Unit 1: Similarity, Congruence, and Proofs

You should be able to:

- enlarge or reduce a geometric figure using a given scale factor.
- given a figure in the coordinate plane, determine the coordinates resulting from a dilation.
- compare geometric figures for similarity and describe similarities by listing corresponding parts.
- use scale factors, length ratios, and area ratios to determine side lengths and areas of similar geometric figures.
- perform basic constructions using a straight edge and compass and describe the strategies used.
- use congruent triangles to justify constructions.
- show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent (CPCTC).
- identify the minimum conditions necessary for triangle congruence (ASA, SAS, and SSS).
- understand, explain, and demonstrate why ASA, SAS, or SSS are sufficient to show congruence.
- prove theorems about lines and angles.
- prove theorems about triangles.
- prove properties of parallelograms

Vocabulary:

✓ Adjacent Angles: Angles in the same plane that have a common vertex and a common side, but no common interior points.

✓ Alternate Exterior Angles: Alternate exterior angles are pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on opposite sides of the transversal and are outside the other two lines. When the two other lines are parallel, the alternate exterior angles are equal.

✓ Alternate Interior Angles: Alternate interior angles are pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on opposite sides of the transversal and are in between the other two lines. When the two other lines are parallel, the alternate interior angles are equal.

✓ Angle: Angles are created by two distinct rays that share a common endpoint (also known as a vertex). \( \angle ABC \) or \( \angle B \) denote angles with vertex B.

✓ Bisector: A bisector divides a segment or angle into two equal parts.

✓ Centroid: The point of concurrency of the medians of a triangle.

✓ Circumcenter: The point of concurrency of the perpendicular bisectors of the sides of a triangle.

✓ Coincidental: Two equivalent linear equations overlap when graphed.

✓ Complementary Angles: Two angles whose sum is 90 degrees.

✓ Congruent: Having the same size, shape and measure. Two figures are congruent if all of their corresponding measures are equal.

✓ Congruent Figures: Figures that have the same size and shape.

✓ Corresponding Angles: Angles that have the same relative positions in geometric figures.

✓ Corresponding Sides: Sides that have the same relative positions in geometric figures

✓ Dilation: Transformation that changes the size of a figure, but not the shape.

✓ Endpoints: The points at an end of a line segment

✓ Equiangular: The property of a polygon whose angles are all congruent.
Equilateral: The property of a polygon whose sides are all congruent.

Exterior Angle of a Polygon: an angle that forms a linear pair with one of the angles of the polygon.

Incenter: The point of concurrency of the bisectors of the angles of a triangle.

Intersecting Lines: Two lines in a plane that cross each other. Unless two lines are coincidental, parallel, or skew, they will intersect at one point.

Intersection: The point at which two or more lines intersect or cross.

Inscribed Polygon: A polygon is inscribed in a circle if and only if each of its vertices lie on the circle.

Line: One of the basic undefined terms of geometry. Traditionally thought of as a set of points that has no thickness but its length goes on forever in two opposite directions. \(\overline{AB}\) denotes a line that passes through point A and B.

Line Segment or Segment: The part of a line between two points on the line. \(\overline{AB}\) denotes a line segment between the points A and B.

Linear Pair: Adjacent, supplementary angles. Excluding their common side, a linear pair forms a straight line.

Measure of each Interior Angle of a Regular n-gon:

Median of a Triangle: A segment is a median of a triangle if and only if its endpoints are a vertex of the triangle and the midpoint of the side opposite the vertex.

Midsegment: A line segment whose endpoints are the endpoint of two sides of a triangle is called a midsegment of a triangle.

Orthocenter: The point of concurrency of the altitudes of a triangle.

Parallel Lines: Two lines are parallel if they lie in the same plane and they do not intersect.

Perpendicular Bisector: A perpendicular line or segment that passes through the midpoint of a segment.

Perpendicular Lines: Two lines are perpendicular if they intersect at a right angle.

Plane: One of the basic undefined terms of geometry. Traditionally thought of as going on forever in all directions (in two-dimensions) and is flat (i.e., it has no thickness).

Point: One of the basic undefined terms of geometry. Traditionally thought of as having no length, width, or thickness, and often a dot is used to represent it.

Proportion: An equation which states that two ratios are equal.

Ratio: Comparison of two quantities by division and may be written as \(r/s\), \(r:s\), or \(r\) to \(s\).

Ray: A ray begins at a point and goes on forever in one direction.

Reflection: A transformation that "flips" a figure over a line of reflection

Reflection Line: A line that is the perpendicular bisector of the segment with endpoints at a pre-image point and the image of that point after a reflection.

Regular Polygon: A polygon that is both equilateral and equiangular.

Remote Interior Angles of a Triangle: the two angles non-adjacent to the exterior angle.

Rotation: A transformation that turns a figure about a fixed point through a given angle and a given direction.
✓ Same-Side Interior Angles: Pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on the same side of the transversal and are between the other two lines. When the two other lines are parallel, same-side interior angles are supplementary.

✓ Same-Side Exterior Angles: Pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on the same side of the transversal and are outside the other two lines. When the two other lines are parallel, same-side exterior angles are supplementary.

✓ Scale Factor: The ratio of any two corresponding lengths of the sides of two similar figures.

✓ Similar Figures: Figures that have the same shape but not necessarily the same size.

✓ Skew Lines: Two lines that do not lie in the same plane (therefore, they cannot be parallel or intersect).

✓ Sum of the Measures of the Interior Angles of a Convex Polygon: 180°(n – 2).

✓ Supplementary Angles: Two angles whose sum is 180 degrees.

✓ Transformation: The mapping, or movement, of all the points of a figure in a plane according to a common operation.

✓ Translation: A transformation that "slides" each point of a figure the same distance in the same direction.

✓ Transversal: A line that crosses two or more lines.

✓ Vertical Angles: Two nonadjacent angles formed by intersecting lines or segments. Also called opposite angles.

Practice Items

1) Figure A'B'C'D' is a dilation of figure ABCD.

a. Determine the center of dilation.
b. Determine the scale factor of the dilation.
c. What is the relationship between the sides of the pre-image and corresponding sides of the image?

