Fleas are some of the animal kingdom’s most amazing athletes. Though they are on average only $\frac{1}{16}$ inch long, they can leap up to seven inches vertically and thirteen inches horizontally. This helps them attach themselves to warm-blooded hosts, such as people, dogs, or cats.

11.1 **BIGGER AND SMALLER**
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11.2 **SAY CHEESE!**
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11.4 **HOUSES FOR OUR FEATHERED FRIENDS!**
Creating Blueprints ......................................................................... 577
Some professional basketball players can jump really high to dunk the ball on a 10-foot-tall goal. But those athletes have got nothing on fleas. These parasitic insects, which spend their time trying to suck the blood from other animals, can jump as high as seven inches.

That doesn’t sound impressive unless you know that a flea is only about \( \frac{1}{16} \) inch long, which means that a flea can jump more than 100 times its own length!

If you could jump like a flea, how high could you jump? What tall buildings could you leap in a single bound?
Problem 1  Scale Drawings

1. Emma enrolled in a sailing class. This diagram of a sailboat is on the first page of her text.

She decided to enlarge the diagram on a separate piece of paper as shown.
2. Determine the geometric shape that best describes each part of the sailboat.
   - Mainsail
   - Hull
   - Centerboard
   - Jib Sheet
   - Rudder

3. Use a centimeter ruler to measure the dimensions of the Mainsail in the text and the Mainsail in Emma’s enlargement of the diagram.

4. The ratio of side lengths in the enlargement to those of the original figure is called the **scale factor**. Determine the scale factor Emma used to create the enlargement of the diagram.

A blueprint is an example of a scale drawing that represents a larger structure. The blueprint shown will be used for the construction of a new house.

5. Use a centimeter ruler to determine the scale factor used to create the blueprint.
Scale drawings are also used to display small objects. The illustration shown is an artist’s drawing of an oxygen atom. It shows eight electrons orbiting a nucleus that contains eight protons (dark spheres) and eight neutrons (light spheres). If the drawing were to scale, the nucleus would be invisible, 10,000 times smaller than it is currently drawn. A more sophisticated depiction of the electrons would show them as pulsating, three-dimensional wavelike clouds rather than little orbiting bullets.

One method for enlarging or shrinking a drawing is to use a grid. The drawing of the sailboat that follows has been made on a grid. Another grid with larger cells is drawn. The idea is to copy each portion of the drawing that is in each square of the original grid into the corresponding square of the new grid.
6. Use this method to enlarge the drawing.
Problem 2  Scale Models

Scale models are also used for three-dimensional models.

1. A model of a C130 airplane has a scale of $\frac{1}{100}$.
   a. If the model plane is one foot long, how long is the actual plane?

   b. If the model’s wingspan is 16 inches, how long is the actual wingspan?

   c. If the width of each of the model’s propellers is 1.62 inches, how wide is an actual propeller?

   d. If the width of the actual tail is 52 feet 8 inches, what is the width of the tail in the model?

   e. If the height of the actual tail is 38 feet 5 inches, what is the height of the tail in the model?
2. This model of a barn has been constructed using a scale of 1 to 48.

a. If the model's barn door is two and one quarter inches high, how high is the actual barn door?

b. If the model's silo is 18 inches high, how high is the actual silo?

c. The actual barn is 80 feet wide, 50 feet deep, and 60 feet to the roof. What are the dimensions of the model?

d. Suppose a dollhouse is built using a scale of 1 : 12. The actual house has 10 foot ceilings in all the rooms. How high are the ceilings in the dollhouse?

e. The porch on the dollhouse is 6 inches high. How high is the actual porch of the house?

Be prepared to share your solutions and methods.
Up until the 1920s, movies did not have any sound with them. These silent films had what were known as intertitles to show dialogue and to tell the story being shown. These movies were far from silent, however. They were often played in a theater and live music was played to the action of the movie. Have you ever seen a silent film?
Problem 1  School Photos

When Timmons Photo Company prints photo packages, they include several sizes of photos that are all mathematically similar. The largest size is $12\text{ in.} \times 16\text{ in.}$ This is read as “12 inches by 16 inches.” The first measure is the width of the photo, and the second measure is the height of the photo.

