

Answers at end
 Work on separate paper

REVIEW—SUM/DIFFERENCE IDENTITIES

NAME key

#1-6. Using the sum & difference identities, condense each of the following and express as a trig function of a single angle.

1. $\sin 97^\circ \cos 43^\circ + \cos 97^\circ \sin 43^\circ$

2. $\cos 72^\circ \cos 130^\circ + \sin 72^\circ \sin 130^\circ$

3. $\frac{\tan 140^\circ - \tan 60^\circ}{1 + \tan 140^\circ \tan 60^\circ}$

4. $\sin \frac{\pi}{5} \cos \frac{2\pi}{3} - \cos \frac{\pi}{5} \sin \frac{2\pi}{3}$

5. $\cos \frac{\pi}{6} \cos \frac{\pi}{7} - \sin \frac{\pi}{6} \sin \frac{\pi}{7}$

6. $\frac{\tan \frac{\pi}{3} + \tan \frac{\pi}{4}}{1 - \tan \frac{\pi}{3} \tan \frac{\pi}{4}}$

#7-8. Use the sum & difference identities with unit circle values to find exact answers for the following:

7. $\tan(-105^\circ)$

8. $\sin 345^\circ$

↑ tricky!

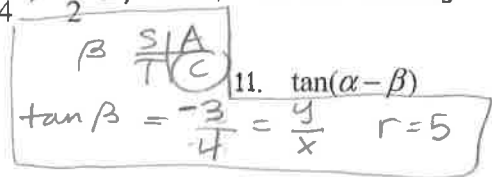
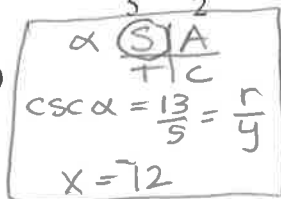
#9-11. Given: $\csc \alpha = \frac{13}{5}$, $\frac{\pi}{2} \leq \alpha \leq \pi$, and $\tan \beta = -\frac{3}{4}$, $\frac{3\pi}{2} \leq \beta \leq 2\pi$, find the following:

9. $\sin(\alpha - \beta)$

10. $\cos(\beta + \alpha)$

11. $\tan(\alpha - \beta)$

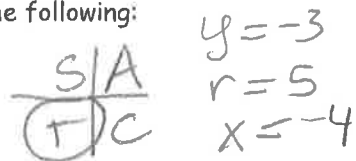
$\csc \alpha =$
 \downarrow
 $\sin \alpha$



#12-13. If $\sin \theta = -\frac{3}{5}$ and θ is in the third quadrant, find the following:

12. $\cos(\theta + \frac{\pi}{3})$

13. $\tan 2\theta$



#14-18. Verify the following identities.

14. $\sin(\pi - x) = \sin x$

15. $\sin(\frac{3\pi}{2} + x) = -\cos x$

16. $\cos(30^\circ - x) + \cos(30^\circ + x) = \sqrt{3} \cos x$

17. $\frac{\sin(\beta - \alpha)}{\sin \alpha \sin \beta} = \cot \alpha - \cot \beta$

18. $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos \alpha \cos \beta$

#19-21. Solve each of the following equations over the interval $[0, 2\pi)$.

$$19. \sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$$

$$20. \tan(x + \pi) + 2\sin(x + \pi) = 0$$

$$21. \sin\left(x + \frac{\pi}{2}\right) - \cos\left(x + \frac{3\pi}{2}\right) = 0$$

Answers: 1. $\sin 140^\circ$ 2. $\cos 58^\circ$ 3. $\tan 80^\circ$ 4. $-\sin\left(\frac{7\pi}{15}\right)$

5. $\cos\left(\frac{13\pi}{42}\right)$ 6. $\tan\frac{7\pi}{12}$ 7. $2 + \sqrt{3}$ 8. $\frac{\sqrt{2} - \sqrt{6}}{4}$ 9. $-\frac{16}{65}$

10. $-\frac{33}{65}$ 11. $\frac{16}{63}$ 12. $\frac{-4 + 3\sqrt{3}}{10}$ 13. $\frac{24}{7}$ 19. $\frac{\pi}{3}, \frac{5\pi}{3}$

20. $0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$ 21. $\frac{\pi}{4}, \frac{5\pi}{4}$