[Key: A. The center of dilation is (4, 2). B. The ratio for each pair of corresponding sides is ½, so the scale factor is ½. C. Each side of the image is parallel to the corresponding side of its pre-image and is ½ the length.]
2) Use this triangle to answer the question.

This is a proof of the statement “If a line is parallel to one side of a triangle and intersects the other two sides at distinct points, then it separates these sides into segments of proportional lengths.”

<table>
<thead>
<tr>
<th>Step</th>
<th>Justification</th>
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<tbody>
<tr>
<td>1</td>
<td>OK is parallel to HJ</td>
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<tr>
<td>2</td>
<td>( \angle HGK \cong \angle HJH ) ( \angle IKG \cong \angle IJH )</td>
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<tr>
<td>3</td>
<td>( \triangle GK \sim \triangle HJU )</td>
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<tr>
<td>4</td>
<td>( \frac{IG}{IH} = \frac{IK}{IJ} )</td>
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<tr>
<td>5</td>
<td>( \frac{HG+IH}{IH} = \frac{JK}{IJ} )</td>
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<tr>
<td>6</td>
<td>( \frac{HG}{IH} = \frac{JK}{IJ} )</td>
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Which reason justifies Step 2?

A. Alternate interior angles are congruent.
B. Alternate exterior angles are congruent.
C. Corresponding angles are congruent.
D. Vertical angles are congruent.

[Key: C]

3) The figure shows quadrilateral \(JKLM\).

What information will NOT be used to prove that \(JKLM\) is a parallelogram?

A. Show that \(\angle JLM \cong \angle LJK\).
B. Show that \(JK \cong LM\).
C. Show that \(\triangle JKL \cong \triangle LMJ\).
D. Show that \(\triangle JKL \cong \triangle JLM\).

[Key: D]
4) Which transformation of $\triangle HIJ$ does NOT result in a congruent triangle?

A. a reflection across the x-axis, followed by a rotation of $180^\circ$ about the origin
B. a rotation by $180^\circ$ about the origin, followed by a translation of 2 units left and 3 units down
C. a translation of 1 unit right and 2 units up, followed by a dilation by a factor of 3
D. a dilation by a factor of 2, followed by a dilation by a factor of 0.5

5) Given in the figure below, line $l$ is the perpendicular bisector of $\overline{AB}$ and of $\overline{CD}$.

a. Show $\overline{AC} \cong \overline{BD}$ using rigid motions.

b. Show $\angle ACD \cong \angle BDC$.

c. Show $\overline{AB} \parallel \overline{CD}$.

6. Given in the figure below, line $l$ is the perpendicular bisector of $\overline{AB}$ and of $\overline{CD}$.

a. Show $\overline{AC} \cong \overline{BD}$ using rigid motions.

Since $l$ is the perpendicular bisector of $\overline{AB}$ and $\overline{CD}$, the reflection through line $l$ brings $A$ to $B$ and $C$ to $D$. Therefore, the two line segments $\overline{AC}$ and $\overline{BD}$ are congruent.

b. Show $\angle ACD \cong \angle BDC$.

The reflection through line $l$ brings $A$ to $B$ and $C$ to $D$. Therefore, $\angle ACD$ goes to $\angle BDC$. The image of $\triangle ACD$ is therefore congruent to $\triangle BDC$.

c. Show $AB \parallel CD$ because the perpendicular bisector intersects the two lines at congruent corresponding angles.
6) The graph of \( \triangle MPN \) is mapped to \( \triangle M'P'N' \) by a dilation with a scale factor of \( \frac{1}{2} \) and centered at point \( M \).

Which statement is true for this dilation?

A. \( \overline{PN} \) would have a slope of \(-3\) and a length of 1.58.

B. \( \overline{PN} \) would have a slope of \(-\frac{3}{2}\) and a length of 1.58.

C. \( \overline{PN} \) would have a slope of \(-3\) and coordinates of \( P'(6, 8) \) and \( N'(8, 2) \).

D. \( \overline{PN} \) would have a slope of \(-\frac{3}{2}\) and coordinates of \( P'(1.5, 2) \) and \( N'(2, 0.5) \).

[Key: A]

7) The coordinate plane shows \( \triangle LJM \) and that \( \overline{LM} \) and \( \overline{KN} \) have the same slope.

Explain how the information given determines that \( \triangle LJM \) and \( \triangle KJN \) are similar.

[Key: The lines with the same slope would be parallel, and this would prove that angles \( JKN \) and \( JLM \) are congruent by corresponding angles between parallel lines. Since both triangles share angle \( J \), the Angle-Angle Similarity Theorem can be used to prove similar triangles.]
8) A group of students are indirectly calculating the height of a bookshelf. They use Kaleb's height of 5.75 feet tall. Kaleb stands parallel to the bookshelf at a distance of 2.5 feet. The group places a long piece of wood on the edge of the bookshelf and angles it down so that it touches Kaleb's head and the ground. The point on the ground where the wood touches is 6.25 feet from Kaleb. The picture shows this situation.

What is the height of the bookshelf? Round your answer to the nearest foot.

[Key: 8 feet]

9) In the diagram shown, line $m$ is the perpendicular bisector of $KL$, line $n$ is the perpendicular bisector of $JK$, and lines $m$ and $n$ intersect at point $P$.

Explain which point(s), $J$, $K$, or $L$, is the closest to point $P$.

[Key: Since $P$ is on a perpendicular bisector of each segment, it is equidistant from each of those points]

10) A student made a construction based on these instructions.

- Construct a circle and draw its diameter with a straight edge.
- Mark the endpoints of the diameter on the circle with $P$ and $Q$.
- Construct the perpendicular bisector of the diameter.
- Mark the intersection of this line with the circle with the points $R$ and $S$.
- Connect the points $P$, $Q$, $R$, and $S$ on the circle with a straightedge.

Which construction did the student create?

