</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>S &gt; 2</td>
<td></td>
</tr>
</tbody>
</table>

7. Choose the pair of fractions that must have a product less than 1. Then compute the exact product.

- $2 \times \frac{7}{8}$
- $\frac{5}{8} \times \frac{2}{3}$
- $1\frac{1}{2} \times 1\frac{1}{2}$
- $3 \times 2\frac{2}{3}$

\(\frac{5}{12}\), work will vary. Example:

\(\frac{5}{6} \times \frac{2}{3} = 1\frac{1}{24} = \frac{5}{12}\)
More Fraction Multiplication

1 Fill in the chart to solve each of the problems below.

<table>
<thead>
<tr>
<th>Multiplication Equation</th>
<th>Labeled Sketch</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ex</strong> ( \frac{2}{3} \times \frac{2}{3} = \frac{4}{9} )</td>
<td><img src="image" alt="Sketch" /></td>
</tr>
<tr>
<td><strong>a</strong> ( \frac{2}{3} \times \frac{6}{7} = \frac{12}{21} = \frac{4}{7} )</td>
<td><img src="image" alt="Sketch" /></td>
</tr>
<tr>
<td><strong>b</strong> ( \frac{1}{2} \times \frac{4}{6} = \frac{4}{12} = \frac{1}{3} )</td>
<td><img src="image" alt="Sketch" /></td>
</tr>
<tr>
<td><strong>c</strong> ( \frac{3}{4} \times \frac{4}{8} = \frac{12}{32} = \frac{3}{8} )</td>
<td><img src="image" alt="Sketch" /></td>
</tr>
</tbody>
</table>

2 Solve each problem.

\[
\begin{align*}
\frac{3}{4} \times \frac{2}{4} &= \frac{6}{16} = \frac{3}{8} \\
\frac{1}{4} \times \frac{3}{6} &= \frac{3}{24} = \frac{1}{8} \\
\frac{5}{6} \times \frac{1}{2} &= \frac{5}{12} \\
\frac{6}{7} \times \frac{3}{5} &= \frac{18}{35} \\
\frac{2}{3} \times \frac{4}{5} &= \frac{8}{15} \\
\frac{6}{8} \times \frac{1}{2} &= \frac{6}{16} = \frac{3}{8} \\
\frac{3}{4} \times \frac{1}{3} &= \frac{3}{12} = \frac{1}{4} \\
\frac{2}{7} \times \frac{2}{4} &= \frac{4}{28} = \frac{1}{7}
\end{align*}
\]
**Work Place Instructions 5B Tic-Frac-Toe**

**Each pair of partners needs:**
- 1 Tic-Frac-Toe Record Sheet
- 1 deck of Number Cards, with the 0s and wild cards removed
- 2 colored pencils, in different colors

1. Decide who will be the dealer. The dealer deals out four cards to each player.

2. Player 1 decides which space he wants to claim on the Tic-Frac-Toe Record Sheet. He arranges his four cards to make a fraction-times-a-fraction multiplication problem with a product that fits the description in the desired space.

   Two cards are numerators and the other two cards are denominators. For example, with the cards 2, 5, 5, and 8, a player could make $\frac{2}{5} \times \frac{8}{5}$. Since $\frac{2}{5} \times \frac{8}{5} = \frac{16}{25}$, Player 1 can fill any space where the product is less than 1, $p < 1$. By rearranging the cards to make $\frac{8}{2} \times \frac{5}{5} = \frac{4}{1}$ instead, Player 1 can fill in any space where the product is greater than 1, $p > 1$.

3. Player 1 writes the equation in the blank in the desired space with a colored pencil.

4. Player 2 arranges her four cards and uses the other colored pencil to write her equation in a space that describes her product.

5. Players continue, using four new cards on each turn and filling spaces until one player has claimed four spaces in a horizontal, vertical, or diagonal row, and wins the game.

**Game Variations**

A. Change some of the spaces to read $p > 2$ for products greater than 2 but less than 3, and $p > 3$ for products greater than 3.

B. Include the wild cards in the deck. A wild card can be any numeral 1–9.
Tic-Frac-Toe Moves

In Tic-Frac-Toe, you draw four number cards and arrange them to make two fractions, then multiply them to get a product greater than 1 or a product less than 1.

1. Bill has the cards 6, 4, 2, and 5. Answers to 1a–1d will vary.
   a. How could he arrange his cards to make two fractions with a product greater than 1?
   b. What would his product be? Write and solve an equation to show.
   c. How could he arrange his cards to make a product less than 1?
   d. What would his product be? Write and solve an equation to show.

2. Jeremiah has the cards 3, 9, 4, and 1. Answers to 2a–2d will vary.
   a. How could he arrange his cards to make two fractions with a product greater than 1?
   b. What would his product be? Write and solve an equation to show.
   c. How could he arrange his cards to make a product less than 1?
   d. What would his product be? Write and solve an equation to show.

3. Find the products.
   \[
   \frac{4}{7} \times \frac{1}{3} = \frac{4}{21} \quad \frac{5}{6} \times \frac{3}{5} = \frac{15}{30} \text{ or } \frac{1}{2} \quad \frac{4}{3} \times \frac{3}{4} = \frac{12}{12} \text{ or } 1 \quad \frac{3}{7} \times \frac{7}{5} = \frac{21}{35} \text{ or } \frac{3}{5}
   \]
1 Decide whether each of the story problems below involves the sharing or the grouping interpretation of division. Circle your choice. You don’t need to solve the problems, but it may help to think about whether the answer will mean how many items are in each group or how many groups can be made.

a Frank picked 12 flowers. He divided the flowers evenly between 3 vases. How many flowers did he put in each vase?

Sharing  Grouping

b Erica had 20 baseball cards. She put them up on her bulletin board in rows containing 5 cards. How many rows was she able to make?

Sharing  Grouping

c Darius and his dad made 28 cupcakes for the bake sale. They put 7 cupcakes on each plate. How many plates of cupcakes were they able to make?

Sharing  Grouping

d Kiara and her sister are collecting pennies. They have 120 pennies so far. They put their pennies into stacks of 10. How many stacks were they able to make?

Sharing  Grouping

e Carlos has 15 toy cars. He wants to share them with 2 of his friends so all three boys have the same number. How many cars will each kid get?

Sharing  Grouping

f Jade and her sister made $5.00 doing chores for their mom. They split the money evenly between themselves. How much money did each girl get?

Sharing  Grouping

(continued on next page)
2 Read each story problem. Then:

• Write an equation (including the answer) for the problem.
• Fill in the bubble to show whether the answer means the size of each group or the number of groups.

a Mai and her mother made 24 invitations to Mai’s birthday party. They put the invitations into stacks of 4. How many stacks did they make?

Equation: \[24 \div 4 = 6\]

The answer means:

○ the size of each group (for example, the number of items each person got)

○ the number of groups

b Troy and his mom got a case of bottled water for the soccer game. They divided the 24 bottles evenly among the 12 boys on the team. How many bottles of water did each boy get?

Equation: \[24 \div 12 = 2\]

The answer means:

○ the size of each group (for example, the number of items each person got)

○ the number of groups

3 Choose one of the expressions below and circle your choice. Write a sharing story problem and a grouping story problem about the same expression.

Students’ choice of expression and story problems will vary. Example:

18 ÷ 6 60 ÷ 12 108 ÷ 4 400 ÷ 25

<table>
<thead>
<tr>
<th>Sharing Story Problem</th>
<th>Grouping Story Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mae and her friends are decorating the cafeteria. They divided 60 balloons evenly among 12 tables. How many balloons did each table get?</td>
<td>James has 60 basketball cards. He wants to put them up on his bulletin board in rows of 12. How many rows can he make?</td>
</tr>
</tbody>
</table>
Skills Review 1

1. Use the standard algorithm to solve the two multiplication problems below.

\[
\begin{align*}
87 & \times 28 \\
\underline{147} & \times 38 \\
696 & + 1740 \\
\underline{2,436} & + 4410 \\
\end{align*}
\]

2. Fill in the ratio table for 32. Then use the information to solve the long division problem. You can add more information to the ratio table if you like.

<table>
<thead>
<tr>
<th>Number of Groups</th>
<th>1</th>
<th>10</th>
<th>5</th>
<th>20</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>32</td>
<td>320</td>
<td>160</td>
<td>640</td>
<td>64</td>
<td>128</td>
</tr>
</tbody>
</table>

\[\begin{array}{c}
24\ R1 \\
\text{Ratio table entries beyond the first four, as well as work, will vary. Example shown.}
\end{array}\]

3. Add. Show your work.

\[3.80 + 17.46 = 21.26\]

Work will vary.

4. Add. Show your work.

\[\frac{3}{5} + \frac{3}{4} = 4 \frac{3}{20}\]

Work will vary.

5. Multiply. Show your work. Simplify your answer to lowest terms.

\[\frac{3}{4} \times \frac{4}{5} = \frac{12}{20} = \frac{1}{2}\]

Work will vary.

6. Multiply. Show your work. Express your answer as a mixed number.

\[4 \times \frac{3}{5} = \frac{12}{5} = 2 \frac{2}{5}\]

Work will vary.
**Skills Review 2**

1. Subtract. Show your work.
\[
18.03 - 4.59 = \text{ } \underline{13.44}
\]

2. Add. Show your work.
\[
4\frac{2}{3} + 5\frac{3}{4} = \underline{10\frac{5}{12}}
\]

Work will vary.

3. Use the array to model and solve this multiplication problem.
\[
\frac{3}{4} \times \frac{5}{6} = \frac{15}{24} = \frac{5}{8}
\]

4. Fill in the ratio table for 27. Then use the information to solve the division problem.

<table>
<thead>
<tr>
<th>Number of Groups</th>
<th>1</th>
<th>10</th>
<th>5</th>
<th>20</th>
<th>30</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>27</td>
<td>270</td>
<td>135</td>
<td>540</td>
<td>810</td>
<td>81</td>
</tr>
</tbody>
</table>

\[
\begin{array}{l}
30 \underline{33} \\
27\underline{810} \\
-810 \\
81 \\
-81 \\
0
\end{array}
\]

\[
33
\]

Ratio table entries beyond the first four, as well as work, will vary. Example shown.

5. The fifth graders are painting the bookshelves in their classroom. It takes \(\frac{3}{4}\) of a quart of paint to paint each bookshelf. There are 8 bookshelves in the room. How many quarts of paint will the kids need to paint all 8 bookshelves?

- Write an expression to represent this problem.
- Solve the problem. Show your work with labeled visual models, numbers, or words.

Expression: \(8 \times \frac{3}{4}\) or \(\frac{3}{4} \times 8\)

6 quarts of paint; work will vary.
1. Solve each of the story problems below. For each problem:
   - Choose and circle one of the numbers in parentheses, depending on how challenging you want the problem to be.
   - Write an expression to represent your problem.
   - Use numbers, labeled visual models, or words to solve the problem and explain your strategy.
   - Complete the sentence below with your solution to the problem

   a. It takes \( \\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{3}{8} \) of a cup of sugar to make a batch of cookies. I have 5 cups of sugar. How many batches of cookies can I make?

   Expression: ________________________________

   The 4 possible expressions, along with the answer to each are shown below. Work will vary.

   \[
   \begin{align*}
   5 \div \frac{1}{2} & : 10 \text{ batches of cookies} \\
   5 \div \frac{1}{4} & : 20 \text{ batches of cookies} \\
   5 \div \frac{3}{4} & : 6 \frac{3}{8} \text{ batches of cookies} \\
   5 \div \frac{3}{8} & : 13 \frac{1}{2} \text{ batches of cookies}
   \end{align*}
   \]

   I can make ________ batches of cookies.

(continued on next page)
b The road-repair crew can fix $\frac{1}{2}$ a mile of road per day. How many days will it take them to fix a stretch of road that is (4 miles, 5 miles, 6 $\frac{1}{2}$ miles, 8 $\frac{3}{4}$ miles)?

Expression: ____________________________________________________________________________

The 4 possible expressions, along with the answer to each are shown below. Work will vary.

\[
\begin{align*}
4 \div \frac{1}{2} & \quad 8 \text{ days} \\
5 \div \frac{1}{2} & \quad 10 \text{ days} \\
6 \frac{1}{2} \div \frac{1}{2} & \quad 13 \text{ days} \\
8 \frac{3}{4} \div \frac{1}{2} & \quad 17 \frac{1}{2} \text{ days}
\end{align*}
\]

It will take ________ days to fix the road.

c How many ($\frac{1}{2}$ cup, $\frac{1}{3}$ cup, $\frac{1}{4}$ cup, $\frac{2}{3}$ cup) servings are there in a quart of ice cream? There are 4 cups in a quart.

Expression ____________________________________________________________________________

The 4 possible expressions, along with the answer to each are shown below. Work will vary.

\[
\begin{align*}
4 \div \frac{1}{2} & \quad 8 \text{ servings} \\
4 \div \frac{1}{3} & \quad 12 \text{ servings} \\
4 \div \frac{1}{4} & \quad 16 \text{ servings} \\
4 \div \frac{2}{3} & \quad 6 \text{ servings}
\end{align*}
\]

There are __________ servings in a quart of ice cream.

(continued on next page)
Little Snail is going to visit his friend over at the next pond, 3 miles away. He can crawl \((\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{3}{4}, \frac{2}{3})\) of a mile per day. How many days will it take him to get there?

Expression: ________________________________

It will take _______ days to get there.

2. Choose one of the expressions below and circle your choice. Write a story problem about the expression you circled. Then solve your own problem using a fractional visual model.

\[
\begin{align*}
10 \div \frac{1}{2} & = 20 \\
12 \div \frac{1}{3} & = 36 \\
20 \div \frac{1}{4} & = 80 \\
24 \div \frac{2}{3} & = 36
\end{align*}
\]

My story problem:

Story problems will vary.

My work:

Work will vary.

The answer to my problem is (Answers shown above)
Skills Review 3

1. Multiply. Show your work.
   \[23.69 \times 14 = 331.66\text{ Work will vary.}\]

2. Subtract. Show your work.
   \[\frac{7}{8} - \frac{5}{6} = \frac{7}{24}\text{ Work will vary.}\]

3. Use the standard algorithm to multiply these numbers.
   \[
   \begin{array}{c}
   \phantom{0}634 \\
   \times \phantom{0}56 \\
   \hline
   \phantom{0}3804 \\
   + 31700 \\
   \hline
   35504
   \end{array}
   \]

4. Round 129.392 to the nearest:
   - a. ten: 130
   - b. one: 129
   - c. tenth: 129.4
   - d. hundredth: 129.39

5. There was \(\frac{1}{2}\) of a cake left over from Hannah’s birthday party. When she and her sister came home from school the next day, they ate \(\frac{2}{3}\) of the leftover cake for a snack. How much of the whole cake did the girls have for their snack?
   - Write an expression to represent this problem.
   - Solve the problem. Show your work with labeled visual models, numbers, or words.

   Expression: \(\frac{2}{3} \times \frac{1}{2} \text{ or } \frac{1}{2} \times \frac{2}{3}\)

The girls ate \(\frac{1}{3}\) of the whole cake. Work will vary.
Skills Review 4

1. Divide. Show your work.
   
   \[225.40 \div 5 = 45.08\]  
   Work will vary.

2. Subtract. Show your work.
   
   \[\frac{8}{5} - \frac{2}{3} = \frac{14}{15}\]  
   Work will vary.

3. Fill in the ratio table for 38. Then use the information to solve the long division problem.

<table>
<thead>
<tr>
<th>Number of Groups</th>
<th>1</th>
<th>10</th>
<th>5</th>
<th>20</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>38</td>
<td>380</td>
<td>190</td>
<td>760</td>
<td>570</td>
</tr>
</tbody>
</table>

   \[38 \div 5 - 570\]

   \[\text{Answer Key: } 15 \text{ R}15\]

4. Choose the best estimate for this combination.
   
   \[\frac{2}{3} + \frac{1}{4}\]
   
   \[\text{□ less than } \frac{1}{2}\]
   \[\text{□ between } \frac{1}{2} \text{ and } \frac{3}{4}\]
   \[\text{□ between } \frac{5}{6} \text{ and } 1\]
   \[\text{□ more than } 1\]

   Why is this the best estimate?
   
   Explanations will vary.

5. Write a story problem to represent the expression below. Then solve your problem.

   Show your work with labeled models, numbers, or words, and write the answer on the line provided.

   \[6 \div \frac{1}{3} = 18\]

   My Story Problem: [Blank line for story problem]

   My work:

   Answer: 18
More Do-It-Yourself Story Problems page 1 of 3

1. Solve each of the story problems below. For each problem:
   - Choose and circle one of the numbers in parentheses.
   - Write an expression to represent your problem.
   - Use numbers, labeled visual model, or words to solve the problem.
   - Complete the sentence below with your solution to the problem

   a. (Two, three, four) dinner guests shared \( \left( \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{3}{4} \right) \) of a pan of cornbread. What fraction of the pan of cornbread did each guest get?

      \[
      \frac{1}{2} \div 2 \hspace{1cm} \frac{1}{4} \text{ pan} \hspace{1cm} \frac{1}{2} \div 3 \hspace{1cm} \frac{1}{6} \text{ pan} \hspace{1cm} \frac{1}{2} \div 4 \hspace{1cm} \frac{1}{8} \text{ pan} \\
      \frac{1}{3} \div 2 \hspace{1cm} \frac{1}{6} \text{ pan} \hspace{1cm} \frac{1}{3} \div 3 \hspace{1cm} \frac{1}{9} \text{ pan} \hspace{1cm} \frac{1}{3} \div 4 \hspace{1cm} \frac{1}{12} \text{ pan} \\
      \frac{2}{3} \div 2 \hspace{1cm} \frac{2}{6} \text{ pan} \hspace{1cm} \frac{2}{3} \div 3 \hspace{1cm} \frac{2}{9} \text{ pan} \hspace{1cm} \frac{2}{3} \div 4 \hspace{1cm} \frac{1}{6} \text{ pan} \\
      \frac{3}{4} \div 2 \hspace{1cm} \frac{3}{8} \text{ pan} \hspace{1cm} \frac{3}{4} \div 3 \hspace{1cm} \frac{1}{4} \text{ pan} \hspace{1cm} \frac{3}{4} \div 4 \hspace{1cm} \frac{3}{16} \text{ pan} \\
      \]

      Each guest got ______ pan of cornbread.

   b. Five cousins shared \( \left( \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{3}{4} \right) \) of a birthday cake. What fraction of the cake did each cousin get?

      Expression: _______________________________________________

      \[
      \frac{1}{2} \div 5 \hspace{1cm} \frac{1}{10} \text{ of the cake} \\
      \frac{1}{3} \div 5 \hspace{1cm} \frac{1}{15} \text{ of the cake} \\
      \frac{1}{4} \div 5 \hspace{1cm} \frac{1}{20} \text{ of the cake} \\
      \frac{3}{4} \div 5 \hspace{1cm} \frac{3}{20} \text{ of the cake} \\
      \]

      Each cousin got ______ of the birthday cake.

(continued on next page)
C  Sara had \( \left( \frac{1}{2}, \frac{1}{3}, \frac{2}{3} \right) \) of a cup of grated cheese. She divided it equally between 4 salads. What fraction of a cup of cheese did each salad get?

Expression: ________________________________

\[
\frac{1}{2} \div 4 \quad \frac{1}{8} \text{ of a cup} \\
\frac{1}{3} \div 4 \quad \frac{1}{12} \text{ of a cup} \\
\frac{2}{3} \div 4 \quad \frac{1}{6} \text{ of a cup}
\]

There was _____ a cup on each salad.

d  Mr. Brown had \( \frac{1}{8} \) of a pack of paper. He divided the pack equally among equally among 3 students. What fraction of the pack of paper did each student get?

Expression: ________________________________

Work will vary. Example:

\[
\frac{1}{8} \\
\frac{1}{24} \rightarrow \\
\frac{1}{24} \rightarrow \\
\frac{1}{24} \rightarrow \\
\]

If you divide \( \frac{1}{8} \) into 3 equal parts, each part is \( \frac{1}{24} \).

Each student got \( \frac{1}{24} \) a pack of paper.

(continued on next page)
**More Do-It-Yourself Story Problems** page 3 of 3

3. There were 504 sheets of paper in the whole pack. How many sheets of paper did each of the 3 students get?

Expression: \( \frac{504}{24} \)

Work will vary. Example:

<table>
<thead>
<tr>
<th>1 [ \times 24 ]</th>
<th>10 [ \times 240 ]</th>
<th>20 [ \times 480 ]</th>
<th>21 [ \times 504 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>240</td>
<td>480</td>
<td>504</td>
</tr>
</tbody>
</table>

\( 24 \times 21 = 504 \), so
\( \frac{504}{24} = 21 \)

Each student got \( 21 \) sheets of paper.

4. Choose one of the expressions below and circle your choice. Write a story problem about the expression you circled. Then solve your own problem.

\( \frac{1}{2} \div 2 = \frac{1}{4} \quad \frac{1}{2} \div 6 = \frac{1}{12} \quad \frac{1}{3} \div 4 = \frac{1}{12} \quad \frac{1}{8} \div 4 = \frac{1}{32} \)

My story problem is:

*Story problems will vary.*

My work:

*Work will vary.*

The answer to my problem is: *(Answers shown above.)*
Skills Review 5

1. Subtract. Show your work.
   \[234.02 - 87.46 = 146.56\] Work will vary.

2. Subtract. Show your work.
   \[7\frac{2}{3} - 5\frac{3}{4} = \frac{69}{20} \text{ or } \frac{3}{10}\] Work will vary.

3. Using the grid below, create an array to model and solve this problem.
   \[\frac{2}{5} \times \frac{3}{4} = 1 \frac{11}{12}\]

4. Add. Show your work.
   \[8\frac{5}{6} + 3\frac{3}{4} = 12 \frac{3}{6} \text{ or } 12 \frac{1}{2}\] Work will vary.

5. Write a story problem to represent the expression below. Then solve your problem. Show your work with labeled models, numbers, or words, and write the answer on the line provided.
   \[\frac{1}{2} \div 3 = \frac{1}{6}\] My Story Problem: ____________________________________________
   ________________________________________________________________
   ________________________________________________________________
   My work:

   Answer: _______________
Fraction Estimate & Check

Before you solve each problem, look carefully at the fractions and write what you know about the sum or difference. Then find the exact sum or difference. Show all your work. If your answer is greater than 1, write it as a mixed number, not an improper fraction.

<table>
<thead>
<tr>
<th>Problem</th>
<th>What You Know Before You Start</th>
<th>Show your work</th>
<th>Exact Sum or Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ex</strong></td>
<td>( \frac{8}{3} + \frac{9}{12} )</td>
<td>The sum is more than 3.</td>
<td>( \frac{32}{12} + \frac{9}{12} = \frac{41}{12} ) and ( \frac{41}{12} = \frac{3}{12} )</td>
</tr>
<tr>
<td>1</td>
<td>( \frac{4}{6} + \frac{8}{12} )</td>
<td>Observations will vary.</td>
<td>Work will vary.</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{12}{8} + \frac{3}{4} )</td>
<td>Observations will vary.</td>
<td>Work will vary.</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{3}{8} + \frac{8}{12} )</td>
<td>Observations will vary.</td>
<td>Work will vary.</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{10}{8} - \frac{9}{12} )</td>
<td>Observations will vary.</td>
<td>Work will vary.</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{5}{6} - \frac{3}{4} )</td>
<td>Observations will vary.</td>
<td>Work will vary.</td>
</tr>
</tbody>
</table>
Fraction & Decimal Story Problems

Solve each problem. Show your work using numbers, labeled sketches, or words.

1 Josie is picking apples. She has 3 identical baskets that she is trying to fill. One basket is \(\frac{3}{5}\) full, another is \(\frac{7}{10}\) full, and the last is \(\frac{2}{3}\) full. What portion of the 3 baskets has Josie filled? Give your answer as a mixed number and as an improper fraction.

\(\frac{59}{30}\) or \(1 \frac{29}{30}\) of the 3 baskets

Work will vary.

2 Tommy picked 2 baskets full of apples. One basket weighed 18.63 kilograms. The other basket weighed 9.97 kilograms. How much more did the first basket weigh?

8.66 kg; work will vary.

3 Kaya filled \(3 \frac{1}{4}\) baskets with apples. On her way home, the baskets spilled and she lost \(2 \frac{1}{3}\) baskets of apples. What portion of the apples did not spill?

\(\frac{11}{12}\) of a basket did not spill. Work will vary.
Zero Patterns Review

1 Fill in the blanks.
   a  $57 \times 10 = 570$
   b  $57 \times 100 = 5700$
   c  $57 \times 1000 = 57000$
   d  $57 \times 10000 = 570000$
   e  $57 \times 100000 = 5700000$

2 What do you notice about the problems above? Explain any patterns you see.
   Observations will vary.

3 Fill in the blanks.
   a  $570 \div 10 = 57$
   b  $570 \div 100 = 5.7$
   c  $570 \div 1000 = 0.57$
   d  $570 \div 10000 = 0.057$
   e  $570 \div 100000 = 0.0057$

4 What do you notice about the problems above? Explain any patterns you see.
   Observations will vary.

5 Evaluate the following expressions.
   a  $25 \times (6 \times 5) = 750$
   b  $67 \times (28 - 9) = 1273$
Graphing the Cube Sequence

1. The picture below shows the first five arrangements in the cube sequence we’ve been working with this session. Record the number of cubes it takes to build each arrangement.

