1.3 Twelve Basic Functions

Twelve Basic Functions

The Identity Function $f(x) = x$

The Squaring Function $f(x) = x^2$

The Cubing Function $f(x) = x^3$

The Reciprocal Function $f(x) = \frac{1}{x}$

The Square Root Function $f(x) = \sqrt{x}$

The Exponential Function $f(x) = e^x$

The Natural Logarithm Function $f(x) = \ln x$

The Sine Function $f(x) = \sin x$

The Cosine Function $f(x) = \cos x$

The Absolute Value Function $f(x) = |x| = \text{abs}\,(x)$

The Greatest Integer Function $f(x) = \text{int}\,(x)$

The Logistic Function $f(x) = \frac{1}{1 + e^{-x}}$

Ex:1 p. 106

In Exercises 13–18, identify which of Exercises 1–12 display functions that fit the description given.

13. The function whose domain excludes zero

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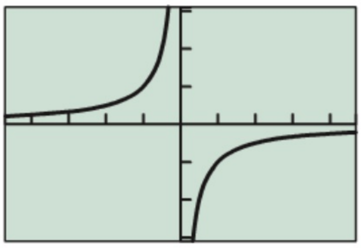

Ex:2 p. 106

In Exercises 13–18, identify which of Exercises 1–12 display functions that fit the description given.

15. The two functions that have at least one point of discontinuity

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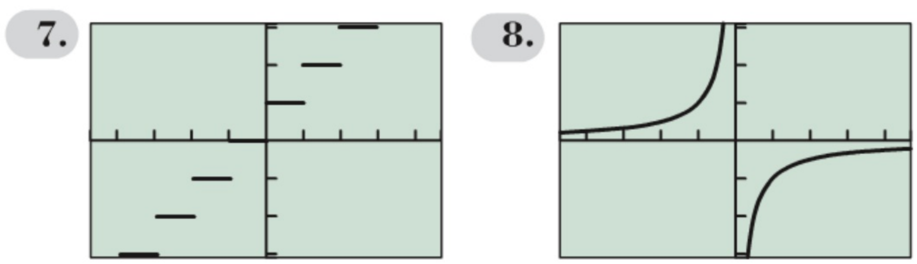

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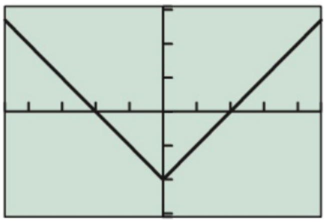

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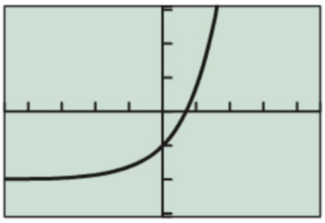

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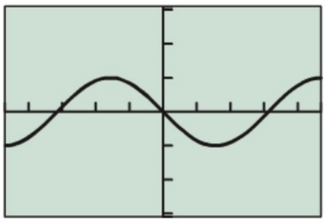

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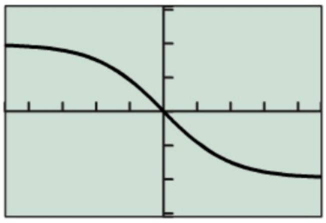

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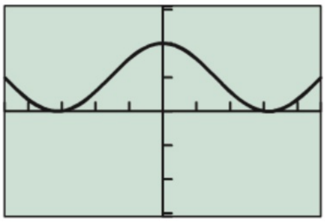

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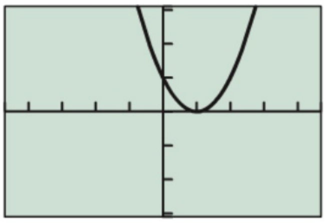

Ex:4 p. 106

In Exercises 19–28, identify which of the twelve basic functions fit the description given.

19. The four functions that are odd

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In Exercises 19–28, identify which of the twelve basic functions fit the description given.

