Can you spot the rock climbers in this picture? Those climbers are scaling El Capitan, in Yosemite National Park, California. In 2008, two rock climbers climbed El Capitan in record time—under 3 hours.

8.1 **SOME PLACES ARE EXPENSIVE; SOME PLACES ARE MORE AFFORDABLE**

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8.2 **PLASTIC CONTAINERS**

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8.3 **JUST ANOTHER SATURDAY**

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8.4 **CLIMBING EL CAPITAN**

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8.6 **EMPTYING A TANK**

Using Multiple Representations to Solve Problems ................. 435
Have you ever heard the phrase “cost of living”? Cost of living is the expenses adults and families have to maintain living. Some cost of living expenses are food, housing, utilities like electricity or heating, and clothing. Do you think that the cost of living is the same throughout your entire state? Do you think that the cost of living is the same in different cities in the United States?
Problem 1  Cost of Living Increase? Prices Are Going Up?

Ms. Jackson translates books for a living. She decides to change her fees to keep up with the cost of living. She will charge an initial fee of $325 to manage each project and $25 per page of translated text. Ms. Jackson does not consider partial pages in her fees.

1. Name the quantities that change in this problem situation.

2. Name the quantities that remain constant.

3. Which quantity depends on the other?
4. Complete the table that shows the various projects that Ms. Jackson has managed recently.

<table>
<thead>
<tr>
<th>Number of Pages</th>
<th>Project Fees (dollars)</th>
<th>Total Cost of the Project (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>400</td>
</tr>
<tr>
<td></td>
<td></td>
<td>425</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>1150</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2100</td>
</tr>
<tr>
<td>92</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. What is the least number of pages that Ms. Jackson could translate? What is the greatest number of pages that Ms. Jackson has translated recently?

6. What are the least and greatest amounts of money that Ms. Jackson has earned?
7. Consider the ranges of values you wrote in Questions 5 and 6 and choose upper bounds (they should be whole numbers) for the $x$- and $y$-axes that are slightly greater than your greatest values. Then, write the lower and upper bound values in the table shown for each quantity.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pages Translated</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earnings</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. Calculate the difference between the upper and lower bounds for each quantity. Then, choose an interval that divides evenly into this number. Doing so will ensure even spacing between the grid lines on your graph. Write these intervals in the table.

9. Label the graph using the bounds and intervals. Then, create a graph of the data from the table in Question 4.

Is there a pattern to the points I plotted?
10. Should the points you graphed be connected by a line? Are the data continuous or discrete? Explain your reasoning.

11. Describe the relationship between the two quantities represented in the graph.

12. Use the graph to answer each question. Explain your reasoning.
   a. Approximately how much money would Ms. Jackson earn if she translated 57 pages?

   b. Approximately how many pages would Ms. Jackson need to translate to earn $750?

13. Write an algebraic equation to represent this situation. Define your variables.
Problem 2  Who’s Correct?

1. Ms. Jackson translated a 23-page technical manual for Technicians Reference Guide Inc. She received a check for $900. Was her check correct? If not, state the correct amount she should have received. Explain your reasoning in terms of the equation and the graph you created in Problem 1, Question 9.

2. Ms. Jackson translated a 42-page year-end report for Sanchez and Johnson Law Office. She received a check for $1050. Was her check correct? If not, state the correct amount she should have received. Explain your reasoning in terms of the equation and the graph you created.

3. Ms. Jackson translated a 35-page product specification document for Storage Pros. She received a check for $2075. Was her check correct? If not, state the correct amount she should have received. Explain your reasoning in terms of the equation and the graph you created.
Problem 3  Marketing

1. Ms. Jackson is planning a reception for potential clients. There is a flat fee of $275 to reserve a room at Mariano’s Restaurant for the evening and a charge of $20 per person for food and a drink. If Ms. Jackson has $1250 to spend, what is the maximum number of potential clients she can invite? Write an equation, and show your work. Make sure to define your variable(s).

2. Ms. Jackson contracts with a marketing firm in hopes of building her business. The marketing firm will put together sets of marketing materials. Each set of materials will include packets to be mailed to 12 businesses. The marketing firm charges $125 to produce one set of materials. If Ms. Jackson has $500 to spend on this type of marketing, how many potential businesses can she contact? Make sure to define your variable(s).
Talk the Talk

Solve each equation. Verify your solution.

1. \(16 = 3x - 4\)

2. \(1.2x + 5.3 = 5.9\)

Be prepared to share your solutions and methods.
Learning Goals
In this lesson, you will:
- Write and use two-step equations.
- Compare two problem situations.