<table>
<thead>
<tr>
<th>Arrangement</th>
<th>Number of Cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
</tr>
</tbody>
</table>

2. Write an ordered pair to represent each cube arrangement. Use the arrangement number for the first number in the pair, and the number of cubes it takes to make the arrangement for the second number in the pair. So, for example, arrangement 1 would be written as (1,2), and arrangement 2 would be written as (2,6).

   Arrangement 1  | Arrangement 2  | Arrangement 3  | Arrangement 4  | Arrangement 5  
   (1 , 2)        | (2 , 6)        | (3 , 10)       | (4 , 14)       | (5 , 18)        

3. Graph and label each of the ordered pairs.

   ![Graph of ordered pairs with points (1,2), (2,6), (3,10), (4,14), (5,18)]
Coordinate Dot-to-Dots

1. On each of the grids below, draw and number a dot at each of the ordered pairs on the list. Connect the dots in order to make a picture. The first dot is drawn for you.

**a**

(1,5)
(1,1)
(5,1)
(5,2)
(4,2)
(4,3)
(3,3)
(3,4)
(2,4)
(2,5)
(1,5)

1
2
3
4
5
6
7
8
9
10
11

**b**

(1,2)
(2,1)
(3,1)
(4,2)
(3,2)
(3,5)
(4,3)
(2,3)
(3,5)
(3,2)
(1,2)

1
2
3
4
5
6
7
8
9
10
11
Graphing Another Cube Sequence

1. The picture below shows the first five arrangements in the new cube sequence we’ve been working with this session. Record the number of cubes it takes to build each arrangement.

<table>
<thead>
<tr>
<th>Arrangement 1</th>
<th>Arrangement 2</th>
<th>Arrangement 3</th>
<th>Arrangement 4</th>
<th>Arrangement 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 1)</td>
<td>(2, 5)</td>
<td>(3, 9)</td>
<td>(4, 13)</td>
<td>(5, 17)</td>
</tr>
</tbody>
</table>

2. Write an ordered pair to represent each cube arrangement. Use the arrangement number for the first number in the pair, and use the number of cubes it takes to make the arrangement for the second number in the pair.

3. Graph and label each of the ordered pairs.
**Short & Tall Towers** page 1 of 2

1. Here are the first three arrangements in the Short Tower sequence.
   - Use your cubes to build the 4th and 5th arrangements in this sequence.
   - In the table below, sketch and label the 4th and 5th arrangements you built. You don’t have to make your drawings look three-dimensional. **Sketches will vary.**
   - Record the number of cubes it took to build each of the 5 arrangements.

<table>
<thead>
<tr>
<th>Arrangement</th>
<th>Sketch</th>
<th>Label</th>
<th>Number of Cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrangement 1</td>
<td><img src="image1" alt="Arrangement 1" /></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Arrangement 2</td>
<td><img src="image2" alt="Arrangement 2" /></td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>Arrangement 3</td>
<td><img src="image3" alt="Arrangement 3" /></td>
<td></td>
<td>11</td>
</tr>
<tr>
<td>Arrangement 4</td>
<td><img src="image4" alt="Arrangement 4" /></td>
<td></td>
<td>16</td>
</tr>
<tr>
<td>Arrangement 5</td>
<td><img src="image5" alt="Arrangement 5" /></td>
<td></td>
<td>21</td>
</tr>
</tbody>
</table>

2. Write an ordered pair to represent each cube arrangement in the Short Tower Sequence. Use the arrangement number for the first number in the pair, and use the number of cubes it takes to make the arrangement for the second number in the pair.

   - Arrangement 1: (1, 1)
   - Arrangement 2: (2, 6)
   - Arrangement 3: (3, 11)
   - Arrangement 4: (4, 16)
   - Arrangement 5: (5, 21)

3. Here are the first three arrangements in the Tall Tower Sequence. Build, sketch, and label the 4th and 5th arrangements in this sequence. Then record the number of cubes it took to build each of the 5 arrangements.

<table>
<thead>
<tr>
<th>Arrangement</th>
<th>Sketch</th>
<th>Label</th>
<th>Number of Cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrangement 1</td>
<td><img src="image6" alt="Arrangement 1" /></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Arrangement 2</td>
<td><img src="image7" alt="Arrangement 2" /></td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>Arrangement 3</td>
<td><img src="image8" alt="Arrangement 3" /></td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>Arrangement 4</td>
<td><img src="image9" alt="Arrangement 4" /></td>
<td></td>
<td>17</td>
</tr>
<tr>
<td>Arrangement 5</td>
<td><img src="image10" alt="Arrangement 5" /></td>
<td></td>
<td>22</td>
</tr>
</tbody>
</table>

4. Write an ordered pair to represent each cube arrangement in the Tall Tower Sequence.

   - Arrangement 1: (1, 2)
   - Arrangement 2: (2, 7)
   - Arrangement 3: (3, 12)
   - Arrangement 4: (4, 17)
   - Arrangement 5: (5, 22)
Graph and label the ordered pairs for both cube sequences—the Short Towers and the Tall Towers. Use a different color for each sequence, and fill in the key to show which is which.

Compare the graphs for the two cube sequences. How are they similar? How are they different? Describe any relationships you see between the patterns in the short tower sequence and the tall tower sequence.

Observations will vary. Examples:
- Each sequence increases by 5 with every new arrangement.
- The first arrangement for the short towers was 1. The first arrangement for the tall towers was 2.
- Every arrangement in the tall tower sequence is 1 more than the corresponding arrangement in the short tower sequence.
- The two sequences are nearly identical; the tall towers each have 1 extra cube on the top.
- If you connected the points on the graph, they would form a straight line for each sequence.
Exploring a New Sequence

1. What do you notice about the first three arrangements in the sequence above?
   Observations will vary.

2. Sketch the 4th and 5th arrangements in this sequence.

3. How many cubes would it take to build the 149th arrangement? Explain your answer using words, numbers, or a labeled sketch.
   596 cubes; explanations will vary.
   Each arrangement is always 4 times the arrangement number.
   
   \[ 4 \times 149 = (4 \times 150) - (4 \times 1) \]
   \[ = 600 - 4 \]
   \[ = 596 \]

4. A certain arrangement takes 124 cubes to build. Which arrangement is it? Explain your answer using words, numbers, or a labeled sketch.
   Arrangement 31; explanations will vary.
   Example: Since it always takes 4 times as many cubes as the arrangement number, you can find out which arrangement this is by dividing 124 by 4.
   
   \[ 124 = 100 + 24 \]
   \[ 100 \div 4 = 25 \]
   \[ 24 \div 4 = 6, \text{ so } 124 \div 4 = 31 \]
Tile Pools page 1 of 2

1. Here are the first five arrangements in the tile pool sequence. In the box below each arrangement, write the number of gray tiles it took to build the border for each.

<table>
<thead>
<tr>
<th>Arrangement</th>
<th>Number of Gray Tiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>24</td>
</tr>
</tbody>
</table>

2. Write an ordered pair to represent each of the pool borders in the sequence. Use the arrangement number for the first number in the pair, and the number of gray tiles it took to make the border around the pool for the second number in the pair.

Arrangement 1: (1, 8)  
Arrangement 2: (2, 12)  
Arrangement 3: (3, 16)  
Arrangement 4: (4, 20)  
Arrangement 5: (5, 24)

3. Here is another picture of the first five arrangements in the tile pool sequence. In the box below each arrangement, write the number of white tiles it took to build the water.

<table>
<thead>
<tr>
<th>Arrangement</th>
<th>Number of White Tiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
</tbody>
</table>

4. Write an ordered pair to represent each of the water areas in the sequence. Use the arrangement number for the first number in the pair, and the number of white tiles it took to make the water in the pool for the second number in the pair.

Arrangement 1: (1, 1)  
Arrangement 2: (2, 4)  
Arrangement 3: (3, 9)  
Arrangement 4: (4, 16)  
Arrangement 5: (5, 25)
5 Graph and label the ordered pairs for both parts of each pool—the borders, and the water. Use a different color for each sequence, and fill in the key to show which is which.

6 In your journal, describe the shape of each graph, and tell why you think the two are so different. Observations will vary.
Tile Pools Challenge page 1 of 2

Here are the first five arrangements in the tile pool sequence.

1

How many tiles would it take to build the water for the 10th arrangement in this sequence? Use numbers, words, or labeled sketches to explain how you got your answer.

100 tiles; explanations will vary. Example: The water part of each arrangement is always the arrangement number times itself, so in the 10th, it would be $10 \times 10 = 100$.

2

How many tiles would it take to build the border for the 10th arrangement in this sequence? Use numbers, words, or labeled sketches to explain how you got your answer.

44 tiles; explanations will vary. It will take 10 tiles on each side of the $10 \times 10$ square the water makes, and then 1 extra tile in each corner. $(4 \times 10) + 4 = 44$

3

What do you have to do to figure out how many tiles it takes to build the water for any arrangement in this sequence? Include a labeled sketch in your explanation.

Explanations will vary. Example: The water is always the arrangement number times itself, and it’s always square, so the water part of the nth arrangement would look like this: $n \times 2 = \text{water}$

(continued on next page)
4 What do you have to do to figure out how many tiles it takes to build the border for any arrangement in this sequence? Include a labeled sketch in your explanation.

Explanations will vary. Example:
Since the water part of any arrangement is the arrangement number times itself, the length of each side of the square has to be the arrangement number. Then you need 1 extra tile in each corner to finish the border. So, the border for the n\textsuperscript{th} arrangement is (4 \times n) + 4 or 4n + 4.

5 It takes exactly 196 tiles to build both the water and the border for a certain arrangement in this sequence. Which arrangement is it? Use numbers, words, or labeled sketches to explain how you got your answer.

The 12\textsuperscript{th} arrangement. Work will vary. Example:
196 - 4 for the corner tiles = 192
I know it takes 144 tiles for the 10\textsuperscript{th} arrangement. I'm going to try 12.

\[(12 \times 12) + (4 \times 12) = 144 + 48 + 4 = 196\]
More Coordinate Dot-to-Dots

1. On the grid below, draw and number a dot at each of the ordered pairs on the list. Connect the dots in order to make a picture. The first dot is drawn for you.

   \[(1, 1) \hspace{1cm} 1\]
   \[(3, 4) \hspace{1cm} 2\]
   \[(5, 1) \hspace{1cm} 3\]
   \[(1, 3) \hspace{1cm} 4\]
   \[(5, 3) \hspace{1cm} 5\]
   \[(1, 1) \hspace{1cm} 6\]

2. Make up your own dot-to-dot picture on the grid below. Use at least 12 points for your picture. List the coordinates for your picture in order.

   \[(\_, \_) \hspace{1cm} 1\]
   \[(\_, \_) \hspace{1cm} 2\]
   \[(\_, \_) \hspace{1cm} 3\]
   \[(\_, \_) \hspace{1cm} 4\]
   \[(\_, \_) \hspace{1cm} 5\]
   \[(\_, \_) \hspace{1cm} 6\]
   \[(\_, \_) \hspace{1cm} 7\]
   \[(\_, \_) \hspace{1cm} 8\]
   \[(\_, \_) \hspace{1cm} 9\]
   \[(\_, \_) \hspace{1cm} 10\]
   \[(\_, \_) \hspace{1cm} 11\]
   \[(\_, \_) \hspace{1cm} 12\]
   \[(\_, \_) \hspace{1cm} 13\]
   \[(\_, \_) \hspace{1cm} 14\]
   \[(\_, \_) \hspace{1cm} 15\]
   \[(\_, \_) \hspace{1cm} 16\]
   \[(\_, \_) \hspace{1cm} 17\]
Anthony is a junior in high school. He decided to get a job this summer so he could put some money in his college savings account. His goal was to put $1,000 into his account, but still have time to rest up before school started again. He is a very good math student who loves computers, and he was lucky to be offered a summer job with two different software companies.

Company 1 offered to pay Anthony $1 on the first day and double the amount each day ($1 the first day, $2 the next day, $4 the third day, $8 the fourth day, and so on).

Company 2 offered to pay Anthony $75 every day.

Which job should Anthony accept if he wants to reach his goal of earning $1,000 as quickly as possible?

1. On the next page, fill in the table for each company’s payment plan. You can stop as soon as the total amount of money reaches or goes over $1,000 for a plan, and then do the other one.

2. On the next page, graph the running totals for each day. Graph each plan in a different color, and mark the key at the bottom of the sheet to show which is which.

3. Which company’s plan turned out to be best? Why?

Company 1’s plan is the best.
Explanations will vary. Example: It only takes 10 days to make over $1,000 with company 1’s plan. With Company 2’s plan, it takes 14 days to reach and go over $1,000.
### Anthony's Problem  page 2 of 2

<table>
<thead>
<tr>
<th>Day</th>
<th>Company 1 Payment Plan</th>
<th>Company 2 Payment Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Daily Amount (Dollars)</td>
<td>Running Total (Dollars)</td>
</tr>
<tr>
<td>1</td>
<td>$1</td>
<td>$1</td>
</tr>
<tr>
<td>2</td>
<td>$2</td>
<td>$3</td>
</tr>
<tr>
<td>3</td>
<td>$4</td>
<td>$7</td>
</tr>
<tr>
<td>4</td>
<td>$8</td>
<td>$15</td>
</tr>
<tr>
<td>5</td>
<td>$16</td>
<td>$31</td>
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<td>9</td>
<td>$256</td>
<td>$511</td>
</tr>
<tr>
<td>10</td>
<td>$512</td>
<td>$1,023</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
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<tr>
<td>12</td>
<td></td>
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<td>17</td>
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<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Graphing the Two Payment Plans

Days

Total Amount of Money Earned (Dollars)

Key

Plan 1  Plan 2
Miranda’s Number Patterns

1. Miranda made a number pattern. She started with 4 and added 3 several times. Continue Miranda’s pattern: 4, 7, 10, 13, 16, 19, 22.

2. Miranda made another number pattern. She started at 30 and subtracted 3 each time. Continue Miranda’s new pattern: 30, 27, 24, 21, 18, 15, 12.

3. Compare Miranda’s patterns. Write two observations about how her number patterns are alike, and two observations about how her number patterns are different.

   Observations will vary.

4. Miranda graphed one of her patterns on the coordinate grid below.
   - Did Miranda graph her first or her second pattern? _____ 2nd pattern _____
   - Label the ordered pairs that Miranda graphed.
   - Graph and label the ordered pairs in Miranda’s other pattern.
Work Place Instructions 6A Dragon’s Treasure

Each pair of players needs:
- 1 Dragon’s Treasure Record Sheet to share
- 1 red and 1 blue game marker
- 1 die numbered 1–6
- scratch paper

1 Each player rolls the die once to determine who gets to start. Player 1 chooses whether to play for red or for blue.

2 Player 1 places his game marker on the coordinate grid at point (1, 0) and rolls the die. He chooses the best move, and then records his path, his score, and the coordinates of the point on which he landed.
- A player moves his marker the number of spaces he rolled, forward, backward, or sideways, but not diagonally.
- A player collects the value of any gold pieces he lands on along the way.
- If there is one star along the path the player takes, he gets to multiply his total for that turn by 10.
- If there are two stars along the path the player takes, he gets to multiply his total for that turn by 100.
- Once the player has decided on the path he will take, he must record it, using numbers and arrows. He must also record the coordinates for the point on which he lands at the end of his path as the Start Point for his next turn.

Player 1 OK, I rolled a 4. I tried some different paths, and I decided to go 2 up and 2 over to the right. On that path, I landed on a gold piece worth $18.25, a star, and another gold piece worth $24.00. I added $18.25 and $24.00 on my scratch paper. I got $42.25, and if you multiply that by 10, you get $422.50.

3 Player 2 places her game marker on the coordinate grid at point (1, 0) and takes her turn.

4 Players take turns until they’ve each had five turns.
- Each time a player takes her next turn, she must start at the coordinate point she landed on at the end of her previous turn.

5 Players add their scores for all five turns. The player with the higher score wins the game.

Game Variations

A Use a calculator to check your addition and multiplication.

B Use a copy of the Challenge Record Sheet, and before the game begins, work with your partner to fill in your own values on the dragon’s gold pieces.

C Multiply by numbers that are more interesting than 10 and 100. If you decide to use this variation, you and your partner have to agree on the numbers. The second number must be 10 times the first number.

D Use a die numbered 4–9 instead of a die numbered 1–6.
Rita's Robot

Pirate Rita built a robot to go out and collect treasure for her. She needs to program the robot so it knows where to go on the map.

The robot can collect only 90 gold coins before it has to come back, and it can travel only along the grid lines (not on the diagonals). It can travel only 30 spaces before it runs out of fuel.

Help Pirate Rita program the robot to collect as much treasure as it can carry and return to the starting point before it runs out of fuel. Draw on the map at right, and keep track of the robot’s moves on the table below.

<table>
<thead>
<tr>
<th>Destination Coordinates</th>
<th>Spaces Moved</th>
<th>Running Total of Spaces Moved</th>
<th>Coins Collected</th>
<th>Running Total of Coins Collected</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F, 2)</td>
<td>7</td>
<td>7</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>(F, 5)</td>
<td>3</td>
<td>10</td>
<td>14</td>
<td>28</td>
</tr>
<tr>
<td>(E, 8)</td>
<td>4</td>
<td>14</td>
<td>15</td>
<td>43</td>
</tr>
<tr>
<td>(D, 10)</td>
<td>3</td>
<td>17</td>
<td>16</td>
<td>59</td>
</tr>
<tr>
<td>(D, 4)</td>
<td>6</td>
<td>23</td>
<td>8</td>
<td>67</td>
</tr>
<tr>
<td>(B, 4)</td>
<td>2</td>
<td>25</td>
<td>12</td>
<td>79</td>
</tr>
<tr>
<td>(A, 0)</td>
<td>5</td>
<td>30</td>
<td>–</td>
<td>79</td>
</tr>
</tbody>
</table>

Responses will vary. Example shown.
Triangles Record Sheet
More Geoboard Triangles

Remember that you can classify and describe triangles in two different ways:

### by the size of their angles

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acute Triangle</td>
<td>All 3 angles are acute.</td>
</tr>
<tr>
<td>Right Triangle</td>
<td>One of the angles is a right angle.</td>
</tr>
<tr>
<td>Obtuse Triangle</td>
<td>One of the angles is obtuse.</td>
</tr>
</tbody>
</table>

### by the length of their sides

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isosceles Triangle</td>
<td>Two sides are the same length.</td>
</tr>
<tr>
<td>Scalene Triangle</td>
<td>Each side is a different length.</td>
</tr>
<tr>
<td>Equilateral Triangle</td>
<td>All 3 sides are the same length.</td>
</tr>
</tbody>
</table>

1. Follow the instructions below each geoboard to draw some different triangles.
   **Hint** Build your triangles on a geoboard first. Then copy them onto the paper.

   - an acute triangle
   - an obtuse triangle
   - a right triangle that is also isosceles
   - a right triangle that is also scalene
   - an obtuse triangle that is also isosceles
   - a scalene triangle that is not obtuse

2. **CHALLENGE** Dana says it is impossible to draw a right triangle that is also acute. Do you agree with her? Why or why not? Use the geoboards below to test your ideas. Explain your ideas in writing.

   Dana is correct. Explanations will vary. (An acute triangle must have 3 acute angles; a right triangle must have 1 right angle. Because there are only 3 angles, if one of them is a right angle it isn’t possible for all 3 of them to be acute.)
Thinking About Quadrilaterals page 1 of 2

1 Fill in the hierarchy of quadrilaterals below.
   a Sketch an example of each shape beside or below its box on the chart.
   Sketches will vary somewhat. Examples shown.
   b Then use the information to help answer the questions on this page and the next.

2 Compare a rhombus and a square. List two ways in which they are alike, and two ways in which they are different.

   Responses will vary. Example:

   **Likenesses**
   - Both have 4 congruent sides.
   - Both have 2 pairs of parallel sides opposite each other.

   **Differences**
   - The square must have 4 right angles.
   - The square must have 4 pairs of adjacent sides that are perpendicular.
3. Why do people say that a square is a special kind of rectangle?

Responses will vary. Example: Because a square is a parallelogram with 4 right angles that just happens to have all 4 sides congruent.

4. Lily says that a square is a special kind of rhombus. Do you agree with her? Explain your answer.

Lily is correct. Explanations will vary. Example: A rhombus is a parallelogram with 4 congruent sides. So is a square. A square just happens to have 4 right angles as well.

5. Jared says that a trapezoid has 2 sides parallel so it must be a parallelogram. Do you agree with him? Explain your answer.

Jared is not correct. Explanations will vary. Example: A parallelogram must have 2 pairs of parallel sides opposite each other. A trapezoid has exactly 1 pair of parallel sides opposite each other, so it can’t be a parallelogram.

6. Square is the most specific name for a quadrilateral with 4 right angles and 4 congruent sides. List all the other names for a square (there are at least 4 of them).

Responses will vary somewhat. Possibilities include:
- Parallelogram
- Rectangle
- Rhombus
- Kite
- Polygon

7. **CHALLENGE** Is a rhombus also a kite? Why or why not?

Yes; explanations will vary. Example: A kite must have 2 pairs of adjacent sides that are congruent, like this:

A rhombus is a special kind of kite where the lengths of 1 pair of adjacent sides are congruent to the lengths of the other pair.

Kite
Rhombus
Perimeters & Trapezoids

This graph represents the perimeters of regular polygons (triangles, squares, pentagons, and hexagons) that have the same side lengths.

1. What does the 4 in the point (4, 8) represent?
   
   The 4 tells how many sides the shape has.

2. What does the 8 in the point (4, 8) represent?
   
   The 8 tells the perimeter of the shape.

3. What does the point (6, 12) represent?
   
   A polygon with 6 sides, and a perimeter of 12.

4. Kevin says, “A trapezoid is a special quadrilateral, so a quadrilateral has all of the properties of a trapezoid.” Do you agree with Kevin? Explain your answer.
   
   Kevin is incorrect. Explanations will vary.
   (Not all quadrilaterals have exactly 1 pair of parallel sides opposite each other. Some quadrilaterals have 0 pairs of parallel sides, while others have 2 pairs of parallel sides.)
The Logic of Quadrilaterals Challenge

1. Label each shape in this diagram with the name that describes it most exactly.

2. Why is the trapezoid inside the quadrilateral but outside the parallelogram?
   Because a trapezoid is a quadrilateral, but not a parallelogram.

3. Why are there a rhombus and a rectangle inside the parallelogram?
   Because both rhombuses and rectangles are types of parallelograms.

4. Why are there two squares, one inside the rhombus and one inside the rectangle?
   Because a square is a rhombus with 4 right angles, and it is also a rectangle with 4 congruent sides.

5. Write at least two other observations to explain why the shapes in this diagram have been placed where they are in relation to each other.
   - All of the shapes inside the quadrilateral have 4 sides.
   - All of the shapes inside the parallelogram have 2 pairs of parallel sides opposite each other.
Quad Construction

Use a ruler marked in inches and the grid lines below to draw the following figures.

1. A trapezoid with at least one right angle, one side length of $1\frac{7}{8}$ inches and one side length of $2\frac{5}{8}$ inches.

2. A parallelogram that is not a rectangle, with an area of 18 square units. (The smallest square on the grid has an area of 1 square unit.) Label your drawing to prove that the area is 18 square units.

3. A parallelogram with 4 right angles and an area of 32 square units. Label your drawing to prove that the area is 32 square units.

4. A parallelogram that is not a rectangle, with an area of 32 square units. Label your drawing to prove that the area is 32 square units.
### Properties of Parallelograms

Use the diagrams below to answer the following questions.

1. List three properties of a parallelogram.
   - Responses will vary somewhat. Possibilities include:
     - Closed
     - 4 Straight sides
     - 2 pairs of parallel sides opposite each other
     - 4 right angles or 2 obtuse and 2 acute angles

2. Fill in the bubbles beside the other names that fit all parallelograms.
   - ○ rectangles  ● quadrilaterals  ● polygons  ○ rhombuses

3. Add one shape to each of the diagrams at the top of the page.
   - Responses will vary. See examples at top of page.

4. Why can't trapezoids be classified as parallelograms?
   - Responses will vary. Example:
     A trapezoid must have exactly one pair of parallel sides, but a parallelogram always has two pairs of parallel sides.

5. Circle the word to tell if each statement below is true or false.
   - a. If the opposite sides on a parallelogram are parallel and congruent, then rectangles are parallelograms.  True  False
   - b. If rectangles have 4 right angles, then all parallelograms must be rectangles.  True  False
   - c. If parallelograms have 4 sides, then all quadrilaterals must be parallelograms.  True  False
Brad’s brother, Matt, wants to ship some marbles. He is going to use some of Brad’s baseball boxes. Brad tells him that he can take some partially labeled boxes, if Matt will finish filling in the labels and then give them to Brad so he’ll know which boxes Matt took. None of the boxes have a dimension that is just one ball.

1. Matt found a box with this label. He could see that one side of the box was 2 balls wide. Complete the label.

   \[
   \frac{12}{\text{# of balls in layer}} \times \frac{6}{\text{# of layers}} = \frac{72}{\text{total number of balls}}
   \]

   \[
   (2 \times 6) \times 6 = 72
   \]

2. Brad found a blank label for a box that was twice as tall as the box in problem 1, and had the same size base. Complete the label.

   \[
   \frac{12}{\text{# of balls in layer}} \times \frac{12}{\text{# of layers}} = \frac{144}{\text{total number of balls}}
   \]

   \[
   (2 \times 6) \times 12 = 144
   \]

3. Complete the labels for these boxes.

   a

   \[
   \frac{20}{\text{# of balls in layer}} \times \frac{3}{\text{# of layers}} = \frac{60}{\text{total number of balls}}
   \]

   \[
   (4 \times 5) \times 3 = 60
   \]
Baseball Box Labels  page 2 of 2

4 **CHALLENGE**  Matt found a label on the floor that had only the total number of balls, 72. What are three completely different boxes that the label could belong to?

