Summer Skills Review Name_____

Entering PreCalculus Level 3 2016-2017

95 points

This packet of material reviews all the topics in Algebra 2 that you covered this past school year that are essential background topics necessary for success in level 3 PreCalculus. This packet will be collected on the first day of school in your PreCalculus class. It is expected that you will complete all the problems with all work shown (use separate paper if necessary). THESE TOPICS WILL NOT BE RETAUGHT IN PRECALCULUS CLASS.

Functions & Function Operations

1) Given: $f(x) = x^2 - 3x$; $g(x) = 5 - 2x$; Find each of the following:

a) $f(-5)$ b) $f(g(4))$ c) $g(f(-1))$

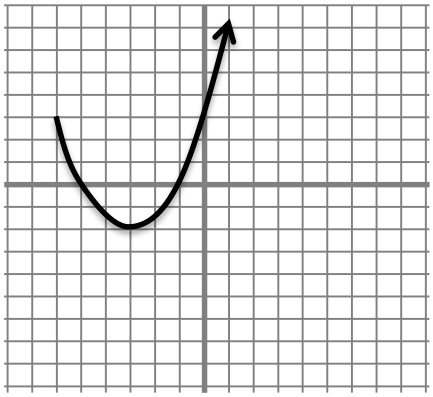
d) $f(g(x))$ *(simplify completely)*

e) $g(f(x))$ *(simplify completely)*

f) $g^{-1}(x)$

Function Characteristics

2) Determine the key characteristics of the function below, including domain, range, increasing/decreasing, LEB and REB (end behavior), extrema and intercepts. Use interval notation when applicable. Also, sketch the inverse of this function on the given graph. Is the inverse also a function?



domain_____

range_____

decreasing_____

increasing_____

LEB _____

REB_____

Minimums _____ maximum(s) _____

Intercepts _____

3) Sketch the graph of the function $f(x) = x^3 - 6x^2 + 3x - 7$. Include domain, range, increasing/decreasing, LEB and REB (end behavior), extrema, and intercepts. Use your graphing calculator to help you, specify the window used. Round values to the thousandth if necessary.

Graph: window_____

Domain_____

Range_____

Decreasing_____

Increasing_____

LEB _____

REB_____

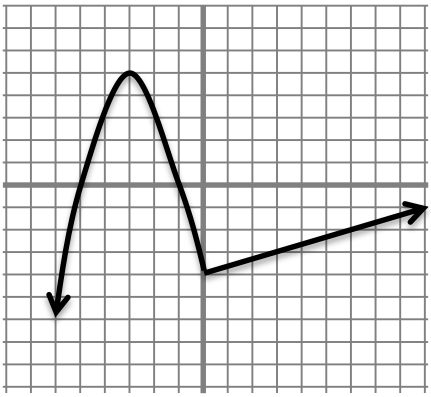
Maximum(s) _____

Minimum(s)_____

Intercepts_____

4) The piecewise function, $p(x)$, is pictured below. We will use this as our "parent function".
We will apply transformations to this parent function resulting in the new function:

$$q(x) = -2p(x-3) + 4$$



List the transformations that are applied to the parent function to create the new function equation:

Then apply the transformations to the parent function and draw the new graph here (use 6 critical points to help you):

Basic Algebra Skills

There are some basic factoring patterns that you should be able to apply to any polynomial. Factor the following (they are all factorable)

1) $x^2 - 16x + 64$

2) $x^2 - 100$

3) $x^2 - 5x - 6$

4) $2x^2 + 7x + 3$

5) $2x^2 + 5x - 3$

6) $18x^2 - 2$

Quadratic Functions

You should be able to solve quadratic polynomials over the real and complex numbers without the aid of a calculator. It is expected that you will use the most efficient method for the problem (factoring over the use of the quadratic formula if possible). Solve the following, show all work. Answers should be in simplest radical form, NOT decimal approximations.

1) $x^2 - 7x + 12 = 0$

2) $x^2 - 5x = 0$

3) $x^2 - 49 = 0$

4) $x^2 + 2x + 7 = 0$

5) $x^2 + 14 = 10x$

Rational Functions – You should be able to do all algebraic skills with rational expressions. Simplify the following:

1) $\frac{x^2-2x}{x^2+2x+1} \cdot \frac{x^2+4x+3}{x^2+3x}$

2) $(x^2 + 10x - 24) \div \frac{x^2-144}{3x-36}$

Rational Functions continued. Simplify or Solve

3) $\dfrac{\left(\frac{1}{x+9}+\frac{1}{5}\right)}{\left(\frac{2}{x^2+10x+9}\right)}$

4) $\dfrac{6}{x}-\dfrac{7x}{5}=\dfrac{x}{10}$

Exponential & Logarithmic Functions

Evaluate each logarithmic expression.

1) $\log_2 \frac{1}{8}$

2) $\log 100$

3) $\ln e^4$

4) $\log_b b$

5) $\log_5 125$

6) $\log_{\frac{1}{2}} 16$

7) $\log_4 \frac{1}{2}$

8) ln 1

Use the properties of logs to condense into an expression with a single log.

8) $2\log x+\log y-\frac{1}{2}\log z$

Use the properties of logs to expand

9) $\ln\left(\frac{10x^3y}{z^2}\right)$

Solve the equation, round to simplest form if possible or else to the nearest thousandth if necessary. Beware of extraneous solutions.