Have you ever gotten a package in the mail or from a delivery company? If you have, chances are that your package contained some type of packaging material. One type of packaging material is packaging peanuts, or foam peanuts. The foam peanuts are poured into a box that contains an item or items being shipped. Even though foam peanuts are lightweight, they offer protection to the item being shipped.

Foam peanuts were invented in 1965 by the Dow Company as a way to cut shipping costs but, at the same time, protect valuables being shipped. Can you think of other materials or devices that can be used to ship items?
Problem 1  Different Shapes and Sizes

Your job at Storage Pros is to create new boxes to ship the company’s plastic containers. Storage Pros makes all different shapes and sizes of plastic containers. To ship the containers, the lids are removed, allowing the containers to be stacked. Storage Pros wants to design its shipping boxes so that they will hold two dozen stacks of the plastic containers without lids in stacks of two dozen, regardless of the size or shape of the container.

The table shows the data gathered from measuring the heights of different-sized stacks of the various plastic containers.

<table>
<thead>
<tr>
<th>Number of Containers</th>
<th>Stack Height (centimeters)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Round</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>9.8</td>
</tr>
<tr>
<td>3</td>
<td>10.6</td>
</tr>
<tr>
<td>4</td>
<td>11.4</td>
</tr>
<tr>
<td>5</td>
<td>12.2</td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

1. What are the variable quantities in this problem situation?

2. What quantity depends on the other?
3. Create a graph for each container shape's stack height in terms of the number of containers used. Determine the bounds and intervals and complete the table shown. Be sure to label your graph clearly. Use the symbols in the legend shown when graphing.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Containers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stack Height</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Legend:
- Square container
- Round container

4. Did you connect the points on your graph? Why or why not?

5. What is the difference in height between a stack of two round containers and a stack of one round container? Explain your reasoning.
6. What is the difference in height between a stack of three round containers and a stack of two round containers?

7. What is the difference in height between a stack of four round containers and a stack of three round containers?

8. What is the difference in height between a stack of five round containers and a stack of four round containers?

9. How does the stack height change when one round container is added to a stack of round containers?

10. How does the stack height change when one square container is added to a stack of square containers? Explain your reasoning.
11. Determine the stack height for 6 and then 7 round and square containers. Then, complete the table and graph, Explain your strategy.

12. Determine the height of each stack of round containers. Show your work.
   a. What is the height of 10 round containers?
   b. What is the height of 15 round containers?
   c. What is the height of 20 round containers?
   d. Explain how you calculated each answer.

13. What does the expression \( r - 1 \) represent if you use \( r \) to represent the number of containers in a stack of round containers? Explain your reasoning.

14. Let \( h \) represent the stack height. Write an equation that represents the stack height of round containers in terms of the number of containers \( c \) in the stack.
15. Let $c$ represent the number of containers in a stack of square containers, and let $h$ represent the stack height. Write an equation that gives the stack height in terms of the number of containers in the stack.

16. How are the two equations you wrote similar? Why are these equations similar? Explain your reasoning.

17. How are the two equations you wrote different? Why are these equations different? Explain your reasoning.

18. Calculate the stack height of two dozen round containers using your equation. Show your work.
19. Calculate the stack height of two dozen square containers. Explain your reasoning.

20. Storage Pros had extra boxes that were 45 centimeters tall.
   a. How many square containers can be in each stack inside the box?
   b. How many round containers can be in each stack inside the box?
Talk the Talk

1. What is the advantage of the first table in Problem 1? What does it show?

2. What is the advantage of the graph in Problem 1? What does it show?

3. What is the advantage of using the equation? What does it show?

Be prepared to share your solutions and methods.
Cutting coupons has been a routine for U.S. citizens since the late 1800s. And who can blame Americans for trying to save money? Many sources agree that in the late 1800s, Atlanta business owner and pharmacist Asa Candler was one of the first to offer handwritten coupons to customers. By 1904, coupons had spread to the cereal market, and by 1930, coupons were a staple of American life. Besides in newspapers, where else have you seen coupons? Have you used coupons before?
Problem 1  Using a Recipe

Cousins Nic and Emily had plans on Saturday to visit the arcade at the mall. Before they could head out for the day, their Aunt Heather asked them for help baking cakes for the family reunion picnic on Sunday.

1. The recipe for the cake calls for $\frac{2}{3}$ of a cup of cocoa powder. The recipe for icing requires $\frac{3}{4}$ of a cup of sugar.

   a. Aunt Heather told the cousins that she has exactly four cups of cocoa powder. How many cakes can they make with the amount of cocoa powder they have? Write and solve an equation.

   b. How much sugar will they need to ice all the cakes? Write and solve an equation.