1. Determine the other possible photo sizes that are mathematically similar.

   a. $2\text{ in.} \times$ _________________
   
   b. _________________ $\times 8\text{ in.}$

   c. $3\text{ in.} \times$ _________________
   
   d. _________________ $\times 2\text{ in.}$

   e. $4\text{ in.} \times$ _________________
   
   f. _________________ $\times 3.5\text{ in.}$
Problem 2  Aspect Ratios

An aspect ratio of an image is the ratio of its width to its height. Aspect ratios are used to determine the screen sizes for movie screens and televisions. Aspect ratios are written as two numbers separated by a colon (width : height).

1. Before 1950, the aspect ratio of all motion pictures and standard definition televisions was 1.33 : 1. This meant that the screen was 1.33 times as wide as it was tall.
   a. Scale this ratio up to a ratio using only whole numbers.
      ____ : ____
   b. What did you use for your scale factor? Explain how you determined what scale factor to use.

2. After 1950, the movie industry wanted to create a different image than what was seen on television, so it adopted the widescreen ratios of 1.85 : 1, which was called the Academy Flat, and 2.35 : 1, which was called Panavision. Explain why these ratios are called widescreen ratios.
3. High definition televisions, or HDTVs, use an aspect ratio of 1.78 : 1. Written as a ratio using whole numbers, the HDTV aspect ratio is 16 : 9. Complete the table to show which similar television screen sizes are appropriate for showing TV shows and movies in high definition.

<table>
<thead>
<tr>
<th>HDTV Sizes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>Height</td>
</tr>
<tr>
<td>8 inches</td>
<td></td>
</tr>
<tr>
<td>18 inches</td>
<td></td>
</tr>
<tr>
<td>48 inches</td>
<td></td>
</tr>
<tr>
<td>3 feet</td>
<td></td>
</tr>
<tr>
<td>4.5 feet</td>
<td></td>
</tr>
</tbody>
</table>

4. Complete the table to show which similar television screen sizes are appropriate for to show movies made in Panavision.

<table>
<thead>
<tr>
<th>Panavision Sizes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>Height</td>
</tr>
<tr>
<td>1 foot</td>
<td></td>
</tr>
<tr>
<td>6 feet</td>
<td></td>
</tr>
<tr>
<td>12 feet</td>
<td></td>
</tr>
<tr>
<td>11.75 feet</td>
<td></td>
</tr>
<tr>
<td>23.5 feet</td>
<td></td>
</tr>
<tr>
<td>20 feet</td>
<td></td>
</tr>
</tbody>
</table>
Problem 3  Flags of the World

Each country of the world has a flag that is designed to a specific ratio of height : length.
All the flags of a particular country must be proportioned in the same ratio.

The table shown lists some countries and the height : length ratio of their flags.

<table>
<thead>
<tr>
<th>Group</th>
<th>Countries</th>
<th>Ratio height : length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group A</td>
<td>Bermuda, Canada, Ethiopia,</td>
<td>1 : 2</td>
</tr>
<tr>
<td></td>
<td>Jamaica, Libya, New Zealand, Nigeria</td>
<td></td>
</tr>
<tr>
<td>Group B</td>
<td>Liberia, United States</td>
<td>10 : 19</td>
</tr>
<tr>
<td>Group C</td>
<td>China, Congo, Egypt, France, Greece, India, Italy, Japan, Kenya, Russia, South Africa, Spain</td>
<td>2 : 3</td>
</tr>
<tr>
<td>Group D</td>
<td>Iran, Mexico</td>
<td>4 : 7</td>
</tr>
<tr>
<td>Group E</td>
<td>England, Germany, Haiti, Nicaragua, Scotland, Wales</td>
<td>3 : 5</td>
</tr>
<tr>
<td>Group F</td>
<td>Switzerland, Vatican City</td>
<td>1 : 1</td>
</tr>
</tbody>
</table>
1. The sizes of flags are given in terms of height × length for each. State which group (A through F) each flag must belong to based on its ratio of height : length.
   a. 2 feet × 4 feet

   b. 10 feet × 15 feet

   c. 20 feet × 20 feet

   d. 12 feet × 21 feet

   e. 5 feet × 9.5 feet

   f. 1.5 feet × 2.5 feet

2. Which group of countries has square flags?

3. Which groups of countries have flags which are slightly different from 1 : 2?
Problem 4  Legoland

Legoland, California, has an area called Miriland, USA with all the famous U.S. buildings built to a 1 : 20 or 1 : 40 scale. One exception is the Empire State Building. The model of the Empire State Building is built using four different scales. The ground floors are built at a 1 : 20 scale to match the size of the model people on the street. The main body of the building is built at a 1 : 40 scale. It then changes to a 1 : 60 scale closer to the top of the model, and the very top tower is built at a 1 : 80 scale. The different scales at the higher levels of the model trick the eye into thinking that the building is much taller than it is. If you were to build a model of the Empire State Building using a 1 : 20 scale for the entire model, it would be over 62 feet tall versus the Legoland version, which is 20 feet tall!

