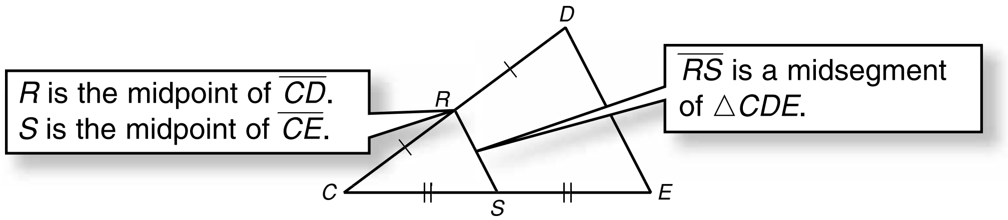
Name ______________________ Date ____________ Class ____________

LESSON 5-4

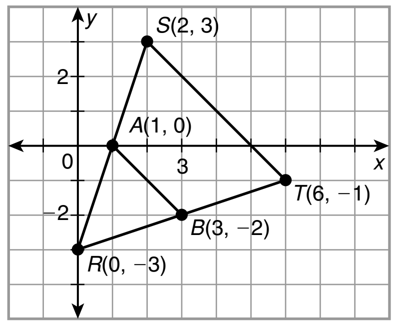
Reteach

The Triangle Midsegment Theorem

A **midsegment** of a triangle joins the midpoints of two sides of the triangle. Every triangle has three midsegments.



Use the figure for Exercises 1–4. $\overline{AB}$ is a midsegment of $\triangle RST$.



1. What is the slope of midsegment $\overline{AB}$ and the slope of side $\overline{ST}$?

____

2. What can you conclude about $\overline{AB}$ and $\overline{ST}$?

____

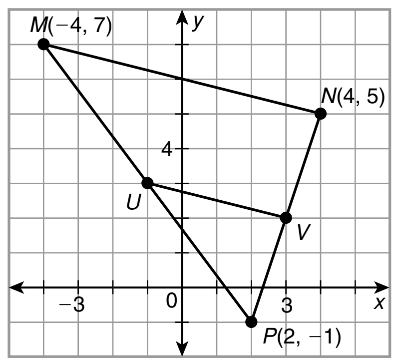
3. Find *AB* and *ST*.

____

4. Compare the lengths of $\overline{AB}$ and $\overline{ST}$.

____

Use $\triangle MNP$ for Exercises 5–7.



5. $\overline{UV}$ is a midsegment of $\triangle MNP$. Find the coordinates of *U* and *V*.

____

6. Show that $\overline{UV} \parallel \overline{MN}$.

____

____

____

7. Show that $UV = \frac{1}{2}MN$.

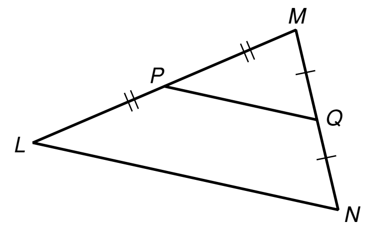
____

Name ______________________ Date __________ Class __________

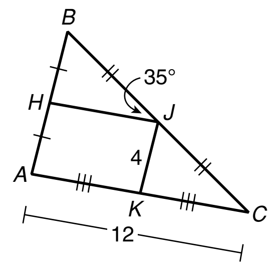
LESSON 5-4

Reteach

The Triangle Midsegment Theorem continued

Theorem	Example
Triangle Midsegment Theorem A midsegment of a triangle is parallel to a side of the triangle, and its length is half the length of that side.	 **Given:** $\overline{PQ}$ is a midsegment of $\triangle LMN$. **Conclusion:** $\overline{PQ} \parallel \overline{LN}$, $PQ = \frac{1}{2}LN$

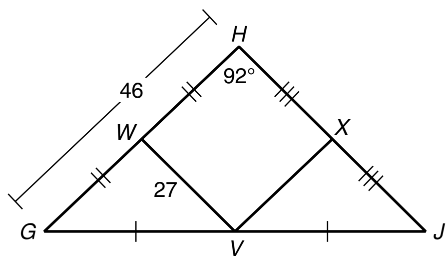
You can use the Triangle Midsegment Theorem to find various measures in $\triangle ABC$.



$HJ = \frac{1}{2}AC$	$\triangle$ Midsegment Thm.
$HJ = \frac{1}{2}(12)$	Substitute 12 for AC.
$HJ = 6$	Simplify.