The one-step equation you solved in Question 1 contained a fractional coefficient. To solve an equation that contains a fractional coefficient, you can use multiplicative inverses. Multiplicative inverses are two numbers that when multiplied together equal 1. When you multiply a term with a fractional coefficient by the multiplicative inverse of the fraction, you can isolate the variable on one side of the equation. This is also known as multiplying by the reciprocal. When you multiply any number by its reciprocal, the result is 1.

A coefficient is a number multiplied by a variable in an algebraic expression. Reciprocals are two numbers that when multiplied together result in a product of 1.
2. Consider the equation \(26 = \frac{2}{3}x\). What number would you multiply \(\frac{2}{3}\) by to get 1?

![Image](221x454 to 299x715)

So, multiplying by the reciprocal IS like dividing the fraction by itself!

Analyze the solution to the equation \(26 = \frac{2}{3}x\).

\[
\begin{align*}
26 &= \frac{2}{3}x \\
\frac{3}{2}(26) &= \frac{3}{2}\left(\frac{2}{3}x\right) \\
13 &= x
\end{align*}
\]

3. Verify that 39 is the solution.

![Image](489x78 to 611x339)

Did you see how the fractions were multiplied?

4. Describe the process used to solve the equation.

![Image](75x186 to 360x69)
5. Write a sentence to describe how to apply operations and inverse operations to solve each equation. Then, solve each equation and verify your solution.

a. $42 = \frac{3}{5}x + 12$

b. $-\frac{7}{3}x - 11 = -25$
c. \( \frac{2}{3}x + \frac{7}{3} = \frac{-5}{3} \)

d. \( \frac{3}{4}(k - 1) + 2x = 15 \)
Problem 2  Can You Do Me A Favor?

After helping bake cakes for the family reunion picnic, Nic and Emily went to the mall. Just as they were leaving, Aunt Heather asked them to stop by the Super Cinema and check the prices of this year’s Frequent Movie Viewer Discount Membership. Aunt Heather is considering buying a discount membership if she can save money throughout the year when she goes to the movies.

The box office associate at the Super Cinema told the cousins that a Frequent Movie Viewer membership costs $40. The membership is good for one year and allows the member to purchase movie tickets for $5.75 at any time with no restrictions. The regular price of a movie ticket is $9.25.

Would you recommend the discount card to Aunt Heather if she went to the movies once every two months? What if Aunt Heather went to the movies once a month? Would you recommend Aunt Heather purchase the discount card if she went to the movies every 3 weeks? What if Aunt Heather went to the movies twice a month?

It appears that you will need to determine the number of times Aunt Heather goes to the movies before you can suggest if she should purchase the discount membership.

1. Write an equation to represent the cost of purchasing a ticket without a membership. Let \( x \) represent the number of movie tickets and \( y \) represent the total cost of the movie tickets.

2. Use the equation you wrote in Question 1 to determine the cost of purchasing the specified number of tickets shown in a year.
   a. 6 tickets
b. 12 tickets

c. 18 tickets

d. 24 tickets

3. Write an equation to represent the cost of purchasing a ticket with a discount membership. Let \( x \) represent the number of movie tickets and \( y \) represent the total cost of the movie tickets.
4. Use your equation to determine the cost of purchasing the specified number of tickets shown in a year.
   a. 6 tickets
   b. 12 tickets
   c. 18 tickets
   d. 24 tickets

5. Write an equation to represent when the cost without a membership and the cost with a membership would be the same.
To solve the equation you wrote in Question 5, you can move all the terms with variables to one side of the equals sign and keep all the constant terms on the other side.

6. If you do this, which term should you move? Describe the mathematical operation that should be used to begin solving this equation. Explain your reasoning.

7. Solve the equation you wrote in Question 5. Show your work.

8. Describe your solution in terms of the problem situation.

9. What recommendation would you provide Aunt Heather about purchasing a discount card?
Problem 3  A Party Surprise

1. While Nic and Emily were off to the mall, Aunt Heather was finalizing the details to rent a moon bounce for the family reunion picnic. She received cost estimates from two different companies that rent inflatable moon bounces. Walkin’ On the Moon charges $55 per hour and a $100 delivery charge to rent a moon bounce. Moo-na Luna charges $65 per hour and a $75 delivery charge.

a. Complete the table by writing an expression to represent the cost of renting a moon bounce from Walkin’ on the Moon and Moo-na Luna. Then, evaluate the expressions for the given number of hours the moon bounce would be rented.

<table>
<thead>
<tr>
<th>Moon Bounce Rental (hours)</th>
<th>Walkin’ On the Moon (dollars)</th>
<th>Moo-na Luna (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. When is the cost of renting a moon bounce from Walkin’ on the Moon the same as when renting from Moo-na Luna?

c. Write an equation that represents the cost for the two companies being equal.
To solve this equation, you can move all the terms with variables to one side of the equals sign, and keep all the constant terms on the other side.