A. a circle inscribed in a kite
B. a square inscribed in a circle
C. a circle inscribed in a square
D. a kite inscribed in a circle

[Key: B]
Unit 2: Right Triangle Trigonometry

You should be able to:

- Make connections between the angles and sides of right triangles
- Select appropriate trigonometric functions to find the angles/sides of a right triangle
- Use right triangle trigonometry to solve realistic problems

Vocabulary and Properties

- Adjacent side: In a right triangle, for each acute angle in the interior of the triangle, one ray forming the acute angle contains one of the legs of the triangle and the other ray contains the hypotenuse. This leg on one ray forming the angle is called the adjacent side of the acute angle.

- For any acute angle in a right triangle, we denote the measure of the angle by $\theta$ and define three numbers related to $\theta$ as follows:

  - sine of $\theta = \sin(\theta) = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$
  - cosine of $\theta = \cos(\theta) = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$
  - tangent of $\theta = \tan(\theta) = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$

- Angle of Depression: The angle below horizontal that an observer must look to see an object that is lower than the observer. Note: The angle of depression is congruent to the angle of elevation (this assumes the object is close enough to the observer so that the horizontals for the observer and the object are effectively parallel; this would not be the case for an astronaut in orbit around the earth observing an object on the ground).

- Angle of Elevation: The angle above horizontal that an observer must look to see an object that is higher than the observer. Note: The angle of elevation is congruent to the angle of depression (this assumes the object is close enough to the observer so that the horizontals for the observer and the object are effectively parallel; this would not be the case for a ground tracking station observing a satellite in orbit around the earth).

- Complementary angles: Two angles whose sum is $90^\circ$ are called complementary. Each angle is called the complement of the other.

- Opposite side: In a right triangle, the side of the triangle opposite the vertex of an acute angle is called the opposite side relative to that acute angle.

- Similar triangles: Triangles are similar if they have the same shape but not necessarily the same size.

Triangles whose corresponding angles are congruent are similar. Corresponding sides of similar triangles are all in the same proportion. Thus, for the similar triangles shown at the right with angles $A$, $B$, and $C$ congruent to angles $A'$, $B'$, and $C'$ respectively, we have that: we have that:

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}.$$
Practice Items:

1) Jane and Mark each build ramps to jump their remote-controlled cars. Both ramps are right triangles when viewed from the side. The incline of Jane's ramp makes a 30-degree angle with the ground, and the length of the inclined ramp is 14 inches. The incline of Mark's ramp makes a 45-degree angle with the ground, and the length of the inclined ramp is 10 inches.

Part A: What is the horizontal length of the base of Jane's ramp and the base of Mark's ramp? Show or explain your work.
Part B: Which car is launched from the highest point? Justify your answer by showing or explaining your work.

2) A radio tower is anchored by long cables called guy wires, such as AB in the figure below. Point A is 250 m from the base of the tower, and \( \angle BAC = 59^\circ \).

   a. How long is the guy wire? Round to the nearest tenth.
   b. How far above the ground is it fastened to the tower?
   c. How tall is the tower, DC, if \( \angle DAC = 71^\circ \)?

3) In right triangle ABC, angle A and angle B are complementary angles. The value of \( \cos A \) is \( \frac{5}{13} \). What is the value of \( \sin B \)?

   A. \( \frac{5}{13} \)  
   B. \( \frac{12}{13} \)  
   C. \( \frac{12}{13} \)  
   D. \( \frac{5}{13} \)
4) In the picture shown, $\triangle TVW$ and $\triangle XYZ$ are similar right triangles.

$\sin \angle ZXY$? Round your final answer to the nearest tenth.

[Key: 0.7]

5) In the figure shown, $\triangle WXY$ is similar to $\triangle WZY$. Let $n$ represent the length, in centimeters, of $\overline{WY}$.

Which ratio represents the correct value of $\sin \angle WYZ$?

A. $\frac{4}{5}$
B. $\frac{9}{16}$
C. $\frac{9}{25}$
D. $\frac{16}{25}$

[Key: A]

6) In the figure shown, $\triangle XYZ$ is a right triangle and $\sin X = \frac{15}{17}$.

Explain how to determine $\cos Z$.

[Key: Angles X and Z are complementary angles, so the sine of one angle would equal the cosine of the other]
7) Right triangle $XYZ$ has a right angle at $X$. The value of $\cos \angle Z = \frac{6}{11}$. What is the value of $\cos \angle Y$?

A. $\frac{5}{11}$
B. $\frac{6}{11}$
C. $\frac{\sqrt{85}}{11}$
D. $\frac{\sqrt{157}}{11}$

[Key: C]

8) Triangle $PRT$ is a right triangle. Segment $QS$ intersects $\overline{PR}$ and $\overline{TR}$ as shown.

Part A
Determine the length of $\overline{QR}$. Round your answer to the nearest whole unit. Show your work.

Part B
Use the relationship between the sine and cosine of complementary angles to explain how to use $\angle SQR$ to determine the length of $\overline{QR}$.
Analytic Geometry Study Guide

Unit 3: Circles and Volume

You should be able to:

- Select an appropriate theorem or formula to use to solve a variety of situations involving circles, their segments, and the angles created, as well as volumes of such solids as the cylinder, cone, pyramid, and sphere.
- Construct Inscribed and Circumscribed Circles of triangles
- Complete a Formal Proof of the opposite angles of an Inscribed Quadrilateral being supplementary.
- Find the Arc Length and Area of any sector of a circle
- Use Cavalieri’s Principle to show that the Volume of an Oblique Solid can be found using Right Solids.

Vocabulary

✓ **Arc**: an unbroken part of a circle; minor arcs have a measure less than 180°; semi-circles are arcs that measure exactly 180°; major arcs have a measure greater than 180°

✓ **Arc Length**: a portion of the circumference of the circle

✓ **Arc Measure**: The angle that an arc makes at the center of the circle of which it is a part.

✓ **Cavalieri’s Principle**: A method, with formula given below, of finding the volume of any solid for which cross-sections by parallel planes have equal areas. This includes, but is not limited to, cylinders and prisms.

  ✓ **Formula**: Volume = Bh, where B is the area of a cross-section and h is the height of the solid.

✓ **Central Angle**: an angle whose vertex is at the center of a circle

✓ **Chord**: a segment whose endpoints are on a circle

✓ **Circumcenter**: The point of intersection of the perpendicular bisectors of the sides of a given triangle; the center of the circle circumscribed about a given triangle.

