answers at end
work on separate paper

REVIEW—SUM/DIFFERENCE IDENTITIES NAME key

#1-6. Using the sum & difference identities, condense each of the following and express as a trig function of a single angle.

1. $\sin 97^o \cos 43^o + \cos 97^o \sin 43^o$

2. $\cos 72^o \cos 130^o + \sin 72^o \sin 130^o$

3. $\dfrac{\tan 140^o - \tan 60^\circ}{1+\tan 140^o \tan 60^\circ}$

4. $\sin\dfrac{\pi}{5}\cos\dfrac{2\pi}{3} - \cos\dfrac{\pi}{5}\sin\dfrac{2\pi}{3}$

5. $\cos\dfrac{\pi}{6}\cos\dfrac{\pi}{7} - \sin\dfrac{\pi}{6}\sin\dfrac{\pi}{7}$

6. $\dfrac{\tan\frac{\pi}{3}+\tan\frac{\pi}{4}}{1-\tan\frac{\pi}{3}\tan\frac{\pi}{4}}$

#7-8. Use the sum & difference identities with unit circle values to find exact answers for the following:

7. $\tan(-105^o)$

tricky!

8. $\sin 345^o$

#9-11. Given: $\csc\alpha = \dfrac{13}{5}$, $\dfrac{\pi}{2} \le \alpha \le \pi$, and $\tan\beta = -\dfrac{3}{4}$, $\dfrac{3\pi}{2} \le \beta \le 2\pi$, find the following:

α S A / T C

$\csc\alpha = \frac{13}{5} = \frac{r}{y}$

$x = -12$

β S A / T C

$\tan\beta = \frac{-3}{4} = \frac{y}{x}$ $r=5$

$\csc\alpha = \frac{1}{\sin\alpha}$

9. $\sin(\alpha-\beta)$

10. $\cos(\beta+\alpha)$

11. $\tan(\alpha-\beta)$

#12-13. If $\sin\theta = -\dfrac{3}{5}$ and θ is in the third quadrant, find the following:

S A / T C

$y=-3$
$r=5$
$x=-4$

12. $\cos(\theta+\dfrac{\pi}{3})$

13. $\tan 2\theta$

#14-18. Verify the following identities.

14. $\sin(\pi - x) = \sin x$

15. $\sin(\dfrac{3\pi}{2}+x) = -\cos x$

16. $\cos(30^0 - x) + \cos(30^o + x) = \sqrt{3}\cos x$

17. $\dfrac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta} = \cot\alpha - \cot\beta$

18. $\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha\cos\beta$

#19-21. Solve each of the following equations over the interval $[0, 2\pi)$.

19. $\sin\left(x+\frac{\pi}{6}\right)-\sin\left(x-\frac{\pi}{6}\right)=\frac{1}{2}$

20. $\tan(x+\pi)+2\sin(x+\pi)=0$

21. $\sin\left(x+\frac{\pi}{2}\right)-\cos\left(x+\frac{3\pi}{2}\right)=0$

$\sin 140^\circ$ $\quad$ 58°

Answers: 1. $\cancel{\sin 190^\circ}$ 2. $\cos \cancel{10^\circ}$ 3. $\tan 80^\circ$ 4. $-\sin\left(\frac{7\pi}{15}\right)$

5. $\cos\left(\frac{13\pi}{42}\right)$ 6. $\tan\frac{7\pi}{12}$ 7. $2+\sqrt{3}$ 8. $\frac{\sqrt{2}-\sqrt{6}}{4}$ 9. $-\frac{16}{65}$

10. $-\frac{33}{65}$ 11. $\frac{16}{63}$ 12. $\frac{-4+3\sqrt{3}}{10}$ 13. $\frac{24}{7}$ 19. $\frac{\pi}{3}, \frac{5\pi}{3}$

20. $0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$ 21. $\frac{\pi}{4}, \frac{5\pi}{4}$

Review - sum & DIFF ID

1) $\sin(97+43) = \boxed{\sin(140)}$

2) $\cos(72-130) = \boxed{\cos 58^\circ}$

3) $\tan(140-60) = \boxed{\tan 80^\circ}$

4) $\sin\left(\frac{\pi}{5} - \frac{2\pi}{3}\right) = \sin\left(\frac{-7\pi}{15}\right)$ odd function thus $= \boxed{-\sin\frac{7\pi}{15}}$

5) $\cos\left(\frac{\pi}{6} + \frac{\pi}{7}\right) = \boxed{\cos\left(\frac{13\pi}{42}\right)}$

