# NORTH HAVEN HIGH SCHOOL

**221 Elm Street**

**North Haven, CT 06473**

# PreCalculus Level 3
# Summer Assignment 2017

June 2017

Dear Parents, Guardians, and Students:

This packet of material reviews all the topics in Algebra 2 that you covered this past school year that are essential background topics necessary for success in level 3 PreCalculus. It is expected that you will complete all the problems with all work shown (use separate paper if necessary). This packet will be collected on the first day of school in your PreCalculus class and graded for completion. There will be a quiz on the material sometime during the first few classes. Please note that these topics will not be retaught in class.

### A Note About Graphing Calculators

Students will be using graphing calculators in mathematics courses such as Algebra I, Algebra II, Pre-calculus, Calculus, and Statistics. Each teacher has enough graphing calculators for every student to use in class, so students are not required to purchase graphing calculators. However, if a student would like to make the investment to use throughout high school and most likely college, we recommend the TI-84 Plus, which is the calculator used in class. (Please note that the TI-84 Plus CE is not necessary.) There are many sales over the summer so if you wish to purchase one it is a good time to do so. These calculators can be found at Amazon, Walmart, Staples, Target, and other stores that sell school supplies.

The mathematics department thanks you for your support and wishes you and your family a happy and restful summer!

Sincerely,

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PreCalculus Teachers

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**Entering PreCalculus Level 3   2017-2018**
*14 points*
This packet of material reviews all the topics in Algebra 2 that you covered this past school year that are essential background topics necessary for success in level 3 PreCalculus. This packet will be collected on the first day of school in your PreCalculus class. It is expected that you will complete all the problems with all work shown (use separate paper if necessary). THESE TOPICS WILL NOT BE RETAUGHT IN PRECALCULUS CLASS.

**Fraction Operations** – fractions are more numerous than whole numbers and thus will be incorporated within any work that we do in PreCalculus. Re-familiarize yourself with all four fraction operations as well as mixed operations. All answers should be in simplest form and exact (no decimal approximations) Examples of problems you will see include:

1) $\dfrac{6\frac{1}{3}}{9}$

2) $\dfrac{\frac{1}{4}}{\frac{3}{8}}$

3) $\dfrac{\frac{3}{4}}{\frac{2}{3}+\frac{5}{9}}$

4) $1-2\left(\dfrac{15}{17}\right)^2$

5) $\dfrac{2\left(\frac{-7}{24}\right)}{1-\left(\frac{-7}{24}\right)^2}$

6) $\dfrac{23\pi}{3}-2\pi$

7) $\sqrt{2}\left(\dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{2}}{2}i\right)$

8) $\dfrac{\frac{4\pi}{3}+2\pi}{3}$

9) $\dfrac{\frac{\pi}{2}+2(3)\pi}{5}$

10) $2\left(\dfrac{\pi}{2}\right)+\dfrac{\pi}{2}$

**Simplifying Radicals** – we will be working with **exact values** in precalculus so a knowledge of how to simplify a variety of radicals expressions is essential. Remember a radical is in simplest form if there are no perfect squares inside the radical, there is no fraction inside the radical and there is no radical in the denominator. It's okay to have a radical inside a radical (as weird as that looks!)

1) $\dfrac{-1}{\sqrt{28}}$

2) $\dfrac{5}{5\sqrt{3}}$

3) $1-2\left(\dfrac{-\sqrt{3}}{3}\right)^2$

4) $\dfrac{\sqrt{2}}{2}\bullet\dfrac{\sqrt{3}}{2}+\dfrac{\sqrt{2}}{2}\bullet\dfrac{1}{2}$

5) $\dfrac{1-\frac{\sqrt{2}}{2}}{\frac{-\sqrt{2}}{2}}$

6) $\sqrt{(-4)^2+\left(4\sqrt{3}\right)^2}$

7) $-\sqrt{\dfrac{1+\frac{3}{5}}{2}}$

8) $\sqrt{\dfrac{1-\frac{\sqrt{3}}{2}}{2}}$

9) $\dfrac{\sqrt{3}-1}{1+\sqrt{3}}$

10) $\dfrac{1+\frac{\sqrt{3}}{3}}{1-\frac{\sqrt{3}}{3}}$

**Functions & Function Operations** – we will expand your understanding of composition & inverse functions in PreCalculus.  Do the following with f(x) = x² – 3x    g(x) = 5 – 2x      h(x) = x² – 5