- **b**
  \[
  \frac{16}{\text{# of balls in layer}} \times \frac{3}{\text{# of layers}} = \frac{48}{\text{total number of balls}}
  \]
  \[
  (2 \times 8) \times 3 = 48
  \]

- **c**
  \[
  \frac{4}{\text{# of balls in layer}} \times \frac{13}{\text{# of layers}} = \frac{52}{\text{total number of balls}}
  \]
  \[
  (2 \times 2) \times 13 = 52
  \]

- **d**
  \[
  \frac{9}{\text{# of balls in layer}} \times \frac{8}{\text{# of layers}} = \frac{72}{\text{total number of balls}}
  \]
  \[
  (3 \times 3) \times 8 = 72
  \]

- **e**
  \[
  \frac{12}{\text{# of balls in layer}} \times \frac{6}{\text{# of layers}} = \frac{72}{\text{total number of balls}}
  \]
  \[
  (3 \times 4) \times 6 = 72
  \]

- **f**
  \[
  \frac{18}{\text{# of balls in layer}} \times \frac{4}{\text{# of layers}} = \frac{72}{\text{total number of balls}}
  \]
  \[
  (3 \times 6) \times 4 = 72
  \]
Baseballs Packed in Cube-Shaped Boxes

Brad likes to package his baseballs in cube-shaped boxes when he can because they’re easy to pack and ship. The graph to the right shows how many baseballs fit into different sizes of cube-shaped boxes. For example, the point at (1,1) shows that a cube-shaped box that is 1 unit long, 1 unit wide, and 1 unit high holds 1 baseball.

Use the information on the graph to answer these questions.

1. What does the point (2,8) represent?
   A cube-shaped box with side lengths of two \((2 \times 2 \times 2)\) holds 8 baseballs.

2. What does the point (4,64) represent?
   A cube-shaped box with side lengths of four \((4 \times 4 \times 4)\) holds 64 baseballs.

3. How many balls does a 3 \(\times\) 3 \(\times\) 3 box hold?
   27 balls

4. Plot the point on the graph that represents how many balls a 3 \(\times\) 3 \(\times\) 3 box holds.

5. Steven says that the point (2,8) is incorrect because \(2 \times 2\) is 4, not 8. Do you agree with Steven? Explain your answer.
   Steven is incorrect. Explanations will vary. Example: A cube has 3 side lengths—width, length, and height—and you have to multiply all 3 to get the volume. \(2 \times 2 \times 2 = 8\) cubic units.
Packing Matt’s Marbles

Matt is ready to pack his marbles into packing cartons (large boxes) he got from Brad yesterday. The illustrations show the size of an individually boxed baseball and an individually boxed marble, along with one example of a packing carton. How many marbles will fit into each large carton with the dimensions (in units of baseball boxes) below? Show your work.

1. \(2 \times 6 \times 6\)
   - 1,944 marbles because the box is 1,944 in\(^3\).
   - Work will vary. Example: \(2 \times 3 = 6 \) and \(6 \times 3 = 18\), so in inches, the packing box is \(6 \times 18 = (6 \times 10) + (6 \times 8) = 60 + 48 = 108\)

2. \(2 \times 6 \times 12\)
   - 3,888 marbles; the box is 3,888 in\(^3\).
   - Work will vary. Example: Just double 1,944 from problem 1 because this box is twice as high as that one. \(1,944 + 1,944 = 3,888\)

3. \(4 \times 5 \times 3\)
   - 1,620 marbles; the box is 1,620 in\(^3\).
   - Work will vary. Example: Each baseball box holds 27 marble boxes, so find the volume in baseballs and multiply it by 27.
   - \((4 \times 5) \times 3 = 20 \times 3 = 60\)

4. \(2 \times 8 \times 3\)
   - 1,296 marbles or 1,296 in\(^3\).
   - Work will vary. Example:
   - \((2 \times 8) \times 3 = 16 \times 3 = 48\)

5. 4 baseballs in one layer, with 13 layers
   - 1,404 marbles or 1,404 in\(^3\).
   - Work will vary. Example:
Graphing & Geometry Review

1. Write the coordinates of the points in the graph. \((0, 3), (3, 2)\)

2. Graph and label these points on the graph: \((2,1), (2,5), \text{ and } (1,4)\).

3. Connect four of the points on the graph to form one of these figures: rectangle, square, rhombus, or kite. Write the name of the figure you formed, and list 3 of its properties.
   - **Rectangle:** properties listed will vary somewhat. Possibilities:
     - Closed
     - 4 straight sides
   - 2 pairs of parallel sides
   - 4 right angles
   - 2 pairs of congruent sides

4. Does a square have all of the properties of a rhombus, or does a rhombus have all of the properties of a square? Use words and labeled sketches to explain.
   - **A square has all the properties of a rhombus; explanations will vary.**

5. How many balls can Brad put in a box that has \(6 \times 6\) balls in the bottom layer and 5 layers? Show your work.
   - **180 balls; work will vary.**
Work Place Instructions 6B Polygon Search  page 1 of 2

Each pair of players needs:
- 2 Polygon Search Record Sheets
- 2 spinner overlays
- 2 regular pencils, 2 red pencils, and 2 blue pencils
- 2 rulers
- 2 privacy screens (such as folders or books)

1 Working behind privacy screens, both players spin three times to determine which polygons they will draw on the left coordinate grid, titled “My Board,” on their record sheets.
   - Players may draw their polygons anywhere on the board.
   - Each shape must have an area of at least 4 square units.
   - Each vertex must be placed where the x- and y-coordinates cross.
   - Polygons must not touch each other or share any vertices.

2 After both players have drawn their shapes without letting their partner see, players decide who will be Player 1 and who will be Player 2.

3 Player 1 calls out an ordered pair. Player 2 marks the ordered pair on his board to see if it falls on any part of one of his polygons, and gives Player 1 one of four responses:
   - Hit—if the ordered pair is on a side of a polygon.
   - Vertex—if the ordered pair is on a vertex of a polygon.
   - Interior—if the ordered pair is inside a polygon.
   - Miss—if the ordered pair does not land on or inside a polygon.

4 Player 1 marks the ordered pair on the right coordinate grid, titled “My Partner’s Board,” with:
   - A red dot for a hit
   - A red “V” for a vertex
   - A red “I” for interior
   - A blue dot for a miss
5 Players take turns repeating steps 4 and 5, calling and marking ordered pairs in an attempt to locate and correctly identify all three polygons on the partner’s coordinate grid.

When a player believes he can identify one of his partner’s polygons, he must give the most specific name for the polygon and report two of its properties. His partner must tell him whether or not he is correct.

7 The game ends when one player has located and correctly identified the names and at least two properties for all three polygons her partner constructed on his game board.

Game Variations

A Players may play for a set time. The player with the most hits wins.

B Two players may play as a team against another pair.
Finding Volume

1. Measure the dimensions of the rectangular prisms below in inches. Then find the volume of each prism in cubic inches. Remember to label your answers.

Volume = 1 cubic inch

\[1 \times 1 \times 1 = 1 \text{ cu. in.}\]

Volume = 6 cubic inches

\[1 \times 3 \times 2 = 6 \text{ cu. in.}\]

2. Matt measured the dimensions of a box and found that the area of the base is 16 in\(^2\) and the height is 64 in. What is the volume of the box? Show your work.

1,024 cubic inches; work will vary.

3. Matt’s friend, Franny, found that the volume of a rectangular prism was 96 in\(^3\). She remembered that the area of the base was 16 in\(^2\). What was the height of the box? Show your work.

6 inches; work will vary.
Volumes Record Sheet

1. Follow the instructions to find and record the volume of 9 different rectangular prisms.
   - Choose a prism.
   - Measure its dimensions (length, width, and height) in centimeters, and record them on this sheet.
   - Use the formula (length × width × height) to find its volume in cubic centimeters (cm).
   - Label all your measurements correctly.

<table>
<thead>
<tr>
<th>Letter of Prism</th>
<th>Dimensions of Prism in cm</th>
<th>Volume of Prism in cm³</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>6 cm</td>
<td>3 cm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 cm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 × 3 × 4 = 72 cm³</td>
</tr>
</tbody>
</table>

2. Did any of the prisms you measured have the same volume? If so, which ones? How do the length, width, and height of those prisms compare?
   Responses will vary

3. Can you find one prism you measured that has twice the volume of another prism you measured? How do the length, width, and height of those prisms compare?
   Responses will vary
Tank Volume

1. At a pet store, the volume of a tank or kennel depends on the size of the pet. Use the information in the pictures to write and solve an equation for the volume of each tank, cage, or kennel. Label your answers with the correct units.

Ex

**Fish Tank Volume**

Equation: \(20 \times 10 \times 12 = 2,400\)

Fish Tank Volume: \(2,400 \text{ in}^3\)

**Ant Farm Volume**

Equation: \(10 \times 2 \times 8 = 160\)

Ant Farm Volume: \(160 \text{ cm}^3\)

**Dog Kennel Volume**

Equation: \(7 \times 7 \times 6 = 294\)

Dog Kennel Volume: \(294 \text{ ft}^3\)

**Scorpion Tank Volume**

Equation: \(9 \times 7 \times 9 = 567\)

Scorpion Tank Volume: \(567 \text{ in}^3\)

**Guinea Pig Cage Volume**

Equation: \(30 \times 18 \times 16 = 8,640\)

Guinea Pig Cage Volume: \(8,640 \text{ in}^3\)
Work Place Instructions 6C Volume Bingo

Each pair of players needs:

- 2 Volume Bingo Game Boards
- 2 Volume Bingo Record Sheets
- 1 spinner overlay
- game markers (about 20 in each of two different colors)

1. Players decide who will be Player 1 and who will be Player 2.
2. Player 1 spins twice and records the digits on her record sheet under “Numbers I spun.”
3. Then, Player 1 thinks of a third factor that will result in a volume listed on her game board when multiplied by the two numbers that she spun.
4. Player 1 marks the volume chosen on her game board with a game marker, then records the following information on her record sheet: number chosen, volume covered on the game board, and the equation created.

5. Player 2 checks Player 1’s work and then takes a turn.
6. If a player cannot think of a third factor that makes an uncovered volume on her game board, the turn is lost.
7. Players take turns until one player covers five spaces in a row vertically, horizontally, or diagonally.

Game Variations

A. Partners team up to play against another pair.
B. Players play a shorter game where they cover three or four spaces in a row.
C. Players use the Challenge Game Board and spinner for a more difficult game.
Volume Problems  page 1 of 3

1. The volume of a rectangular prism is 140 cubic centimeters. The area of the base of the prism is 20 square centimeters.
   - Do you have enough information to find the height of the prism?  Yes
   - If not, what information do you need? If so, what is the height?
     \[
     \text{Height} = 7 \text{ cm} \\
     \text{Work will vary. Example:} \\
     140 \div 20 = 7
     \]

2. The base of a rectangular prism is 147 centimeters by 346 centimeters.
   - Do you have enough information to find the volume of the prism?  No
   - If not, what information do you need? If so, what is the volume?
     You need the height to find the volume of this rectangular prism.

3. The volume of a rectangular prism with a square base is 200 cubic centimeters and the height is 8 centimeters.
   - Do you have enough information to find the length of the rectangular prism?  Yes
   - If not, what information do you need? If so, what is the length of the rectangular prism?
     \[
     5 \text{ cm} \\
     \text{Work will vary. Example:} \\
     200 \div 8 = 25, \text{ and } 5 \times 5 = 25
     \]

4. Rectangular prism A has a volume of 38 cm³ and rectangular prism B has a volume of 168 cm³. What is the volume of the figure formed by putting A and B together? Show your work.
   \[
   206 \text{ cm}^3 \\
   \text{Work will vary. Example:} \\
   38 + 168 = 206
   \]

(continued on next page)
**Volume Problems** page 2 of 3

5 Miguel says you only need to measure one edge of a cube to find its volume. Do you agree with him? Why or why not? Use numbers, labeled sketches, and words to explain your answer.

Miguel is correct. Explanations will vary. Example: In order to be a cube, a rectangular prism must have width, length, and height congruent. So if you know the measure of one edge, you can multiply it by itself to get the area of the base, and then again to get the volume.

6 Mia has already measured the dimensions of this packing box. Help her find the volume. Show your work.

192 cubic inches; work will vary. Example:

\[4 \times 6 = 24\]
\[24 \times 8 = (20 \times 8) + (4 \times 8)\]
\[= 160 + 32\]
\[= 192\]

7 Brandon is going on a fishing trip with his family. He wants to find the volume of the family’s ice chest. Which expression should he use?

- ○ 2 × 3
- ○ 3 × 2 × 2
- ○ 3 + 2 + 2
- ○ (3 × 2) – 2

8 Jeff’s little brother is trying to find out how many alphabet blocks will fit into a shoebox. He is measuring:

- ○ the volume of the shoebox
- ○ the area of the shoebox
- ○ the length of the shoebox

*(continued on next page)*
**Volume Problems** page 3 of 3

9  Which of these situations is about volume?
- determining the amount of fencing it takes to go around a square garden
- determining how many tiles it will take to cover the kitchen floor
- determining how many rectangular containers of food will fit into a freezer

10  Vanesa wants to find the volume of her lunchbox. Which of these units should she use?
- cubic feet
- cubic inches
- cubic yards

11  The volume of this rectangular solid is 40 cubic feet. What is its height? Show your work.

![Rectangular solid diagram]

Its height is 4 ft.
Work will vary. Example:
5 × 2 = 10
40 ÷ 10 = 4

12  **CHALLENGE**  The volume of this cube is 216 cubic inches. What is the length of each edge? Show your work.

![Cube diagram]

6 inches; work will vary. Example:
I started with 10, but 10 × 10 × 10 = 1,000 so that was too much.
I tried 5, but 5 × 5 × 5 = 125 so that wasn't enough.
Then I tried 6 and that worked.

\[
\begin{align*}
6 \times 6 &= 36 \\
36 \times 6 &= (30 \times 6) + (6 \times 6) \\
&= 180 + 36 \\
&= 216
\end{align*}
\]
The pet store sells boxes of pet food and snacks in four sizes. Write and solve an equation to find the volume of each box. Label your answers with the correct units.

### a

![Box a]

- **Equation**: $9 \times 9 \times 9 = 729$
- **Volume**: $729 \text{ cm}^3$

### b

![Box b]

- **Equation**: $17 \times 3 \times 9 = 459$
- **Volume**: $459 \text{ cm}^3$

### c

![Box c]

- **Equation**: $19 \times 30 \times 6 = 3,420$
- **Volume**: $3,420 \text{ cm}^3$

### d

![Box d]

- **Equation**: $16 \times 21 \times 5 = 1,680$
- **Volume**: $1,680 \text{ cm}^3$

2. Use the information above to solve these problems.

   a. How much more is the volume of box c than the volume of box a? Show your work.

   $2,691 \text{ cm}^3$; work will vary.

   b. What is the combined volume of boxes b and d? Show your work.

   $2,139 \text{ cm}^3$; work will vary.

   c. Jenny bought 3 boxes of Yummy Dog Snacks, the same size as shown in d above. What is the total volume of the 3 boxes put together? Show your work.

   $5,040 \text{ cm}^3$; work will vary.
Caleb just got an order for 2 more flags. Use his sketch plans to figure out how many square feet of cloth it will take to make each flag.

1. The first order is for a flag that is 2 1/2 feet wide and 4 1/2 feet long.
   
a. Write an expression to represent the area of this flag.
      \[ 2 \frac{1}{2} \times 4 \frac{1}{2} \]

   b. Here are three different estimates for the answer. Choose the one that is most reasonable, and explain why it is most reasonable.
      
      \[ \bigcirc \ 9 \text{ square feet} \quad \bigcirc \ 10 \text{ square feet} \quad \bigcirc \ 11 \text{ square feet} \]
      
      *Explanations will vary. Example: It will be \( 2 \times 4 = 8 \), but then it will also be \( \frac{1}{2} \times 4 \) and \( \frac{1}{2} \times 2 \), so that’s 3 more square feet, and there will be a little bit more for \( \frac{1}{2} \times \frac{1}{2} \).*

   c. Here is the plan Caleb has sketched out for this flag. Use his sketch to solve the problem. Show all of your work.

   ![Diagram of flag sketch]

   \[
   2 \times 4 = 8 \quad 2 \times \frac{1}{2} = 1 \\
   \frac{1}{2} \times 4 = 2 \quad \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}
   \]

   \[
   8 + 1 + 2 + \frac{1}{4} = 11 \frac{1}{4}
   \]

   2 1/2 \times 4 1/2 \(= 11 \frac{1}{4} \) sq. ft.

   It will take 11 1/4 square feet of cloth to make this flag.

   (continued on next page)
2. The second order is for a flag that is $3 \frac{1}{2}$ feet wide and $4 \frac{3}{4}$ feet long.

   a. Write an expression to represent the area of this flag.

   $$3 \frac{1}{2} \times 4 \frac{3}{4}$$

   b. Here are three different estimates for the answer. Choose the one that is most reasonable, and explain why it is most reasonable.

   - 12 square feet
   - 14 square feet
   - 16 square feet

   **Explanations will vary. Example:** It will be $3 \times 4 = 12$, but also $\frac{1}{2} \times 4$ and $\frac{3}{4} \times 3$, which is like adding about another 4 sq. ft, and that’s $12 + 4 = 16$.

   c. Here is the plan Caleb has sketched out for this flag. Use his sketch to solve the problem. Show all of your work.

   ![Diagram of flag dimensions]

   It will take $16 \frac{5}{8}$ square feet of cloth to make this flag.

3. **CHALLENGE** Find at least two different pairs of dimensions for a flag that has an area of $8 \frac{1}{2}$ square feet. Show your work.

   **Answers and work will vary. Possible pairs of dimensions include:**

   - $1' \times 8 \frac{1}{2}'$
   - $2 \frac{1}{2}' \times 3 \frac{3}{8}'$
   - $2' \times 4 \frac{3}{4}'$
   - $\frac{3}{4}' \times 11 \frac{1}{3}'$
   - $4' \times 2 \frac{1}{8}'$
   - $1 \frac{1}{2}' \times 5 \frac{3}{8}'$
Aaron’s Arrays

1. Aaron is setting up an array to solve $2\frac{1}{3} \times 4\frac{1}{4}$.
   a. Fill in the blanks on the array.

   
   $2\frac{1}{3} \times 4\frac{1}{4}$

   
   $\begin{array}{c|c|c}
   \frac{4}{4} & \frac{1}{4} \\
   \hline
   2 \times \frac{4}{4} = 8 & 2 \times \frac{1}{4} = \frac{1}{2} \\
   \frac{1}{3} \times 4 = 1 \frac{1}{3} & \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}
   \end{array}$

   b. $2\frac{1}{3} \times 4\frac{1}{4} = 8 + \frac{1}{2} + 1 \frac{1}{3} + \frac{1}{12} = 9 \frac{11}{12}$

2. Aaron needs to solve $1\frac{4}{5} \times 2\frac{1}{2}$.
   a. Sketch and label an array that shows $1\frac{4}{5} \times 2\frac{1}{2}$.

   $\begin{array}{c|c|c}
   1 \frac{4}{5} & 2 \\
   \hline
   \frac{1}{4} \times 2 = 2 & \frac{1}{2} \\
   \frac{1}{4} \times 2 = 1 \frac{3}{5} & \frac{1}{10} \\
   \frac{1}{4} \times 1 \frac{3}{5} = \frac{1}{12} & \frac{1}{4} \times 1 \frac{3}{5} = \frac{4}{10}
   \end{array}$

   b. $1\frac{4}{5} \times 2\frac{1}{2} = 2 + \frac{1}{2} + 1 \frac{3}{5} + \frac{1}{10} = 4 \frac{1}{2}$

3. Fill in the blanks:
   a. $3\frac{1}{2} \times 14 = 7 \times 7 = 49$
   b. $32 \times 2\frac{1}{4} = 16 \times 4\frac{1}{2} = 72$
   c. $24 \times 7\frac{1}{2} = 12 \times 15 = 180$

Review

4. Solve. Use the strategy that makes the most sense to you.

   $\begin{array}{c|c|c|c|c}
   49.5 & 27.25 & 30.01 & 62.50 \\
   + 53.6 & \times 16 & - 26.49 & \times 24 \\
   103.1 & 436 & 3.52 & 1,500
   \end{array}$
**Thinking About Flags**

Every country in the world has its own flag. Every state in the United States also has its own flag. Cities, sports teams, colleges, and high schools often have their own flags as well. Flags are designed to tell a story about a group of people or an organization. A country’s flag often tells something about the history of that country. For example, the American flag still has 13 red and white stripes, which stand for the original 13 colonies. There are also 50 stars on the American flag, one for each state.

Flag makers use special words to describe the dimensions of a flag. The side of the flag nearest the flag pole—its width—is called the *hoist*. The side of the flag that extends from the hoist to the free end—its length—is called the *fly*.

Flags come in all different sizes, from the tiny flags children wave at parades to the huge flags people march out onto football fields at halftime. You may have noticed that the flag for a particular country is usually the same shape, no matter how large or small it is. That is because flag makers use the same ratio for the hoist to the fly, no matter what the size of the flag. The ratio of the hoist to the fly for the American flag is 10:19. If the hoist is 10 inches, the fly is 19 inches. If the hoist is doubled to 20 inches, the fly must also be doubled. Fill in the ratio table below to see some different sizes for the U.S. flag.

<table>
<thead>
<tr>
<th>Hoist</th>
<th>10”</th>
<th>20”</th>
<th>100”</th>
<th>50”</th>
<th>1”</th>
<th>2”</th>
<th>5”</th>
<th>6”</th>
<th>36” = 1 yard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fly</td>
<td>19”</td>
<td>38”</td>
<td><strong>190”</strong></td>
<td><strong>95”</strong></td>
<td>1.9”</td>
<td><strong>3.8”</strong></td>
<td><strong>9.5”</strong></td>
<td><strong>11.4”</strong></td>
<td><strong>68.4”</strong></td>
</tr>
</tbody>
</table>
Caleb’s U.S. Themed Flags page 1 of 2

Caleb has decided to design and make a new set of flags. Each of these flags will feature a theme that is uniquely American. Naturally, Caleb wants to use the hoist-to-fly ratio of the American flag, 10:19.

He wants to make the largest of the flags in this new set 10 feet by 19 feet. Then he plans to cut the dimensions in half, and then in half again, and then in half again to make smaller and smaller flags.

1. Fill in the ratio table below to help Caleb plan the different sizes. Use fractions, rather than decimals, when the answers are not whole numbers.

<table>
<thead>
<tr>
<th>Hoist</th>
<th>Fly</th>
</tr>
</thead>
<tbody>
<tr>
<td>a 10’</td>
<td>b 19’</td>
</tr>
<tr>
<td></td>
<td>÷ 2</td>
</tr>
<tr>
<td></td>
<td>÷ 2</td>
</tr>
<tr>
<td></td>
<td>÷ 2</td>
</tr>
<tr>
<td></td>
<td>÷ 2</td>
</tr>
</tbody>
</table>

2. Caleb uses the area of a flag to help figure out how much to charge for it. The area of the largest flag on the ratio table is 10’ × 19’ = 190 ft². Help him find the area of each of the other flags on the ratio table in square feet. Label each sketch with the dimensions of the flag. Then use the sketch to find the area of that flag.

Flag b is 5’ × 9 1/2’.

Area = 45 + 2 1/2 = 47 1/2 sq. ft.
Caleb’s U.S. Themed Flags page 2 of 2

Flag c is \(2 \frac{1}{2} \times 4 \frac{3}{4}\).

\[
\begin{array}{c|c|c}
\text{4}' & \text{3/4}' & \\
\hline
2' & \hline
2 \frac{1}{2} & 2 \times 4 = 8 & 1 \frac{1}{2} & 2 \times \frac{3}{4} = 1 \frac{1}{2} & \\
\hline
\frac{1}{2}' & \hline
\frac{1}{2} \times 4 = 2 & \frac{3}{8} & \frac{1}{2} \times \frac{3}{4} = \frac{3}{8} & \\
\hline
\hline
\end{array}
\]

Area = \(11 \frac{7}{8}\) ft\(^2\)

Flag d is \(1 \frac{1}{4} \times 2 \frac{3}{8}\).

\[
\begin{array}{c|c|c}
\text{2}' & \text{3/8}' & \\
\hline
1' & \hline
1 \frac{1}{4} & 1 \times 2 = 2 & 1 \frac{3}{8} = \frac{3}{8} & \\
\hline
\frac{1}{4}' & \hline
\frac{1}{4} \times 2 = \frac{1}{2} & \frac{1}{4} \times \frac{3}{8} = \frac{3}{32} & \\
\hline
\hline
\end{array}
\]

Area = \(2 \frac{31}{32}\) ft\(^2\)

**CHALLENGE** Flag e is \(\frac{5}{8} \times 1 \frac{3}{16}\).

\[
\begin{array}{c|c|c}
\text{1}' & \text{3/16}' & \\
\hline
\frac{5}{8}' & \hline
\frac{5}{8} \times 1 = \frac{5}{8} & \frac{15}{128} & \frac{5}{8} \times \frac{3}{16} = \frac{15}{128} & \\
\hline
\hline
\end{array}
\]

Area = \(\frac{95}{128}\) ft\(^2\)
Sophia’s Work

1 Sophia solved $2\frac{1}{6} - 1\frac{2}{3}$ like this:

\[
2\frac{1}{6} - \left| \frac{2}{3} \right| = \\
2 - 1 = 1 \\
\frac{2}{3} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6} = \frac{3}{6} \\
2\frac{1}{6} - \left| \frac{2}{3} \right| = \frac{3}{6}
\]

a Sophia did not get the correct answer. Can you explain why?