Review - SUM + DIFF ID

$$1) \sin(97+43) = \boxed{\sin(140)}$$

$$2) \cos(72-136) = \boxed{\cos 58^\circ}$$

$$3) \tan(140-60) = \boxed{\tan 80^\circ}$$

$$4) \sin\left(\frac{\pi}{5} - 2\frac{\pi}{3}\right) = \sin\left(-\frac{7\pi}{15}\right) \begin{array}{l} \text{odd} \\ \text{function} = \\ \text{thus} \end{array} \boxed{-\frac{\sin 7\pi}{15}}$$

$$5) \cos\left(\frac{\pi}{6} + \frac{\pi}{7}\right) = \boxed{\cos\left(\frac{13\pi}{42}\right)}$$

$$6) \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \boxed{\tan\left(\frac{7\pi}{12}\right)}$$

$$7) \tan(-105^\circ) \text{ tricky I converted into a positive angle first, then used a unit circle image of angles to see which two might add or subtract to equal } 255^\circ$$

$$\tan(120+135) \quad \tan 120 = -\sqrt{3} \quad \tan 135 = -1$$

$$\tan 120 + \tan 135$$

$$1 - \tan 120 \tan 135$$

$$= \frac{-\sqrt{3} + -1}{1 - (-\sqrt{3})(-1)}$$

$$= \frac{-\sqrt{3} - 1}{1 - \sqrt{3}}$$

must rationalize the denominator, no radical allowed!

multiply by conjugate

rationalize

$$\frac{-\sqrt{3} - 1}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}} \text{ conjugate}$$

$$\frac{-\sqrt{3} - 3 - 1 - \sqrt{3}}{1 - 3} = \frac{-4 - 2\sqrt{3}}{-2}$$

$$= \frac{-2(2 + \sqrt{3})}{-2}$$

$$= \boxed{2 + \sqrt{3}} \text{ phew!}$$

$$300 = 5\pi/3$$

8) $\sin 345$ look at unit circle image to help
 $\sin(300+45)$ $\sin 300 = -\sqrt{3}/2$ $\cos 300 = 1/2$
 $\sin 45 = \sqrt{2}/2$ $\cos 45 = \sqrt{2}/2$

$$\sin 300 \cos 45 + \sin 45 \cos 300$$

$$-\sqrt{3}/2 \cdot \sqrt{2}/2 + \sqrt{2}/2 \cdot 1/2$$

$$-\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{-\sqrt{6} + \sqrt{2}}{4}} \text{ or } \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

9) $\sin(\alpha - \beta)$

$$\sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\frac{5}{13} \cdot \frac{4}{5} - \frac{-12}{13} \cdot \frac{-3}{5}$$

$$\frac{20}{65} - \frac{36}{65} = \boxed{\frac{-16}{65}}$$

$$\sin \alpha = 5/13$$

$$\cos \alpha = -12/13$$

$$\sin \beta = -3/5$$

$$\cos \beta = 4/5$$

$$\tan \alpha = -5/12$$

$$\tan \beta = -3/4$$

10) $\cos(\beta + \alpha)$

$$\cos \beta \cos \alpha - \sin \beta \sin \alpha$$

$$4/5 \cdot -12/13 - (-3/5) \cdot 5/13$$

$$-\frac{48}{65} + \frac{15}{65} = \boxed{\frac{-33}{65}}$$

11) $\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$

$$1 + \tan \alpha \tan \beta$$

$$= \frac{-5/12 - (-3/4)}{1 + (-5/12)(-3/4)}$$

$$1 + (-5/12)(-3/4)$$

$$= \frac{1/3}{21/16} = 1/3 \cdot 16/21$$

$$= \boxed{\frac{16}{63}}$$

$$12) \cos\left(\theta + \frac{\pi}{3}\right)$$

$$\cos\theta \cos\frac{\pi}{3} - \sin\theta \sin\frac{\pi}{3}$$

$$-\frac{4}{5} \cdot \frac{1}{2} - \left(-\frac{3}{5} \cdot \frac{\sqrt{3}}{2}\right)$$

$$-\frac{4}{10} + \frac{3\sqrt{3}}{10} = \boxed{\frac{-4 + 3\sqrt{3}}{10}}$$

$$\cos\theta = -\frac{4}{5}$$

$$\sin\theta = -\frac{3}{5}$$

$$\cos\frac{\pi}{3} = \frac{1}{2}$$

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{3}{4}$$

$$13) \tan 2\theta \text{ tricky!} = \tan(\theta + \theta) \text{ aha!}$$

$$\frac{\tan\theta + \tan\theta}{1 - \tan\theta \tan\theta} = \frac{\frac{3}{4} + \frac{3}{4}}{1 - \frac{3}{4} \cdot \frac{3}{4}} =$$

$$= \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{2} \cdot \frac{16}{7}$$

$$= \boxed{\frac{24}{7}}$$

$$14) \sin(\pi - x) = \boxed{\sin x}$$

$$\sin\pi \cos x - \sin x \cos\pi$$

$$0 \cdot \cos x - \sin x \cdot -1$$

$$= \boxed{\sin x} \checkmark$$

(sine diff ID)