1) $f(g(x))$  *(simplify completely)*          2) $g(f(x))$   *(simplify completely)*          3) $h^{-1}(x)$

**Algebraic Problem Solving (Alg One) –** this is a classic quadratic application problem that is covered in Algebra One. We do a number of performance tasks in PreCalculus that involve this type of problem solving.

You have 1200 feet of fencing with which you need to enclose a rectangular field along a straight river bank for your small herd of cows. (there is no need to have fence along the river, your cows can't swim).

1) Draw a picture of this situation.
2) Let's represent the dimensions of the fenced field algebraically.
    a. Let "x" represent the width of the field (the two dimensions perpendicular to the river). Label them in your picture.
    b. Considering the total fence length is 1200 ft (and you are going to use it all) and the width is "x" – what is an expression you can write for the length that uses x & 1200 in some way? Label the length above with this expression.
3) Write a formula for the area of the cattle pen, write your formula in standard form.
4) What does "x" represent in your formula?
5) What does "y" (or A(X)) represent in your formula?
6) What is the maximum area that you can have using all the fencing? What are the dimensions of this cattle pen?

**Function Characteristics**

1) Determine the key characteristics of the function below, including domain, range, increasing/decreasing intervals, LEB and REB (end behavior), extrema and intercepts. Use interval notation when applicable. Also, sketch the inverse of this function on the same graph.
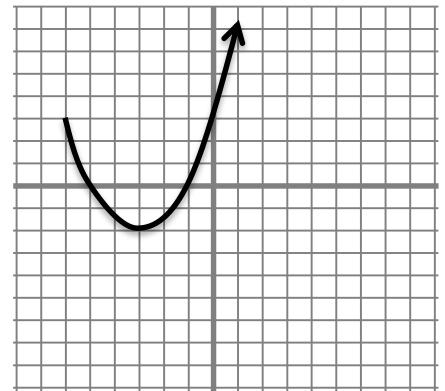
domain_____

range_____

decreasing_____

increasing_____

LEB _____

REB_____

Minimums _____ maximum(s) _____

Intercepts _____

Is the inverse of the above also a function? _____How do you know?



2) Sketch the graph of the function $f(x) = x^3 - 6x^2 + 3x - 7$. Include domain, range, increasing/decreasing intervals, LEB and REB (end behavior), extrema, and both x & y intercepts. Use your graphing calculator to help you, <u>specify the window used on your calculator</u>. Round values to the <u>thousandth</u> if necessary. Use interval notation when appropriate.

3) The piecewise function, $p(x)$, is pictured below. We will use this as our "parent function".
We will apply transformations to this parent function resulting in the new function:
$$q(x) = -2p(x - 3) + 4$$
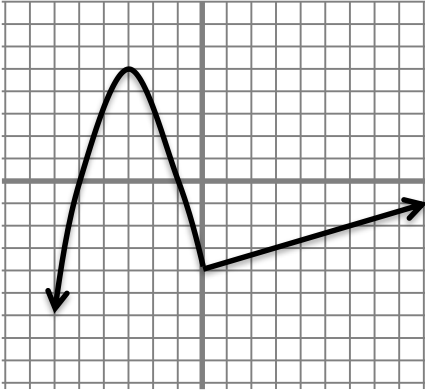
List the transformations that are applied to the parent function to create the new function equation:

Then apply the transformations to the parent function and draw the new graph on graph paper. (use 6 critical points to help you)

**Basic Algebra Skills**
There are some basic factoring patterns that you should be able to apply to any polynomial **without the aid of a calculator or quadratic formula.** Factor the following (they are all factorable)