1. Approximately how tall is the Empire State Building? Use the fact that a 1 : 20 scale model would be over 62 feet tall. Show and explain your work.

2. Complete the table to represent the heights of actual buildings and the heights of their models at a 1 : 20 scale.

<table>
<thead>
<tr>
<th>Name of Building</th>
<th>Height of the Actual Building</th>
<th>Height of the Scale Model at a 1 : 20 Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Washington Monument</td>
<td>555.5 feet</td>
<td></td>
</tr>
<tr>
<td>Washington, D.C.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S. Capitol Building</td>
<td>4.4 meters</td>
<td></td>
</tr>
<tr>
<td>Washington, D.C.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Willis Tower (formerly the Sears Tower)</td>
<td>1451 feet</td>
<td></td>
</tr>
<tr>
<td>Chicago, Illinois</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transamerica Pyramid</td>
<td>850 feet</td>
<td></td>
</tr>
<tr>
<td>San Francisco, CA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>191 Peachtree Tower</td>
<td>13.25 m</td>
<td></td>
</tr>
<tr>
<td>Atlanta, GA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modis Tower</td>
<td>163.07 m</td>
<td></td>
</tr>
<tr>
<td>Jacksonville, FL</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Problem 5  Gulliver’s Travels

Maybe you have read or seen *Gulliver’s Travels*, written by Jonathan Swift and published in 1726. In the story, Lemuel Gulliver visits two lands in his travels: Lilliput, the land of tiny people, and Brobdingnag, the land of the giants. The Lilliputians are $\frac{1}{12}$ of Lemuel’s size, and the Brobdingnagians are 12 times his size.

1. Complete the measurements in the table to compare your world, which is the same as Lemuel’s, with the worlds of the Lilliputians and the Brobdingnagians.

<table>
<thead>
<tr>
<th></th>
<th>Your World</th>
<th>Lilliput World</th>
<th>Brobdingnag World</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Pencil Length</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Your Height</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. Math Book Length and Width</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. Your Foot Length</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e. Paper Clip Length</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f. Postage Stamp Length and Width</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Be sure to label your measurements.
Problem 6  Models

1. The scale factor for a model car is 1 : 24. What does this mean?

2. The scale factor for a model train is 1 : 87. What does this mean?

Be prepared to share your solutions and methods.
What do surveyors, mapmakers, architects, engineers, and builders all have in common? All of these people use scale drawings. Scale drawings are representations of real objects or places that are in proportion to the real objects or places they represent. The scale in a scale drawing is given as a ratio. Maps and blueprints are examples of scale drawings.

Why do you think scale drawings are important?
The purpose of a scale drawing is to represent either a very large or very small object.

The scale of a drawing might be written as:

\[ 1 \text{ cm} : 4 \text{ ft} \]

Drawing : Actual
Length : Length

This scale means that every 1 centimeter of length in the drawing represents 4 feet of the length of the actual object.

The scale of a map might look like this:

\[ 1 \text{ in.} : 200 \text{ mi} \]

Map : Actual
Distance : Distance

This scale means that every 1 inch of distance on the map represents 200 miles of actual distance.
1. Write a sentence to describe the meaning of each.

   a. A scale on a map is 1 in. : 2 ft

   b. A scale on a drawing is 1 cm : 4 cm

   c. A scale on a drawing is 2 in. : 1 in.

   d. A scale on a drawing is 1 cm : 1 cm.
Problem 2  A Map of Washington, D.C.

A partial map of Washington, D.C., is provided. A scale is included on the map.

1. Complete the table to help tourist groups plan their visits to our nation’s capital.

<table>
<thead>
<tr>
<th>Sights</th>
<th>Approximate Distance Using Roads and Paths</th>
</tr>
</thead>
<tbody>
<tr>
<td>White House to Lincoln Memorial</td>
<td></td>
</tr>
<tr>
<td>Lincoln Memorial to Arlington Cemetery (Visitor Center)</td>
<td></td>
</tr>
<tr>
<td>Arlington Cemetery (Visitor Center) to Jefferson Memorial</td>
<td></td>
</tr>
<tr>
<td>Jefferson Memorial to Washington Monument</td>
<td></td>
</tr>
<tr>
<td>Washington Monument to U.S. Capitol</td>
<td></td>
</tr>
<tr>
<td>U.S. Capitol to Union Station</td>
<td></td>
</tr>
</tbody>
</table>
2. Why does it make sense to use roads and paths instead of measuring directly from one sight to the next sight?