10) $8 = 2^{2x-9}$

11) $3^x \cdot 9^{x-5} = 3^{6-x}$

12) $10 + 0.1e^{3x} = 18$

13) $3\log_2(x^2 - 4) + 5 = 17$

14) $\ln(x - 3) + \ln(x + 4) = 3 \ln 2$

Trigonometry

Find the exact trigonometric value without using a calculator. Results should be in simplest radical form (rationalize the denominator if necessary)

1) $\sin 60^o$

2) $\tan 30^o$

3) $\cos 45^o$

4) cos 60°

5) tan 60°

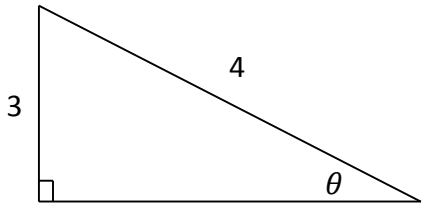
6) sin 30°

7) sin 45°

8) tan 45°

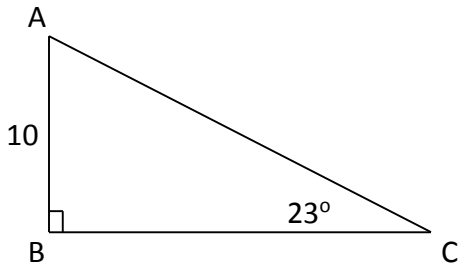
9) cos 30°

10) Find the sine, cosine and tangent for the angle shown. (simplest radical form, do not find θ)



11) If $\tan\theta = \frac{12}{5}$, find $\sin\theta$.

12) Solve the right triangle (use degree mode). Show all work and round answers to the nearest whole number.



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Trigonometric Problem Solving

Solve the following using trig ratios (not law of sines or cosines) – show your work

13) Mr. Willis is cleaning his gutters. If he places his ladder so that the foot of the ladder is 6 feet from the his house and the ladder makes an angle of 68° with the ground, how long is the ladder that he is using? Round to the nearest whole number.

14) A plane takes off at an angle of elevation of 15°. After traveling 1 mile along its flight path, how high (to the nearest foot) is the plane above the ground? (1 mile = 5,280 feet)

15) A woman stands 12 feet from a statue. The angle of elevation from eye level to the top of the statue is 30°, and the angle of depression to the base of the statue is 25°. How tall is the statue? Round final result to the tenth.

Law of Sines and Cosines

Find the missing sides and/or angles in each of the following triangles. Avoid using rounded found values to find other missing values. Round final answers to the nearest thousandth.

1) In ΔABC, m < A = 30°, m < B = 45°, a = 20 in.

m < C =_____

b = _____

c = _____

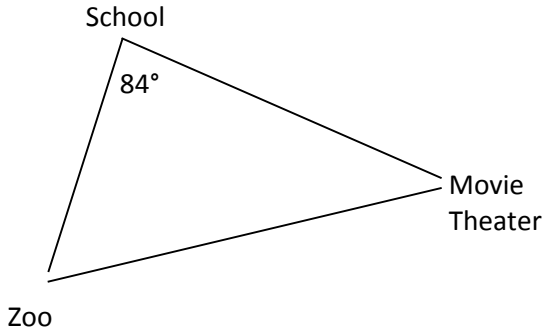
2) In ΔXYZ, x = 6 cm, y = 8 cm, z = 12 cm

m < X = _____

m < Y = _____

m < Z = _____

3) A zoo is 6.5 miles from a school and the school is 12 miles from a movie theater. The included angle between them is 84°. How far is the zoo from the movie theater? Round to the nearest mile.



4) Two observers are 150 feet apart on opposite sides of a hot air balloon. The observers are holding ropes which are attached to the top of the basket of the balloon. The angles of elevation from the observers to the basket are 23° and 32°. Round to the nearest tenth. Show all work, provide pictures of scenarios.

a. Find the lengths of the ropes.

b. Find the height of the hot air balloon.

If you had difficulty with any of the problems in this packet up to this point, you MUST go back & get some help and extra practice on this material over the summer. Fluency in all these topics is essential for a successful year in PreCalculus.

PROBLEM SOLVING – You should be able to apply any skills from previous math courses in solving a real life problem. This can be challenging in that you need to make a model (draw a picture) and consider any algebra or geometry you've learned over your high school mathematics career. Your ability to model a situation with mathematics/algebra and to persevere in solving a challenging problem is an important indicator of success in higher level mathematics classes. Consider the following problem situation and solve the problem. Show all work and explain what process you used. Guess & Check is not a valid algebraic process. You may use various resources to help you but DO NOT just copy a method from the internet or other resource. Caution – some internet methods use CALCULUS (finding the derivative) – there is a much easier way using methods from Algebra 2. Do not use the derivative! Remember you must understand and explain what you did in solving.

PROBLEM – A Norman window is going to be built with the perimeter of 30 ft. A Norman window has the shape of a rectangle with a semi-circle attached at the top. The diameter of the circle is equal to the width of the rectangle. Your job is to determine the dimensions of the Norman window that allows the maximum amount of light to pass through.

You may use separate paper.