$JK = \frac{1}{2}AB$	$\triangle$ Midsegment Thm.	$\overline{HJ} \parallel \overline{AC}$	Midsegment Thm.
$4 = \frac{1}{2}AB$	Substitute 4 for JK.	$m\angle BCA = m\angle BJH$	Corr. $\angle$s Thm.
$8 = AB$	Simplify.	$m\angle BCA = 35°$	Substitute 35° for $m\angle BJH$.

Find each measure.



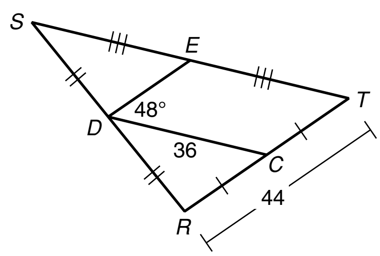
8. $VX =$ ____________________

9. $HJ =$ ____________________

10. $m\angle VXJ =$ ____________________

11. $XJ =$ ____________________

Find each measure.



12. $ST =$ ____________________

13. $DE =$ ____________________

14. $m\angle DES =$ ____________________

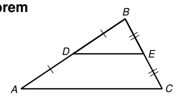
15. $m\angle RCD =$ ____________________

LESSON 5-4

Practice A

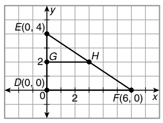
The Triangle Midsegment Theorem

Use the Triangle Midsegment Theorem to name parts of the figure for Exercises 1–5.



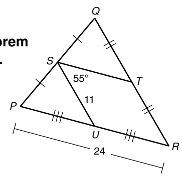
1. a midsegment of $\triangle ABC$ — $\overline{DE}$
2. a segment parallel to $\overline{AC}$ — $\overline{DE}$
3. a segment that has the same length as $\overline{BD}$ — $\overline{AD}$
4. a segment that has half the length of $\overline{AC}$ — $\overline{DE}$
5. a segment that has twice the length of $\overline{EC}$ — $\overline{BC}$

Complete Exercises 6–13 to show that midsegment $\overline{GH}$ is parallel to $\overline{DF}$ and that $GH = \frac{1}{2}DF$.



6. Use the Midpoint Formula to find the coordinates of G. (0 , 2)
7. Use the Midpoint Formula to find the coordinates of H. (3 , 2)
8. Use the Slope Formula to find the slope of $\overline{DF}$. 0
9. Use the Slope Formula to find the slope of $\overline{GH}$. 0
10. If two segments have the same slope, then the segments are parallel. Are $\overline{DF}$ and $\overline{GH}$ parallel? yes
11. Use the Distance Formula to find DF. 6
12. Use the Distance Formula to find GH. 3
13. Does $GH = \frac{1}{2}DF$? yes

Use the Triangle Midsegment Theorem and the figure for Exercises 14–19. Find each measure.



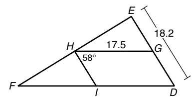
14. ST 12
15. QR 22
16. PU 12
17. $m\angle SUP$ 55°
18. $m\angle SUR$ 125°
19. $m\angle PRQ$ 55°

LESSON 5-4

Practice B

The Triangle Midsegment Theorem

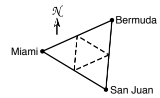
Use the figure for Exercises 1–6. Find each measure.



1. HI 9.1
2. DF 35
3. GE 9.1
4. $m\angle HIF$ 58°
5. $m\angle HGD$ 122°
6. $m\angle D$ 58°

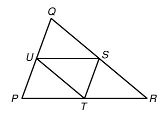
The Bermuda Triangle is a region in the Atlantic Ocean off the southeast coast of the United States. The triangle is bounded by Miami, Florida; San Juan, Puerto Rico; and Bermuda. In the figure, the dotted lines are midsegments.

	Dist. (mi)
Miami to San Juan	1038
Miami to Bermuda	1042
Bermuda to San Juan	965



7. Use the distances in the chart to find the perimeter of the Bermuda Triangle. 3045 mi
8. Find the perimeter of the midsegment triangle within the Bermuda Triangle. 1522.5 mi
9. How does the perimeter of the midsegment triangle compare to the perimeter of the Bermuda Triangle?
   It is half the perimeter of the Bermuda Triangle.

Write a two-column proof that the perimeter of a midsegment triangle is half the perimeter of the triangle.