6) $\tan\left(\pi/3 + \pi/4\right) = \boxed{\tan(7\pi/12)}$

7) $\tan(-105^\circ)$ tricky I converted into a positive angle first, then used a unit circle image of angles to see which two might add or subtract to equal 255°

$= \tan(255^\circ)$

$\tan(120+135)$ $\tan 120 = -\sqrt{3}$ $\tan 135 = -1$

$$\frac{\tan 120 + \tan 135}{1 - \tan 120 \tan 135} = \frac{-\sqrt{3} + -1}{1-(-\sqrt{3})\cdot(-1)}$$

$$= \frac{-\sqrt{3}-1}{1-\sqrt{3}}$$ must rationalize the denominator, no radical allowed!

multiply by conjugate

rationalize

$$\frac{-\sqrt{3}-1}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}}$$ conjugate

$$\frac{-\sqrt{3}-3-1-\sqrt{3}}{1-3} = \frac{-4-2\sqrt{3}}{-2}$$

$$= \frac{\cancel{-2}(2+\sqrt{3})}{\cancel{-2}}$$

$$= \boxed{2+\sqrt{3}}$$ phew!

$300 = 5\pi/3$

8) sin 345 look at unit circle image to help

$\sin(300+45)$

$\sin 300 = -\sqrt{3}/2 \quad \cos 300 = 1/2$

$\sin 45 = \sqrt{2}/2 \quad \cos 45 = \sqrt{2}/2$

$\sin 300 \cos 45 + \sin 45 \cos 300$

$-\sqrt{3}/2 \cdot \sqrt{2}/2 + \sqrt{2}/2 \cdot 1/2$

$$\frac{-\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{-\sqrt{6}+\sqrt{2}}{4}} \text{ or } \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

9) $\sin(\alpha - \beta)$

$\sin\alpha\cos\beta - \cos\alpha\sin\beta$

$5/13 \cdot \frac{4}{5} - \frac{-12}{13} \cdot {-3/5}$

$$\frac{20}{65} - \frac{36}{65} = \boxed{\frac{-16}{65}}$$

$\sin\alpha = 5/13$

$\cos\alpha = -12/13$

$\sin\beta = -3/5$

$\cos\beta = 4/5$

$\tan\alpha = -5/12$

$\tan\beta = \frac{-3}{4}$

10) $\cos(\beta + \alpha)$

$\cos\beta\cos\alpha - \sin\beta\sin\alpha$

$4/5 \cdot -12/13 - -3/5 \cdot 5/13$

$$\frac{-48}{65} + \frac{15}{65} = \boxed{\frac{-33}{65}}$$

11) $$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

$$= \frac{-5/12 - (-3/4)}{1 + (-5/12)(-3/4)}$$

$$= \frac{1/3}{21/16} = 1/3 \cdot 16/21$$

$$= \boxed{\frac{16}{63}}$$

12) $\cos\left(\theta + \frac{\pi}{3}\right)$

$\cos\theta = -4/5$
$\sin\theta = -3/5$
$\cos \pi/3 = 1/2$
$\sin \pi/3 = \sqrt{3}/2$
$\tan\theta = 3/4$

$\cos\theta \cos \pi/3 - \sin\theta \sin \pi/3$

$-4/5 \cdot 1/2 - (-3/5 \cdot \sqrt{3}/2)$

$-\frac{4}{10} + \frac{3\sqrt{3}}{10} = \boxed{\frac{-4+3\sqrt{3}}{10}}$

13) $\tan 2\theta$ tricky! $= \tan(\theta + \theta)$ aha!

$$\frac{\tan\theta + \tan\theta}{1 - \tan\theta \tan\theta} = \frac{3/4 + 3/4}{1 - 3/4 \cdot 3/4} =$$

$$= \frac{3/2}{7/16} = \frac{3}{\cancel{2}} \cdot \frac{\cancel{16}\,8}{7}$$

$$= \boxed{\frac{24}{7}}$$

14) $\sin(\pi - x) = \boxed{\sin x}$

$\underbrace{\sin \pi}\cos x - \sin x \cos \pi$

$0 \cdot \cos x - \sin x \cdot -1$

$= \boxed{\sin x}$ ✓

(sine diff ID

(evaluate

(zero property mult and multiply

15) $\sin\left(\frac{3\pi}{2} + x\right) = \boxed{-\cos x}$

$\underbrace{\sin \frac{3\pi}{2}} \cos x + \cos \frac{3\pi}{2} \cdot \sin x$

$-1 \cdot \cos x + 0 \cdot \sin x$

$\boxed{-\cos x}$ ✓

sine sum ID
evaluate
zero prop mult and multiply

I prefer to turn into radians...