3. Explain how you estimated the distances between sights.

4. Why are your answers approximate distances?

5. What is the total miles traveled between sights?
Problem 3 A Map of the United States

A map of the United States is shown. A scale is included on the map.

Determine the approximate distances between the locations. State the distances in miles and kilometers.

1. Washington, D.C., to San Francisco, California

2. Washington, D.C., to Seattle, Washington
3. Washington, D.C., to your state capital ____________

4. Chicago, Illinois, to Los Angeles, California

5. Augusta, Maine, to Austin, Texas

6. Which is longer, a mile or a kilometer? How can you tell?

7. How many kilometers make one mile? Explain how you determined your answer.
8. How many days would it take to travel from Washington, D.C., to San Francisco, California, traveling at 60 miles per hour for 8 hours per day? Show your work.


Problem 4  Interpreting Scales

1. Which scale would produce the largest scale drawing of an object when compared to the actual object? Explain your reasoning.
   - 1 in. : 25 in.
   - 1 cm : 1 m
   - 1 in. : 1 ft
2. Which scale would produce the smallest scale drawing of an object when compared to the actual object? Explain your reasoning.
   - 1 in. : 10 in.
   - 1 cm : 10 cm
   - 1 mm : 1 m

3. The scale of a drawing is 6 cm : 1 mm. Is the scale drawing larger or smaller than the actual object or place? Explain your reasoning.

4. Given a scale of $\frac{5}{4}$, explain how you can tell whether the drawing is bigger or smaller than the actual object.

   So since scales are ratios, you can write them in fraction form just like any other ratio.
5. In the 1989 movie *Honey I Shrunk the Kids*, a professor accidentally shrinks his kids to \( \frac{1}{4} \) of an inch with a shrink ray. The kids then get accidentally sent out to the backyard. To the tiny kids, the backyard seems to have giant ants, giant bees, and grass as tall as trees!

Each ant and bee were actually these sizes in real life:

<table>
<thead>
<tr>
<th></th>
<th>Length</th>
<th>Height</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ant</td>
<td>12 mm</td>
<td>3 mm</td>
<td>1 mm</td>
</tr>
<tr>
<td>Bee</td>
<td>0.5 in.</td>
<td>0.25 in.</td>
<td>0.25 in.</td>
</tr>
</tbody>
</table>

The special effects team used a scale of 1 : 40 to create models of giant ants and bees. One unit of actual length corresponded to 40 units of length on each model. Complete the table to show the sizes of the models built by the team.

<table>
<thead>
<tr>
<th></th>
<th>Length</th>
<th>Height</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ant</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bee</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. A microscope has a scale of 100 : 1. A microorganism appears to be 0.75 inch in length under the microscope.
   
   **a.** How long is the microorganism? Show your work.

   **b.** A microorganism is 0.085 millimeter long. How long will it appear under the microscope? Show your work.
7. A different microscope has a scale of 1000 : 1. An amoeba has a length of 25 millimeters under the microscope. What is the actual length of the amoeba? Show your work.

8. A 0.035-centimeter-long paramecium appears to be 17.5 millimeters long under a microscope. What is the power of the microscope? Show your work.

9. The height of a building in an architectural drawing is 12 inches. The actual height of the building is 360 feet. What is the scale of the drawing? Show your work.

10. A poster was enlarged and made into a billboard. The billboard was 20.5 feet by 36 feet. The scale used was 5 : 1. What was the size of the original poster? Explain your reasoning.

11. How do you determine the scale if a statue is 60 feet high and its scale drawing shows the height as 1 foot high?
12. Explain how to calculate the actual distance between two cities if you know the distance between them on a map and the scale of the map.

13. Draw a scale drawing of your math classroom. Give the dimensions of the room and the scale. Remember, you will need to determine the actual size of the room before you can draw it to scale.
Problem 5  Blueprints

A blueprint is a technical drawing, usually of an architectural or engineering design. An example of a blueprint is shown.