2. Describe the steps that you will use to solve this equation. Explain your reasoning.

3. Solve the equation. Show your work.

4. Describe your solution in terms of the problem situation.

5. Aunt Heather decides to rent the moon bounce for 3 hours. Which company should she call? Explain your reasoning.
Nic and Emily arrive at the arcade and head to their favorite games. Nic plays Dinosaur Tag, which costs $0.75 per game, and Emily plays Comet Avoidance, which costs $0.50 per game.

Nic has $15 to spend.

1. Write an equation to determine how much money Nic will have after playing Dinosaur Tag. Let \( x \) represent the number of games and let \( y \) represent the amount of money remaining.

2. Use your equation to determine how much money Nic has remaining after:
   a. 4 games.
   b. 6 games.

Emily has $13 to spend.

3. Write an equation to determine how much money Emily will have after playing Comet Avoidance. Let \( x \) represent the number of games, and let \( y \) represent the amount of money remaining.
4. Use your equation to determine how much money Emily has remaining after:
   a. 4 games.
   b. 6 games.

5. After how many games played will Nic and Emily have the same amount of money? Write an equation and solve. Explain your solution in terms of the problem situation.

6. How much money will they each have left?
Problem 5  Just the Math

Solve each equation. Then check your answer.

1. $5x = 3x + 18$
2. $14x - 13 = 9x + 2$

3. $6x - 24 = 30$
4. $5x + 10 = 3x - 4$

5. $5x + 8 = 2(3x + 1)$
6. $4(3x - 1) = 2(x + 3)$

Be prepared to share your solutions and methods.
El Capitan is a 3000-foot vertical rock formation in Yosemite National Park in California. The granite cliff is one of the most popular challenges for experienced rock climbers. On July 3, 2008, Hans Florine and Yuji Hirayama scaled El Capitan in a record time of 2 hours 43 minutes and 33 seconds.

What type of equipment do you think these climbers used to climb El Capitan? Do you think that they used a map similar to a coordinate plane?
Problem 1  Rock Climbing

Two new climbers want to attempt to break the record by climbing El Capitan in 2 hours and 30 minutes.

1. If these climbers are to reach their goal, on average, how fast in feet per minute will they have to climb?

2. On average, about how fast in feet per minute did the record holders climb on average?

You want to watch the climbers attempt to break the record for climbing El Capitan. On the morning of the climb, you arrive late at 11:30 AM. When you arrive, the climbers are exactly halfway to the top.

3. How many feet high are the climbers?

4. Assuming they are climbing at the average rate needed, how many feet up the cliff will the climbers be:
   a. in two more minutes?
b. in a quarter of an hour?

c. in one hour?

5. What are the two quantities that are changing in this problem situation?

6. Which quantity depends on the other?

7. Identify and define the independent and dependent variables with their units of measure for this situation.
8. Write the units of measure and the variables in the table. Then, complete the first four rows for this situation.

<table>
<thead>
<tr>
<th>Quantities</th>
<th>Time</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units of Measure</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9. Write an equation or rule for calculating the value of the dependent variable when the value of the independent variable is given.
10. Use your equation to determine how long after 11:30 AM it will take the climbers to reach the top at 3000 feet. Make sure to show your work.

11. What time would the climbers reach the top?

12. Use your equation to determine when the climbers are 1400 feet up the cliff. Make sure to show your work.

13. What does this answer mean in terms of the problem situation?
14. Use your equation to determine how high up the cliff the climbers were:
   a. two minutes before 11:30.
   b. a half hour before 11:30.

15. Use your equation to determine how many minutes before 11:30 the climbers started
to climb.

16. What time of day was that?
17. Write the values from Questions 10–15 in the table used in Question 8.

18. Plot the points from the table on the coordinate plane shown. Label the axes with the units of measure, and draw the graph of your equation.

19. At what point should the graph of your equation begin? Explain your reasoning.

20. At what point should the graph of your equation end? Explain your reasoning.

21. Locate the point at \( x = -60 \).
   a. What is the height of the climbers at this point?
   b. Interpret the meaning of this ordered pair in terms of the problem situation.
22. This analysis of the climbers’ progress assumes that the climbers would climb at a steady rate of 20 feet per minute. In reality, would the climbers be able to do this during the whole climb?

23. Because of the first assumption that the climbers would climb at a steady rate of 20 feet per minute, the graph representing their height above the ground is a straight line. In reality, would this be a straight line?

24. The negative values of time on the graph represent “time ago” or “time before 11:30.” Does this make sense?

Be prepared to share your solutions and methods.
To explore one of the last unknown regions on our planet, companies are starting to produce single-person, submersible deep-sea submarines like the Deep Flight I.