✓ **Circumscribed Circle**: a circle containing an inscribed polygon; for this unit the polygon will be a triangle and so the center of the circle will be the circumcenter of the triangle.

✓ **Composite Figures**: If a figure is made from two or more geometric figures, then it is called a Composite Figure.

✓ **Inscribed**: an inscribed planar shape or solid is one that is enclosed by and “fits snugly” inside another geometric shape or solid.

✓ **Inscribed Angle**: an angle whose vertex is on the circle and whose sides contain chords of a circle

✓ **Inscribed Circle**: a circle enclosed in a polygon, where every side of the polygon is a tangent to the circle; specifically for this unit the polygon will be a triangle and so the center of the Inscribed Circle is the incenter of the triangle.

✓ **Inscribed Polygon**: a polygon whose vertices all lie on a circle

✓ **Lateral Area**: The sum of the areas of the lateral (vertical) faces of a cylinder, cone, frustum or the like.
✓ **Major and Minor Arcs**: Given two points on a circle, the minor arc is the shortest arc linking them. The major arc is the longest.

✓ **Point of Tangency**: the point where a tangent line touches a circle.

✓ **Secant Line**: a line in the plane of a circle that intersects a circle at exactly two points

✓ **Secant Segment**: a segment that contains a chord of a circle and has exactly one endpoint outside of the circle

✓ **Sector**: the region bounded by two radii of the circle and their intercepted arc

✓ **Slant Height**: The diagonal distance from the apex of a right circular cone or a right regular pyramid to the base.

✓ **Tangent Line**: a line in the plane of a circle that intersects a circle at only one point, the point of tangency

**Practice Items**

1) In the circle shown, $\overline{BC}$ is a diameter and $m\angle B = 120^\circ$.

![Diagram of a circle with diameter BC and angle B as 120°]

What is the measure of $\angle ABC$?

A. 15°

B. 30°

C. 60°

D. 120°

[Key; B]

2) Circle $E$ is shown.

![Diagram of a circle with arc CD and angle C as 145°]

What is the length of $\overline{CD}$?

A. $\frac{29}{72}\pi$ yd.

B. $\frac{29}{6}\pi$ yd.

C. $\frac{29}{3}\pi$ yd.

D. $\frac{29}{2}\pi$ yd.

[Key; C]
3) Billy is creating a circular garden divided into 8 equal sections. The diameter of the garden is 12 feet.

What is the area of one section of the garden? Explain how you determined your answer.

[Key: I can find the area of the entire circle and divide by 8. This equals 4.5π.]

4) Circle Y is shown.

What is the area of the shaded part of the circle?

A. \(\frac{57}{4}\pi \text{ cm}^2\)
B. \(\frac{135}{8}\pi \text{ cm}^2\)
C. \(\frac{405}{8}\pi \text{ cm}^2\)
D. \(\frac{513}{8}\pi \text{ cm}^2\)

[Key: D]

5) Jason constructed two cylinders using solid metal washers. The cylinders have the same height, but one of the cylinders is slanted as shown.

Which statement is true about Jason’s cylinders?

A. The cylinders have different volumes because they have different radii.
B. The cylinders have different volumes because they have different surface areas.
C. The cylinders have the same volume because each of the washers has the same height.
D. The cylinders have the same volume because they have the same cross-sectional area at every plane parallel to the bases.

[Key: D]
6) Which polygon inscribed in a circle has an area closest to π square feet?

[Key: D]

7) Which statement is true for any two circles?

A The ratio of the areas of the circles is the same as the ratio of their radii.
B The ratio of the circumferences of the circles is the same as the ratio of their radii.
C The ratio of the areas of the circles is the same as the ratio of their diameters.
D The ratio of the areas of the circles is the same as the ratio of their circumferences.

[Key: B]

8) The diagram shows the vertices of quadrilateral $PQRS$ which lie on circle $T$.

Which conjecture can be proven to be true about quadrilateral $PQRS$?

A. Arc $PS$ is congruent to arc $QR$.
B. The diagonals of $PQRS$ intersect at point $T$.
C. The measures of $\angle S$ and $\angle Q$ are congruent.
D. The measures of $\angle P$ and $\angle R$ are supplementary.

[Key: D]
9) A line segment has endpoints $X(-3, 1)$ and $Y(-1, -7)$. Segment $KL$ is a diameter of circle $C$ and a radius of circle $L$ as shown.

The points $P(-9, -9)$ and $R(1, -15)$ lie on circle $L$. The point $F(-6, -4)$ lies on circle $C$.

**Part A**

What is the measure of $\angle XPR$? What is the measure of $\angle XFY$? Show your work or explain your answer.

Describe the relationship between each angle and the diameter of the circle the angle is inscribed in.

**Part B**

Determine the lengths of $EY$ and $TR$. Show your work.

Describe the relationship between the length of the arc and the radius of a circle. Explain your answer.

[Key:](#)
10) Sphere $S$ is placed inside a right cone so that point $R$ on the sphere is the center of the base of the cone. Point $V$ represents the vertex of the cone. The sphere is tangent to the cone at points $X$ and $Y$. Points $R$, $S$, and $V$ are collinear. The diagram represents this situation.

**Part A**
What is the radius, in centimeters, of the sphere? Show your work and explain your answer.

**Part B**
What is the percentage of the volume of the cone occupied by the sphere? Show your work and explain your answer.
You should be able to:

- Make connections between radicals and fractional exponents
- Distinguish between real and imaginary numbers. For example, \((-81)^{\frac{3}{4}}\) as opposed to \((-81)^{\frac{3}{4}}\)
- Results of operations performed between numbers from a particular number set does not always belong to the same set. For example, the sum of two irrational numbers \((2 + \sqrt{3})\) and \((2 - \sqrt{3})\) is 4, which is a rational number; however, the sum of a rational number 2 and irrational number \(\sqrt{3}\) is an irrational number \((2 + \sqrt{3})\)
- Realize that irrational numbers are not the same as complex numbers

