

Prove each identity:

1. $\sec x - \tan x \sin x = \frac{1}{\sec x}$

$$\frac{1}{\cos x} - \frac{\sin x \cdot \sin x}{\cos x} =$$

$$\frac{1 - \sin^2 x}{\cos x} =$$

$$\frac{\cos^2 x}{\cos x} =$$

$$\cos x = \frac{1}{\sec x} =$$

2. $\frac{1 + \cos x}{\sin x} = \csc x + \cot x$

$$\frac{1}{\sin x} + \frac{\cos x}{\sin x} =$$

$$\csc x + \cot x = \checkmark$$

3. $\frac{\sec \theta \sin \theta}{\tan \theta + \cot \theta} = \sin^2 \theta$

$$\frac{\frac{1}{\cos \theta} \cdot \frac{\sin \theta}{1}}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} = \frac{\tan \theta}{\tan \theta + \cot \theta}$$

$$\frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}} = \frac{\tan \theta}{\frac{1}{\sin \theta \cos \theta}}$$

4. $\frac{\sec \theta}{\cos \theta} \frac{\tan \theta}{\cot \theta} = 1$

$$\frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1 - \sin^2 \theta}{\cos^2 \theta} =$$

$$\frac{\cos^2 \theta}{\cos^2 \theta} =$$

$$1 = 1$$

5. $\cos^2 y - \sin^2 y = 1 - 2\sin^2 y$

$$1 - \sin^2 y - \sin^2 y =$$

$$1 - 2\sin^2 y = \checkmark$$

6. $\csc^2 \theta \tan^2 \theta - 1 = \tan^2 \theta$

$$(1 + \cot^2 \theta)(\tan^2 \theta) - 1 =$$

$$\tan^2 \theta + \cot^2 \theta \cdot \tan^2 \theta - 1 = 0$$

$$\tan^2 \theta + 1 - 1 = 0$$

$$\tan^2 \theta = \checkmark$$

7. $\frac{\sec^2 \theta}{\sec^2 \theta - 1} = \csc^2 \theta$

$$\frac{\sec^2 \theta}{\tan^2 \theta} =$$

$$\frac{1}{\cos^2 \theta} \cdot \frac{\cos^2 \theta}{\sin^2 \theta} =$$

$$\csc^2 \theta = \checkmark$$

8. $\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$

$$(\sec^2 x - 1)(\sin^2 x) =$$

$$\sec^2 x \cdot \sin^2 x - \sin^2 x =$$

$$\frac{1}{\cos^2 x} \cdot \sin^2 x - \sin^2 x =$$

$$\tan^2 x - \sin^2 x = \checkmark$$

$$9. (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$$

$$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta =$$

$$1 + 1 = 2$$

$$10. (\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \csc\theta$$

$$\sin\theta \tan\theta + \sin\theta \cot\theta + \cos\theta \tan\theta + \cos\theta \cot\theta =$$

$$\left(\frac{\sin^2\theta}{\cos\theta} + \cos\theta\right) + \left(\sin\theta + \frac{\cos^2\theta}{\sin\theta}\right)$$

$$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta} + \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$$

$$11. \frac{\tan\theta - 1}{\tan\theta + 1} = \frac{1 - \cot\theta}{1 + \cot\theta}$$

$$\frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}} = \frac{\frac{1}{\cot\theta} - \cot\theta}{\frac{1}{\cot\theta} + \cot\theta}$$

$$\frac{1 - \cot^2\theta}{\cot\theta + \cot\theta} = \frac{1 - \cot\theta \cdot \cot\theta}{\cot\theta + \cot\theta}$$

$$= \frac{1 - \cot\theta}{1 + \cot\theta}$$

$$12. \frac{1 - \tan^2 x}{1 + \tan^2 x} = 1 - 2\sin^2 x$$

$$\frac{1 - \tan^2 x}{\sec^2 x} = \frac{1 - \tan^2 x}{1 + \tan^2 x} \cdot \frac{1}{1} = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$\frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}} = \frac{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$$

$$= \frac{1 - 2\sin^2 x}{1} = 1 - 2\sin^2 x$$

$$13. \frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$$

$$\frac{\cos x + 1}{\sin x (1 - \cos^2 x)} = \frac{1}{\sin x (1 - \cos x)(1 + \cos x)}$$

$$\frac{1}{\sin x (1 - \cos x)} = \frac{1}{\sin x} \cdot \frac{1}{1 - \cos x}$$

$$14. \csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$$

$$(\csc^2 x - \cot^2 x)(\csc^2 x + \cot^2 x) = (1 + \cot^2 x)(\csc^2 x + \cot^2 x)$$

$$1(\csc^2 x + \cot^2 x) = \csc^2 x + \cot^2 x$$

$$15. \frac{\tan\theta}{\sec\theta} + \frac{\cot\theta}{\csc\theta} = \sin\theta + \cos\theta$$

$$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} = \sin\theta + \cos\theta$$

$$16. \frac{\csc x (1 - \cos x)}{1 - \cos x} = \frac{\sin y + \tan y}{1 + \sec y} = \sin y$$

$$\frac{\sin y + \frac{\sin y}{\cos y}}{1 + \frac{1}{\cos y}} = \sin y$$

$$\frac{\frac{\sin y \cdot \cos y + \sin y}{\cos y}}{\frac{\cos y + 1}{\cos y}} = \frac{\sin y (\cos y + 1)}{\cos y + 1} = \sin y$$