10. **Given:** $\overline{US}$, $\overline{ST}$, and $\overline{TU}$ are midsegments of $\triangle PQR$.
    Prove: The perimeter of $\triangle STU = \frac{1}{2}(PQ + QR + RP)$.

Possible answer:

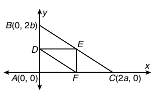
Statements	Reasons
1. $\overline{US}$, $\overline{ST}$, and $\overline{TU}$ are midsegments of $\triangle PQR$.	1. Given
2. $ST = \frac{1}{2}PQ$, $TU = \frac{1}{2}QR$, $US = \frac{1}{2}RP$	2. Midsegment Theorem
3. The perimeter of $\triangle STU = ST + TU + US$.	3. Definition of perimeter
4. The perimeter of $\triangle STU = \frac{1}{2}PQ + \frac{1}{2}QR + \frac{1}{2}RP$.	4. Substitution
5. The perimeter $\triangle STU = \frac{1}{2}(PQ + QR + RP)$	5. Distributive Property of =

LESSON 5-4

Practice C

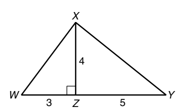
The Triangle Midsegment Theorem

Pedro has a hunch about the area of midsegment triangles. He is a careful student, so he investigates in a methodical manner. First Pedro draws a right triangle because he knows it will be easy to calculate the area.



1. Find the area of $\triangle ABC$. $2ab$
2. Find the coordinates of the midpoints D, E, and F. $D(0, b)$, $E(a, b)$, $F(a, 0)$
3. Pedro knows it will be easy to find the area of $\triangle EFD$ if $\angle DEF$ is a right angle. Write a proof that $\angle DEF \cong \angle A$.
   Possible answer: F is the midpoint of $\overline{AC}$, so $AF = \frac{1}{2}AC$. $DE = \frac{1}{2}AC$ by the Midsegment Theorem, so $\overline{AF} \cong \overline{DE}$. $\overline{DE} \parallel \overline{AF}$ by the Midsegment Theorem and $\angle EDF$ and $\angle AFD$ are alternate interior angles, so $\angle EDF \cong \angle AFD$. $\overline{DF} \cong \overline{DF}$ by the Reflexive Property, thus $\triangle EFD \cong \triangle ADF$. By CPCTC, $\angle DEF \cong \angle A$.
4. Find the area of $\triangle EFD$. $\frac{1}{2}ab$
5. Compare the areas of $\triangle ABC$ and $\triangle EFD$.
   $\triangle ABC$ has four times the area of $\triangle EFD$.
6. Pedro has already shown that $\triangle EFD \cong \triangle ADF$. Calculate the area of $\triangle ADF$. $\frac{1}{2}ab$
7. Write a conjecture about congruent triangles and area.
   Possible answer: Congruent triangles have equal area.

Pedro already knows some things about the area of the midsegment triangle of a right triangle. But he thinks he can expand his theorem. Before he can get to that, however, he has to show another property of triangles and area.



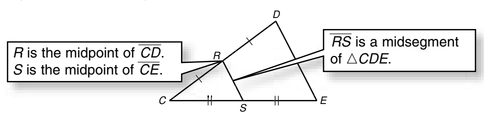
8. Find the area of $\triangle WXY$, $\triangle WXZ$, and $\triangle YXZ$.
   16; 6; 10
9. Compare the total of the areas of $\triangle WXZ$ and $\triangle YXZ$ to the area of $\triangle WXY$.
   Possible answer: The total of the areas of $\triangle WXZ$ and $\triangle YXZ$ is equal to the area of $\triangle WXY$.
10. Write a conjecture about the areas of triangles within a larger triangle.
    Possible answer: The area of a larger triangle is the sum of areas of the triangles within it.

LESSON 5-4

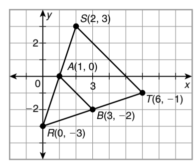
Reteach

The Triangle Midsegment Theorem

A **midsegment** of a triangle joins the midpoints of two sides of the triangle. Every triangle has three midsegments.

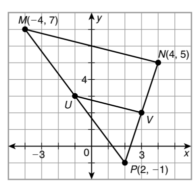


Use the figure for Exercises 1–4. $\overline{AB}$ is a midsegment of $\triangle RST$.