Vocabulary

- **Binomial Expression**: An algebraic expression with two unlike terms.
- **Complex Conjugate**: A pair of complex numbers, both having the same real part, but with imaginary parts of equal magnitude and opposite signs. For example, \(3 + 4i\) and \(3 - 4i\) are complex conjugates.
- **Complex number**: A complex number is the sum of a real number and an imaginary number (a number whose square is a real number less than zero), i.e. an expression of the form \(a + bi\), where \(a\) and \(b\) are real numbers and \(i\) is the imaginary unit, satisfying \(i^2 = -1\).
- **Exponential functions**: A function of the form \(y = a^x\) where \(a > 0\) and either \(0 < b < 1\) or \(b > 1\).
- **Expression**: A mathematical phrase involving at least one variable and sometimes numbers and operation symbols.
- **Monomial expression**: An algebraic expression with one term.
- **Nth roots**: The number that must be multiplied by itself \(n\) times to equal a given value. The \(n\)th root can be notated with radicals and indices or with rational exponents, i.e. \(a^{\frac{1}{n}}\) means the cube root of \(a\).
- **Polynomial function**: A polynomial function is defined as a function, \(f(x) =\), where the coefficients are real numbers.
- **Rational exponents**: For \(a > 0\), and integers \(m\) and \(n\), with \(n > 0\), \(a^{\frac{m}{n}} = (\sqrt[n]{a})^m\); \(a^{\frac{1}{n}}\) is a real number.
- **Rational expression**: A quotient of two polynomials with a non-zero denominator.
- **Rational number**: A number expressible in the form \(a/b\) or \(-a/b\) for some fraction \(a/b\). The rational numbers include the integers.
- **Standard Form of a Polynomial**: To express a polynomial by putting the terms in descending exponent order.
- **Trinomial**: An algebraic expression with three unlike terms.
- **Whole numbers**: The numbers 0, 1, 2, 3, ....
- **The properties of operations**: Here \(a, b\) and \(c\) stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

    Associative property of addition \((a + b) + c = a + (b + c)\)

    Commutative property of addition \(a + b = b + a\)

    Additive identity property of 0 \(a + 0 = 0 + a = a\)

    Existence of additive inverses For every \(a\) there exists \(-a\) so that \(a + (-a) = (-a) + a = 0\).

    Associative property of multiplication \((a \times b) \times c = a \times (b \times c)\)

    Commutative property of multiplication \(a \times b = b \times a\)

    Distributive property of multiplication over addition \(a \times (b + c) = a \times b + a \times c\)
Practice Items:

1) The volume of a box is \( a^3 + 8a^2 + 19a + 12 \). Its length is \( a + 4 \) units and its width is \( a + 3 \) units. What is its height?
   a. \( a + 4 \)
   b. \( a + 3 \)
   c. \( a + 2 \)
   d. \( a + 1 \)

   [Key: D]

2) Which expression is equivalent to \((12 - 5i) / (2 - i)\)?
   a. \( \frac{19}{3} - \frac{22}{3}i \)
   b. \( \frac{29}{3} + \frac{2}{3}i \)
   c. \( \frac{19}{5} - \frac{22}{5}i \)
   d. \( \frac{29}{5} + \frac{2}{5}i \)

   [Key: D]

3) The figure shows a rectangular region divided into four rectangular regions, three of which have areas \( 5x \), \( 5x \), and \( x^2 \), respectively. What is the area of the blank rectangle?

   a. 9
   b. 25
   c. 30
   d. 34

   [Key: B]

4) Evaluate \((-64)^{3/2}\)
   a. 512
   b. -512i
   c. 512i
   d. -512

   [Key: B]

5) A train travels at a rate of \((4x + 5)\) miles per hour. How many miles can it travel at that rate in \( (x - 1) \) hours?
   a. \( 3x - 4 \) miles
b. \(5x - 4\) miles

c. \(4x^2 + x - 5\) miles

d. \(4x^2 - 9x - 5\) miles

6) Cassandra claims that the number \(\left(\frac{\sqrt{16}}{4}\right)^3\) is an irrational number. Explain whether Cassandra's claim is true.

[Key: Her claim is not correct because the expression is equal to 4 and 4 is a rational number]

7) Explain why any of the operations \((+, -, \times, +)\) between the two terms \(8\) and \(-2\sqrt{6}\) will result in an irrational number.

[Key: The square root of 6 will remain part of the solution regardless of the operation performed]

8) Brianna stated that all four numbers listed below were part of the complex number system. Antonia stated that at least one of the numbers was not part of the complex number system.

\[
\begin{align*}
n_1 &= \sqrt{4} \\
n_2 &= 6 - \sqrt{4} \\
n_3 &= 5i \\
n_4 &= \sqrt{25} + 3i
\end{align*}
\]

Explain which person is correct.

[Key: Brianna is correct because every number in the complex number system can be written in the form \(a + bi\), where \(a\) and \(b\) are real numbers]

9) The picture shows the dimensions, in centimeters, of two rectangles.

Write an expression in simplest form that represents the difference in the areas, in square centimeters, of rectangle A and rectangle B.

[Key: \(x^2 + 7x - 19\)]

10) What is the conjugate of the complex number \(\frac{5}{4} + \frac{1}{7}\)i? Explain your answer.

[Key: \(\frac{5}{4} - \frac{1}{7}i\)]
Unit 5: Quadratic Functions

You should be able to:

- Given raw data, graph and interpret a quadratic function.
- Given raw data, graph and interpret a quadratic inequality.
- Translate the graph of \( f(x) = x^2 \) as directed.
- Determine and use the most advantageous method of finding the zeros of a quadratic equation or inequality.
- Determine the variable rate of change of a quadratic function.
- Model data by finding intercepts, relative maxima and minima, and other important information.