Explanations will vary. (She subtracted $\frac{1}{6}$ from $\frac{2}{3}$ instead of rewriting the minuend and the subtrahend as fractions with a common denominator.)

b How would you solve $2\frac{1}{6} - 1\frac{2}{3}$?

Responses will vary. Example:

\[
2\frac{1}{6} - 1\frac{2}{3} = 13/6 - 10/6 = 3/6 = 1/2
\]

2 Sophia has to read 5 books each month. By the middle of April, she had read $1\frac{5}{8}$ books. How many more books does Sophia need to read before the end of April?

3 \( \frac{3}{8} \) books

3 Write a story problem for this expression: $2\frac{1}{4} \times 1\frac{3}{8}$. Then solve the problem.

3 \( 3\frac{3}{32} \); story problems will vary.

4 Fill in the blanks.

\[
\begin{align*}
\frac{8}{9} \times \underline{12} &= 12 \\
\frac{18}{9} \times \underline{5} &= 10 \\
\frac{5}{5} \times 5 &= \underline{5}
\end{align*}
\]
Design-a-Flag Challenge page 1 of 2

Most countries have flags with ratios that are easier to work with than 10:19. Here are some of the more common ratios, and some of the countries that use those ratios.

<table>
<thead>
<tr>
<th>1:2</th>
<th>2:3</th>
<th>3:4</th>
<th>3:5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>Ethiopia</td>
<td>Aruba</td>
<td>Greece</td>
</tr>
<tr>
<td>Canada</td>
<td>Fiji</td>
<td>Belize</td>
<td>Namibia</td>
</tr>
<tr>
<td>Cuba</td>
<td>Philippines</td>
<td>Cambodia</td>
<td>Spain</td>
</tr>
<tr>
<td>Ecuador</td>
<td>Zimbabwe</td>
<td>Botswana</td>
<td>Viet Nam</td>
</tr>
</tbody>
</table>

You are going to design your own flag today. Here are your instructions:

1. Choose one of the ratios above for your flag.

2. No matter which ratio you choose, the hoist of your flag will be 9 inches. What fraction of a foot is 9 inches? Show your work.

   \[
   9'' = \frac{3}{4} \text{ of a foot; work will vary. Example:} \quad \frac{9 \div 3}{12 \div 3} = \frac{3}{4}
   \]

3. Fill in the ratio table to figure out how long your flag’s fly will be if the hoist is 9”.

   a. My ratio is _________.

   Think: What do I have to multiply the first number in my ratio by to make 9? When you figure that out, multiply the second number in your ratio by the same number. Use the example above to help you. Draw a flag and label the ratios or dimensions if you like.

   The hoist of each flag will be 9”. The fly will vary, depending on the ratio selected. Answers for all 4 ratios shown.

   (continued on next page)
Design-a-Flag Challenge page 2 of 2

4. The hoist of my flag will be 9 inches. The fly of my flag will be ____ inches.

a. What is the fly of your flag in feet? Show your work.

   Answers shown for each ratio; work will vary.
   1:2 Fly = 18” or 1 ½’
   2:3 Fly = 13 ½” or 1 ½’
   3:4 Fly = 12” or 1’
   3:5 Fly = 15” or 1 ¼’

b. In the space below, draw an outline of your flag. Use a scale of 1 centimeter per inch. For example, if your flag is going to be 9 inches by 15 3/4 inches, measure and draw a rectangle that is 9 cm by 15 3/4 cm.

c. Label your sketch with its dimensions given in feet.

d. Find the area of your flag in square feet. Show all your work.

   Answers shown for each ratio; work will vary somewhat. Note that the scale used for sketches below is ½ cm per inch rather than 1 cm per inch, so students’ sketches will be twice the size of those.

   1:2 Ratio
   
<table>
<thead>
<tr>
<th>1’</th>
<th>½’</th>
</tr>
</thead>
<tbody>
<tr>
<td>¾’</td>
<td>¼ × 1 = ¾</td>
</tr>
<tr>
<td>¾ × ½ = ¾/8</td>
<td></td>
</tr>
</tbody>
</table>
   
   Area = ¾ + ¾/8 = ⁹/₈ = 1 ¹/₈ sq. ft.

   2:3 Ratio
   
<table>
<thead>
<tr>
<th>1’</th>
<th>½’</th>
</tr>
</thead>
<tbody>
<tr>
<td>¾’</td>
<td>¼ × 1 = ¾</td>
</tr>
<tr>
<td>¾ × ½ = ¾/32</td>
<td></td>
</tr>
</tbody>
</table>
   
   Area = ¾ + ¾/32 = ²⁷/₃₂ sq. ft.

   3:4 Ratio
   
<table>
<thead>
<tr>
<th>1’</th>
</tr>
</thead>
<tbody>
<tr>
<td>¾’</td>
</tr>
<tr>
<td>¾ × 1 = ¾</td>
</tr>
</tbody>
</table>
   
   Area = ¾ sq. ft.

   3:5 Ratio
   
<table>
<thead>
<tr>
<th>1’</th>
<th>¼’</th>
</tr>
</thead>
<tbody>
<tr>
<td>¾’</td>
<td>¼ × 1 = ¾</td>
</tr>
<tr>
<td>¾ × ¼ = ¾/₁₆</td>
<td></td>
</tr>
</tbody>
</table>
   
   Area = ¾ + ¾/₁₆ = ¹⁵/₁₆ sq. ft.

   The area of my flag is ______ ft².
Boxes & Banners

1. Ebony’s cousin Jada is away at college this year. Ebony wants to send her a package with some candy in it. She has the three boxes shown below. Which box should she use if she wants to send Jada as much candy as possible?

- **Box A**: 8 cm x 22 cm x 52 cm
- **Box B**: 22 cm x 22 cm x 22 cm
- **Box C**: 22 cm x 17 cm x 15 cm

**a** What do you need to know about the boxes in order to answer the question above?

- **Their volume**

**b** Solve the problem. Show all your work.

- **Jada should use Box B. Work will vary.** Volume of each box shown here.
  - **Box A**: $52 \times 22 \times 8 = 9,152$ cm$^3$
  - **Box B**: $22 \times 22 \times 22 = 10,648$ cm$^3$
  - **Box C**: $22 \times 17 \times 15 = 5,610$ cm$^3$

2. Ebony also made a banner for Jada to hang on the door of her dormitory room. The banner is $1 \frac{1}{4}$ feet wide and $2 \frac{1}{2}$ feet long.

**a** Mark the bubble to show which flag-making ratio Ebony used.

- ○ 2:3
- ○ 3:5
- ● 1:2
- ○ 3:4

**b** What is the area of the banner? Make a labeled sketch to model and solve this problem. Show all of your work.

- **3 $\frac{1}{8}$ sq. feet; work will vary.**

  Example:

<table>
<thead>
<tr>
<th>1'</th>
<th>2'</th>
<th>$\frac{1}{2}'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \times 1 = 2$</td>
<td>$1 \times \frac{1}{2} = \frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>$2 \times \frac{1}{4} = \frac{1}{2}$</td>
<td>$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$</td>
<td></td>
</tr>
</tbody>
</table>

  **Area** = $2 + \frac{1}{2} + \frac{1}{2} + \frac{1}{8} = 3 \frac{1}{8}$ sq. ft.
Simplifying Fractions Review

1. Divide the numerator and denominator of each fraction by the largest factor they have in common (the greatest common factor) to show each fraction in its simplest form. A fraction is in its simplest form when its numerator and denominator have no common factor other than 1. Some of the fractions below may already be in simplest form.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Factors of the Numerator (top number)</th>
<th>Factors of the Denominator (bottom number)</th>
<th>Greatest Common Factor</th>
<th>Divide</th>
<th>Simplest Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>ex ( \frac{21}{24} )</td>
<td>1, 3, 7, 21</td>
<td>1, 2, 4, 6, 8, 12, 24</td>
<td>3</td>
<td>( \frac{21}{24} \div 3 = \frac{7}{8} )</td>
<td>( \frac{7}{8} )</td>
</tr>
<tr>
<td>a ( \frac{14}{16} )</td>
<td>1, 2, 7, 14</td>
<td>1, 2, 4, 8, 16</td>
<td>2</td>
<td>( \frac{14}{16} \div 2 = \frac{7}{8} )</td>
<td>( \frac{7}{8} )</td>
</tr>
<tr>
<td>b ( \frac{16}{21} )</td>
<td>1, 2, 4, 8, 16</td>
<td>1, 3, 7, 21</td>
<td>1</td>
<td>( \frac{16}{21} \div 1 = \frac{16}{21} )</td>
<td>( \frac{16}{21} )</td>
</tr>
<tr>
<td>c ( \frac{27}{36} )</td>
<td>1, 3, 9, 27</td>
<td>1, 2, 3, 4, 6, 9, 12, 18, 36</td>
<td>9</td>
<td>( \frac{27}{36} \div 9 = \frac{3}{4} )</td>
<td>( \frac{3}{4} )</td>
</tr>
<tr>
<td>d ( \frac{15}{36} )</td>
<td>1, 3, 5, 15</td>
<td>1, 2, 3, 4, 6, 9, 12, 18, 36</td>
<td>3</td>
<td>( \frac{15}{36} \div 3 = \frac{5}{12} )</td>
<td>( \frac{5}{12} )</td>
</tr>
</tbody>
</table>

2. Write two fractions that are equal to the fraction shown.  

- **Example:**
  - \( \frac{3}{4} = \frac{6}{8} \) and \( \frac{3}{4} = \frac{9}{12} \)
  - \( \frac{6}{21} = \frac{2}{7} \) and \( \frac{6}{21} = \frac{12}{42} \)
  - \( \frac{3}{15} = \frac{1}{5} \) and \( \frac{3}{15} = \frac{6}{30} \)
  - \( \frac{7}{12} = \frac{14}{24} \) and \( \frac{7}{12} = \frac{21}{36} \)
Rob’s Review

1 Rob was solving multiplication problems. Write an expression with parentheses to record his thinking for each of the two problems below.

a Rob solved $97 \times 50$ by multiplying 100 by 50 and then removing 3 groups of 50.

$$\left(100 \times 50\right) - \left(3 \times 50\right)$$

b Rob solved $25 \times 44$ by finding $\frac{1}{4}$ of 44 and then multiplying by 100.

$$\left(\frac{1}{4} \times 44\right) \times 100 \text{ or } (44 \div 4) \times 100$$

2 Evaluate Rob’s two expressions.

a 4,850

b 1,100

3 Rob saw a friend use the standard algorithm to solve the problem $290 \times 14$.

a Solve the problem using the standard algorithm.

$$\begin{array}{c}
3
\ \ \ \ 290 \\
\times \ 14 \\
1160 \\
+ 2900 \\
\underline{4,060}
\end{array}$$

b Rob said he thought there was a more efficient way to solve this problem, and suggested his friend use the ratio table below. Fill in Rob’s ratio table.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>300</th>
<th>10</th>
<th>290</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>28</td>
<td>42</td>
<td>4,200</td>
<td>140</td>
<td>4,060</td>
</tr>
</tbody>
</table>
Array Work

Fill in the blanks on each array. Then write two equations—one multiplication, and one division—equation, to match the array.

1

\[
\begin{array}{c|c}
11 & 20 \\
\hline
220 & 2
\end{array}
\]

\[11 \times 22 = 242\]
\[242 \div 11 = 22\]

2

\[
\begin{array}{c|c}
2 & 2 \frac{1}{2} \\
\hline
3 & 7 \frac{1}{2}
\end{array}
\]

\[3 \frac{1}{2} \times 2 \frac{1}{2} = 8 \frac{3}{4}\]
\[8 \frac{3}{4} \div 3 \frac{1}{2} = 2 \frac{1}{2}\]

3

\[
\begin{array}{c|c|c|c}
2 & \frac{1}{2} & \hline
4 & 1 & \\
\hline
\frac{1}{2} & 1 & \frac{1}{4}
\end{array}
\]

\[2 \frac{1}{2} \times 2 \frac{1}{2} = 6 \frac{1}{4}\]
\[6 \frac{1}{4} \div 2 \frac{1}{2} = 2 \frac{1}{2}\]

4

\[
\begin{array}{c|c|c}
11 & 30 & 3 \\
\hline
330 & 33 & \\
\hline
\end{array}
\]

\[11 \times 33 = 363\]
\[363 \div 11 = 33\]
Work Place Instructions 7A Roll Five

Each pair of players needs:

- math journals
- 2 dice numbered 1–6
- 2 dice numbered 4–9
- 1 die numbered 0–5
- 1 Roll Five Record Sheet to share

1. Each player rolls a die numbered either 1–6 or 4–9.

2. The product of the two dice rolled becomes the target number, and both players write the target number on their sides of the record sheet.

3. All five dice are rolled, and players write the digits rolled on their sides of the record sheet. 
   Note: Both players write the same five digits.

4. Working separately in their math journals, players use any combination of operations (addition, subtraction, multiplication, and/or division) to make an equation that results in the target number.
   - Players do not have to use all five digits rolled, but they must make an equation that results in the target number.
   - Each digit may be used only once.
   - Players keep track of their thinking in their math journals.

5. Players write their equations on the record sheet, using parentheses as necessary. Players check each other’s equations and work for accuracy.
   - Players can have different equations; they just need to reach the same target number.

6. Players determine their scores and write them on the record sheet.
   - 1 point for each digit used
   - 1 additional point for using multi-digit division where the divisor is two or more digits
   - 1 additional point for using fractions or decimals
     - For example, given 5, 7, 3, 2, 5, and the target number 15, a player could earn an extra point for $375 ÷ 25 = 15$.
     - Given 5, 6, 5, 5, 1, and the target number 10, a player could earn an extra point for $(6 \times 5) \times 5 ÷ 15 = 10$.
     - Given 8, 3, 5, 1, 2, and the target number 25, a player could earn an extra point for $((8 \times 3) \times .5) \times 2 + 1 = 25$.
   - A player who is unable to create an equation for the target number earns zero points for that round.

7. The player with the higher total score after four rounds wins the game.

Game Variations

A. Players work together to create one equation for each round.

B. Players must use all five digits in order for the equation to earn any points.

C. Players use a calculator to help create equations that result in their target number.
   Note: Check that the calculator uses the standard order of operations. Enter $1 + 3 \times 2$. If the answer is 7, that calculator will work for this game.
Roll Five & Ratio Tables

Destiny and Jesse are playing Roll Five. They want to add, subtract, multiply and/or divide any of the digits on their five dice to reach their target number.

1  Destiny’s target number was 24. She rolled the digits 2, 5, 6, 9, and 1.
   a  Destiny thinks she can use 6, 2 and 9 to reach her target number. Record an expression she could use with these three digits, and then evaluate the expression.
       Expressions may vary slightly. Example:
       \[6 + (2 \times 9) = 24\]
   b  What is her score?
       3 points (1 point for each digit used)
   c  Jesse encourages Destiny to try for a higher score. Record an equation Destiny could make with 2, 5, 6, 9, and 1 to reach the target number of 24.
       Expressions may vary slightly. Example:
       \[((6 \times 9) − (5 + 1)) ÷ 2 = (54 − 6) ÷ 2 = 48 ÷ 2 = 24\]
   d  What is her new score?
       5 points

2  Jesse’s target number was 16. He rolled the digits 3, 4, 4, 6, and 8.
   a  Jesse says that 6 times 8 is 48, divided by 3 is 16. Write an expression to record Jesse’s thinking, and then evaluate the expression.
       \[(6 \times 8) ÷ 3 = 48 ÷ 3 = 16\]
   b  Jesse thinks he can get a higher score if he multiplies 6 and 8, divides the product by 4, and then adds 4. Write an expression to record Jesse’s thinking, and then evaluate the expression.
       \[(6 \times 8) ÷ 4) + 4
       = (48 ÷ 4) + 4
       = 12 + 4
       = 16\]

3  Mariah was solving the problem 342 ÷ 19. She started the ratio table below. Complete the ratio table to find the quotient. Add to Mariah’s ratio table as needed.

<table>
<thead>
<tr>
<th>Use of ratio table will vary. Example:</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>19</td>
<td>38</td>
</tr>
<tr>
<td>380 − 38</td>
<td></td>
</tr>
</tbody>
</table>
Fruit Pizza

Lisa’s parents own a bakery. They sell fruit pizzas made of granola crust with yogurt filling and one fruit topping. Many fruit topping choices are available.

Lisa wants to make a few fruit pizzas to eat with her friends, and she needs to figure out how much fruit to buy for each kind of pizza she makes. She found a chart that her parents made for their latest pizza order that shows how much of each kind of fruit was used on different numbers of pizzas.

Use the information on the chart below to help Lisa figure out how many cups of fruit she would need for each different kind of pizza, so she can decide if she wants to make a blueberry pizza, a strawberry pizza, a peach pizza, or some of the other kinds on the list.

Note: The cups of fruit are divided evenly among the number of pizzas for that kind of fruit.

1. Solve each problem in your math journal, and show all of your work. Write your answers on the chart below.

<table>
<thead>
<tr>
<th>Type of Fruit</th>
<th>Cups of Fruit</th>
<th>Number of Pizzas</th>
<th>Cups of Fruit per Pizza</th>
</tr>
</thead>
<tbody>
<tr>
<td>blueberries</td>
<td>237</td>
<td>79</td>
<td>3 cups</td>
</tr>
<tr>
<td>strawberries</td>
<td>352</td>
<td>88</td>
<td>4 cups</td>
</tr>
<tr>
<td>peaches</td>
<td>91</td>
<td>182</td>
<td>½ cup</td>
</tr>
<tr>
<td>blackberries</td>
<td>176</td>
<td>44</td>
<td>4 cups</td>
</tr>
<tr>
<td>kiwi</td>
<td>42</td>
<td>28</td>
<td>1 ½ cups</td>
</tr>
<tr>
<td>raspberries</td>
<td>88</td>
<td>22</td>
<td>4 cups</td>
</tr>
<tr>
<td>mandarin oranges</td>
<td>120</td>
<td>48</td>
<td>2 ½ cups</td>
</tr>
<tr>
<td>grapes</td>
<td>1</td>
<td>¼</td>
<td>4 cups</td>
</tr>
<tr>
<td>pineapple</td>
<td>2</td>
<td>½</td>
<td>4 cups</td>
</tr>
<tr>
<td>mango</td>
<td>1</td>
<td>⅓</td>
<td>3 cups</td>
</tr>
</tbody>
</table>
Thinking About Division

1. Show your strategy and answer in three ways:
   - with a model such as an array or ratio table
   - in words
   - with an equation

   a. How can you use $1100 \div 11$ to help you find $1089 \div 11$?
   
   Responses will vary. Example:
   
   $1,100 \div 11$ is 100 because $100 \times 11 = 1,100$.
   $1,089$ is 11 less than 1,100 so you know
   $99 \times 11 = 1,089$
   $1,089 \div 11 = 99$

   b. How can you use $900 \div 9$ to help you find $936 \div 9$?
   
   Responses will vary. Example:
   
   $100 + 4$ is 104, so $936 \div 9 = 104$

2. Look at the five division problems below. Before you solve them, decide if any of the problems have the same quotient. Explain how you know, and then solve each one.

   a. $42 \div 7 = 6$
   
   b. $420 \div 7 = 60$

   Problems a and c will have the same quotient because the dividend and the divisor have both been multiplied by 10. The same is true for problems b and d.

   c. $420 \div 70 = 6$
   
   d. $4,200 \div 70 = 60$
   
   e. $4,620 \div 70 = 66$

3. Mariana needs to solve the problem $588 \div 42$. She prefers to think about division as multiplication. She begins by thinking, “I know 10 groups of 42…” Create a ratio table to model Mariana’s thinking and solve the problem.

   Responses will vary. Example:
**Food Project**

The Sellwood Community Center wants to donate hot dogs to the Southeast Portland Food Project, and they want to find the best deal.

1. At Food Mart, hot dogs can be purchased in packages of 8 for $2.40 or packages of 12 for $3.00. Which is a better buy? Explain how you know.

   **Packages of 12 for $3.00 are the better buy. Explanations will vary. (At 8 for $2.40, each hot dog costs 30¢. At 12 for $3.00, each hot dog costs 25¢.)**

2. At Food World, hot dogs are sold in packages of 24 for $5.76 or packages of 50 for $12.00. Which is a better buy? Explain how you know.

   **Neither is the better buy. Explanations will vary. (In both cases, each hot dog costs 24¢.)**

3. The community center organizers want to buy 600 hot dogs. What will the cost be if they purchase the packages with the best buy? Show your thinking.

   **$144; work will vary.**

4. Six blocks of neighbors in Sellwood and Westmoreland donate food to the Southeast Portland Food Project. The table below shows how many pounds of food each block donated during the last 12 months. Fill in the table to show how many pounds of food each block donated per month.

<table>
<thead>
<tr>
<th>Block</th>
<th>Pounds of Food Donated in 12 Months</th>
<th>Pounds of Food Donated per Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>192</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>216</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>144</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>138</td>
<td>11 1/2</td>
</tr>
<tr>
<td>5</td>
<td>174</td>
<td>14 1/2</td>
</tr>
<tr>
<td>6</td>
<td>210</td>
<td>17 1/2</td>
</tr>
</tbody>
</table>
Division with Fractions

1. Alice is filling candy molds with chocolate for her brother’s birthday party. It takes her 6 minutes to fill $\frac{1}{5}$ of the molds. How long will it take her to fill all of the molds? Complete the ratio table to show the answer.

<table>
<thead>
<tr>
<th>minutes</th>
<th>6</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>molds filled</td>
<td>$\frac{1}{5}$ of the molds</td>
<td>all the molds</td>
</tr>
</tbody>
</table>

2. Alice is filling cupcake molds with old, melted crayons to make new crayons for her brother’s party. If it takes her 9 minutes to fill $\frac{1}{4}$ of the molds, how long will it take her to fill all the molds? Show your thinking on a ratio table.

<table>
<thead>
<tr>
<th>Minutes</th>
<th>9</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molds Filled</td>
<td>$\frac{1}{4}$ of the molds</td>
<td>all the molds</td>
</tr>
</tbody>
</table>

3. Write a story problem for the expression $14 \div \frac{1}{2}$. Then solve the problem.

**NOTE** Remember that $14 \div \frac{1}{2}$ means, “How many halves are there in 14?”

28; story problems will vary.
**Story Problems** page 1 of 2

Solve each problem. Show your work. Write an equation to represent the problem and the solution.

**Work will vary.**

1. Trading cards come in packs of 12 cards. Chris bought 300 cards. How many packs did he buy?

   \[300 \div 12 = 25 \text{ packs}\]
   \[300 = 240 + 60\]
   \[240 \div 12 = 20\]
   \[60 \div 12 = 5\]
   so \[300 \div 12 = 25\]

2. Chris dealt out all the trading cards to himself and 11 friends so they could play a game. How many cards did each person get?

   \[300 \div 12 = 25 \text{ cards for each friend}\]
   \[(10 \times 12) + (10 \times 12) + (5 \times 12)\]
   \[= 120 + 120 + 60\]
   \[= 300, \text{ so } 300 \div 12 = 25\]

3. Anna earned $270 over the summer by mowing lawns. If she mowed 18 lawns and earned the same amount each time, how much did Anna earn per lawn?

   \[270 \div 18 = \$15 \text{ per lawn}\]

4. Mr. Foley is setting up chairs for a meeting. Each row has 18 chairs. 270 people are expected to come to the meeting. How many rows of chairs should he set up?

   \[270 \div 18 = 15 \text{ rows}\]
5. There is \( \frac{1}{2} \) of a pan of lasagna to feed Marta and her three cousins. How much of the whole pan of lasagna do they each get?

\[
\frac{1}{2} \div 4 = \frac{1}{8} \text{ of the pan}
\]

If you divide \( \frac{1}{2} \) a pan into 4 equal parts, each part is \( \frac{1}{8} \) of the whole pan.

6. Marta has 4 cups of grated cheese. She needs \( \frac{1}{2} \) a cup of cheese for each mini-pizza. How many mini-pizzas can she make?

\[
4 \div \frac{1}{2} = 8 \text{ mini-pizzas}
\]

There are 2 halves in every whole cup, so there must be \( 4 \times 2 = 8 \) halves in 4 cups.

\[
\frac{1}{2} + \frac{1}{2} = 1
\]

7. **Challenge** Craig is organizing a neighborhood party for his scout troop. He needs to deliver fliers to all of the houses in the neighborhood. There are 13 streets with 12 houses, 14 streets with 11 houses, and 8 streets with 13 houses. If there are 18 boys in his scout troop helping Craig deliver the fliers, how many fliers should each pair of boys get?

**Note** The group of 18 boys who will be working in pairs to deliver the fliers does not include Craig, because he will be busy keeping everyone organized.