1. What is the slope of midsegment $\overline{AB}$ and the slope of side $\overline{ST}$?
   -1; -1
2. What can you conclude about $\overline{AB}$ and $\overline{ST}$?
   Since the slopes are the same, $\overline{AB} \parallel \overline{ST}$.
3. Find AB and ST.
   $AB = 2\sqrt{2}$, $ST = 4\sqrt{2}$
4. Compare the lengths of $\overline{AB}$ and $\overline{ST}$.
   $AB = \frac{1}{2}ST$ or $ST = 2AB$

Use $\triangle MNP$ for Exercises 5–7.

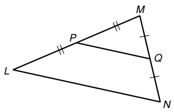


5. $\overline{UV}$ is a midsegment of $\triangle MNP$. Find the coordinates of U and V.
   $U(-1, 3)$, $V(3, 2)$
6. Show that $\overline{UV} \parallel \overline{MN}$.
   The slope of $\overline{UV} = -\frac{1}{4}$ and the slope of $\overline{MN} = -\frac{1}{4}$. Since the slopes are the same, $\overline{UV} \parallel \overline{MN}$.
7. Show that $UV = \frac{1}{2}MN$.
   $UV = \sqrt{17}$ and $MN = 2\sqrt{17}$. Since $\sqrt{17} = \frac{1}{2}(2\sqrt{17})$, $UV = \frac{1}{2}MN$.

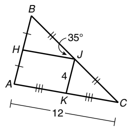
LESSON 5-4

Reteach

The Triangle Midsegment Theorem continued

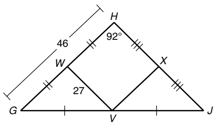
Theorem	Example
Triangle Midsegment Theorem A midsegment of a triangle is parallel to a side of the triangle, and its length is half the length of that side.	 **Given:** $\overline{PQ}$ is a midsegment of $\triangle LMN$. **Conclusion:** $\overline{PQ} \parallel \overline{LN}$, $PQ = \frac{1}{2}LN$

You can use the Triangle Midsegment Theorem to find various measures in $\triangle ABC$.



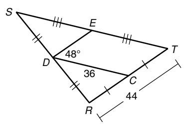
$HJ = \frac{1}{2}AC$	$\triangle$ Midsegment Thm.		
$HJ = \frac{1}{2}(12)$	Substitute 12 for AC.		
$HJ = 6$	Simplify.		
$JK = \frac{1}{2}AB$	$\triangle$ Midsegment Thm.	$\overline{HJ} \parallel \overline{AC}$	Midsegment Thm.
$4 = \frac{1}{2}AB$	Substitute 4 for JK.	$m\angle BCA = m\angle BJH$	Corr. ∠s Thm.
$8 = AB$	Simplify.	$m\angle BCA = 35°$	Substitute 35° for $m\angle BJH$.

Find each measure.



8. $VX =$ 23
9. $HJ =$ 54
10. $m\angle VXJ =$ 92°
11. $XJ =$ 27

Find each measure.



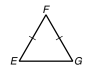
12. $ST =$ 72
13. $DE =$ 22
14. $m\angle DES =$ 48°
15. $m\angle RCD =$ 48°

LESSON 5-4

Challenge

Consider Different Cases

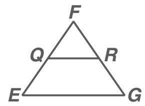
When solving a problem, it is sometimes necessary to consider more than one possible case. It is helpful to make a drawing of each case.



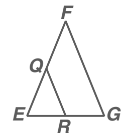
Triangle EFG is an isosceles triangle with $EF = FG$ and with the perimeter equal to 22 units. A midsegment, $\overline{QR}$, of $\triangle EFG$ is equal to 4 units.

1. Describe two possible cases and make a drawing of each.

Case 1: The midsegment connects the two congruent sides EF and FG.



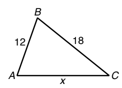
Case 2: The midsegment connects the base EG and one of the congruent sides of $\triangle EFG$.



2. Find the lengths of the triangle's sides for each of the cases in Exercise 1.

case 1: $EF = FG = 7$ and $EG = 8$; case 2: $EF = FG = 8$ and $EG = 6$

Use $\triangle ABC$ for Exercises 4 and 5. A midsegment of the triangle is 9.



3. How many cases are there to consider when making a conclusion about the third side of the triangle? Explain.

Two cases; the midsegment joins the sides with lengths 12 and 18. The midsegment joins the side with lengths 12 and x.