16) $\cos(\pi/6 - x) + \cos(\pi/6 + x) = \boxed{\sqrt{3}\cos x}$

(cos diff + sum ID) $\cos\pi/6\cos x + \cancel{\sin\pi/6\sin x} + \cos\pi/6\cos x - \cancel{\sin\pi/6\sin x}$ (Inverse add and addition)

$2\underbrace{\cos\pi/6}\cos x$

evaluate $2 \cdot \frac{\sqrt{3}}{2}\cos x$

Inverse mult $\boxed{\sqrt{3}\cos x}$ ✓

17) $\frac{\sin(\beta - \alpha)}{\sin\alpha\sin\beta} = \cot\alpha - \cot\beta$

sin diff ID $\frac{\sin\beta\cos\alpha - \sin\alpha\cos\beta}{\sin\alpha\sin\beta}$

separate fractions $\frac{\cancel{\sin\beta}\cos\alpha}{\sin\alpha\cancel{\sin\beta}} - \frac{\cancel{\sin\alpha}\cos\beta}{\cancel{\sin\alpha}\sin\beta}$

def one (OR mult. inverse) $\frac{\cos\alpha}{\sin\alpha} - \frac{\cos\beta}{\sin\beta}$

Quotient ID. $\boxed{\cot\alpha - \cot\alpha}$ ✓

18) $\cos(\alpha + \beta) + \cos(\alpha - \beta) = \boxed{2\cos\alpha\cos\beta}$

Cos sum + diff ID $\cos\alpha\cos\beta - \sin\alpha\sin\beta + \cos\alpha\cos\beta + \sin\alpha\sin\beta$

add inverse $\cos\alpha\cos\beta + \cos\alpha\cos\beta$

add $\boxed{2\cos\alpha\cos\beta}$ ✓

19) $\sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6} - \left(\sin x \cos\frac{\pi}{6} - \cos x \sin\frac{\pi}{6}\right) = \frac{1}{2}$

$$\cancel{\sin x \cos\frac{\pi}{6}} + \cos x \sin\frac{\pi}{6} - \cancel{\sin x \cos\frac{\pi}{6}} + \cos x \sin\frac{\pi}{6} = \frac{1}{2}$$

$$2\cos x \sin\frac{\pi}{6} = \frac{1}{2}$$

$$2\cos x \cdot 1/2 = 1/2$$

$$\cos x = 1/2$$

S | A
T | C

$\pi/3$

$$\boxed{\pi/3,\ 5\pi/3}$$

20) $\frac{\tan x + \tan\pi}{1 - \tan x \tan\pi} + 2(\sin x \cos\pi + \sin\pi \cos x) = 0$

$$\frac{\tan x + 0}{1 - \tan x \cdot 0} + 2\sin x(-1) + 2 \cdot 0 \cdot \cos x = 0$$

$$\tan x - 2\sin x = 0$$

$$\frac{\sin x}{\cos x} - 2\sin x = 0$$

$$\sin x\left(\frac{1}{\cos x} - 2\right) = 0$$

$\sin x = 0$

$$\boxed{0,\ \pi}$$

$$\frac{1}{\cos x} - 2 = 0$$

$$\frac{1}{\cos x} = \frac{2}{1}$$

many ways to proceed...

$$2\cos x = 1$$

$$\cos x = 1/2$$

S | A
T | C

$\pi/3$

$$\boxed{\pi/3,\ 5\pi/3}$$

21) $\sin\left(x+\frac{\pi}{2}\right) - \cos\left(x+\frac{3\pi}{2}\right) = 0$

$\sin x \cos \pi/2 + \sin \pi/2 \cos x - \left(\cos x \cos \frac{3\pi}{2} - \sin x \sin \frac{3\pi}{2}\right)$

$\cancel{\sin x \cdot 0} + 1 \cdot \cos x - \cancel{\cos x \cdot (0)} + \sin x \cdot -1$

$\cos x - \sin x = 0$

$\cos x = \sin x$

weird one again! When are they equal? both equal $\frac{\sqrt{2}}{2}$ at $\frac{\pi}{4}$ $-\sqrt{2}/2$ at $\frac{5\pi}{4}$

$\boxed{\pi/4 \ , 5\pi/4}$