This submersible can dive to a depth of 3300 feet below sea level at a rate of 480 feet per minute.
### Problem 1 Deep Flight 1

1. Suppose Deep Flight I is going to do a dive starting at sea level. Identify the independent and dependent quantities and their units of measure, and define variables for these quantities. Then, write an equation to represent Deep Flight I’s depth.

2. Use your equation to complete the table shown for this problem situation. Do not forget to define the quantities, units of measures, and variables for this situation.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantities</td>
<td></td>
</tr>
<tr>
<td>Units of Measure</td>
<td></td>
</tr>
<tr>
<td>Variables</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Depths in feet below sea level can be represented by negative numbers.
3. Why does the table end at 6 minutes for this problem situation?

4. Examine your table. What do you notice about each depth value in relation to the one before and the one after?

The unit rate of change is the amount that the dependent value changes for every one unit that the independent value changes.

5. In this problem, what is the unit rate of change?

6. How deep would the submersible be after:
   a. 2.5 minutes?
   b. 90 seconds?
   c. 45 seconds?
7. How many minutes would it take Deep Flight I to be:
   a. 1400 feet below sea level?
   b. 2100 feet below sea level?
8. Construct a graph of this problem situation. Label the units on each axis. Then, plot all the points from the table and from Questions 6 and 7. Finally, draw the graph to represent the problem situation.

9. Use the graph to decide what times Deep Flight I will be:
   a. above 1400 feet below sea level.
   b. below 2100 feet below sea level.
10. Write an inequality and solve it to determine the time Deep Flight I is:
   a. above 1400 feet below sea level.
   b. below 2100 feet below sea level.

11. How do your answers using the graph compare to those when you wrote and solved inequalities?

Problem 2  Deep Flight 2

Deep Flight I can dive to a depth of 3300 feet below sea level and can ascend to the surface at a rate of 650 feet per minute.

1. Suppose Deep Flight I is going to ascend to sea level starting at its maximum depth of 3300 feet below sea level. Identify the independent and dependent quantities, define variables for these quantities, and write an equation to represent Deep Flight I’s depth.
2. Use your equation to complete the table shown for this problem situation.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantities</td>
<td></td>
</tr>
<tr>
<td>Units of Measure</td>
<td></td>
</tr>
<tr>
<td>Variables</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

3. Why does the table end at 5 minutes for this problem situation?

4. Examine your table. What do you notice about each depth value in relation to the one before and the one after?
5. In this problem, what is the unit rate of change?

6. How deep would the submersible be after ascending for:
   a. 2.5 minutes?
   b. 90 seconds?
   c. 45 seconds?
7. How many minutes would it take Deep Flight I to ascend to:
   a. 1000 feet below sea level?
   b. 2100 feet below sea level?
   c. sea level?

How do I represent "sea level" as a number?
8. Use this information to construct a graph of this problem situation. First, label the units of measure on each axis. Then, plot all the points from the table and from Questions 6 and 7. Finally, draw the graph to represent the problem situation.

9. Write an inequality and solve it to determine the time Deep Flight I is above 1000 feet below sea level.

10. Write an inequality and solve it to determine the time Deep Flight I is below 2000 feet below sea level.

Be prepared to share your solutions and methods.
Emptying a Tank
Using Multiple Representations to Solve Problems

Learning Goal
In this lesson, you will:
- Use multiple representations to analyze problem situations.

Celsius and Fahrenheit seem to be always different. When it's freezing out, the Celsius temperature will be 0°, but the Fahrenheit temperature will be 32°. On a warm day, when it's about 75°F outside, the Celsius temperature will be about 24°.

There is one temperature where both Fahrenheit and Celsius are the same number. And you could use the formula $F = \frac{9}{5}C + 32$ to figure out what that number is. Can you do it?
A tank that currently contains 2500 gallons of oil is being emptied at a rate of 25 gallons per minute. The capacity of this tank is 3000 gallons.

1. How many gallons are currently in the tank?

2. How fast is the tank being emptied?

3. What are the two quantities that are changing?

4. Define variables for these quantities. Then, identify which is the independent variable and which is the dependent variable.

5. What is the unit rate of change in this situation? Explain your reasoning.

6. Write an equation that relates the two quantities.
7. How many gallons will be in the tank after:
   a. a quarter of an hour?
   
   b. five and a half minutes?
   
   c. an hour and a half?

8. When will the tank be:
   a. half full?
b. empty?

9. How long ago did the tank contain 2600 gallons?

10. How long ago was the tank full?
11. Complete the table for this problem situation using your results from Questions 7–10.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
</table>

Quantities

Units of Measure

Variables
12. Label the units of measure on each axis and plot all the points from the table. Then, graph the equation for this situation. Make sure to label the units on the axes.