Each pair should get 46 fliers

\[
13 \times 12 = 156
\]

\[
14 \times 11 = 154
\]

\[
8 \times 13 = 104
\]

414 houses

\[
\begin{array}{c|c|c|c|c}
1 & 10 & 40 & 6 & 46 \\
9 & 90 & 360 & 54 & 414
\end{array}
\]

\[6 \div 9 = 46 \]

\[9 \div 414 - 360 \]

\[9 \div 414 - 54 \]

\[9 \div 414 - 0 \]

It's \( 414 \div 9 \) because there are 9 pairs of boys.
Thinking About Money

Ebony was putting her loose change into rolls to take to the bank.

1. Ebony discovered she had $8 in quarters.
   a. How many quarters are there in $8? 32
   b. Choose the division expression you would use to find the number of quarters in $8.
      - $8 ÷ 4
      - $8 ÷ \frac{1}{4}
      - $8 ÷ \frac{1}{2}

2. Ebony had $6 in dimes.
   a. How many dimes are in $6? 60
   b. Write the division expression you would use to find the number of dimes in $6.
      $6 ÷ \frac{1}{10}$

3. Ebony also had $6 in nickels.
   a. How many nickels are in $6? 120
   b. Write the division expression you would use to find the number of nickels in $6.
      $6 ÷ \frac{1}{20}$

4. Ebony deposited $64.32 when she went to the bank. If she already had $487.99 in her account, what is her new balance (total)? Show your work.
   $552.31; work will vary.$

5. How much more does Ebony need to reach her goal of $1,000.00 in her account? Show your work.
   $447.69; work will vary.$
# Multiplication Round & Check

1. Multiply the numbers.

<table>
<thead>
<tr>
<th></th>
<th>20</th>
<th>30</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>100</th>
<th>100</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td></td>
<td></td>
<td></td>
<td>400</td>
<td></td>
<td>2,000</td>
<td>3,000</td>
<td>6,000</td>
</tr>
</tbody>
</table>

2. Think about rounding to estimate the answers to the problems below. Then rewrite each problem vertically and solve it using the standard algorithm.

Hint: Use the answers above to help with your estimates.

<table>
<thead>
<tr>
<th>Problem</th>
<th>9 × 39</th>
<th>9 × 41</th>
<th>9 × 32</th>
<th>12 × 18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solution</td>
<td>369</td>
<td>3288</td>
<td></td>
<td>216</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem</th>
<th>18 × 28</th>
<th>22 × 33</th>
<th>18 × 103</th>
<th>32 × 123</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td></td>
<td></td>
<td>Estimates will vary.</td>
<td></td>
</tr>
<tr>
<td>Solution</td>
<td>504</td>
<td>726</td>
<td>1,854</td>
<td>3,936</td>
</tr>
</tbody>
</table>
### Fraction Division on a Clock

<table>
<thead>
<tr>
<th>Problem</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Fraction Division on a Clock" /></td>
<td><img src="image2.png" alt="Fraction Division on a Clock" /></td>
</tr>
<tr>
<td><img src="image3.png" alt="Fraction Division on a Clock" /></td>
<td><img src="image4.png" alt="Fraction Division on a Clock" /></td>
</tr>
<tr>
<td><img src="image5.png" alt="Fraction Division on a Clock" /></td>
<td><img src="image6.png" alt="Fraction Division on a Clock" /></td>
</tr>
<tr>
<td><img src="image7.png" alt="Fraction Division on a Clock" /></td>
<td><img src="image8.png" alt="Fraction Division on a Clock" /></td>
</tr>
</tbody>
</table>
**Work Place Instructions 7B Quotients Race to One Hundred**

Each pair of players needs:
- math journals
- 1 deck of Number Cards, wild cards removed
- 1 Quotients Race to One Hundred Record Sheet to share

1. Players decide who will be the dealer and who will go first. The dealer gives 5 cards to each player.

2. Player 1 arranges her 5 cards so that 2 cards form a divisor and 3 cards form a dividend. Then, Player 1 writes the division problem in the Problem column on her side of the record sheet.

Players need to think strategically about how to arrange the cards to produce the greatest quotient possible.

*Player 1* I drew 9, 6, 4, 3, and 7. Hmm... I think I’ll try 976 and 34, so my problem is 976 ÷ 34.

3. Player 1 works in her math journal to solve the division problem formed by the cards. She can disregard any remainder.

4. Player 1 shows her work to Player 2 and Player 2 says whether he agrees with the quotient.
   - If Player 2 agrees with the quotient, Player 1 records the whole number portion of the quotient in the Quotient/Score column on the record sheet.
   - If Player 2 disagrees with the quotient, the two players rework the problem together. Then, Player 1 records the whole number part of the quotient in the Quotient/Score column on the record sheet.

*Player 1* I made a ratio table for 34, and subtracted off the amounts, like this.

*Player 2* I agree with your work, so you can put 28 for your first quotient.

5. Player 2 takes a turn.

6. Players continue to take turns. After each turn, a player records her newest quotient, adds that quotient to her previous quotients to get her new score, and records her new score. The player whose quotients total 100 or more first is the winner.
Work Place Instructions 7B Quotients Race to One Hundred

Game Variations

A  Players can play individually and see how many turns it takes them to get to 100, or they can play together, supporting each other as they see how many turn it takes them to get to 100.

B  Players can deal out 6 cards and make 4-digit dividends.

C  Players can play Remainders Race to 3. In this version, players find the remainder in each division problem and record the remainder as a fraction. As they play, they add the remainders instead of the whole number quotients. The first player who gets a total of 3 wins the game.

D  If players need more practice with 1-digit divisors, they can deal 3 or 4 cards and arrange the problem so there is a 1-digit divisor and a 2- or 3-digit dividend.

E  Players can add the wild cards to the deck of Number Cards and, upon receiving a wild card, choose a number for that card to represent.
Dividing Fractions & Whole Numbers

1. Fill in the bubble to show what each expression below means. Then use labeled sketches and numbers to model and solve each problem. Show your work and remember to write the answer at the bottom of each box.

\[ 4 \div \frac{1}{5} \]
- How many groups of 4 are there in \( \frac{1}{5} \)?
- How many groups of \( \frac{1}{5} \) are there in 4?
- What is \( \frac{1}{5} \) of 4?

\[ \frac{1}{2} \div 4 \]
- How many groups of \( \frac{1}{2} \) are there in 4?
- What is \( \frac{1}{2} \) of 4?
- If you split \( \frac{1}{2} \) into 4 equal shares, how big is each share?

Answer: _______ 20 ______

Answer: _______ \( \frac{1}{6} \) ______

2. Mr. Ortega had \( \frac{1}{2} \) of a box of felt markers. He divided the box equally among 3 small groups of students. What fraction of the box of felt markers did each group get?

a. Choose the expression that best represents this problem.

- \( \frac{1}{2} \div 3 \)
- \( 3 \div \frac{1}{2} \)
- \( \frac{1}{2} \times 3 \)

b. Solve the problem. Show your work.

Work will vary.

Each group of students got \( \frac{1}{6} \) of a box of felt markers.
**Dividing Marbles**

1. Peter has 320 marbles. He is going to divide them into 32 bags. How many marbles will be in each bag?
   - **a** Write an equation for the problem.
     \[ 320 \div 32 = m \]
   - **b** Solve the problem. Show your work.
     10 marbles; work will vary.

2. Sophia has 352 marbles. She is also going to divide them into 32 bags. How many marbles will be in each bag?
   - **a** Write an equation for the problem.
     \[ 352 \div 32 = m \]
   - **b** Solve the problem. Show your work.
     11 marbles; work will vary.

3. Cole has 288 marbles. He is also going to divide them into 32 bags. How many marbles will be in each bag?
   - **a** Write an equation for the problem.
     \[ 288 \div 32 = m \]
   - **b** Solve the problem. Show your work.
     9 marbles; work will vary.
Dealing with Remainders page 1 of 3

1 A bakery donated 273 cookies for the end of the year fifth grade school party. How many cookies does each of the 84 students get if they share the cookies fairly?

a Solve the problem. Show your work.

3 ½ cookies each; work will vary.

Example:

\[
\begin{array}{c|c|c|c|c}
1 & 2 & 3 & 3 R21 \\
84 & 168 & 252 & \\
\end{array}
\]

There are 21 cookies left over. 21 × 4 = 84, so each kid can have an extra ¼ of a cookie.

b Write an equation to represent the problem and the answer.

\[273 ÷ 84 = 3 \frac{1}{4}\]

c Explain what you did with the remainder, if any, and why.

Explanations will vary. (Cookies can be cut, so it makes sense to show the remainder as a fraction.)

2 There are 814 students going on a field trip, and they have 31 buses. How many students should go on each bus if they split up evenly on the buses?

a Solve the problem. Show your work.

26 students on each of 23 buses;
27 students on each of 8 buses

\[
\begin{array}{c|c|c|c|c|c|c}
1 & 10 & 20 & 5 & 1 \boxed{26 R8} \\
31 & 310 & 620 & 155 & \\
\end{array}
\]

b Write an equation to represent the problem and the answer.

\[814 ÷ 31 = 26 R8\]

c Explain what you did with the remainder, if any, and why.

Explanations will vary. (It makes best sense to show the remainder as a whole number.)
Dealing with Remainders page 2 of 3

3  Green Lake School is serving cake to their 840 students on the last day of school. Each large cake serves 48 students. How many cakes does the school need?

a  Solve the problem. Show your work.

\[
\begin{array}{ccc}
10 & 5 & 2 \\
\hline
48 & 480 & 240 & 96 \\
\hline
840 & 480 & 240 & 96 \\
\hline
360 & 360 & 120 & 24 \\
\hline
120 & 120 & 24 & 24 \\
\hline
96 & 96 & 24 & 24 \\
\hline
24 & 24 & 24 & 24 \\
\hline
24 & 24 & 24 & 24 \\
\hline
0 & 0 & 0 & 0 \\
\end{array}
\]

17 ½ cakes. Work will vary. Example:

\[840 \div 48 = 17 \frac{24}{48} = 17 \frac{1}{2}\]

b  Write an equation to represent the problem and the answer.

\[840 \div 48 = 17 \frac{24}{48} = 17 \frac{1}{2}\]

c  Explain what you did with the remainder, if any, and why.

Explanations will vary. (It makes good sense to show the remainder as a fraction because cakes can be cut, but some students may agree that 18 cakes are needed because the store won’t sell half a cake.)

4  Green Lake school wants to improve the landscaping around the school. The principal collected $476 from 56 parents to plant new trees. If all of the parents donated the same amount, how much did they each give?

a  Solve the problem. Show your work.

\[
\begin{array}{ccc}
1 & 2 & 8 \\
\hline
56 & 476 & 28 \\
\hline
56 & 560 & 112 \\
\hline
\end{array}
\]

$8.50; work will vary. Example:

\[476 \div 56 = 8.5\]

b  Write an equation to represent the problem and the answer.

\[476 \div 56 = 8.5\]

c  Explain what you did with the remainder, if any, and why.

Explanations will vary. (It makes best sense to show the remainder as a decimal in money-related problems.)

(continued on next page)
Dealing with Remainders  page 3 of 3

5  During a play, 744 students will sit in rows in the auditorium. If each row can seat 32 students, how many rows do they need?

a  Solve the problem. Show your work.

24 rows; work will vary. Example:

\[
\begin{array}{cccc}
1 & 10 & 20 & 3 \\
32 & 320 & 640 & 96 \\
\end{array}
\]

\[
32 \div 744 = 23 \text{ R}8
\]

b  Write an equation to represent the problem and the answer.

\[744 \div 32 = 23 \text{ R}8\]

c  Explain what you did with the remainder, if any, and why.

Explanations will vary. (Since each row only seats 32 students, another row will be needed for the 8 remaining students.)

6  CHALLENGE  The Friends and Family Committee sold tickets for a school dance. They used the money to pay for music, decorations, and snacks. Tickets cost $4.50, and 259 students bought tickets. The committee spent \(\frac{1}{3}\) of the money they made on food. They bought 15 platters of food. How much did each platter cost?

a  Solve the problem. Show your work.

Each platter cost $25.90; work will vary.

\[
\begin{align*}
\text{Each platter cost} & \quad \text{work will vary.} \\
($4.50 \times 259) &= 1,165.50 \\
1,165.50 \times 3 &= 388.50 \\
388.50 \div 15 &= 25.90
\end{align*}
\]

b  Write an equation to represent the problem and the answer.

\[((4.50 \times 259) \div 3) \div 15 = 25.90\]

c  Explain what you did with the remainder, if any, and why.

Explanations will vary. (Decimals make sense in a money-related situation.)
Story Problems: Division with Remainders

Solve each problem below. Show your work. Use the context of each story to help you decide what to do with the remainder.

1. The entire Harrisville Middle School is taking a field trip to the symphony. All 282 people will ride the subway to the theater. Each subway has 8 subway cars. If an equal number of people ride in each subway car, how many people will ride in each?

   35 people in each of 6 subway cars; 36 people in each of 2 subway cars. Work will vary. (282 ÷ 8 = 35 R2; in this case, it makes sense to leave the remainder as a whole number.)

2. Mrs. Smith’s class has 24 students. The tickets for her class cost $162. How much did each student’s ticket cost? (Teachers get free tickets.)

   $6.75; work will vary.

3. Olivia made brownies for Mrs. Smith’s class. She made 30 brownies and divided the brownies evenly. How many brownies will each student get? (Mrs. Smith says she doesn’t want any.)

   1 ¼ brownies each; work will vary.

4. Olivia wants Mrs. Smith to have an equal portion of brownies as well (to take home to her niece). Now how many brownies will each person get?

   1 ½ brownies each; work will vary.
   (Some students may decide that it’s too hard to divide brownies into fifths, and respond that each person can have 1 brownie, and there will be 5 left over.)
**Division Practice**

1. Mr. Lee’s classroom has 966 markers. His 28 students need to share the markers equally so everyone can work on an art project. How many markers should each student get?

   **a** Solve the problem. Show your work.
   
   34 marker each; work will vary. \(966 \div 28 = 34 \text{ R}14\), so some students may note that half of Mr. Lee’s students could have 35 markers each.

   **b** Between which two whole numbers does your answer lie? **34** and **35**

   **c** Write an equation to represent the problem and the answer.
   
   \(966 \div 28 = 34 \text{ R}14\)

   **d** Explain what you did with the remainder, if any, and why.
   
   **Explanations will vary.**

2. Mr. Lee brought 70 granola bars for his students to share. Three students were absent. How many granola bars can each of his 25 students have?

   **a** Solve the problem. Show your work.
   
   2 \(\frac{2}{5}\) granola bars each; work will vary. (Some students may decide to leave the remainder of 20 bars as a whole number, arguing that it’s too hard to cut granola bars into fifths.)

   **b** Between which two whole numbers does your answer lie? **2** and **3**

   **c** Write an equation to represent the problem and the answer.
   
   \(70 \div 25 = 2 \frac{2}{5}\) or \(2 \text{ R}20\)

   **d** Explain what you did with the remainder, if any, and why.
   
   **Explanations will vary.**
**Exponents & Powers of Ten**

When you multiply 10s together, like $10 \times 10 \times 10$, the product is called a power of 10. You can use an exponent to show a power of 10. The exponent tells how many times to multiply 10 by itself.

1. Complete the chart below to show different powers of 10.

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Factor Form</th>
<th>Exponential Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10 × 10</td>
<td>$10^1$</td>
</tr>
<tr>
<td>100</td>
<td>$10 \times 10$</td>
<td>$10^2$</td>
</tr>
<tr>
<td>1,000</td>
<td>$10 \times 10 \times 10$</td>
<td>$10^3$</td>
</tr>
<tr>
<td>10,000</td>
<td>$10 \times 10 \times 10 \times 10$</td>
<td>$10^4$</td>
</tr>
<tr>
<td>100,000</td>
<td>$10 \times 10 \times 10 \times 10 \times 10$</td>
<td>$10^5$</td>
</tr>
<tr>
<td>1,000,000</td>
<td>$10 \times 10 \times 10 \times 10 \times 10 \times 10$</td>
<td>$10^6$</td>
</tr>
</tbody>
</table>

2. Multiply each whole number by powers of 10, using the steps shown in the example.

   **ex** $43 \times 10^2 = 43 \times (10 \times 10) = 43 \times 100 = 4,300$

   a. $79 \times 10^3 = 79,000$

   b. $105 \times 10^2 = 10,500$

   c. $4,568 \times 10^4 = 45,680,000$

   d. $17 \times 10^5 = 1,700,000$

3. Multiply each decimal by powers of 10, using the steps shown in the example.

   **ex** $5.8 \times 10^3 = 5.8 \times (10 \times 10 \times 10) = 5.8 \times 1,000 = 5,800$

   a. $4.7 \times 10^3 = 4,700$

   b. $0.68 \times 10^2 = 68$

   c. $5.16 \times 10^4 = 51,600$

   d. $12.63 \times 10^5 = 1,263,000$
Patterns in Multiplying by Powers of Ten  page 1 of 2

1. The post office sells 1¢ stamps. Fill out the table below to show how much it would cost to buy different quantities of 1¢ stamps.

<table>
<thead>
<tr>
<th>Number of Stamps</th>
<th>Decimal Equation</th>
<th>Fraction Equation</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 stamp</td>
<td>$1 \times 0.01 = 0.01$</td>
<td>$1 \times \frac{1}{100} = \frac{1}{100}$</td>
<td>$0.01$</td>
</tr>
<tr>
<td>2 stamps</td>
<td>$2 \times 0.01 = 0.02$</td>
<td>$2 \times \frac{1}{100} = \frac{2}{100}$</td>
<td>$0.02$</td>
</tr>
<tr>
<td>10 stamps</td>
<td>$10 \times 0.01 = 0.10$</td>
<td>$10 \times \frac{1}{100} = \frac{10}{100}$</td>
<td>$0.10$</td>
</tr>
<tr>
<td>20 stamps</td>
<td>$20 \times 0.01 = 0.20$</td>
<td>$20 \times \frac{1}{100} = \frac{20}{100}$</td>
<td>$0.20$</td>
</tr>
<tr>
<td>45 stamps</td>
<td>$45 \times 0.01 = 0.45$</td>
<td>$45 \times \frac{1}{100} = \frac{45}{100}$</td>
<td>$0.45$</td>
</tr>
<tr>
<td>321 stamps</td>
<td>$321 \times 0.01 = 3.21$</td>
<td>$321 \times \frac{1}{100} = \frac{321}{100}$</td>
<td>$3.21$</td>
</tr>
<tr>
<td>404 stamps</td>
<td>$404 \times 0.01 = 4.04$</td>
<td>$404 \times \frac{1}{100} = \frac{404}{100}$</td>
<td>$4.04$</td>
</tr>
</tbody>
</table>

a. What do you notice about multiplying by 0.01?

Responses will vary. Example: When you multiply a number by 0.01, the product is 100 times as small as the original number, because multiplying a number by one-hundredth is the same as dividing it by 100.

2. Amelia feeds her pet lizard crickets. The pet store sells crickets for 10¢ each. Fill out the table below to show how much it would cost to buy different quantities of crickets.

<table>
<thead>
<tr>
<th>Number of Crickets</th>
<th>Decimal Equation</th>
<th>Fraction Equation</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cricket</td>
<td>$1 \times 0.10 = 0.10$</td>
<td>$1 \times \frac{1}{10} = \frac{1}{10}$</td>
<td>$0.10$</td>
</tr>
<tr>
<td>2 crickets</td>
<td>$2 \times 0.10 = 0.20$</td>
<td>$2 \times \frac{1}{10} = \frac{2}{10}$</td>
<td>$0.20$</td>
</tr>
<tr>
<td>10 crickets</td>
<td>$10 \times 0.10 = 1.00$</td>
<td>$10 \times \frac{1}{10} = \frac{10}{10}$</td>
<td>$1.00$</td>
</tr>
<tr>
<td>20 crickets</td>
<td>$20 \times 0.10 = 2.00$</td>
<td>$20 \times \frac{1}{10} = \frac{20}{10}$</td>
<td>$2.00$</td>
</tr>
<tr>
<td>45 crickets</td>
<td>$45 \times 0.10 = 4.50$</td>
<td>$45 \times \frac{1}{10} = \frac{45}{10}$</td>
<td>$4.50$</td>
</tr>
<tr>
<td>321 crickets</td>
<td>$321 \times 0.10 = 32.10$</td>
<td>$321 \times \frac{1}{10} = \frac{321}{10}$</td>
<td>$32.10$</td>
</tr>
<tr>
<td>404 crickets</td>
<td>$404 \times 0.10 = 40.40$</td>
<td>$404 \times \frac{1}{10} = \frac{404}{10}$</td>
<td>$40.40$</td>
</tr>
</tbody>
</table>

a. What do you notice about multiplying by 0.10?

Responses will vary. Example: When you multiply a number by 0.10 (which is one tenth), the answer is 10 times as small as the original number, because you’re finding one-tenth of it.

(continued on next page)
Alfonso’s company sells T-shirts to soccer teams. Each T-shirt costs $10. Fill out the table below to show how much it would cost to buy different quantities of T-shirts.

<table>
<thead>
<tr>
<th>Number of Shirts</th>
<th>Equation</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 shirt</td>
<td>$1 \times 10 = 10$</td>
<td>$10$</td>
</tr>
<tr>
<td>2 shirts</td>
<td>$2 \times 10 = 20$</td>
<td>$20$</td>
</tr>
<tr>
<td>10 shirts</td>
<td>$10 \times 10 = 100$</td>
<td>$100$</td>
</tr>
<tr>
<td>20 shirts</td>
<td>$20 \times 10 = 200$</td>
<td>$200$</td>
</tr>
<tr>
<td>45 shirts</td>
<td>$45 \times 10 = 450$</td>
<td>$450$</td>
</tr>
<tr>
<td>321 shirts</td>
<td>$321 \times 10 = 3,210$</td>
<td>$3,210$</td>
</tr>
<tr>
<td>404 shirts</td>
<td>$404 \times 10 = 4,040$</td>
<td>$4,040$</td>
</tr>
</tbody>
</table>

What do you notice about multiplying by 10?

Responses will vary. Example: the product is 10 times as much as the number you multiplied by 10.
Multiplying by Powers of Ten Practice

106 × 0.01 = 1.06
47 × 0.01 = 0.47
3 × 0.01 = 0.03

0.6 × 0.01 = 0.006
0.32 × 0.01 = 0.0032
0.1 × 0.01 = 0.001

10 × 0.01 = 0.1

452 × 0.1 = 45.2
302 × 0.1 = 30.2
64 × 0.1 = 6.4

0.9 × 0.1 = 0.09
0.57 × 0.1 = 0.057
0.04 × 0.1 = 0.004

0.1 × 0.1 = 0.01

360 × 10 = 3,600
23 × 10 = 230
4 × 10 = 40

0.7 × 10 = 7
0.54 × 10 = 5.4
0.01 × 10 = 0.1

0.32 × 100 = 32
4.3 × 100 = 430
4 × 100 = 400

45 × 100 = 4,500
309 × 100 = 30,900
0.1 × 100 = 10

0.17 × 1,000 = 170
0.34 × 1,000 = 340
9.6 × 1,000 = 9,600

603 × 1,000 = 603,000
0.01 × 1,000 = 10
More Exponents & Powers of Ten

1. Multiply each number by powers of 10, using the steps shown in the example.
   - **ex** $29 \times 10^3 = 29 \times (10 \times 10 \times 10) = 29 \times 1,000 = 29,000$
   - **ex** $7.2 \times 10^3 = 7.2 \times (10 \times 10 \times 10) = 7.2 \times 1,000 = 7,200$
   - a $62 \times 10^3 = 62 \times (10 \times 10 \times 10) = 62 \times 1,000 = 62,000$
   - b $2,078 \times 10^2 = 2,078 \times (10 \times 10) = 2,078 \times 100 = 207,800$
   - c $47 \times 10^4 = 47 \times (10 \times 10 \times 10 \times 10) = 47 \times 10,000 = 470,000$
   - d $6.8 \times 10^3 = 6.8 \times (10 \times 10 \times 10) = 6.8 \times 1,000 = 6,800$
   - e $0.098 \times 10^2 = 0.098 \times (10 \times 10) = 0.098 \times 100 = 9.8$
   - f $26.75 \times 10^4 = 26.75 \times (10 \times 10 \times 10 \times 10) = 26.75 \times 10,000 = 267,500$

2. Sara solved the problems below, and got all the answers correct.
   - $76 \times 10^1 = 76$
   - $76 \times 10^2 = 7,600$
   - $76 \times 10^3 = 76,000$
   - $76 \times 10^4 = 760,000$
   - a Describe what is happening to the number of zeros in these problems.
     - Responses will vary.
   - b Explain why it works that way.
     - Explanations will vary.

In 2012, the estimated population of Chicago, Illinois, was 2,714,856. The chart below shows four different ways to write this number.

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>2,714,856</th>
</tr>
</thead>
<tbody>
<tr>
<td>Word Form</td>
<td>two million, seven hundred fourteen thousand, eight hundred fifty-six</td>
</tr>
<tr>
<td>Expanded Form</td>
<td>$(2 \times 1,000,000) + (7 \times 100,000) + (1 \times 10,000) + (4 \times 1,000) + (8 \times 100) + (5 \times 10) + (6 \times 1)$</td>
</tr>
<tr>
<td>Exponential Form</td>
<td>$(2 \times 10^6) + (7 \times 10^5) + (1 \times 10^4) + (4 \times 10^3) + (8 \times 10^2) + (5 \times 10) + (6 \times 10^0)$</td>
</tr>
</tbody>
</table>

3. Use the information shown above to help write the 2012 estimated population of Los Angeles, California, in word, expanded, and exponential form. (The standard form is written in for you.)