1)  $x^2 - 16x + 64$

2)  $x^2 - 100$

3)  $x^2 - 5x - 6$

4)  $2x^2 + 7x + 3$

5)  $2x^2 + 5x - 3$

6)  $18x^2 - 2$

Use either synthetic or long division to find the quotient. Our focus in precalculus will be synthetic division.
7)  $(x^3 + 2x^2 - x + 6) \div (x + 3)$

**Quadratic Functions**
You should be able to solve quadratic polynomials over the <u>real and complex numbers</u> without the aid of a calculator.  You are expected to use the most efficient method for the problem (factoring or square root method over the use of the quadratic formula if possible). Solve the following, using the method indicated and <u>show all work.</u> Answers should be in simplest radical form, NOT decimal approximations. None of the following are "no solution" – they all have solutions within the set of complex numbers.

USE FACTORING & ZERO PRODUCT PROPERTY:

1) $x^2 - 7x + 12 = 0$

2) $x^2 - 5x = 0$

3) $x^2 - 49 = 0$

4) $x^2 = x + 1 - x^2$

USE THE SQUARE ROOT METHOD:
5) $2x^2 - 1 = 0$

6) $1 - 5x^2 = -x^2$

USE THE QUADRATIC FORMULA:

7) $x^2 + 2x + 7 = 0$

8) $x^2 + 14 = 10x$

**Rational Functions** – You should be able to do all algebraic skills with rational expressions. What you will see most in precalculus is simplifying rational expressions. Be able to do problems such as the following:

1) $\dfrac{\left(\frac{1}{x+9}+\frac{1}{5}\right)}{\left(\frac{2}{x^2+10x+9}\right)}$

2) $\dfrac{9}{\dfrac{9}{x}+\dfrac{2}{3x}}$

3) $\dfrac{\left(\frac{1}{x-1}\right)+1}{\left(\frac{1}{x-1}\right)}$

4) $\dfrac{2\left(\frac{x+3}{x-2}\right)+3}{\left(\frac{x+3}{x-2}\right)}$

**Trigonometry** – more than half of PreCalculus involves the study or application of Trigonometry. You learned the basics in Algebra 2, we expand upon that without a review of what you've already learned. The following topics should be solid for you:

**Convert between radians & degrees** – our primarily focus will be on radian measure in PreCalculus. You should be able to convert back & forth between radians & degrees.

1) Convert each of the degree measures to radians.     a) 30°     b) 90°

2) Convert each of the radian measures to degrees.     a) $\dfrac{2\pi}{3}$     b) $4\pi$
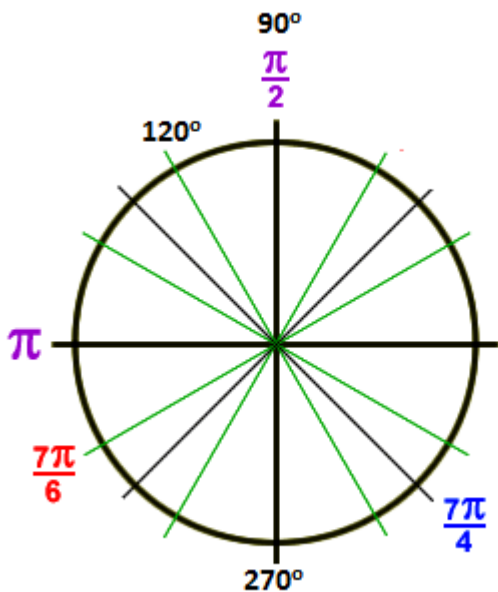
You should know all the radian conversions for angles around a circle. Fill in the degree and radian measures for the special angles in the following circle. (A few have been filled in, either partially or completely, to get you started).