<table>
<thead>
<tr>
<th>x</th>
<th>-40</th>
<th>-20</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>300</td>
<td>600</td>
<td>900</td>
<td>1200</td>
<td>1500</td>
<td>1800</td>
<td>2100</td>
<td>2400</td>
</tr>
</tbody>
</table>

Problem 2 Start with an Equation

The equation that converts a temperature in degrees Celsius to a temperature in degrees Fahrenheit is \( F = \frac{9}{5}C + 32 \), where \( F \) is the temperature in degrees Fahrenheit, and \( C \) is the temperature in degrees Celsius.

1. What is the temperature in degrees Fahrenheit if the temperature is:
   a. 36 degrees Celsius?
   b. -20 degrees Celsius?
2. What is the temperature in degrees Celsius if the temperature is:
   a. 32 degrees Fahrenheit?

   b. 212 degrees Fahrenheit?
3. What is the unit rate of change? Explain your reasoning.

4. At what temperature are both the Fahrenheit and Celsius temperatures equal? Show your work.
5. Complete the table with the information you calculated in Question 1 through Question 4.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantities</td>
<td></td>
</tr>
<tr>
<td>Units of Measure</td>
<td></td>
</tr>
<tr>
<td>Variables</td>
<td></td>
</tr>
</tbody>
</table>

6. Label the units of measure on each axis and plot all the points from the table. Then, graph the equation for this situation.
Herman and Melville found this table.

<table>
<thead>
<tr>
<th>Time in Minutes</th>
<th>Total Cost in Dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>23</td>
</tr>
<tr>
<td>2</td>
<td>26</td>
</tr>
<tr>
<td>3</td>
<td>29</td>
</tr>
<tr>
<td>5</td>
<td>35</td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

The bottom three entries in the second column were smudged, and the boys couldn’t read them.

Let’s see if you can calculate the unknown values.

1. What is the unit rate of change shown in the table? Explain your reasoning.

2. Define variables for the quantities in the table, and write an equation that relates the two quantities.
3. Use your equation to complete the table. Show your work.

4. Use the completed table to construct a graph.

Problem 4 Start with a Graph

This graph shows the relationship between two quantities.
1. Complete the table using the information in the graph.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. What is the unit rate of change? Explain your reasoning.

3. Write an equation for this relationship.

4. Write a problem situation that can be represented by this graph, table, and equation.

Be prepared to share your solutions and methods.
Using a Table to Represent a Two-Step Problem Situation

To create a table to represent a problem situation, first decide which quantities change, which remain constant, and which quantity depends on the other. Label the independent quantity in the left column (or top row), the constant quality in the middle column (or middle row), and the dependent quantity in the right column (or bottom row). Label the units of measure for each quantity. Then, choose several values for the independent quantity, and determine the corresponding dependent quantity.

Example

A floral shop charges $2 per stem of flowers in an arrangement. It also charges a $4.50 delivery fee.

<table>
<thead>
<tr>
<th>Number of Flowers (stem)</th>
<th>Delivery Fee (in dollars)</th>
<th>Total Cost of Arrangement (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4.50</td>
<td>24.50</td>
</tr>
<tr>
<td>12</td>
<td>4.50</td>
<td>28.50</td>
</tr>
<tr>
<td>15</td>
<td>4.50</td>
<td>34.50</td>
</tr>
<tr>
<td>17</td>
<td>4.50</td>
<td>38.50</td>
</tr>
<tr>
<td>20</td>
<td>4.50</td>
<td>44.50</td>
</tr>
</tbody>
</table>

Getting some sleep helps your brain learn complicated tasks. Work hard during the day and your brain will work hard for you at night!
Using a Graph to Represent a Two-Step Problem Situation

To create a graph to represent a problem situation, first decide which quantities change, which remain constant, and which quantity depends on the other. The dependent quantity is written along the y-axis and the independent quantity is written along the x-axis. Then, determine the upper and lower bounds, or greatest and least values, of the x- and y-axes. Calculate the difference between the upper and lower bounds for each quantity, and choose an interval that divides evenly into this number to ensure even spacing between the grid lines on your graph. Plot ordered pairs on the graph, and connect them in a line if they represent continuous data.

Example

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Flowers</td>
<td>10</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>Total Cost of Arrangement</td>
<td>5</td>
<td>50</td>
<td>5</td>
</tr>
</tbody>
</table>

![Graph Example]

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8.1 Using a Two-Step Equation to Represent a Problem Situation

Write a problem situation that is written in sentence form and requires two operations to solve as a two-step equation. Define unknown amounts in the situation as variables.

Example

Let $C$ represent the total cost of an arrangement, and $f$ represent the number of flowers in the arrangement.