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>3,857,799</th>
</tr>
</thead>
<tbody>
<tr>
<td>Word Form</td>
<td>Three million, eight hundred fifty-seven thousand, seven hundred ninety-nine.</td>
</tr>
<tr>
<td>Expanded Form</td>
<td>$(3 \times 1,000,000) + (8 \times 100,000) + (5 \times 10,000) + (7 \times 1,000) + (7 \times 100) + (9 \times 10) + (9 \times 1)$</td>
</tr>
<tr>
<td>Exponential Form</td>
<td>$(3 \times 10^6) + (8 \times 10^5) + (5 \times 10^4) + (7 \times 10^3) + (7 \times 10^2) + (9 \times 10) + (9 \times 10^0)$</td>
</tr>
</tbody>
</table>
Patterns in Dividing by Powers of Ten  page 1 of 3

1 Alfonso’s company sells T-shirts to soccer teams. Each T-shirt costs $10.

   a If you spent $1,030, how many shirts could you buy?

      103 T-shirts

   b Fill out the table below to show how many T-shirts you could buy with different amounts of money.

<table>
<thead>
<tr>
<th>Total Cost</th>
<th>Equation</th>
<th>Number of Shirts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10</td>
<td>$10 ÷ 10 = 1</td>
<td>1</td>
</tr>
<tr>
<td>$20</td>
<td>$20 ÷ 10 = 2</td>
<td>2</td>
</tr>
<tr>
<td>$100</td>
<td>$100 ÷ 10 = 10</td>
<td>10</td>
</tr>
<tr>
<td>$200</td>
<td>$200 ÷ 10 = 20</td>
<td>20</td>
</tr>
<tr>
<td>$450</td>
<td>$450 ÷ 10 = 45</td>
<td>45</td>
</tr>
<tr>
<td>$3,210</td>
<td>$3,210 ÷ 10 = 321</td>
<td>321</td>
</tr>
<tr>
<td>$1,020</td>
<td>$1,020 ÷ 10 = 102</td>
<td>102</td>
</tr>
</tbody>
</table>

   c What do you notice about dividing by 10?

      Responses will vary. Example: The quotient is 10 times as small as the dividend.

(continued on next page)
Patterns in Dividing by Powers of Ten  page 2 of 3

2  Amelia feeds her pet lizard crickets. The pet store sells crickets for 10¢ each.

   a  If Amelia spent $1.30 on crickets last week, how many crickets did she buy?

      13 crickets

   b  Fill out the table below to show how much it would cost to buy different quantities of crickets.

<table>
<thead>
<tr>
<th>Total Cost</th>
<th>Decimal Equation</th>
<th>Fraction Equation</th>
<th>Number of Crickets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.10</td>
<td>$0.10 ÷ 0.10 = 1</td>
<td>$\frac{1}{10} ÷ \frac{1}{10} = 1$</td>
<td>1 cricket</td>
</tr>
<tr>
<td>$0.20</td>
<td>$0.20 ÷ 0.10 = 2</td>
<td>$\frac{2}{10} ÷ \frac{1}{10} = 2$</td>
<td>2 crickets</td>
</tr>
<tr>
<td>$1.00</td>
<td>$1.00 ÷ 0.10 = 10$</td>
<td>$\frac{10}{10} ÷ \frac{1}{10} = 10$</td>
<td>10 crickets</td>
</tr>
<tr>
<td>$2.00</td>
<td>$2.00 ÷ 0.10 = 20$</td>
<td>$\frac{20}{10} ÷ \frac{1}{10} = 20$</td>
<td>20 crickets</td>
</tr>
<tr>
<td>$3.30</td>
<td>$3.30 ÷ 0.10 = 33$</td>
<td>$\frac{33}{10} ÷ \frac{1}{10} = 33$</td>
<td>33 crickets</td>
</tr>
<tr>
<td>$5.20</td>
<td>$5.20 ÷ 0.10 = 52$</td>
<td>$\frac{52}{10} ÷ \frac{1}{10} = 52$</td>
<td>52 crickets</td>
</tr>
</tbody>
</table>

   c  What do you notice about dividing by 0.10?

      Responses will vary. Example: When you divide a number by 0.10 (Which is one-tenth), the quotient is 10 times greater than the dividend. This is because you're asking how many tenths there are in the number. Since there are 10 tenths in 1, dividing a number by $\frac{1}{10}$ (or 0.10) is like multiplying the number by 10.
3 The post office sells 1¢ stamps.

a If you spent $2.08, how many 1¢ stamps could you buy?  

208 stamps

b Fill out the table below to show how many stamps you could buy with different amounts of money.

<table>
<thead>
<tr>
<th>Total Cost</th>
<th>Decimal Equation</th>
<th>Fraction Equation</th>
<th>Number of Stamps</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.01</td>
<td>$0.01 ÷ 0.01 = 1</td>
<td>$\frac{1}{100} ÷ \frac{1}{100} = 1$</td>
<td>1 stamp</td>
</tr>
<tr>
<td>$0.02</td>
<td>$0.02 ÷ 0.01 = 2</td>
<td>$\frac{2}{100} ÷ \frac{1}{100} = 2$</td>
<td>2 stamps</td>
</tr>
<tr>
<td>$0.10</td>
<td>$0.10 ÷ 0.01 = 10$</td>
<td>$\frac{10}{100} ÷ \frac{1}{100} = 10$</td>
<td>10 stamps</td>
</tr>
<tr>
<td>$0.40</td>
<td>$0.40 ÷ 0.01 = 40$</td>
<td>$\frac{40}{100} ÷ \frac{1}{100} = 40$</td>
<td>40 stamps</td>
</tr>
<tr>
<td>$0.86</td>
<td>$0.86 ÷ 0.01 = 86$</td>
<td>$\frac{86}{100} ÷ \frac{1}{100} = 86$</td>
<td>86 stamps</td>
</tr>
<tr>
<td>$2.47</td>
<td>$2.47 ÷ 0.01 = 247$</td>
<td>$\frac{247}{100} ÷ \frac{1}{100} = 247$</td>
<td>247 stamps</td>
</tr>
<tr>
<td>$3.05</td>
<td>$3.05 ÷ 0.01 = 305$</td>
<td>$\frac{305}{100} ÷ \frac{1}{100} = 305$</td>
<td>305 stamps</td>
</tr>
</tbody>
</table>

c What do you notice about dividing by 0.01?

Responses will vary. Example: When you divide a number by 0.01 (or one hundredth), you are finding out how many hundredths there are in that number. Since there are 100 hundredths in 1, dividing by a hundredth is the same as multiplying by 100. So, when you divide a number by 0.01, the quotient is 100 times as much as the divided.
Dividing by Powers of Ten Practice

Complete the following equations.

3000 ÷ 1000 = \(3\)  
2504 ÷ 1000 = \(2.504\)

372 ÷ 1000 = \(0.372\)  
0.6 ÷ 1000 = \(0.0006\)

0.03 ÷ 1000 = \(0.00003\)

900 ÷ 100 = \(9\)  
406 ÷ 100 = \(4.06\)

7 ÷ 100 = \(0.07\)  
3.2 ÷ 100 = \(0.032\)

0.08 ÷ 100 = \(0.0008\)

405 ÷ 10 = \(40.5\)  
0.63 ÷ 10 = \(0.063\)

87 ÷ 0.1 = \(870\)  
6 ÷ 0.1 = \(60\)

0.5 ÷ 0.1 = \(5\)  
0.48 ÷ 0.1 = \(4.8\)

3 ÷ 0.01 = \(300\)  
6.9 ÷ 0.01 = \(690\)

0.8 ÷ 0.01 = \(80\)  
409 ÷ 0.01 = \(40,900\)
Multiplying & Dividing by Powers of Ten

1. Solve the multiplication problems below.
   \[34 \times 0.01 = 0.34\]  \[34 \times 0.10 = 3.4\]  \[34 \times 1 = 34\]
   \[34 \times 10 = 340\]  \[34 \times 100 = 3,400\]  \[34 \times 1,000 = 34,000\]

2. Solve the division problems below.
   \[34 \div 0.01 = 3,400\]  \[34 \div 0.10 = 340\]  \[34 \div 1 = 34\]
   \[34 \div 10 = 3.4\]  \[34 \div 100 = 0.34\]  \[34 \div 1,000 = 0.034\]

3. What patterns do you notice in the equations you completed above?
   Responses will vary.

4. Solve the multiplication and division problems below.
   \[62 \div 100 = 0.62\]  \[3.4 \times 1000 = 3,400\]  \[7.89 \div 0.10 = 78.90\]
   \[0.43 \times 100 = 43\]  \[0.08 \times 0.01 = 0.0008\]  \[123.05 \div 100 = 1.2305\]

5. Ramon bought erasers shaped like animals to give away at Family Night at his school. Each eraser costs $0.10. If he spent $25.60, how many erasers did he buy?
   a. Write a division equation to represent this situation.
      \[25.60 \div 0.10 = e\]
   b. Solve the problem using a strategy that makes sense to you. Show all your work.
      256 erasers; work will vary.
Using the Area Model to Multiply Decimal Numbers

1. A piece of paper measures 0.3 m by 0.65 m.
   a. Estimate the total area of the piece of paper.
      Estimates will vary.
   b. Make a labeled sketch of the piece of paper and use it to calculate an exact answer.
      \[0.195 \text{ m}^2\]
      \[0.3 \text{ m} \times 0.65 \text{ m} = 0.195 \text{ sq. m}\]

2. The city park measures 1.2 km by 0.63 km.
   a. Estimate the total area of the park.
      Estimates will vary.
   b. Make a labeled sketch of the park and use it to calculate an exact answer.
      \[0.756 \text{ km}^2\]
      \[1.2 \text{ km} \times 0.63 \text{ km} = 0.756 \text{ sq. km}\]

3. Use an algorithm or sketch arrays to find the products below.
   \[
   \begin{array}{c}
   1.6 \\
   \times 0.7 \\
   \hline
   1.12
   \end{array}
   \quad \text{Work will vary.}
   \quad
   \begin{array}{c}
   4.5 \\
   \times 2.3 \\
   \hline
   10.35
   \end{array}
   \quad \text{Work will vary.}
   \]
Multiplying Two Decimal Numbers

1. The memory card for Steve’s camera measures 0.82 inches by 1.25 inches.
   a. What do you estimate the total area of the memory card is?

   Estimates will vary.

   b. Find the exact area of the memory card. Show all your work. Fill in the array below if it helps you.

   \[ 0.82 \times 1.25 = 1.025 \text{ sq. inches} \]

   Work will vary.

   Example:

   \[ 0.800 \]
   \[ 0.160 \]
   \[ 0.040 \]
   \[ 0.020 \]
   \[ 0.004 \]
   \[ + 0.001 \]
   \[ 1.025 \]

2. What is the place value of the smallest unit of area in the array above?

   One-thousandth

2. Fill in an estimate and the exact answer for the problems below. Show your work.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40 \times 0.56 \quad 240 \quad \frac{+ 2000}{0.2240}</td>
<td>2.06 \times 1.42 \quad 412 \quad \frac{+ 740}{1.036}</td>
<td>3.7 \times 0.28 \quad \frac{296}{1.036}</td>
</tr>
<tr>
<td>Examples shown</td>
<td>Examples shown</td>
<td>Examples shown</td>
</tr>
</tbody>
</table>

   Exact Answer: 0.224  Exact Answer: 2.9252  Exact Answer: 1.036
More Decks

1 Andre, Raven, and their mom are building a deck in the backyard. Here is a sketch of their deck.

Deck Area = 16.75 sq. m

a Estimate the area of this deck in square meters. Then explain your estimate.

Estimates and explanations will vary. Example: about 17 or 18 sq. meters because 6.7 rounds to 7. Then 2 × 7 = 14, and 0.5 × 7 = 3.5, and that’s 17.5 sq. m.

b Label the area of each region on the sketch above. Then add the areas of all 4 regions to find the total area of the deck. Show your work beside the sketch.

c Use the partial products method and the standard multiplication algorithm to find the area of the deck.

<table>
<thead>
<tr>
<th>Partial Products</th>
<th>Standard Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.7 × 2.5</td>
<td>6.7 × 2.5</td>
</tr>
<tr>
<td>2 × 6 = 12</td>
<td>335</td>
</tr>
<tr>
<td>2 × 0.7 = 1.4</td>
<td>+ 1340</td>
</tr>
<tr>
<td>0.5 × 6 = 3.0</td>
<td>16.75</td>
</tr>
<tr>
<td>0.5 × 0.7 = 0.35</td>
<td></td>
</tr>
<tr>
<td>16.75</td>
<td></td>
</tr>
</tbody>
</table>
Decimals, Powers of Ten & Exponents

In our number system, the value of every place is a different power of 10. Powers of 10 can be represented using exponents, as shown in the chart here.

<table>
<thead>
<tr>
<th>Ten-Thousands</th>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10,000 = 10 \times 10 \times 10 \times 10$</td>
<td>$1,000 = 10 \times 10 \times 10$</td>
<td>$100 = 10 \times 10$</td>
<td>$10$</td>
</tr>
<tr>
<td>$10,000 = 10^4$</td>
<td>$1,000 = 10^3$</td>
<td>$100 = 10^2$</td>
<td>$10 = 10^1$</td>
</tr>
</tbody>
</table>

Places less than 1, such as tenths, hundredths, and thousandths are also powers of 10. These are represented using negative exponents, as shown here.

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.01</td>
<td>0.001</td>
</tr>
<tr>
<td>$1 = 10^0$</td>
<td>$0.1 = 10^{-1}$</td>
<td>$0.01 = 10^{-2}$</td>
<td>$0.001 = 10^{-3}$</td>
</tr>
</tbody>
</table>

Most people know that it takes 1 year, or 365 days, for the earth to make one entire trip, or orbit, around the sun. Scientists tell us that the amount of time it actually takes is 365.25 days. Here are four different ways to write the number 365.25.

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>365.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Word Form</td>
<td>three hundred sixty-five and twenty-five hundredths</td>
</tr>
<tr>
<td>Expanded Form</td>
<td>$(3 \times 100) + (6 \times 10) + (5 \times 1) + (2 \times 0.1) + (5 \times 0.01)$</td>
</tr>
<tr>
<td>Exponential Form</td>
<td>$(3 \times 10^2) + (6 \times 10^1) + (5 \times 10^0) + (2 \times 10^{-1}) + (5 \times 10^{-2})$</td>
</tr>
</tbody>
</table>

1. It takes Mars 686.971 days to orbit the sun one time. Write this number in word, expanded, and exponential form. (The standard form is written in already.)

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>686.971</th>
</tr>
</thead>
<tbody>
<tr>
<td>Word Form</td>
<td>six hundred eighty-six and nine hundred seventy-one thousandths</td>
</tr>
<tr>
<td>Expanded Form</td>
<td>$(6 \times 100) + (8 \times 10) + (6 \times 1) + (9 \times 0.01) + (7 \times 0.01) + (1 \times 0.001)$</td>
</tr>
<tr>
<td>Exponential Form</td>
<td>$(6 \times 10^2) + (8 \times 10^1) + (6 \times 10^0) + (6 \times 10^{-1}) + (6 \times 10^{-2}) + (6 \times 10^{-3})$</td>
</tr>
</tbody>
</table>

2. It takes the moon 27.322 days—a little less than a month—to orbit the earth one time. Write this number in word, expanded, and exponential form. (The standard form is written in already.)

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>27.322</th>
</tr>
</thead>
<tbody>
<tr>
<td>Word Form</td>
<td>twenty-seven and three hundred twenty-two thousandths</td>
</tr>
<tr>
<td>Expanded Form</td>
<td>$(2 \times 10) + (7 \times 1) + (3 \times 0.1) + (2 \times 0.01) + (2 \times 0.001)$</td>
</tr>
<tr>
<td>Exponential Form</td>
<td>$(2 \times 10^1) + (7 \times 10^0) + (3 \times 10^{-1}) + (2 \times 10^{-2}) + (2 \times 10^{-3})$</td>
</tr>
</tbody>
</table>
1. Kait spent $9.12 on 6 granola bars. How much did she pay for each one?

\[ \frac{9.12}{6} = 1.52 \]

Estimates will vary. Example:

\[
\begin{array}{c}
6) 9.12 \\
- 6.00 \\
\hline
3.12 \\
- 3.00 \\
\hline
0.12 \\
- 0.12 \\
\hline
0
\end{array}
\]

Work, including ratio table entries, will vary.

Example shown.

2. The cash register recorded $3.44 for 4 packages of markers. Kale was going to pay for one of the packages. How much does he owe?

\[ \frac{3.44}{4} = 0.86 \]

Estimates will vary. Example:

\[
\begin{array}{c}
4) 3.44 \\
- 2.00 \\
\hline
1.44 \\
- 1.00 \\
\hline
0.44 \\
- 0.44 \\
\hline
0
\end{array}
\]

Work, including ratio table entries, will vary.

Example shown.

(continued on next page)
3. A group of 12 girls went to see a new movie on its opening night. Altogether their entrance fees were $88.20. How much did each girl pay?

$88.20 \div 12 = \$7.35$

Estimates will vary.
Example:
$\$7.20$

Work, including ratio table entries, will vary.
Example shown.

<table>
<thead>
<tr>
<th>Number of Groups</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>12.00</td>
</tr>
<tr>
<td>10.00</td>
<td>120.00</td>
</tr>
<tr>
<td>5.00</td>
<td>60.00</td>
</tr>
<tr>
<td>2.00</td>
<td>24.00</td>
</tr>
<tr>
<td>0.50</td>
<td>6.00</td>
</tr>
<tr>
<td>0.25</td>
<td>3.00</td>
</tr>
<tr>
<td>0.10</td>
<td>1.20</td>
</tr>
</tbody>
</table>

4. Eight families decided to chip in to buy their team’s practice soccer balls. The bill was $103.92, which was half the regular price. How much should each family pay, if everyone pays a fair share?

$103.92 \div 8 = \$12.99$

Estimates will vary.
Example:
$\$12.50$

Work, including ratio table entries, will vary.
Example shown.

<table>
<thead>
<tr>
<th>Number of Groups</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>8.00</td>
</tr>
<tr>
<td>10.00</td>
<td>80.00</td>
</tr>
<tr>
<td>5.00</td>
<td>40.00</td>
</tr>
<tr>
<td>2.50</td>
<td>20.00</td>
</tr>
<tr>
<td>0.50</td>
<td>4.00</td>
</tr>
<tr>
<td>0.25</td>
<td>2.00</td>
</tr>
<tr>
<td>0.10</td>
<td>0.80</td>
</tr>
<tr>
<td>0.20</td>
<td>1.60</td>
</tr>
<tr>
<td>0.04</td>
<td>0.32</td>
</tr>
</tbody>
</table>
# Comparing & Multiplying Fractions & Decimals

1. Use one of the following symbols to make each expression below true.

   - > (greater than)
   - < (less than)
   - = (equal to)

<table>
<thead>
<tr>
<th>Expression 1</th>
<th>Expression 2</th>
<th>Expression 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{2}{4} &lt; \frac{3}{5}$</td>
<td>$\frac{11}{16} &lt; \frac{3}{4}$</td>
<td>$\frac{3}{4} &gt; \frac{3}{5}$</td>
</tr>
<tr>
<td>$0.34 &lt; \frac{1}{2}$</td>
<td>$0.58 &lt; \frac{3}{4}$</td>
<td>$0.50 = \frac{1}{2}$</td>
</tr>
<tr>
<td>$0.25 &lt; 0.256$</td>
<td>$0.103 &gt; 0.099$</td>
<td>$5.618 &lt; 5.621$</td>
</tr>
</tbody>
</table>

2. Solve these combinations using the strategies that make the most sense to you right now. Show your work.

<table>
<thead>
<tr>
<th>Expression 1</th>
<th>Expression 2</th>
<th>Expression 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.025 \times 12 = 0.3$</td>
<td>$56 \times 0.50 = 28$</td>
<td>$24 \times 0.25 = 6$</td>
</tr>
<tr>
<td>$0.25 \times 7 = 1.75$</td>
<td>$48 \times 0.25 = 12$</td>
<td>$36 \times 0.75 = 27$</td>
</tr>
<tr>
<td>$3 \times 0.25 = 0.75$</td>
<td>$0.25 \times 9 = 2.25$</td>
<td>$8 \times 0.50 = 4$</td>
</tr>
</tbody>
</table>
Using Models & Strategies

1. If school lunches cost $112.50 per quarter (9 weeks), about how much would each week of lunches cost?

\[ \frac{112.50}{9} = 12.50 \]

Estimates will vary. Example:
\[
\begin{array}{c|c}
9 & 112.50 \\
-90.00 & 22.50 \\
-18.00 & 4.50 \\
-4.50 & 0
\end{array}
\]

Work, including ratio table entries, will vary.
Example shown.

<table>
<thead>
<tr>
<th>Number of Groups</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>9.00</td>
</tr>
<tr>
<td>2.00</td>
<td>18.00</td>
</tr>
<tr>
<td>10.00</td>
<td>90.00</td>
</tr>
<tr>
<td>20.00</td>
<td>180.00</td>
</tr>
<tr>
<td>0.50</td>
<td>4.50</td>
</tr>
</tbody>
</table>

2. A fifth grader earned $94.00 gardening this month for a neighbor. If she worked 8 hours this month, then how much did she earn per hour?

\[ \frac{94.00}{8} = 11.75; \text{ work will vary.} \]

3. Marcy joined the school track team and ran a total of 231.80 miles in practice over 61 days. How many miles did she average per day?

\[ \frac{231.80}{61} = 3.8 \text{ miles per day; work will vary.} \]

4. A store owner had 7.11 lbs. of nuts left in the bin. If he divided the nuts evenly into 9 jars, how much did the nuts in each jar weigh?

\[ 7.11 \div 9 = 0.79 \text{ pounds; work will vary.} \]

5. There are 2.54 centimeters in one inch. How many centimeters are in 38.10 inches?

\[ 38.10 \div 2.54 = 15 \text{ cm; work will vary.} \]
Common Division Mistakes

Help the following students sort out their misunderstandings about division.

1. Su has to solve $8 \div \frac{1}{7}$. She says the answer is $\frac{8}{7}$. What would you tell Su about her thinking? What is $8 \div \frac{1}{7}$?

$8 \div \frac{1}{7} = 56$; advice to Su will vary. Example: It looks like you multiplied 8 by $\frac{1}{7}$ instead of dividing 8 by $\frac{1}{7}$. Dividing 8 by $\frac{1}{7}$ is like asking how many sevenths there are in 8. Since there are 7 sevenths in 1, you know there are 8 times 7, or 56.

2. Zane has to solve $\frac{1}{2} \div 4$. He says the answer is 2, because that's half of 4. What would you tell Zane about his thinking? What is $\frac{1}{2} \div 4$?

$\frac{1}{2} \div 4 = \frac{1}{8}$; advice to Zane will vary. Example: Maybe you don't understand that $\frac{1}{2} \div 4$ means you have to divide $\frac{1}{2}$ into 4 equal parts. Here's a picture, and you can see that the answer is $\frac{1}{8}$.

3. Irene wants to use equivalent ratios to solve $3,712 \div 64$, but she can't remember how. Show Irene what to do.

$3,712 \div 64 = 58$
Reviewing Decimal Addition & Subtraction

1. Complete the following addition problems.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3.034</td>
<td>4.067</td>
<td>1.437</td>
<td>7.63</td>
<td>4.803</td>
</tr>
<tr>
<td>+ 1.886</td>
<td>+ 3.290</td>
<td>+ 1.042</td>
<td>+ 4.592</td>
<td>+ 1.420</td>
</tr>
<tr>
<td>4.920</td>
<td>7.357</td>
<td>2.479</td>
<td>12.222</td>
<td>6.223</td>
</tr>
</tbody>
</table>

2.45 + 1.469 = 3.919
3.043 + 1.588 = 4.631

2. Complete the following subtraction problems.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3.046</td>
<td>2.405</td>
<td>3.437</td>
<td>5.26</td>
<td>4.513</td>
</tr>
<tr>
<td>- 1.273</td>
<td>- 0.512</td>
<td>- 2.106</td>
<td>- 3.40</td>
<td>- 1.382</td>
</tr>
<tr>
<td>1.773</td>
<td>1.893</td>
<td>1.331</td>
<td>1.86</td>
<td>3.131</td>
</tr>
</tbody>
</table>

5.604 – 3.025 = 2.579
6.045 – 2.039 = 4.006

3. Circle the pairs of numbers whose sums are greater than 2.

1.26 + 0.773
1.255 + 0.094
1.53 + 0.458
1.502 + 0.6
**Temperature Conversions**

The formulas for converting between Fahrenheit and Celsius are as follows:

\[
F = (C \times \frac{9}{5}) + 32
\]

\[
C = (F - 32) \times \frac{5}{9}
\]

1. Use the formulas to answer the following questions.

   a. A cold wintery day is about 35°F. What is that temperature in degrees Celsius?
      \[1 \frac{6}{9}° C\] or \[1 \frac{2}{3}° C\] or \[1.66° C\]. Work will vary.

   b. The temperature of a dog is about 101°F. What is the dog’s temperature in degrees Celsius?
      \[38 \frac{3}{9}° C\] or \[38 \frac{1}{3}° C\] or \[38.33° C\]. Work will vary.

2. The temperature in the Sahara Desert can reach 78°C. What is that temperature in degrees Fahrenheit?
   \[172 \frac{2}{5}° F\] or \[172.4° F\]. Work will vary.