Formulas and Definitions:

- **Complete factorization over the integers**: Writing a polynomial as a product of polynomials so that none of the factors is the number 1, there is at most one factor of degree zero, each polynomial factor has degree less than or equal to the degree of the product polynomial, each polynomial factor has all integer coefficients, and none of the factor polynomial can written as such a product.
- **Completing the Square**: Completing the Square is the process of converting a quadratic equation into a perfect square trinomial by adding or subtracting terms on both sides.
- **Difference of Two Squares**: A squared (multiplied by itself) number subtracted from another squared number. It refers to the identity \( a^2 - b^2 = (a + b)(a - b) \) in elementary algebra.
- **Discriminant of a quadratic equation**: The discriminant of a quadratic equation of the form \( ax^2 + bx + c = 0 \), \( a \neq 0 \), is the number \( b^2 - 4ac \).
- **Horizontal shift**: A rigid transformation of a graph in a horizontal direction, either left or right.
- **Perfect Square Trinomial**: A trinomial that factors into two identical binomial factors.
- **Quadratic Equation**: An equation of degree 2, which has at most two solutions.
- **Quadratic Function**: A function of degree 2 which has a graph that “turns around” once, resembling an umbrella-like curve that faces either rightside up or upside down. This graph is called a parabola.
- **Root**: The x-values where the function has a value of zero.
- **Standard Form of a Quadratic Function**: \( ax^2 + bx + c \)
- **Vertex**: The maximum or minimum value of a parabola, either in terms of y if the parabola is opening up or down, or in terms of x if the parabola is opening left or right.
- **Vertex form of a quadratic function**: A formula for a quadratic equation of the form \( f(x) = a(x - h)^2 + k \), where \( a \) is a nonzero constant and the vertex of the graph is the point \((h, k)\).

Theorems:

- For \( h = \frac{-b}{2a} \) and \( k = f\left(\frac{-b}{2a}\right) \), \( f(x) = a(x - h)^2 + k \) is the same function as \( f(x) = ax^2 + bx + c \).

- The graph of any quadratic function can be obtained from transformations of the graph of the basic function \( f(x) = x^2 \).

- Quadratic formula: The solution(s) of the quadratic equation of the form \( ax^2 + bx + c = 0 \), where \( a, b, \) and \( c \) are real numbers with \( a \neq 0 \), is
  \[
  x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
  \]

- The discriminant of a quadratic equation is positive, zero, or negative if and only if the equation has two real solutions, one real solution, or two complex conjugate number solutions respectively.
Practice Items

1) This equation can be used to determine a number such that the square of the number is 25 less than 10 times the number.

\[ x^2 + 25 = 10x \]

Which of the following is equivalent to the equation?

A. \( x^2 = 5x \)
B. \( x^2 = -15x \)
C. \( (x-5)^2 = 0 \)
D. \( (x+5)(x-5) = 0 \)

[Key: C]

2) This graph shows a set of data from a study of the nitrogen concentration in plants with multiple base stems.

Which equation BEST fits the data in the graph?

A. \( y = 0.38x + 18.08 \)
B. \( y = -0.38x + 18.08 \)
C. \( y = 0.00058x^2 + 0.38x + 18.08 \)
D. \( y = -0.00058x^2 + 0.38x + 18.08 \)

[Key: D]

3) Which function best represents a quadratic function of \( x \)?

A. \[ \begin{array}{c|c|c|c|c|c} x & 0 & 1 & 2 & 3 & 4 \\ \hline f(x) & 0 & 2 & 4 & 6 & 8 \end{array} \]
B. \[ \begin{array}{c|c|c|c|c|c} x & 0 & 1 & 2 & 3 & 4 \\ \hline g(x) & 1 & 3 & 9 & 19 & 33 \end{array} \]
C. \[ \begin{array}{c|c|c|c|c|c} x & 0 & 1 & 2 & 3 & 4 \\ \hline h(x) & 1 & 2 & 4 & 8 & 16 \end{array} \]
D. \[ \begin{array}{c|c|c|c|c|c} x & 0 & 1 & 2 & 3 & 4 \\ \hline k(x) & 0 & 10 & 20 & 30 & 40 \end{array} \]

[Key: B]
4) As the value of $x$ increases, which function has the greatest rate of growth?

A $f(x) = x^2 + 7$
B $g(x) = 2x^4$
C $h(x) = 7 - x^2$
D $k(x) = 2x^2 + 7$

[Key: B]

5) Use these functions to answer this question.

$P(x) = x^2 - x - 6$
$Q(x) = x - 3$

What is $P(x) - Q(x)$?

A $x^2 - 3$
B $x^2 - 9$
C $x^2 - 2x - 3$
D $x^2 - 2x - 9$

[Key: C]

6) Miranda correctly solved for the zeros of the function $f(x) = x^2 + x + 2$. What conclusion can she make about the graph of the function based on those zeros?

[Key: Since the zeros of the function are non-real, the graph of the function does not intersect the x-axis]

7) To represent the area of a walkway around a square garden, an architect used the function $f(x) = (2x + 8)(2x + 8) - (8)(8)$, where the garden has a length of 8 feet and the walkway was $x$ feet wide.

Part A
Write the function as a difference of two squares.

Part B
Factor the function as a difference of two squares.

Part C
If the area of the walkway is 192 square feet, determine the roots of the function. Show your work.

Part D
Explain what your answer to Part C represents in terms of the situation it models.
8) The graph of a quadratic equation is shown on the coordinate plane.

![Graph Image]

Write the equation, in standard form, that represents the graph.

\[ y = -x^2 + 6x - 3 \]

9) The equations of a line and a parabola are shown.

\[ y = 3x - 4 \]
\[ y = 2(x - 1.5)^2 - 12.5 \]

Which expression represents the \( x \)-coordinate of a point of intersection of the graphs of the equations?

A. \( \frac{3 - \sqrt{41}}{4} \)
B. \( \frac{3 + \sqrt{105}}{4} \)
C. \( \frac{9 + \sqrt{113}}{4} \)
D. \( \frac{9 - \sqrt{185}}{4} \)

[Key: C]

10) Answer the questions using the two quadratic functions represented by the graph and the table.

![Graph Image]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-7</td>
<td>-7</td>
</tr>
</tbody>
</table>

Part A
What is the maximum value of \( f(x) \)? Explain your answer.

Part B
Write an equation to represent \( g(x) \). Show or explain your work.

Part C
Which of the two functions has the greater maximum value? Show or explain your work.
Exemplar

Part A
8 since (1, 8) is the highest point on the graph.