The equation is $C = 2f + 4.50$.

8.1 Using an Equation and Graph to Determine if a Solution Is Correct

Substitute a value for the independent variable into the equation to see if the solution given is correct. You can also look at the graph to see if the solution given is represented as a point along the line in the graph of the equation.

Example

Victor takes an order for two dozen roses to be delivered tomorrow. He tells the customer that the arrangement will cost $54.50.

Victor’s calculation is incorrect. The customer will owe $52.50. You can verify that Victor’s calculation is incorrect by writing an equation.

$2(24) + 4.50 = 52.50$

The ordered pair (24, 54.50) is not a point along the line in the graph.
**8.1 Solving Two-Step Equations**

To solve a two-step equation, the variable must be isolated by performing two inverse operations. Perform the inverse operations in the reverse order of operations found in the original equation. To determine if your solution is correct, substitute the value of the variable back into the original equation. If the equation remains balanced, then you have calculated the solution of the equation.

**Example**

\[ 9x - 14 = 94 \]
\[ 9x - 14 + 14 = 94 + 14 \]
\[ 9x = 108 \]
\[ \frac{9x}{9} = \frac{108}{9} \]
\[ x = 12 \]

**Check the solution.**

\[ 9(12) - 14 = 94 \]
\[ 108 - 14 = 94 \]
\[ 94 = 94 \]

**8.2 Writing and Solving Two-Step Equations**

Look at a table of values for a situation. Decide which set of values is the independent quantity, and which is the dependent quantity. Calculate the difference between consecutive independent quantities in the dependent quantity. Define the variables, and write the equation. Solve to answer questions about the situation.

**Example**

The table shows the cost to rent a car.

<table>
<thead>
<tr>
<th>Number of Days</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$240</td>
</tr>
<tr>
<td>4</td>
<td>$269</td>
</tr>
<tr>
<td>5</td>
<td>$298</td>
</tr>
<tr>
<td>6</td>
<td>$327</td>
</tr>
<tr>
<td>7</td>
<td>$356</td>
</tr>
</tbody>
</table>

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Write an equation that represents the total cost, \( t \), of renting a car from U Rent It in terms of the number of days rented, \( d \).

\[
\begin{align*}
269 - 240 &= 29 \\
298 - 269 &= 29 \\
327 - 298 &= 29 \\
356 - 327 &= 29 \\
t &= 240 + 29(d - 3)
\end{align*}
\]

Comparing Two Problem Situations

To compare two problem situations, substitute the same value into two similar equations and compare their outcomes.

Example

Ronald needs to hire a caterer for the school picnic. He is comparing catering companies to find the most affordable option. The total cost, \( t \), for a picnic lunch from Callie's Catering in terms of the number of students, \( s \), is represented by the equation \( t = 120 + 5(s - 20) \).

The total cost, \( t \), for a picnic lunch from Paco's Picnics in terms of the number of students, \( s \), is represented by the equation \( t = 100 + 7.50(s - 15) \). Calculate the total cost for each company if Ronald estimates 200 students. Ronald can determine which company is more affordable for providing catering for 200 students.

Total Cost from Callie's Catering

\[
\begin{align*}
t &= 120 + 5(s - 20) \\
&= 120 + 5(200 - 20) \\
&= 120 + 900 \\
&= 1020
\end{align*}
\]

Total Cost from Paco's Picnics

\[
\begin{align*}
t &= 100 + 7.50(s - 15) \\
&= 100 + 7.50(200 - 15) \\
&= 100 + 1387.50 \\
&= 1487.50
\end{align*}
\]

For 200 students, Callie's Catering is more affordable.
8.3 Solving Equations with Fractions

To solve an equation that contains a fractional coefficient, you can use multiplicative inverses to isolate the variable on one side. This is also known as multiplying by the reciprocal.

Example
Solve the equation $\frac{4}{5}x = 16$.

\[
\frac{4}{5}x = 16 \\
5\left(\frac{4}{5}x\right) = 5(16) \\
x = 20
\]

Verify the solution.

\[
\frac{4}{5}(20) \neq 16 \\
16 = 16
\]

8.3 Solving Equations with Variables on Both Sides

To solve an equation with variables on both sides, you must move all the terms with variables to one side of the equation and keep all the constant terms on the other side.

Example
Solve the equation $2x + 7 = 8(4 - x)$.

\[
2x + 7 = 8(4 - x) \\
2x + 7 = 32 - 8x \\
2x + 8x + 7 = 32 - 8x + 8x \\
10x + 7 - 7 = 32 - 7 \\
10x = 25 \\
x = 2.5
\]

Verify the solution.

\[
2(2.5) + 7 \neq 8(4 - 2.5) \\
5 + 7 \neq 8(1.5) \\
12 = 12
\]
8.3 Verifying Solutions in a Problem Context

To determine when the outcomes of two similar situations will be the same, set the two equations equal to each other and solve. Verify the exact value of the outcome by substituting the solution for the variable in each equation.