(evaluate zero property mult and multiply)

$$15) \sin\left(\frac{3\pi}{2} + x\right) = \boxed{-\cos x}$$

$$\sin\frac{3\pi}{2} \cos x + \cos\frac{3\pi}{2} \cdot \sin x$$

$$-1 \cdot \cos x + 0 \cdot \sin x$$

$$\boxed{-\cos x} \checkmark$$

sine sum ID

evaluate

zero prop mult and multiply

I prefer to turn into radians...

$$16) \cos(\pi/6 - x) + \cos(\pi/6 + x) = \boxed{\sqrt{3} \cos x}$$

(cos diff + sum ID)

$$\cos \pi/6 \cos x + \sin \pi/6 \sin x + \cos \pi/6 \cos x - \sin \pi/6 \sin x$$

$$2 \cos \pi/6 \cos x$$

(inverse add and addition)

evaluate $2 \cdot \frac{\sqrt{3}}{2} \cos x$

inverse mult

$$\boxed{\sqrt{3} \cos x} \checkmark$$

$$17) \frac{\sin(\beta - \alpha)}{\sin \alpha \sin \beta} = \cot \alpha - \cot \beta$$

sin diff ID

$$\frac{\sin \beta \cos \alpha - \sin \alpha \cos \beta}{\sin \alpha \sin \beta}$$

separate fractions

$$\frac{\sin \beta \cos \alpha}{\sin \alpha \sin \beta} - \frac{\sin \alpha \cos \beta}{\sin \alpha \sin \beta}$$

cancel one

(or mult. inverse)

$$\frac{\cos \alpha}{\sin \alpha} - \frac{\cos \beta}{\sin \beta}$$

Quotient ID

$$\boxed{\cot \alpha - \cot \beta} \checkmark$$

$$18) \cos(\alpha + \beta) + \cos(\alpha - \beta) = \boxed{2 \cos \alpha \cos \beta}$$

cos sum + diff ID

$$\cos \alpha \cos \beta - \sin \alpha \sin \beta + \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

add inverse

$$\cos \alpha \cos \beta + \cos \alpha \cos \beta$$

add

$$\boxed{2 \cos \alpha \cos \beta} \checkmark$$

$$19) \sin x \cos \frac{\pi}{6} + \cos x \sin \frac{\pi}{6} - (\sin x \cos \frac{\pi}{6} - \cos x \sin \frac{\pi}{6}) = \frac{1}{2}$$

$$\cancel{\sin x \cos \frac{\pi}{6}} + \cos x \sin \frac{\pi}{6} - \cancel{\sin x \cos \frac{\pi}{6}} + \cos x \sin \frac{\pi}{6} = \frac{1}{2}$$

$$2 \cos x \sin \frac{\pi}{6} = \frac{1}{2}$$

$$2 \cos x \cdot \frac{1}{2} = \frac{1}{2}$$

$$\cos x = \frac{1}{2} \quad \begin{matrix} \text{S} \text{ (A)} \\ \text{T} \text{ (C)} \end{matrix} \quad \pi/3$$

$$\boxed{\pi/3, 5\pi/3}$$

$$20) \frac{\tan x + \tan \pi}{1 - \tan x \tan \pi} + 2(\sin x \cos \pi + \sin \pi \cos x) = 0$$

$$\frac{\tan x + 0}{1 - \tan x \cdot 0} + 2 \sin x (-1) + 2 \cdot 0 \cdot \cos x = 0$$

$$\tan x - 2 \sin x = 0$$

$$\frac{\sin x}{\cos x} - 2 \sin x = 0$$

$$\sin x \left(\frac{1}{\cos x} - 2 \right) = 0$$

$$\sin x = 0$$

$$\boxed{0, \pi}$$

$$\frac{1}{\cos x} - 2 = 0$$

~~$$\frac{1}{\cos x} = 2$$~~

many ways to proceed...

$$2 \cos x = 1$$

$$\cos x = \frac{1}{2}$$

$$\begin{matrix} \text{S} \text{ (A)} \\ \text{T} \text{ (C)} \end{matrix} \quad \pi/3$$

$$\boxed{\pi/3, 5\pi/3}$$

$$21) \sin\left(x + \frac{\pi}{2}\right) - \cos\left(x + \frac{3\pi}{2}\right) = 0$$

$$\sin x \cos \frac{\pi}{2} + \sin \frac{\pi}{2} \cos x - \left(\cos x \cos \frac{3\pi}{2} - \sin x \sin \frac{3\pi}{2}\right)$$

$$\cancel{\sin x \cdot 0} + 1 \cdot \cos x - \cos x \cdot \cancel{0} + \sin x = 1$$

$$\cos x - \sin x = 0$$

$$\cos x = \sin x$$

Weird one
again!

When are they
equal?

both equal

$$\frac{\sqrt{2}}{2} \text{ at } \frac{\pi}{4}$$

$$-\frac{\sqrt{2}}{2} \text{ at } \frac{5\pi}{4}$$

$$\boxed{\frac{\pi}{4}, \frac{5\pi}{4}}$$