4. Find the length of the third side of $\triangle ABC$ by considering both cases.

If the midsegment joins the sides with lengths 12 and 18, then the third side is 18. If the midsegment joins the side with lengths 12 and x, then it is impossible to find the length of the third side.

LESSON 5-4

Problem Solving

The Triangle Midsegment Theorem

1. The vertices of $\triangle JKL$ are $J(-9, 2)$, $K(10, 1)$, and $L(5, 6)$. $\overline{CD}$ is the midsegment parallel to $\overline{JK}$. What is the length of $\overline{CD}$? Round to the nearest tenth.

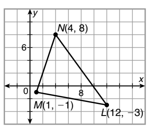
9.5

2. In $\triangle QRS$, $QR = 2x + 5$, $RS = 3x - 1$, and $SQ = 5x$. What is the perimeter of the midsegment triangle of $\triangle QRS$?

$5x + 2$

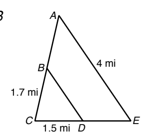
3. Is XY a midsegment of $\triangle LMN$ if its endpoints are $X(8, 2.5)$ and $Y(6.5, -2)$? Explain.

Yes; X is the midpoint of $\overline{LN}$, and Y is the midpoint of $\overline{ML}$.



4. The diagram at right shows horseback riding trails. Point B is the halfway point along path $\overline{AC}$. Point D is the halfway point along path $\overline{CE}$. The paths along $\overline{BD}$ and $\overline{AE}$ are parallel. If riders travel from A to B to D to E, and then back to A, how far do they travel?

9.2 mi



Choose the best answer.

5. Right triangle FGH has midsegments of length 10 centimeters, 24 centimeters, and 26 centimeters. What is the area of $\triangle FGH$?

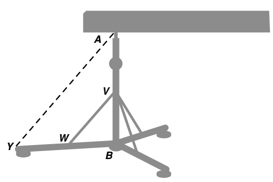
A 60 cm^2 C 240 cm^2
B 120 cm^2 (D) 480 cm^2

6. In triangle HJK, $m\angle H = 110°$, $m\angle J = 30°$, and $m\angle K = 40°$. If R is the midpoint of $\overline{JK}$, and S is the midpoint of $\overline{HK}$, what is $m\angle JRS$?

(F) 150° H 110°
G 140° J 30°

Use the diagram for Exercises 7 and 8.

On the balance beam, V is the midpoint of $\overline{AB}$, and W is the midpoint of $\overline{YB}$.



7. The length of $\overline{VW}$ is $1\frac{7}{8}$ feet. What is AY?

A $\frac{7}{8}$ ft (C) $3\frac{3}{4}$ ft
B $\frac{15}{16}$ ft D $7\frac{1}{2}$ ft

8. The measure of $\angle AYW$ is 50°. What is the measure of $\angle VWB$?

F 45° H 90°
(G) 50° J 130°

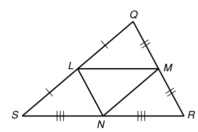
LESSON 5-4

Reading Strategies

Identify Relationships

A **midsegment** of a triangle is a segment that joins the midpoints of two sides of the triangle.

A **midsegment triangle** is formed from the three midsegments of a triangle.



1. Name the midsegments in $\triangle QRS$.

$\overline{LM}$; $\overline{MN}$; $\overline{NL}$

2. What is the midsegment triangle in $\triangle QRS$?

$\triangle LMN$

The Triangle Midsegment Theorem: A midsegment of a triangle is parallel to a side of the triangle, and its length is half the length of that side.

3. Which midsegment is parallel to side $\overline{QS}$?

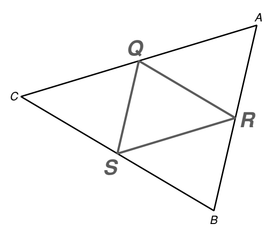
$\overline{MN}$

4. If side $\overline{RS}$ is 12 cm, how long is $\overline{LM}$?

6 cm

5. $\overline{LN} \cong$ $\overline{QM}$ $\cong$ $\overline{MR}$

6. Draw the midsegments in $\triangle ABC$.



7. What are the names of the midsegments?

Answers will vary based on students' choice of letters: $\overline{QR}$, $\overline{RS}$, $\overline{QS}$.

8. What is the midsegment triangle?

$\triangle QRS$