1. A ladder is leaning against the outside wall of a building. The figure at the right shows the view from the end of the building, looking directly at the side of the ladder. The ladder is exactly 10 feet long and makes an angle of 60° with the ground. If the ground is level, what angle does the ladder make with the side of the building? How far up the building does the ladder reach (give an exact value and then approximate to the nearest inch)? Hint: Use a known trigonometric ratio in solving this problem.

Since one angle is 60°, the missing angle is 30°. Because this is a special 30°-60°-90° right angle and the hypotenuse is 10, this means the shorter side is 5. Then the measure of how high the ladder reaches on the wall is the long leg of the triangle and is therefore $5\sqrt{3}$.

Let’s look again at this problem using our calculators. First press Shift, Mode, 3. This puts our calculator in degrees. We were told the angle at the base of the ladder is 60° and the ladder is 10 feet long. We are asked to find how high on the wall the ladder reaches. This measure would represent the side opposite the given angle while the ladder represents the hypotenuse. Thus we can set up the sine ratio:

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$$

$$\sin(60°) = \frac{x}{10}$$

$$x = 10\sin(60°)$$

What did you get? $5\sqrt{3}$!
2. The main character in a play is delivering a monologue, and the lighting technician needs to shine a spotlight onto the actor's face. The light being directed is attached to a ceiling that is 10 feet above the actor's face. When the spotlight is positioned so that it shines on the actor's face, the light beam makes an angle of 20° with a vertical line down from the spotlight. How far is it from the spotlight to the actor's face? How much further away would the actor be if the spotlight beam made an angle of 32° with the vertical?

\[
\cos(20) = \frac{10}{x} \\
x = \frac{10}{\cos(20)} = 10.64\text{ft}
\]

\[
\cos(32) = \frac{10}{x} \\
x = \frac{10}{\cos(32)} = 11.79\text{ft} \\
11.79 - 10.64 = 1.15\text{ft}
\]

Consider the following problem. An ant (A) is on the ground and looks up to see a bird (B) looking down at him.

Since the ant elevated his eyes to see the bird, this is called an angle of elevation. The bird looked down to see the ant and this is called the angle of depression.

Assuming the ground is flat and parallel to the horizon, what can you say about \( \angle \text{HBA} \) and \( \angle \text{GAB} \)? They are congruent by the Alternate Interior Angle Postulate.

3. A forest ranger is on a fire lookout tower in a national forest. His observation position is 214.7 feet above the ground when he spots an illegal campfire. The angle of depression of the line of site to the campfire is 12°. What is the distance he viewed to see the fire?

\[
\sin(12) = \frac{214.7}{x} \\
x = \frac{214.7}{\sin(12)} \\
x = 1032.6\text{ft}
\]

4. An airport is tracking the path of one of its incoming flights. If the distance to the plane is 850 ft. (from the ground) and the altitude of the plane is 400 ft, then

a. What is the cosine of the angle of elevation?

Since cosine uses the adjacent side so we need to find it first. Let it be \( x \).

\[
x^2 + 400^2 = 850^2 \\
x^2 = 722500 - 160000 = 562500 \\
x = 750
\]

\[
\cos(\theta) = \frac{750}{850} = \frac{15}{17}
\]
4. An airport is tracking the path of one of its incoming flights. If the distance to the plane is 850 ft. (from the ground) and the altitude of the plane is 400 ft, then

b. Now using \( \cos(\theta) = \frac{15}{17} \)
to find the angle \( \theta \).
What did you get?
\[ \cos(\frac{15}{17}) = 28.07^\circ \]

c. Use \( \sin(\theta) = \frac{8}{17} \)
to find the angle \( \theta \).
What did you get?
\[ \sin(\frac{8}{17}) = 28.07^\circ \]

d. Why did you get the same answer each time?
Since the ratios are based from the same angle, you get the same angle solution.

5. The top of a billboard is 40 feet above the ground. What is the angle of elevation of the sun when the billboard casts a 30-foot shadow on level ground?

\[ \tan(\theta) = \frac{40}{30} = \frac{4}{3} \]
\[ \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ \]

6. An observer in a lighthouse sees a sailboat out at sea. The angle of depression from the observer to the sailboat is 6°. The base of the lighthouse is 50 feet above sea level and the observer’s viewing level is 84 feet above the base. (See the figure, which is not to scale.)
What is the distance from the sailboat to the observer? Let distance = \( d \).

Consider the triangle that includes a side that is 84+50, 134 ft. Since the angle of depression is 6°, the angle of elevation is also 6°.

\[ \sin(6) = \frac{134}{d} \]
\[ d = \frac{134}{\sin(6)} = 1282 \text{ ft} \]

7. The angle of elevation as Jeff looks up at a building is 56°. If Jeff is 140 ft away from the building and he is 6 ft tall, how tall is the building?

\[ \tan(56) = \frac{x}{140} \]
\[ x = 140\tan(56) \]
\[ x = 208 \]

Building = 208 + 6
Building = 214 ft
8. A ladybug looks up at an angle of 70° to see a bird. 25 feet behind the ladybug, another ladybug sees the same bird at an angle of 50°. How high is the bird to the nearest foot?

Let x represent the distance the first ladybug is from the point below the bird. Then:

\[ \tan(50) = \frac{h}{25 + x}, \quad \tan(70) = \frac{h}{x} \]

\[ h = (25 + x)\tan(50) = (x)\tan(70) \]

\[ (25 + x)1.1918 = (x)2.7475 \]

\[ 1.1918x + 29.7950 = 2.7475x \]

\[ -1.1918x = -1.5557x \]

\[ 29.7950 = 1.5557x \]

\[ x = 19.1522 \]

9. Students were challenged to use trig functions to find the length of AC.

Anne: \( \sin(48) = \frac{y}{12} \)

Bert: \( \cos(48) = \frac{x}{y} \)

Chad: \( \tan(42) = \frac{12}{x} \)

Debb: \( \sin(48) = \frac{y}{12} \)

Ella: \( \sin(42) = \frac{x}{18} \)

Fred: \( \cos(42) = \frac{x}{18} \)

Gina: \( \tan(48) = \frac{y}{12} \)

Huda: \( 18^2 - 12^2 = x^2 \)

Name those that are correct!

Chad, Debb, Fred and Gina.

Huda is correct but just doesn't listen
(Pythagorean Theorem is not a trig function)

10. A 20 ft tree throws a 7 foot shadow. What is the degree of elevation to the nearest degree?

\[ \tan(x) = \frac{20}{7} \]

\[ x = \tan^{-1}\left(\frac{20}{7}\right) = 71° \]