Example

Kylie is buying balloons for the school dance. Better Balloons charges $0.10 per balloon plus a $6.00 inflating fee per order. Bouncy Balloons charges $0.18 per balloon plus a $0.04 inflating fee per balloon. Let \( t \) represent the total cost and \( b \) represent the number of balloons ordered. What number of balloons can Kylie order so that the cost of the order from both balloon stores will be the same? What is the total cost for that number of balloons?

\[
0.1b + 6 = 0.18b + 0.04b \\
6 = 0.18b + 0.04b - 0.1b \\
6 = 0.12b \\
\frac{6}{0.12} = \frac{0.12b}{0.12} \\
50 = b
\]

If Kylie buys 50 balloons, the cost will be the same from either store. Fifty balloons will cost $11.00.

8.4 Identifying Independent and Dependent Variables to Write and Solve Algebraic Equations

Algebraic equations can be used to represent and analyze problem situations. Identify and define the variables as the quantities that change in a problem situation. The dependent variable is the quantity that depends on the other. When given the dependent variable, an algebraic equation can be solved to determine the independent variable.

Example

Ling is competing in a 26-mile marathon. She hopes to run at an average rate of 5 miles per hour. It is now 3 hours after the beginning of the race, and Ling is 15 miles into the race. You can write and solve an equation to determine when Ling will reach the 20 mile mark.
Independent variable \( t \) = time in hours
Dependent variable \( d \) = total distance in miles

\[
d = 15 + 5t \\
20 = 15 + 5t \\
5 = 5t \\
1 = t
\]

Ling will reach the 20 mile mark of the race 1 hour from now, or 4 hours after the beginning of the race.

8.4 Interpreting Negative Solutions

Often a negative solution will arise when evaluating an algebraic equation with a valid dependent value. In the case of time, the solution can represent “time ago” or “time before.” In the case of height or elevation, the solution can represent a value below the current level.

Example

Ling is competing in a 26-mile marathon. She hopes to run at an average rate of 5 miles per hour. It is now 3 hours after the beginning of the race, and Ling is 15 miles into the race. You can write and solve an equation to determine when Ling reached the halfway point of the race.

Independent variable \( t \) = time in hours
Dependent variable \( d \) = total distance in miles

\[
d = 15 + 5t \\
13 = 15 + 5t \\
-2 = 5t \\
-0.4 = t
\]

Ling reached the halfway point of the race 0.4 hour, or 24 minutes, ago.
8.5 Identifying Rate of Change

Algebraic equations can be used to represent problem situations. The unit rate of change is the amount the dependent value changes for every one unit the independent value changes.

Example

A helicopter is rising at 9.5 meters per second. You can write and solve an equation to determine how long it will take the helicopter to reach a height of 855 meters.

Independent variable \( t \) = time in seconds
Dependent variable \( h \) = height in meters
Unit rate of change = 9.5 meters per second

\[
\begin{align*}
9.5t &= 855 \\
90 &= t
\end{align*}
\]

90 seconds = 1.5 minutes
The helicopter will take 1.5 minutes to reach 855 meters.

8.6 Using Multiple Representations

A problem situation can be represented in multiple ways. A verbal description, an algebraic equation, a table, or a graph can represent a problem situation.

Example

Manuel is using a garden hose to fill his backyard pool at 8 gallons per minute. After 5 minutes, the pool has 40 gallons of water. The capacity of the pool is 400 gallons. You can write and solve an equation to determine how many gallons of water will be in the pool after an additional 10 minutes.

Independent variable \( t \) = time in minutes
Dependent variable \( w \) = amount of water in gallons
Unit rate of change = 8 gallons per minute

\[
\begin{align*}
w &= 40 + 8t \\
w &= 40 + 8(10) \\
w &= 40 + 80 \\
w &= 120
\end{align*}
\]
8.6

There will be 120 gallons of water in the pool after an additional 10 minutes.

You can use a table to represent this problem situation.

<table>
<thead>
<tr>
<th>Quantities</th>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>Time</td>
<td>Water</td>
</tr>
<tr>
<td>Units of Measure</td>
<td>Minutes</td>
<td>Gallons</td>
</tr>
<tr>
<td>Variables</td>
<td>$t$</td>
<td>$w$</td>
</tr>
<tr>
<td>0</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>160</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>280</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>400</td>
<td></td>
</tr>
</tbody>
</table>

You can also plot the points and graph the equation for this problem situation.