3. Write two of your own temperature conversion questions and share them with another student. Make sure you find the answers before you share.
   Questions will vary.
Absorbing & Reflecting Solar Energy  page 1 of 3

When solar energy strikes objects, some of the energy is absorbed and converted into heat and some is reflected. Some objects absorb more radiant energy than others.

1. Predict what will happen when you place one thermometer on black paper and another thermometer on white paper and leave them in the sun for 5 minutes. Predictions will vary.

Prepare to test your prediction.
- Cut a 1” × 3” strip from black paper.
- Cut a 1” × 3” strip from white paper.
- Wrap the black strip of paper around the bottom of a thermometer and tape it together on the back.
- Repeat with the white strip of paper on another thermometer.
- Save the remaining pieces of paper. They will also be used in the experiment.

(continued on next page)
Now you’ll test your prediction.

- Go outside and place the white-wrapped thermometer in the center of the white paper and the black-wrapped thermometer in the center of the black paper.
- On the table below, record the starting temperature. Then, read the temperature on each thermometer every minute for five minutes and record your readings on the table.
- Give your data table a title.

**Data and title will vary.**

<table>
<thead>
<tr>
<th>Thermometer on Black</th>
<th>Thermometer on White</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start Temperature</td>
<td></td>
</tr>
<tr>
<td>1 minute</td>
<td></td>
</tr>
<tr>
<td>2 minutes</td>
<td></td>
</tr>
<tr>
<td>3 minutes</td>
<td></td>
</tr>
<tr>
<td>4 minutes</td>
<td></td>
</tr>
<tr>
<td>5 minutes</td>
<td></td>
</tr>
<tr>
<td>Change in Temperature from Start to Finish</td>
<td></td>
</tr>
</tbody>
</table>

2. What do you notice about the data? Write at least three observations. Use your math journal if you need more room.

**Observations will vary.**
Absorbing & Reflecting Solar Energy page 3 of 3

Plot the data you collected on the line graph (see your teacher if you do not yet have a sheet for the line graph).

- Give the graph a title and label each axis.
- Then use your graph to help answer the questions below.

**Line graphs and responses to these prompts will vary according to student data and observations.**

3 Explain how to find the change in temperature by reading the graph.

4 During which 1-minute period did the temperature rise the most for each thermometer? How can you tell by looking at your graph?

5 Describe at least two things you learned about the reflection and absorption of solar energy by conducting today’s experiment.

6 **CHALLENGE** Predict the temperature of the thermometer on the black paper after 15 minutes and explain your reasoning.
Reading a Line Graph

One pair of students in Mr. Ivy’s class collected the data in the graph below.

1. What was the change in temperature for the black-covered thermometer after 5 minutes? \(17^\circ\) increase \((83 - 66 = 17)\)

2. What was the difference between the black and the white-covered thermometers’ ending temperatures? \(11^\circ\) difference \((83 - 72 = 11)\)

3. During which 1-minute period did the temperature rise the most for the black-covered thermometer? From 0–1 minute (the first minute)

4. Mr. Ivy’s class covered a thermometer in foil. They collected the change in temperature at the same time they collected data for the white and black-covered thermometers.
   
   a. Predict what the change in temperature for the foil-covered thermometer would look like. Show your prediction on the graph at the top of the page.

   b. Explain the reasoning for your prediction.

      Predictions and explanations will vary.
Today teams will set up thermometers just like in the last session, except you will concentrate solar energy on two of the thermometers, one wrapped in black and one wrapped in white. The other thermometers will not have concentrators.

1. Predict what will happen to each of the four thermometers when they are placed in the sun. **Predictions will vary.**

2. Follow these directions with your partner to make a solar concentrator.
   - Get 1 piece of card stock and 1 piece of aluminum foil from your teacher.
   - Fold the card stock into thirds, and then open it again.
   - Wrap the edges of the foil around the card stock and tape it to the back.
   - You now have a solar concentrator!

3. Go outside and work with another pair to conduct the experiment.
   - Place your two white-wrapped thermometers in the center of the white pieces of paper and your two black-wrapped thermometers in the center of the black pieces of paper.
   - Position a concentrator behind one of the black thermometers and the other concentrator behind one of the white thermometers, so each reflects and concentrates the sun directly onto each thermometer.

(continued on next page)
4 Measure and record the starting temperature of each thermometer. Continue to measure and record the temperature every minute for 5 minutes.

<table>
<thead>
<tr>
<th>Start Temperature</th>
<th>Black with Concentrator</th>
<th>White with Concentrator</th>
<th>Black No Concentrator</th>
<th>White No Concentrator</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 minute</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 minutes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 minutes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 minutes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 minutes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Change in Temperature from Start to Finish |                         |                         |                       |                       |

5 Plot the data you collected on the graph below. Give the graph a title and label each axis. **Graphs will vary.**
6. What do you notice about the data? Write at least two observations.

7. What is the difference in temperature after 5 minutes between the thermometer that absorbed the most solar energy and the one that absorbed the least?

8. Look at your graph. During which 1-minute period did the temperature rise the most for each thermometer?

9. Describe at least two things you learned about concentrating solar energy by conducting today’s experiment.

10. **Challenge** Pose some of your own questions for your classmates to answer.
**Solar Concentration**

A team of students in Mr. Ivy’s class concentrated solar energy on thermometers and collected data. The graph below shows the data the students collected.

### Concentrating Solar Energy

**KEY**
- blue: black with concentrator
- red: white with concentrator
- green: black, no concentrator
- purple: white, no concentrator

1. What was the change in temperature from start to finish for the white thermometer with the concentrator? **Increase of 14° (81 – 67 = 14)**

2. What is the difference in temperature after 5 minutes between the thermometer that absorbed the most solar energy and the one that absorbed the least? **25° (95 – 70 = 25)**

3. How long did the white thermometer without the concentrator remain at a constant temperature? **2 ½ (2.5) minutes (5 – 2.5 = 2.5)**

4. During which half-minute period did the temperature rise the most for each thermometer?
   - **White, no concentrator:** 1–1.5 minutes
   - **Black, no concentrator:** 3.5–4 minutes
   - **White with concentrator:** 3.5–4 minutes
   - **Black with concentrator:** 4.5–5 minutes

5. Did the concentrator make a difference in the amount of solar energy that was absorbed by the thermometers? Explain your reasoning.
   **Yes. Explanations will vary.**
Solar collectors absorb solar energy, convert it into heat and then hold the heat, rather than letting it escape rapidly back into the atmosphere. During today’s experiment, you will explore some conditions that best hold the heat.

1. Follow these directions to set up the experiment.
   - Cut out four trays, two from white paper and two from black paper, that each have a base of 3” by 3” and height of 1”. Use clear tape to tape the edges.
   - Get four plastic cups.
   - Add $\frac{1}{3}$ cup of water to each cup.
   - Place a thermometer in each cup.
   - Cover two cups with plastic wrap held in place with rubber bands.
   - Place one covered cup in a white tray and one uncovered cup in a white tray. Place one covered cup in a black tray and one uncovered cup in a black tray.
   - When your teachers directs you to, record the temperature of the water in each cup. Then go outside and place the cups and trays in a sunny place so that the sun is directly over them and there are no shadows covering them.
   - Leave the cups in the sun for 30 minutes and then find and record the temperature of each cup of water.

(continued on next page)
Collecting Solar Energy page 2 of 3

2  Predict what will happen. Explain your reasoning.
   **Predictions and explanations will vary.**

3  In the following table, record the starting temperature of the water in each cup before you go outside. Then record the water temperature after the cups have been in the sun for 30 minutes. Calculate the change in temperature for each cup of water. When you are finished, give your table a title.

   **Data and titles will vary.**

<table>
<thead>
<tr>
<th></th>
<th>Black, with Cover</th>
<th>White, with Cover</th>
<th>Black, No Cover</th>
<th>White, No Cover</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start Temperature</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temperature after 30 minutes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in Temperature from Start to Finish</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4 Look at your graph. What do you notice about the data?

5 What is the difference in temperature between the collector that absorbed the most solar energy and the one that absorbed the least?

6 Describe at least two things you learned about collecting and storing solar energy by conducting today’s experiment.

7 **CHALLENGE** What could you change in these solar collectors to make them absorb more solar energy? Explain your reasoning.
Solar Collection

A pair of students in Mr. Ivy’s class collected the data in the table below.

<table>
<thead>
<tr>
<th>Solar Collection Data</th>
<th>Black, with Cover</th>
<th>White, with Cover</th>
<th>Black, No Cover</th>
<th>White, No Cover</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 min.</td>
<td>58° F 14.4° C</td>
<td>58° F 14.4° C</td>
<td>58° F 14.4° C</td>
<td>58° F 14.4° C</td>
</tr>
<tr>
<td>15 min.</td>
<td>64° F</td>
<td>62° F</td>
<td>60° F</td>
<td>59° F</td>
</tr>
<tr>
<td>30 min.</td>
<td>71° F</td>
<td>68° F</td>
<td>65° F</td>
<td>60° F</td>
</tr>
<tr>
<td>45 min.</td>
<td>78° F</td>
<td>73° F</td>
<td>69° F</td>
<td>60° F</td>
</tr>
<tr>
<td>60 min.</td>
<td>86° F 30° C</td>
<td>78° F 25.6° C</td>
<td>74° F 23.33° C</td>
<td>60° F 15.6° C</td>
</tr>
</tbody>
</table>

1. Create a graph to show their data. Be careful to set up a scale along the vertical axis that goes from the lowest to the highest temperature they recorded. Remember to title and label the graph. **Graph titles, labels, and scales will vary. Example shown.**

![Solar Collection Graph](image)

2. What do you notice about the data these students collected? Write at least two observations. **Observations will vary.**

3. What is the difference in temperature between the collector that absorbed the most solar energy after 60 minutes and the one that absorbed the least? **26° (86 – 60 = 26)**

4. During which 15-minute period did the temperature rise the most for each cup of water? **Black w/cover: 45–60**  **White w/cover: 15–30**  **Black, no cover: 15–30 & 45–60**  **White, no cover: 0–15 & 15–30**

5. **CHALLENGE** Convert the starting temperature and the temperature of each cup after 60 minutes to degrees Celsius. Use the formula $C = (F – 32) \times \frac{5}{9}$.

See table above. Decimal values are shown, but students may use fractions.
Solar Boxes page 1 of 2

1. Write your prediction for which of the boxes with a volume of 24 cubic inches will collect the most solar energy in 20 minutes. Explain your reasoning.

   Predictions and explanations will vary.

2. In the following table, record the starting temperature of each box and then its temperature after 20 minutes of being in the sun. Calculate the change in temperature for each box. Remember to give your table a title.

   Data and title will vary.

<table>
<thead>
<tr>
<th>Box Dimensions</th>
<th>Start Temperature</th>
<th>Temperature after 20 minutes</th>
<th>Change in Temperature from Start to Finish</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(continued on next page)
Find the average (mean) temperature change for each box placed in the sun. Use the data from the chart your class created. Show your work.

<table>
<thead>
<tr>
<th>Box Dimensions (in inches)</th>
<th>Average (Mean) Temperature Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 6 x 4</td>
<td></td>
</tr>
<tr>
<td>1 x 2 x 12</td>
<td></td>
</tr>
<tr>
<td>1 x 3 x 8</td>
<td></td>
</tr>
<tr>
<td>2 x 4 x 3</td>
<td></td>
</tr>
<tr>
<td>2 x 2 x 6</td>
<td></td>
</tr>
<tr>
<td>1 x 1 x 24</td>
<td></td>
</tr>
</tbody>
</table>

Which box had the greatest average change in temperature? Why do you think it worked out that way?

If we had a collector with a volume of 48 cubic inches, which box dimensions should we expose to the sun? Explain your reasoning.
More Solar Boxes

Mr. Ivy’s class made solar collector boxes with a volume of 40 cubic inches, then gathered solar collection data for 10 minutes. Their data is in the table below.

<table>
<thead>
<tr>
<th>Box Dimensions</th>
<th>Team Data (Change in Temperature, in Degrees F)</th>
<th>Average (Mean) Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Team 1</td>
<td>Team 2</td>
</tr>
<tr>
<td>1 × 1 × 40</td>
<td>22</td>
<td>20</td>
</tr>
<tr>
<td>1 × 2 × 20</td>
<td>19</td>
<td>18</td>
</tr>
<tr>
<td>1 × 4 × 10</td>
<td>21</td>
<td>19</td>
</tr>
<tr>
<td>1 × 5 × 8</td>
<td>18</td>
<td>17</td>
</tr>
<tr>
<td>2 × 2 × 10</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>2 × 4 × 5</td>
<td>12</td>
<td>15</td>
</tr>
</tbody>
</table>

1. Calculate the mean temperature change for each collector, and record it in last column.
   Reminder: To find the mean temperature change for each box, add the 5 data entries in the row, and divide the total by 5. Do the work in your math journal.

2. The temperature of the collectors at the start of the experiment was 70° F. Create a graph to show the mean temperature change for each box. Give the graph a title and label the axes. Use different colored pencils to represent each collector.

   **Solar Box Graph**

3. What is the difference in mean average temperature change between the collector that absorbed the most solar energy and the one that absorbed the least?
   7 (20.2 – 13.2 = 7)
Changing Surface Area

1 Mr. Ivy’s class conducted an experiment to see whether changing the surface area of black paper exposed to the sun would make a difference in the amount of solar energy the paper absorbed. They set up the experiment like this:

They placed the pieces of black paper and thermometers in the sun and read their temperatures after 5, 10, and 15 minutes. The data they collected is in the table below.

<table>
<thead>
<tr>
<th>Surface Area (in square inches)</th>
<th>Start Temp.</th>
<th>5 min.</th>
<th>10 min.</th>
<th>15 min.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>68</td>
<td>70</td>
<td>74</td>
<td>78</td>
</tr>
<tr>
<td>$4 \times 4 = 16$</td>
<td>68</td>
<td>72</td>
<td>78</td>
<td>84</td>
</tr>
<tr>
<td>$4 \times 8 = 32$</td>
<td>68</td>
<td>75</td>
<td>82</td>
<td>90</td>
</tr>
<tr>
<td>$8 \times 8 = 64$</td>
<td>68</td>
<td>79</td>
<td>91</td>
<td>100</td>
</tr>
</tbody>
</table>

a Make a graph of their data. Give the graph a title and label the axes.

**Surface Area Experiment**

b Maya says that when they made the surface area four times bigger, the paper absorbed twice as much solar energy. Look at the data. Is she correct? Explain your reasoning in your math journal. Yes; explanations will vary.

(The change in temperature for the 16 sq. in. paper was 16°. The change in temperature for the 64 sq. in. paper was twice that: 32°.)

2 What do you predict the temperature of a thermometer placed under a $6'' \times 8''$ piece of black paper exposed to the sun would read after 15 minutes? Explain your reasoning. Predictions and explanations will vary.
Earth Materials page 1 of 2

1 What do you predict will happen to the temperature of earth materials when you place cups of four different earth materials in the sun and then move them to the shade? Explain your reasoning.

Predictions and explanations will vary.

2 Follow these directions to test your prediction.
   • Each team member choose a different earth material to test—dry soil, wet soil, water, or rocks.
   • Get a plastic cup and mark a line on the cup 2 inches from the bottom with a marking pen.
   • Place a thermometer in the cup and then carefully fill your cup to the line with the material you chose. If you are testing wet soil, pour \( \frac{1}{4} \) cup water into the cup, add the dry soil to the line, and stir before adding the thermometer.

3 When you go outside, place the cups close together, but do not let them cast a shadow on each other. In the table on the next page, record the starting time and temperature of all of your team's cups. Then return inside to start creating a temperature/time graph.
4 Return outside in 20 minutes and record the temperature of your earth material. Then transfer the cup to the shade, take another temperature reading, and record it in the table.

5 Take a temperature reading every three minutes in the shade and record it in the table. While you are waiting, record the temperatures of all of your team’s earth materials.

Data and table titles will vary.

<table>
<thead>
<tr>
<th>Time</th>
<th>Dry Soil</th>
<th>Wet Soil</th>
<th>Water</th>
<th>Rocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>38</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>41</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>44</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6 What do you notice about the temperature of the earth materials as they sit in the shade?

Observations will vary. (All of the materials should cool over time in the shade.)
Earth Materials in Boxes

The students in Ms. Vega’s class wanted to use boxes without lids instead of cups to test how earth materials collect and store solar energy. They decided to build all the different boxes with a volume of 96 cubic centimeters.

1 What are the dimensions of all the boxes the students made? List all of the dimensions on the left side of the table below.

   For this task, list each set of dimensions only one time, regardless of order. For example, 2 × 4 × 12 would be considered the same as 12 × 2 × 4 or 4 × 12 × 2.

   All possible boxes with a volume of 96 are included below. Order of dimensions may vary.

2 The students filled each box with an earth material. They placed each box in the sun so that the side with the greatest surface area was most exposed to the sun. For each box, list the surface area of the side that the students chose to expose in the table below.

3 Predict what happened to the temperature when the students placed dry soil in each box. Explain your reasoning.

   Predictions and explanations will vary.

4 Predict what happened when they placed water in each box. Explain your reasoning.

   Predictions and explanations will vary.
Earth Materials Questions  Responses will vary.
Use your Earth Materials Graph to help answer these questions.

1. Look at the graph. What do you notice about the data? Write at least two observations.

2. Which material collected the most solar energy? How do you know?

3. Which material collected the least solar energy? How do you know?

4. Which material lost the most heat? How do you know?

5. Which material stored the heat the longest? How do you know?

6. Describe at least two things you learned about collecting and storing solar energy in earth materials by conducting this experiment.
Mr. Ivy’s class conducted an experiment to test the collection and storage of solar energy in earth materials. They left the materials in the sun for 20 minutes, then moved them to the shade and took temperature readings every 3 minutes. The graph below shows their results.

1. What was the temperature change in the shade for dry soil?
   18° decrease (100 – 82 = 18)

2. Which material collected the least solar energy? How do you know?
   Water. Explanations will vary.

3. Which material lost the most heat? How do you know?
   Dry soil. Explanations will vary.

4. How does sand compare to the other earth materials the class tested?
   Responses will vary. Possible observations include:
   • The sand collected more solar energy than the water, but less than dry soil or rocks.
   • The sand lost more heat than the water, but less than the dry soil or rocks.
Students in Mr. Ivy’s class wanted to conduct an experiment about the effect of the size and placement of windows on the amount of heat a house collected. They remembered reading that passive solar houses have large windows facing south to allow sunlight to enter in the winter and that awnings can protect the south facing windows from the sun when it is high in the sky in the summer. The class made model houses with different-size windows. They placed the large window on one house toward the sun. On another house, they placed the small window toward the sun. On a third house, they placed an awning over the large window and faced it toward the sun. On a fourth house, they added a solar water heater on the floor of the house, faced the large window toward the sun, and let the sun shine directly onto the solar water heater.

They left the houses in the sun for two hours and took temperature readings every half-hour. Then they removed all of the houses from the sun and continued to take temperature readings. The class then created a graph of their data:
Window Orientation page 2 of 2

1. What do you notice about the data? List at least three observations.
   
   Observations will vary. Possibilities include:
   - House D, with the large window and the solar water heater, heated up the most.
   - House B, with the small window, collected the least heat.
   - House A, with the large window, heated up most quickly, but didn't gain much more after the first 30 minutes and lost the most heat the most quickly.

2. Which house is best for the summer? Explain your reasoning.
   - House B or House C. Explanations will vary. Example: If you don't mind having a small window, House B would be the best, because it heated up the least. House C has a large window with an awning and it didn't collect much more heat than House B; the amount they collected was almost equal after 2 hours.

3. Which house is best for the winter? Explain your reasoning.
   - House D. Explanations will vary. Example: House D heats up the most, even though it takes more than an hour longer to get warmer than House A. It also holds heat better than the others. After an hour and a half out of the sun, it was still about 98°, while House A dropped from 108° all the way down to 69°.
Session 3

Making Windows

Solve each problem. Show your work using words, numbers, or labeled sketches.

1. Mr. Ivy’s class made model houses with the dimensions of 11” wide by 10” long by 8” tall. What is the volume of one of their model houses?
   
   **880 cubic inches. Work will vary.**

2. What is the combined surface area of the four walls?
   
   **336 sq. inches. Work will vary. Example:**
   
   \[(2 \times (10 \times 8)) + (2 \times (11 \times 8)) =
   \]
   
   \[(2 \times 80) + (2 \times 88) = 160 + 176 = 336\]

3. The students need to cut out windows that will take up \(\frac{1}{6}\) of the surface area of the four walls. How many square inches of windows do they need to cut out?
   
   **They need to cut out 56 square inches of windows. Work will vary.**

4. Draw a sketch of each of the four walls with the windows cut out. Label the dimensions of each of the windows.
   
   **Sketches will vary; students will choose differing ways to assign window space to the faces of the model. Example:**

   ![Sketches of the four walls with windows cut out]
Insulation Materials page 1 of 3

1 Which insulator do you think will keep a cup of warm water warmest the longest? Explain your reasoning.

Predictions and explanations will vary.

2 Follow these directions to test your prediction.
   - Each team member choose a different insulation material to test.
     One person will be in charge of the control cup and will not use insulation material.
   - Surround your cup with your insulation material.
   - Fill your cup with $\frac{1}{3}$ cup hot water.
   - Place a thermometer in the cup and record the temperature.
   - Seal the cup with plastic wrap and a rubber band.
   - Place a thermometer in one uninsulated, uncovered cup to act as the control.
   - Record the temperature of your cup every four minutes in the table that follows.
   - Get the data from your other team members after each temperature reading and record it in the table, too.
   - Stop recording temperatures after 16 minutes.
Record your data in the table below. Enter the data for the other insulation materials next to your data.

<table>
<thead>
<tr>
<th>Materials</th>
<th>Starting Temperature</th>
<th>Temperature at 4 minutes</th>
<th>Temperature at 8 minutes</th>
<th>Temperature at 12 minutes</th>
<th>Temperature at 16 minutes</th>
<th>Change in Temperature</th>
</tr>
</thead>
</table>

Plot the data your team collected on a graph. Give the graph a title and label each axis.
5. What do you notice about the data? Record at least two observations.

6. How does the temperature change over time?

7. Which was the best insulator and which was the worst? How do you know?

8. **Challenge**: Using the same materials, how could you make an even better insulator?
### Amount of Insulation Materials

1. The students in Ms. Vega’s class wanted to test the insulating qualities of different materials using boxes instead of cups. They built boxes, open at the top, with walls that were 3” tall by $5 \frac{1}{4}$” wide.

   **a** What is the surface area of one of the four walls? Show your work.
   
   $15 \frac{3}{4}$ square inches. Work will vary. Example:
   
   $3 \times 5 = 15$
   
   $3 \times \frac{1}{4} = \frac{3}{4}$
   
   $15 + \frac{3}{4} = 15 \frac{3}{4}$
   
   so
   
   $3 \times 5 \frac{1}{4} = 15 \frac{3}{4}$

   **b** What is the surface area of the bottom of the box? Show your work.
   
   $27 \frac{9}{16}$ square inches. Work will vary. Example:
   
   $5 \times 5 = 25$
   
   $5 \times \frac{1}{4} = \frac{5}{4}$
   
   $25 + \frac{5}{4} + \frac{5}{4} + \frac{1}{16} = 27 + \frac{9}{16} + \frac{9}{16} + \frac{1}{16} = 27 \frac{9}{16}$

   **c** The floor and all four walls need to be insulated. What is the total surface area the class needs to cover with insulation materials? Show your work.
   
   $90 \frac{9}{16}$ square inches. Work will vary. Example:
   
   $(4 \times 15 \frac{3}{4}) + 27 \frac{9}{16} = 63 + 27 \frac{9}{16} = 90 \frac{9}{16}$
   
   $(4 \text{ walls}) \quad (\text{floor})$

2. The class has styrofoam that comes in sheets with the dimensions of 9.5” × 9.5”. Is one sheet enough to cover one box? If it is, draw a sketch showing how. If it isn’t, explain why.

   **No.** Explanations (and sketches, for students who claim that it’s enough material) will vary.
   
   $9.5 \times 9.5 = 90.25$ sq. inches, or $90 \frac{1}{4}$ square inches, which is less than $90 \frac{9}{16}$ inches.
**Insulating Our House  Students’ plans will vary.**

1. Follow these requirements to insulate your house.
   - You can insulate your door, but it must open and close.
   - You may not cover your windows with anything but curtains or storm windows.
   - Insulation on the floor and walls cannot exceed $\frac{1}{2}$ inch in thickness. The ceiling cannot exceed 1 inch.

2. Examine the insulation materials available and their cost.
   - You may not buy more than $4.00 of insulation materials.
   - As a team, determine the type and amount of the materials you want to use.
   - Use scratch paper to draw sketches to help you determine the amount of each material you need.
   - Record the amount and cost of each insulation material on the cost sheet below.
   - Calculate the total cost for all the materials and write it below the table.
   - Check your calculations to make sure you have enough of every material you need.
   - When you have completed the cost sheet, send a team member to buy the materials from your teacher.

### Insulation Costs

<table>
<thead>
<tr>
<th>Insulation Material</th>
<th>Cost per Unit</th>
<th>Amount Needed</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(piece, sheet, etc.)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Total Cost of All Materials**
Buying Window Materials

The students in Mr. Ivy’s class plan to buy some insulation materials for their model houses. The costs are listed in the table below.