**Find the _exact_ trigonometric value _without using a calculator_.** Results should be in simplest radical form (rationalize the denominator if necessary).
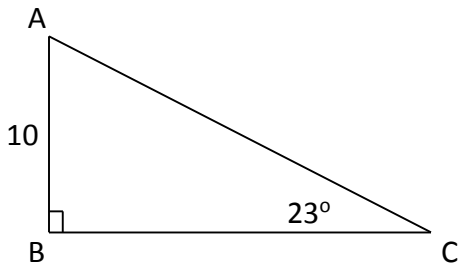
1) $\sin 60^o$

2) $\tan\dfrac{\pi}{6}$

3) $\cos 45^o$

4) $\cos\dfrac{\pi}{3}$

5) $\tan\dfrac{\pi}{2}$

6) $\sin\dfrac{\pi}{2}$

7) $\sin\dfrac{\pi}{4}$

8) $\tan 45°$

9) $\cos 30°$

10) $\cos\pi$

11) $\sin\dfrac{\pi}{6}$

12) $\tan 60°$

11) **Find the sine, cosine and tangent for the angle shown.** (simplest radical form, do not find θ)
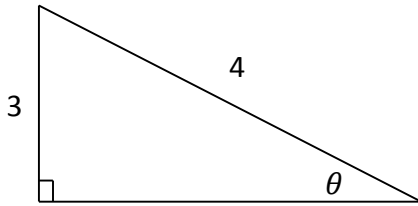


11) If $\tan\theta = \frac{12}{5}$, find $\sin\theta$. Do not find $\theta$.

12) Solve the right triangle (use degree mode).  Show all work and round answers to the hundredth.



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**Trigonometric Problem Solving**
Solve the following <u>using trig ratios</u> (<u>not</u> law of sines or cosines) – show your work

13)  Mr. Willis is cleaning his gutters.  If he places his ladder so that the foot of the ladder is 6 feet from the his house and the ladder makes an angle of 68° with the ground, how long is the ladder that he is using? Round to the <u>nearest whole number.</u>

14)  A plane takes off at an angle of elevation of 15°.  After traveling 1 mile along its flight path, how high (to the nearest foot) is the plane above the ground?  (1 mile = 5,280 feet)

15)  A woman stands 12 feet from a statue.  The angle of elevation from eye level to the top of the statue is 30°, and the angle of depression to the base of the statue is 25°.  How tall is the statue?  Round final result to the tenth.

**Law of Sines and Cosines –** you learned these in Alg 2. We will not use them in PreCalculus. They are something you should know for the math subject area SAT test if you choose to take it.

**If you had difficulty with any of the problems in this packet up to this point, you MUST go back & get some help and extra practice on this material over the summer. Fluency in all these topics is essential for a successful year in PreCalculus.**

## Pre-assessment Benchmark

Please do this following problem ON YOUR OWN. We are measuring what you are able to figure out yourself without outside assistance. This helps us in determining what problem solving skills our new students come to class with. Show all your work on separate paper. This will be scored but WILL NOT BE CALCULATED INTO YOUR COURSE GRADE. The score is a benchmark measurement for us to work on improving!

**PROBLEM SOLVING** – You should be able to apply any skills from previous math courses in solving a real life problem. This can be challenging in that you need to make a model (draw a picture) and consider any algebra or geometry you have learned over your high school mathematics career. Your ability to model a situation with mathematics/algebra and to persevere in solving a challenging problem is an important indicator of success in higher-level mathematics classes. Consider the following problem situation and solve the problem. Show all work and explain what process you used. Guess & Check is not a valid algebraic process. You may use various resources to help you but DO NOT just copy a method from the internet or other resource. However, please WORK ON YOUR OWN – do not work with other students, an adult or a tutor on this problem.

 Caution – some internet methods use CALCULUS (finding the derivative) on this type of problem – there is a much easier way using methods from Algebra 2.  Do not use the derivative!  Remember you must understand and explain what you did in solving.

**PROBLEM – A Norman window is going to be built with the perimeter of 60 ft. A Norman window has the shape of a rectangle with a semi-circle attached at the top.  The diameter of the circle is equal to the width of the rectangle.  Your job is to determine the dimensions of the Norman window that allows the maximum amount of light to pass through.**

*Please work through this problem on separate paper that is separate from the work paper in the rest of this packet (the Norman Window problem will be scored separately as a benchmark). Remember to explain in words your process.*