1. Design a courtyard for your school using this blueprint and the scale \( \frac{1}{8} \) inch = 1 foot. Include:
   - features appropriate for a courtyard that would enhance the environment
   - features that would be popular for students, teachers, and parents
   - at least 10 features in the space provided (multiples of the same feature are acceptable)

All features should be:
   - drawn to scale
   - positioned on the blueprint keeping scale in mind
   - drawn directly on the blueprint or cut out of paper and taped to the blueprint
   - labeled, either directly on the item or by using a key

Be prepared to share your solutions and methods.
The swallows of San Juan Capistrano are famous. They leave Argentina at about the end of October and arrive at the same church every year in California on March 19. How far do these birds travel to their summer vacations? Not far. Just 6000 miles! Do you think there are other creatures that travel long distances at different times of years? Do you think there are any other reasons animals would migrate from one part of the world to another?
Problem 1  Rectangular Wren Houses

Wren houses are built in several sizes and shapes. One example of a square wren house is shown.

1. Label the boards with appropriate measures.

2. One example of a rectangular wren house is shown.

Think about how tall and wide you want the birdhouse to be.

Draw the different boards used for this wren house. Include measurements.
3. You can construct a birdhouse using only nails and a single 1 ft by 6 ft board. Some of the measurements were not included. Label the boards and determine the unknown measurements. (The front and back are made from two pieces.)
Problem 2  Design Your Own Bird Hotel!

Draw a scale model of a bird hotel. The hotel should have several rooms and separate openings such that each bird can enter its own room.

Create a blueprint that includes the measurements necessary to build the birdhouse and include the scale used to draw the model. You may be able to search the Internet for ideas.

Be prepared to share your solutions and methods.
Dilating Scale Drawings
Scale drawings are used to display very large or very small objects. Maps and blueprints are examples of scale drawings. The ratio of lengths in an enlargement to those of the original figure is called the scale factor. One way to dilate, or enlarge or shrink, a scale drawing is to use a grid.

Example
The drawing of the car is enlarged on the grid.
Using Scale Models to Calculate Measurements

Scale models are three-dimensional dilations of actual objects. The scale factor can be used to calculate actual measurements.

Example

Suppose that a scale model of an Apache helicopter was constructed using a scale factor of $\frac{1}{48}$. The model is 3.5 inches tall, and each of the four rotating blades on an actual Apache helicopter is 300 inches long.

The height of an actual Apache helicopter is $3.5 \times 48 = 168$ inches, or $168 \div 12 = 14$ feet. Each blade on the model is $300 \times \frac{1}{48} = 6.25$ inches long.
**11.2 Exploring Aspect Ratio**

An aspect ratio of an image is the ratio of its width to its height. Aspect ratios are written as two numbers separated by a colon (width : height).

**Example**

Gwen has a photo that is 8 inches wide by 10 inches high. She would like to enlarge the photo into a poster that is 36 inches wide and has the same aspect ratio as the photo. First, the aspect ratio is determined by following the steps shown.

\[
\text{aspect ratio} = \frac{8}{10} = \frac{4}{5} \quad \text{aspect ratio} = 4 : 5.
\]

Now that the aspect ratio is determined, you can calculate the height of the poster.

\[
\frac{4}{36} = \frac{5}{x}
\]

\[
(4)(x) = (36)(5)
\]

\[
4x = 180
\]

\[
x = 45
\]

poster = 36 in. \times 45 in.

The poster will be 36 inches wide by 45 inches high.
Exploring Scale Drawings

Scale drawings are representations of real objects or places that are in proportion to the real objects or places they represent. The scale is given as a ratio of drawing length to actual length.

Example
The height of the Statue of Liberty is 93 meters. Althea would like to create a scale model for her history class. The model must be no taller than 0.5 meters. Althea can determine the scale of her model by using the maximum height of her model and the height of the Statue of Liberty.

\[
\frac{0.5}{93} = \frac{5}{930} = \frac{1}{186}
\]

Althea should build the model at a scale of 1 : 186.

Interpreting Scales
It can be determined if an actual object is larger or smaller than the drawing because scales are written as drawing length: actual length or \( \frac{\text{drawing length}}{\text{actual length}} \). If the drawing length value is larger, then the real object is smaller and vice versa.

Example
A photo is enlarged using a scale of 8 : 1. The resulting photo is 8" \( \times \) 10". The original photo was 1" \( \times \) 1\( \frac{1}{4} \)".

\[
\begin{align*}
8 \div 8 &= 1 \\
10 \div 8 &= 1.25
\end{align*}
\]
11.4 Drawing a Blueprint Given an Illustration of an Object

A blueprint is a technical drawing, usually of an architectural or engineering design. Measurements in a blueprint are drawn to scale.

Example

A blueprint is drawn for the dog house shown.