<table>
<thead>
<tr>
<th>Insulation Material</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>weatherstripping (electrician's tape)</td>
<td>½ yard @ $0.25</td>
</tr>
<tr>
<td>storm windows (transparency film)</td>
<td>8.5” × 11” @ $0.50 each</td>
</tr>
<tr>
<td>newspaper</td>
<td>1 sheet @ $0.20</td>
</tr>
<tr>
<td>felt</td>
<td>8.5” × 5.5” @ $0.35 each</td>
</tr>
<tr>
<td>polyester-blend fabric (curtains)</td>
<td>$0.40 per 42 square inches</td>
</tr>
<tr>
<td>masking tape</td>
<td>1 roll @ $0.40</td>
</tr>
<tr>
<td>caulking (tacky glue)</td>
<td>2 feet @ $0.25</td>
</tr>
</tbody>
</table>

1 Ramon’s team wants to use weatherstripping. How many inches of weatherstripping will one window of their house get if two windows share ½ yard equally? Show your work.

9” of weatherstripping. (½ yard = 18 inches; 18 ÷ 2 = 9)

2 Suki’s team’s house has 12 windows, each with a perimeter of 9 inches. How many yards of weatherstripping do they need to buy? Show your work.

3 yards. Work will vary.

3 Sarah’s team needs ¼ foot of caulking per small window. Sarah says that 2 feet is enough for their 4 small windows. Is she correct? Why or why not?

Yes. Explanations will vary. (¼ × 4 = 4/3 = 1 1/3 feet, so 2 feet is more than enough.)

4 CHALLENGE Sarah’s team wants to make curtains for their windows. Their windows have 56 square inches of total surface area. How much do they need to spend on curtain material? How much material will be left? Use pictures, numbers, and words to show your answer.

Expenditure: $0.80 if they buy in units shown in the chart. (Answers may vary if students suggest that the team buy a fraction of a piece of fabric. See example.) Material left over: 28 sq. inches if they buy in units shown in the chart (and cover each window exactly).
In this experiment, your team will fill the container with hot water and place it inside your insulated house. The container of hot water will act as a furnace to heat the house.

1. What do you think will happen to the temperature of your house when you add the hot water furnace and let it sit for two hours?

   **Predictions will vary.**

2. Follow these directions to test your prediction.
   - Cut a small hole in your roof just large enough to slide one of your thermometers through.
   - Align the thermometer at the 0°F mark.
   - Fill a container with two cups of hot water.
   - Use your second thermometer to find the temperature of the water, and record it in the table on the next page.
   - Remove the second thermometer from the container of water.
   - Carefully place the container of water in the middle of your insulated house.
   - Place the roof on top of your house.
   - Record the temperature inside your house in the table on the next page.
   - Measure and record the temperature inside your house every 15 minutes.
Record your data in the table below. **Data and titles will vary.**

<table>
<thead>
<tr>
<th>Minutes Passed</th>
<th>Actual Clock Time</th>
<th>House Temperature (in Degrees Fahrenheit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 Minutes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 Minutes</td>
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<td></td>
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<tr>
<td>30 Minutes</td>
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<tr>
<td>45 Minutes</td>
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<tr>
<td>60 Minutes</td>
<td></td>
<td></td>
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<td>75 Minutes</td>
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<td>90 Minutes</td>
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<tr>
<td>105 Minutes</td>
<td></td>
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</tr>
<tr>
<td>120 Minutes</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Based on the data you've collected in the first 30 minutes, do you want to change your prediction? Explain your reasoning.

**Choices, revised predictions, and explanations will vary.**
### Insulation Experiment

1. Mr. Ivy’s class tested the insulation efficiency of their model houses. The class data is in the table below.

<table>
<thead>
<tr>
<th>Time</th>
<th>Team 1</th>
<th>Team 2</th>
<th>Team 3</th>
<th>Team 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>66</td>
<td>66</td>
<td>67</td>
<td>66</td>
</tr>
<tr>
<td>15</td>
<td>82</td>
<td>81</td>
<td>83</td>
<td>82</td>
</tr>
<tr>
<td>30</td>
<td>82</td>
<td>81</td>
<td>84</td>
<td>81</td>
</tr>
<tr>
<td>45</td>
<td>81</td>
<td>81</td>
<td>84</td>
<td>79</td>
</tr>
<tr>
<td>60</td>
<td>80</td>
<td>81</td>
<td>84</td>
<td>78</td>
</tr>
<tr>
<td>75</td>
<td>79</td>
<td>81</td>
<td>83</td>
<td>77</td>
</tr>
<tr>
<td>90</td>
<td>77</td>
<td>80</td>
<td>82</td>
<td>75</td>
</tr>
<tr>
<td>105</td>
<td>75</td>
<td>80</td>
<td>80</td>
<td>74</td>
</tr>
<tr>
<td>120</td>
<td>72</td>
<td>79</td>
<td>78</td>
<td>73</td>
</tr>
</tbody>
</table>

Plot the data on the graph below. Give the graph a title and label the axes.

![Graph of Insulation House Experiment]

For the next three questions, explain your reasoning in your math journal.

2. Which team had the most efficient house? **Team 2 (their house lost the least heat).**

3. Which team had the least efficient house? **Team 1 (their house lost the most heat).**

4. Predict what the temperature in each house was after 3 hours. **Predictions will vary.**
Insulation Graph Questions Responses will vary.

After you create your graph, answer the following questions.

1. Once your house heated up, how efficient was it at maintaining its temperature? What was the temperature change from the highest temperature until your team stopped taking readings?

2. What would you do differently if you could insulate your house again? Record at least two changes you would make.

Look at another team’s graph and your own together.

3. What was the other team’s temperature change, from the highest temperature until they stopped taking readings? How does that compare with yours?

4. Look at your line’s slope and the other team’s line’s slope as the temperature decreases. Which one is steeper? What does this mean?

5. Examine the other team’s house. Look at the insulation they used, and compare it with yours. Write at least three reasons why there might be a difference in the insulation efficiency of the two houses.
Using Energy

1. Cole’s neighbors are interested in incorporating several solar energy features into their home. They gathered information and found that they can save approximately $\frac{1}{5}$ on basic electricity costs and an additional $\frac{1}{4}$ on heating costs.

   a. What fraction of their utility bill will Cole’s neighbors save by using solar energy? Show your work.

      They can save $\frac{9}{20}$ of their current bill ($\frac{1}{4} + \frac{1}{5}$).
      Work will vary.

   b. If Cole’s neighbors’ monthly utility bill is approximately $200, how much money can they save on their bill each month? Show your work.

      $90$ each month ($200 \times \frac{9}{20} = 90$).
      Work will vary.

2. The graph below shows the sources of energy used throughout the world.

   a. What fraction of the energy used is petroleum? Show your work.

      $\frac{39}{100}$
      ($\frac{100}{100} - \frac{25}{100} - \frac{20}{100} - \frac{5}{100} - \frac{11}{100} = \frac{39}{100}$)
      Work will vary.

   b. How much more petroleum is used than renewable energy sources? Show your work.

      $\frac{28}{100}$ ($\frac{39}{100} - \frac{11}{100}$)
      Work will vary.
      According to this chart, of all energy used, petroleum accounts for $28\%$ more than renewables.
Solar Devices page 1 of 3 Data will vary.

1 Set up your house to test the efficiency of your solar collector.
   • Place the thermometer in the slot and line it up with the 0°F line.
   • Read the temperature, and record the temperature and time in the table below.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td></td>
</tr>
<tr>
<td>End</td>
<td></td>
</tr>
</tbody>
</table>

   • Figure out the time 20 minutes from now, and record the end time in the table.

2 After 20 minutes, remove the thermometer and record the temperature. What is the change in temperature?

3 Photovoltaic (PV) cells absorb solar energy and convert it to electricity. A motor converts electricity into motion. Follow these directions to see how a PV cell can run a fan for your model house.
   • Place the motor stem in the slot you made on the roof (where the thermometer usually goes).
   • Open up your house and carefully attach the binder clip to the stem of the motor inside your house. This will serve as a fan. Make sure the clip is not touching the top of your ceiling.
   • Close the house and place the PV panel on top.
   • Carefully connect the PV panel to the motor by attaching the black and red clips to each metal piece on the motor.
   • Look through a window. What happens?
   • Cover the PV cell with your hand. What happens?

(continued on next page)
4. Now that your house is set up for testing, follow these directions to conduct experiments with your PV cells and record your observations.

   a. Place the PV cell in bright sunlight. Write at least two observations about the rate of spin of the fan.

   b. Have a team member cover half of the PV cell with a piece of paper. Write at least two observations about the rate of spin of the fan.

   c. Now try covering $\frac{3}{4}$ of the PV cell with the paper. Write at least two observations about the rate of spin of the fan.

   d. How much of the PV cell can you cover before the motor stops?

   e. Hold the PV cell at different angles to the sun. Write at least two observations about the rate of spin of the fan.

   f. What do you think is the best angle to point the PV cell? Explain your reasoning.
5 When the sunlight hits the PV cell, it produces watts of electricity. The more surface area exposed, the more watts the PV cell generates. One PV cell can generate 0.18 watts per square inch of surface area. How many watts can your PV cell generate? (Hint: the dimensions are $1 \frac{1}{2}$ inches by 3 inches.)

0.81 watts. Work will vary. Example shown.

\[
\begin{array}{c|c}
3" & 1 \frac{1}{2} \times 3 = 4 \frac{1}{2}, \text{ so the area of the PV cell is } 4 \frac{1}{2} \text{ square inches.} \\
1" & 1 \times 3 = 3 \\
\frac{1}{2}" & \frac{1}{2} \times 3 = 1 \frac{1}{2}
\end{array}
\]

\[4 \frac{1}{2} \times 0.18 = (4 \times 0.18) + (\frac{1}{2} \times 0.18) = 0.72 + 0.09 = 0.81\]

6 If you connect PV cells together, they can generate more electricity than one alone. If 1 PV cell can generate 0.18 watts per square inch, how many watts could 5 PV cells like yours generate?

4.05 watts ($0.81 \times 5 = 4.05$)

7 **CHALLENGE** When you covered $\frac{3}{4}$ of your PV cell with paper, how many watts was it generating? 0.2025 watts. Work will vary. Examples shown.

If we cover $\frac{3}{4}$ of the cell, only $\frac{1}{4}$ is exposed, so the watts should be $\frac{1}{4}$ of the amount generated by the whole cell.

\[0.81 \div 4 = (0.8 \div 4) + (0.01 \div 4) = 0.2 + (\frac{1}{100} \div 4) = 0.2 + \frac{25}{10,000} = 0.2 + 0.0025 = 0.2025\]

\[0.81 \div 4 = 0.2025 \text{ watts.}\]
Solar PV Cells

1 Sage’s team added 8 photovoltaic (PV) cells to the roof of their model house. Each PV cell has dimensions of $2 \times 3\frac{1}{2}$ inches. If each PV cell can provide 0.18 watts per square inch, how many total watts can these 8 PV cells produce? Show your work.

10.08 watts. Work will vary. (2 × 3 ½ = 7, so each cell has an area of 7 sq. inches. $7 \times 8 = 56$ sq. inches. 56 sq. inches × 0.18 watts per sq. inch = 10.08 watts.)

2 Satellites use PV cells to run their instruments. The cells are attached to the outer surface of the satellite.

a Look at the picture of the satellite below. The PV cells on the surface of the satellite can provide 0.18 watts per square inch, and the satellite needs 580 watts. Is there enough surface area to meet this satellite’s electrical needs? Show your work. Yes. Work will vary. Example shown.

(697.5 × 2) + 1860 = 3255 sq. inches
3255 × 0.18 = 585.90 watts, which is a bit more than the 580 watts needed.

b This satellite has two holes that do not have PV cells. The remaining PV cells on the surface of the satellite can provide 0.18 watts per square inch, and the satellite needs 600 watts. Is there enough surface area to meet this satellite’s electrical needs? Show your work. No. Work will vary. Example shown.

Area without the two holes: 3,600 sq. in. – 90 sq. in – 180 sq. in. = 3,330 sq. in.
3,330 sq. in. × 0.18 watts/sq. in. = 599.4 watts. Not quite enough!
Solar Collector Experiment

Mr. Ivy’s class tested their model houses with the added solar collectors. The class data is in the table shown here.

Graph titles and labels will vary. Example shown.

1. Plot the data on a graph, then answer the questions. Title and label the graph.

<table>
<thead>
<tr>
<th>Time</th>
<th>Team 1</th>
<th>Team 2</th>
<th>Team 3</th>
<th>Team 4</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>66</td>
<td>66</td>
<td>67</td>
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<td>15</td>
<td>87</td>
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<td>105</td>
<td>84</td>
<td>88</td>
<td>91</td>
<td>79</td>
</tr>
<tr>
<td>120</td>
<td>82</td>
<td>88</td>
<td>91</td>
<td>78</td>
</tr>
</tbody>
</table>

2. Which team had the most efficient solar collector? Explain your reasoning.
   Team 3; their collector gathered the most heat (and lost it very slowly). Students might instead choose Team 4, whose collector gathered heat the fastest but didn’t gather as much as Team 3’s. Students might also choose Team 2, as their house lost the least heat, but this reflects best overall model efficiency rather than best collector efficiency.

3. Based on the data and your own experiments, what type of solar collector did Team 3 build? Explain your reasoning.
   Responses will vary. Team 3’s house collected a great deal of heat at a moderate pace and lost it very slowly, so it had a good collector, but also good insulation. Some options that might fulfill both functions include black paper or plastic, soil, and rocks. They might also have used a more efficient collector in conjunction with separate insulation (such as foil with foam). This latter scenario is more likely if students take into account the data from Mr. Ivy’s students’ model houses shown on Student Book page 332, which suggests a well-insulated model.
Choosing Our Materials page 1 of 2

1 Determine the minimum amount of cardboard you need for the house, and list the size of each piece you’ll need to cut.

Responses will vary. (The model houses will need to have a volume of 1,152 cubic inches, if adhering to the standards set on the Solar House Model Guidelines Teacher Master.)

2 Determine the minimum size of the piece of window material you need for the windows, and list the size of each piece you’ll need to cut. Remember that you need to make each piece $\frac{1}{6}$” larger than the window opening, all the way around.

Responses will vary. (1/6 of the house’s surface area must be in windows, if adhering to the standards set on the Solar House Model Guidelines Teacher Master.)

3 Determine the materials you need for any solar collection devices you are incorporating and list them below.

Responses will vary.

(continued on next page)
4 Determine the insulation you need.

- As a team, determine the type and amount of the materials you want to use.
- Record the amount and cost of each insulation material on the cost sheet below.
- Calculate the total cost for all the materials and write it below the table.
- Check your calculations to make sure you have enough of every material you need.
- When you have completed the cost sheet, send a team member to buy the materials from your teacher.

### Insulation Costs

<table>
<thead>
<tr>
<th>Insulation Material</th>
<th>Cost per Unit (piece, sheet, etc.)</th>
<th>Amount Needed</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>

Total Cost of All Materials
Another Solar House

1 One team in Mr. Ivy’s class made a model house with dimensions 12” wide by 13 1/2” long by 8” tall. What is the volume of their model house? Show your work.

1,296 cubic inches \((12 \times 13.5 \times 8 = 1,296)\). Work will vary.

2 What is the total surface area of the model house’s four walls? Show your work.

408 square inches. Work will vary.
\((2 \times (12 \times 8)) + (2 \times (13 \frac{1}{2} \times 8)) = 408\)

3 The team needs to cut out windows that take up \(\frac{1}{6}\) of the surface area of the four walls. How many square inches of windows do they need to cut out? Show your work.

68 square inches of windows. Work will vary. \((408 ÷ 6 = 68)\)

4 Find and list at least three more sets of dimensions a team could use to make a model house with the same volume as the team in problem 1 above. Show your work.

Possibilities include, but are not limited to:

- \(6” \times 27” \times 8”\)
- \(12” \times 27” \times 4”\)
- \(12” \times 6 \frac{3}{4}” \times 16”\)
- \(24” \times 6 \frac{3}{4}” \times 8”\)
- \(18” \times 9” \times 8”\)
- \(6” \times 9” \times 24”\)
- \(18” \times 18” \times 4”\)
- \(12” \times 18” \times 6”\)
- \(12” \times 12” \times 9”\)

5 Which house dimensions would you choose to make? Explain your reasoning.

Responses and explanations will vary.
Determining House Materials

One team in Mr. Ivy’s class made a model house with dimensions 12” wide by 18” long by 6” tall. They want to buy some insulation materials. The costs are in the table below.

<table>
<thead>
<tr>
<th>Insulation Material</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weatherstripping (electrician’s or duct tape)</td>
<td>$0.25 per yard</td>
</tr>
<tr>
<td>Storm windows (transparency film)</td>
<td>$0.50 per sheet</td>
</tr>
<tr>
<td>Foam sheeting</td>
<td>$0.20 per sheet</td>
</tr>
<tr>
<td>Newspaper</td>
<td>$0.20 per roll</td>
</tr>
<tr>
<td>Paper towels</td>
<td>$0.20 per roll</td>
</tr>
<tr>
<td>Felt</td>
<td>$0.35 per sheet</td>
</tr>
<tr>
<td>Polyester/blend fabric (insulation or curtains)</td>
<td>$0.50 per 72 sq. in.</td>
</tr>
<tr>
<td>Masking tape</td>
<td>$0.40 per roll</td>
</tr>
<tr>
<td>Caulking (glue)</td>
<td>$0.50 per bottle</td>
</tr>
</tbody>
</table>

1. The team decided to buy foam sheeting for the floor, ceiling, and one of the small walls without windows. How much foam sheeting do they need to buy? Show your work.
   **They need to buy 11 pieces of foam sheeting. Each piece is 46.75 sq. inches.**
   They need 504 sq. inches to cover floor, ceiling, and one small wall:
   \[(2 \times (12 \times 18)) + (12 \times 6) = 10 \times 46.75 = 467.5, \text{ and they need a bit more than that, so they need 11 sheets.}\]

2. The house has 8 windows that each have an area of \(4\frac{1}{2}\) square inches, and 1 more window with an area of 24 square inches. Ava says they only need half of one transparency film for storm windows. Is she correct? Explain your reasoning.
   **No, Ava is incorrect. Explanations will vary. 1 sheet of transparency film has an area of 93.5 sq. inches. Total window area is \((8 \times 4.5) + 24 = 60 \text{ sq. inches. That’s more than half of 93.5 sq. inches.}\)**

3. The team decided to use weatherstripping. How many inches of weatherstripping will each of their small windows get if 2 windows share \(\frac{1}{2}\) yard equally? Show your work. If you need more room, write in your math journal.
   **9 inches of weatherstripping (\(\frac{1}{2} \text{ yard} = 18 \text{ inches; } 18 \div 2 = 9\)). Work will vary.**
With your team, set up your house to test its efficiency. Then record and analyze data on this page individually.

- Place the thermometer in the slot, and line it up with the 0°F line.
- Read the Start (in sun) temperature and record it and the start time in the table below. Remember to label the time a.m. or p.m.

<table>
<thead>
<tr>
<th>Total Minutes</th>
<th>Time</th>
<th>Temperature in degrees Fahrenheit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start (in sun)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>End (in sun)</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>After 20 minutes inside</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>After 40 minutes inside</td>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>

- Figure out the time it will be 20 minutes from now, and record the End (in sun) time in the table. Remember to come back and check the temperature at this time.
- After 20 minutes in the sun, take the temperature reading of your house and record it in the table.

Carefully take your house inside the classroom.

- Figure out the times you need to check the temperatures again and record these times in the table. Remember to check the temperatures at these times.
- After 20 minutes inside, take the temperature reading and record it in the table.

What was the temperature change of your house in the sun?

What was the temperature change from the time you brought your house inside until it was inside for 20 minutes?

What was the total temperature change of your house at that time?

(continued on next page)
6. Plot the data your team collected on a graph. Give the graph a title and label each axis.

---

7. What do you notice about the data? Record at least three observations.
After 40 minutes inside, take one last temperature reading and add the data to your graph. Then answer the following questions:

a  Did your house continue to lose its heat at the same rate? Explain how you know and why you think this happened.

b  What temperature do you think your house will be after 2 hours? Explain your reasoning.

c  Are you happy with your results? What could you change to keep your house warmer for a longer period of time?
Floor Plan page 1 of 2 Scales and drawings will vary. Example shown.

1 One team in Mr. Ivy’s class made a model house 9” long by 12” wide by 4” tall. Draw a bird’s-eye view of their model to scale. Label the scale you choose.

Scale 1:2

2 Now draw and label a floor plan for the model, using your scaled drawing above. Remember to scale the dimensions of each room as you add it.

- The bedroom is \( \frac{1}{4} \) the area of the whole model.
- The bathroom is \( \frac{1}{8} \) the area of the whole model.
- The kitchen is 4 1/2 inches by 6 inches.
- The living room is the space left over.

The example shown meets these requirements. Students’ solutions will vary. (continued on next page)
If the dimensions of the entire floor of the model house are $9'' \times 12''$, what is the area of each room? Use numbers, sketches, or words to show your work.

<table>
<thead>
<tr>
<th>Bedroom</th>
<th>Bathroom</th>
</tr>
</thead>
</table>
| 27 sq. inches  
Work will vary.  
The bedroom is $\frac{1}{4}$ of the area of the whole model, so it’s $\frac{1}{4}$ of $9 \times 12$. Students may also notice that it has the same area as the kitchen. | 13 $\frac{1}{2}$ sq. inches  
Work will vary.  
The bathroom is $\frac{1}{8}$ of the area of the whole model, so it’s $\frac{1}{8}$ of $9 \times 12$ (or half the size of the bedroom). |

<table>
<thead>
<tr>
<th>Kitchen</th>
<th>Living Room</th>
</tr>
</thead>
</table>
| 27 sq. inches  
Work will vary.  
The kitchen is $4 \frac{1}{2}'' \times 6''$ according to problem 2. Students may also notice that it has the same area as the bedroom. | 40 $\frac{1}{2}$ sq. inches  
Work will vary.  
The remaining space is for the living room, so:  
$(9 \times 12) - (2 \times 27) - 13 \frac{1}{2} = 108 - 54 - 13 \frac{1}{2} = 40 \frac{1}{2}$. |
Room Volume

1. One team in Mr. Ivy’s class made a model house with dimensions 12” wide by 12” long by 8” tall. They want to make a two-story house, but they aren’t sure they have enough room. What is the height of each floor in the real house if they use a scale factor of 30 inches to 1 inch? Do they have enough room to make two stories? Why or why not?

   The height of each floor in the real house would be $30 \times 4 = 120”$ or 10 feet. Students will vary in their thoughts about whether this room height is adequate.

2. The team decides to make a kitchen, living room, and bathroom on the main floor. They made a drawing like the one here. What is the volume of each room in their model? Show your work.

   Living Room: $10.5 \times 7 \times 4 = 294$ cubic inches
   Kitchen: $9.5 \times 5 \times 4 = 190$ cubic inches
   Bathroom: $2.5 \times 5 \times 4 = 50$ cubic inches

   Students might choose to calculate the volume of the stairwell. Since the stairs have to use both floors, their volume would be $1.5 \times 6 \times 8 = 72$ cubic inches, but only $36$ cubic inches of that is first floor space.

3. Challenge. What is the volume of each room in the actual house if the scale factor is 30” to 1”? Show your work.

   One way to calculate these answers is to scale each dimension, then figure the volume:

   Living Room: $315 \times 210 \times 120 = 7,938,000$ in$^3$ (4,593 3/4 ft$^3$)
   Kitchen: $285 \times 150 \times 120 = 5,130,000$ in$^3$ (2,968 3/8 ft$^3$)
   Bathroom: $75 \times 150 \times 120 = 1,350,000$ in$^3$ (781 1/4 ft$^3$)
More Solar Features

1 A team in Mr. Ivy’s class wanted to add more solar energy features to their house. The house is 9” long by 12” wide by 5” tall. One large window takes up 2/3 of the large wall.

a What is the area of their large window? Show your work.

40 sq. inches. Work will vary.

b List two pairs of possible dimensions for the large window. Tell which pair you would choose, and explain why. Choices and explanations will vary.

Possibilities include, but are not limited to:
1 × 40 4 × 10 2 1/2 × 16 = 40
2 × 20 5 × 8

b The team wants to add an awning to provide shade in the summer. The material sells for $1.50 for every 2” by 2 1/2” piece. How much do they need to spend on an awning to cover their window? Show your work using numbers, words, or labeled sketches. Use your math journal if you need more room.

Assuming that the awning will be as large as the window:* $12.00 (2 × 2 1/2 = 5 square inches

40 ÷ 5 = 8, so they need 8 pieces.

8 × $1.50 = $12.00)

* An awning generally does not need to be as large as the window to provide adequate shade, so student results may vary.

2 The team wants to cover 1/2 of the area of their house’s roof with photovoltaic (PV) cells. Each PV cell has dimensions of 2 × 3 inches.

a How many PV cells will it take to cover 1/2 of the area of the roof on this house? Show your work.

9 PV cells (Assuming a flat roof, roof area is 9 × 12 = 108 sq. in., and 108 ÷ 2 = 54. Area of each PV cell is 6 sq. in. 54 ÷ 6 = 9 PV cells.)

b If PV cells can provide 0.18 watts of electricity per square inch, how many total watts can the cells on this house produce? Show your work.

9.72 watts (9 PV cells at 6 sq. in. each = 54 sq. inches. 54 × 0.18 = 9.72)