Part B
Since \( g(x) = 0 \) for \( x = -7 \) and \( x = -1 \), then \(-7\) and \(-1\) are zeros of the function, meaning \((x + 7)\) and \((x + 1)\) are factors.
\[ g(x) = a(x + 7)(x + 1) \]
Since \((0, -7)\) is on the graph of the function, I can put 0 in for \( x \) and \(-7\) in for \( g(x) \) and solve for \( a \).
\[ -7 = a(0 + 7)(0 + 1) \]
\[ -7 = a(7)(1) \]
\[ -7 = 7a \]
\[ a = -1 \]
Now I substitute \( a \) back into the formula.
\[ g(x) = -(x + 7)(x + 1) \]

Part C
\[ g(x) = -(x^2 + 7x + x + 7) = -x^2 - 8x - 7 \]
\[ x = \frac{-b}{2a} = \frac{-(8)}{2(-1)} = \frac{8}{2} = -4 \]
Or
Find the midpoint of the \( x \)-intercepts:
\[ \frac{-7 + (-1)}{2} = -4 \]

Substitute \( x = -4 \) into the equation:
\[ g(x) = -(-4 + 7)(-4 + 1) = -(3)(-3) = 9 \]

[Key: \( 9 > 8 \), so the max for \( g(x) \) is greater.]
You should be able to:

- Write the equation for a circle given information such as a center, radius, point on the circle, etc.
- Demonstrate the process for deriving the general form equation of a circle from a center and a point.
- Complete the square to find the center and radius of a circle given by an equation.
- Write the equation of a parabola given information such as vertex, focus, directrix, or a point on the parabola.
- Demonstrate the process for deriving the general form parabola equation from a focus and directrix.
- Prove simple geometric properties using coordinates.

Vocabulary:

- Center of a Circle: The point inside the circle that is the same distance from all of the points on the circle.
- Circle: The set of all points in a plane that are the same distance, called the radius, from a given point, called the center.
  
  Standard form: 
  \[
  (x - h)^2 + (y - k)^2 = r^2
  \]

- Conic Section: A figure formed by the intersection of a plane and a right circular cone. Depending on the angle of the plane with respect to the cone, a conic section may be a circle, an ellipse, a parabola, or a hyperbola.

- Diameter: The distance across a circle through its center. The line segment that includes the center and whose endpoints lie on the circle.

- Directrix of a Parabola: every point on a parabola is equidistant from a fixed point (focus) and a fixed line (directrix).

- Focus of a Parabola: every point on a parabola is equidistant from a fixed point (focus) and a fixed line (directrix).

- General Form of a Circle: 
  \[
  Ax^2 + By^2 + Cx + Dy + E = 0
  \]

- Parabola: the set of all points in a plane equidistant from a fixed line, the directrix, and a fixed point, the focus, in the plane.

- Pythagorean Theorem: A theorem that states that in a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs.

- Radius: The distance from the center of a circle to any point on the circle. Also, the line segment that has the center of the circle as one endpoint and a point on the circle as the other endpoint.

- Standard Form of a Circle: 
  \[
  (x - h)^2 + (y - k)^2 = r^2
  \]
  where \((h, k)\) is the center and \(r\) is the radius.

Practice Items:

1) Which coordinate plane shows the graph of a parabola that has a focus at \((3, 3)\) and a directrix of \(y = -1\)?

[Key: A]
2) The graph of a circle has its center at (2, 3) with a radius of 10 units. Which point does NOT lie on the circle?
   - A. (-4, -5)
   - B. (8, 11)
   - C. (-2, 6)
   - D. (-4, 11)

   [Key: C]

3) What is the radius of the circle represented by the equation $x^2 + y^2 - 18x + 12y + 68 = 0$?

   [Key: 7 units]

4) The focus of a parabola is (5, 3) and the directrix is $y = -3$. Which equation represents the parabola?
   - A. $y = \frac{1}{8}(x - 5)^2 + 1$
   - B. $y = \frac{1}{8}(x - 5)^2 - 3$
   - C. $y = \frac{1}{4}(x - 5)^2 + 1$
   - D. $y = \frac{1}{4}(x - 5)^2 - 3$

   [Key: A]

5) What is the equation of the parabola shown on the coordinate plane?

   [Key: $y = \frac{1}{24}x^2$]

6) The picture shows a circle and Belinda's work to prove whether the point $(-2, \sqrt{24})$ lies on the circle.

   Explain whether or not Belinda's work is correct.

   {Key: Belinda used the wrong x-coordinate for the vertex in her equation]
7) Danielle created a graph to solve the system of equations shown.

\[
\begin{align*}
(x - 3)^2 + (y + 1)^2 &= 4 \\
y &= \frac{1}{2}x - 3
\end{align*}
\]

Which statement is true about the graph?

A. There are no points of intersection.

B. Two points of intersection lie in Quadrant IV.

C. One point of intersection lies in Quadrant I and the other in Quadrant IV.

D. One point of intersection lies in Quadrant III and the other in Quadrant IV.

[Key: C]

8) A parabola has a focus at the point \((6, 0)\), and the equation of the directrix is \(x = 2\).

**Part A**
Determine the vertex of the parabola. Explain your answer.

**Part B**
Prove that point \((12, 8)\) is on the parabola. Show your work.

\[
\begin{align*}
\text{Part A} \\
\text{Determine the vertex of the parabola. Explain your answer.} \\
\text{Part B} \\
\text{Prove that point (12, 8) is on the parabola. Show your work.}
\end{align*}
\]

[Key: ]
Unit 7: Applications of Probability

You should be able to:

- use set notation to represent a set of events mathematically.
- use the addition rule for two events and/or their probabilities.
- read and interpret a two-way frequency table.
- determine conditional probabilities given sufficient information.
- use the formula for conditional probability.
- use the formula for the probability independent events.
- confirm whether or not two events are independent using probability.

Vocabulary:

- **Addition Rule:**
  \[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]

- **Complement:** Given a set A, the complement of A, denoted \( \overline{A} \) or \( A' \), is the set of elements that are not members of A.

- **Conditional Probability:** The probability of an event A, given that another event, B, has already occurred; denoted \( P(A \mid B) \).

\[ P(A \mid B) = \frac{P(A \cap B)}{P(B)} \]

- **Dependent Events:** Two or more events in which the outcome of one event affects the outcome of the other event or events.

- **Element:** A member or item in a set.

- **Independent Events:** Events whose outcomes do not influence each other.

- **Intersection of Sets:** The set of all elements contained in all of the given sets, denoted \( \cap \).

\[ P(A \cap B) = P(A) \cdot P(B) \]

- **Mutually Exclusive Events:** Two events that cannot occur simultaneously, meaning that the probability of the intersection of the two events is zero; also known as disjoint events.

- **Outcome:** A possible result of an experiment.

- **Overlapping Events:** Events that can occur simultaneously – they have an intersection.

- **Sample Space:** The set of all possible outcomes from an experiment.

- **Set:** A collection of numbers, geometric figures, letters, or other objects that have some characteristic in common.

- **Subset:** A set in which every element is also contained in a larger set.

- **Union of Sets:** The set of all elements that belong to at least one of the given two or more sets denoted \( \cup \).

- **Venn Diagram:** A picture that illustrates the relationship between two or more sets.
1) In a particular state, the first character on a license plate is always a letter. The last character is always a digit from 0 to 9. If $V$ represents the set of all license plates beginning with a vowel, and $O$ represents the set of all license plates that end with an odd number, which license plate belongs to the set $V \cap \overline{O}$?

A. E23 PC8
B. MG4 3F5
C. AR8 8X9
D. P7M Z56

[Key: A]

2) Assume that the following events are independent:
- The probability that a high school senior will go to college is 0.72.
- The probability that a high school senior will go to college and live on campus is 0.46.
What is the probability that a high school senior will live on campus, given that the person will go to college?

A. 0.26
B. 0.33
C. 0.57
D. 0.64

[Key: D]

3) Each letter of the alphabet is written on a card using a red ink pen and placed in a container. Each letter of the alphabet is also written on a card using a black ink pen and placed in the same container. A single card is drawn at random from the container. What is the probability that the card has a letter written in black ink, the letter A, or the letter Z?

A. $\frac{1}{2}$
B. $\frac{7}{13}$
C. $\frac{15}{26}$
D. $\frac{8}{13}$

[Key: B]

4) In soccer, a shutout is a game where the winning team does not allow the other team to score a goal.

If the set $W$ represents all wins, and $S$ represents all shutouts, which set describes the set of shutout wins?

A. $W \cap S$
B. $W \cup S$
C. $W \cap S'$
D. $(W \cup S')$

[Key: A]
5) Stephanie works for a department store that sells products at its location as well as online. Some products are discounted. Stephanie’s job is to determine the availability of products. The diagram shows part of her recent report.

![Venn Diagram]

What does the shaded region of the diagram represent in terms of the products sold by the department store?

[Key: The area represents the products that are available both at the store’s location and online, but which are not discounted]

6) Garret sells laptop and desktop computers. He asks his customers what the computer will be primarily used for—school, home, or work. The table shows Garret’s yearly summary giving the proportion of each type of computer he sold and its primary use.

<table>
<thead>
<tr>
<th>Garret’s Yearly Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Laptop</td>
</tr>
<tr>
<td>Desktop</td>
</tr>
</tbody>
</table>

Part A
What proportion of Garret’s sales are computers primarily used for school or work? Show your work and explain your answer.

Part B
Consider the following two situations:
- A customer buys a computer from Garret for work.
- A customer buys a desktop computer from Garret.

Based on the data in the table, which of the two situations is most likely to occur? Show your work and explain your answer.

Part C
If Garret randomly selects a customer from those who purchased a desktop computer, what is the probability the customer uses the computer primarily for work? Show your work and explain your answer.

Part D
If Garret randomly selects a customer from those who purchased a computer primarily for school, what is the probability the customer bought a laptop? Show your work and explain your answer.
## Analytic Geometry Study Guide

### Geometry Resource

**Properties of Congruence**

<table>
<thead>
<tr>
<th>Property</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive Property</td>
<td>If and only if $\overline{AB} \cong \overline{CD}$, then $\overline{AB} \cong \overline{CD}$</td>
</tr>
<tr>
<td>Symmetric Property</td>
<td>If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$</td>
</tr>
<tr>
<td>Transitive Property</td>
<td>If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$</td>
</tr>
<tr>
<td>Addition Postulate</td>
<td>If three points, $A$, $B$, and $C$ are on a same line, then $\overline{AB} = \overline{BC} + \overline{CA}$</td>
</tr>
<tr>
<td>Angle Addition Postulate</td>
<td>If point $P$ is in the interior of $\angle AOC$, then $\angle AOP + \angle COP = \angle AOC$</td>
</tr>
</tbody>
</table>

### Triangle Relationships

<table>
<thead>
<tr>
<th>Triangle Relationships</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pythagorean Theorem</td>
<td>In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse: $a^2 + b^2 = c^2$, where $a$ and $b$ are the lengths of the legs and $c$ is the length of the hypotenuse of the triangle.</td>
</tr>
<tr>
<td>Converse of the Pythagorean Theorem</td>
<td>If $a^2 + b^2 = c^2$, then the triangle is a right triangle, where $a$ and $b$ are the lengths of the shorter sides and $c$ is the length of the longest side of the triangle.</td>
</tr>
<tr>
<td>Pythagorean Inequality Theorems</td>
<td>If $a^2 + b^2 &gt; c^2$, then the triangle is acute, where $a$ and $b$ are the lengths of the shorter sides and $c$ is the length of the longest side of the triangle.</td>
</tr>
<tr>
<td>Side-Side-Side Congruence (SSS)</td>
<td>If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.</td>
</tr>
<tr>
<td>Angle-Side-Side Congruence (ASS)</td>
<td>If an angle and the sides included of one triangle are congruent to an angle and the sides included of another triangle, then the triangles are congruent.</td>
</tr>
<tr>
<td>Hypotenuse-Leg Congruence (HL)</td>
<td>If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, the triangles are congruent.</td>
</tr>
<tr>
<td>CPCTC</td>
<td>Corresponding parts of congruent triangles are congruent.</td>
</tr>
<tr>
<td>Angle-Angle Similarity (AA)</td>
<td>If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.</td>
</tr>
</tbody>
</table>

### Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

### Exterior Angle Theorem

The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.