19. The four functions that are odd

$$y = x, \quad y = x^3, \quad y = \frac{1}{x}, \quad y = \sin x$$

Ex:5 p. 106

In Exercises 35–42, graph the function. Then answer the following questions:

35. $r(x) = \sqrt{x - 10}$

(a) On what interval, if any, is the function increasing? Decreasing?

(b) Is the function odd, even, or neither?

(c) Give the function's extrema, if any.

(d) How does the graph relate to a graph of one of the twelve basic functions?

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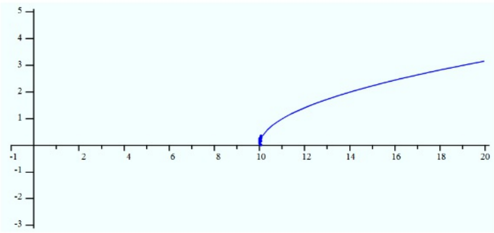

a) Increasing: $[10, \infty)$ Decreasing: None

b) Neither

c) Absolute minimum at $y = 0$ when $x = 10$

d) Square Root Function that was shifted 10 units to the right.

Ex:6 p. 107

In Exercises 45–52, sketch the graph of the piecewise-defined function. (Try doing it without a calculator.) In each case, give any points of discontinuity.

45. $f(x) = \begin{cases} x & \text{if } x \leq 0 \\ x^2 & \text{if } x > 0 \end{cases}$

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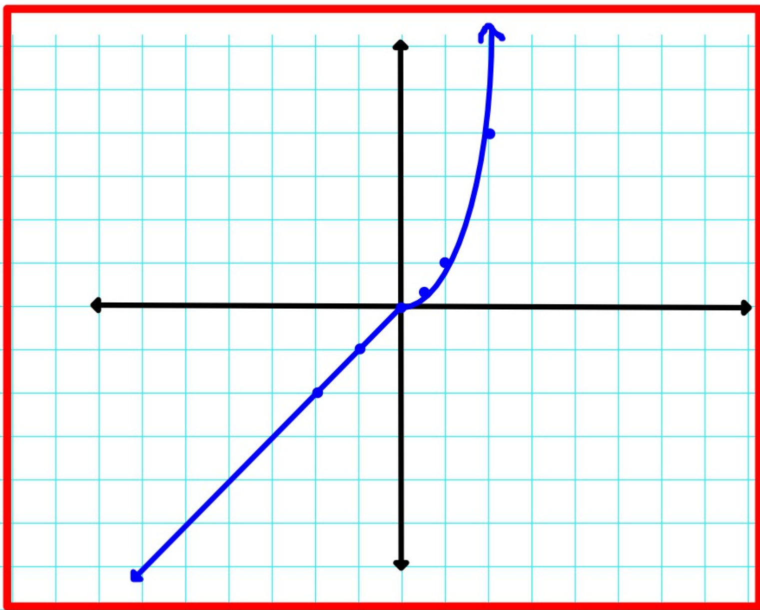
x	y
-2	-2
-1	-1
0	0

$y = x$

x	y
0	0
1	1
2	4
3	9

$y = x^2$



Ex:7 p. 107

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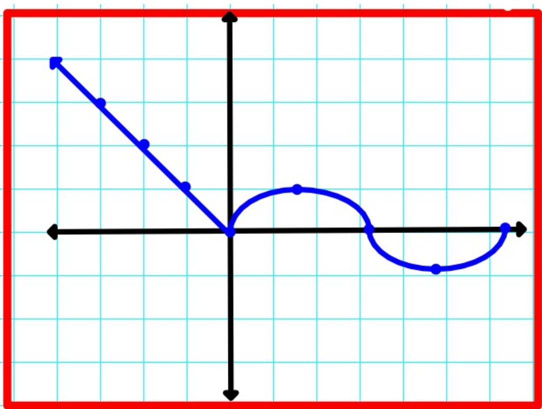

Ex:8 p. 107

55. **Writing to Learn** The function $f(x) = \ln(e^x)$ is one of our twelve basic functions written in another form.

(a) Graph the function and identify which basic function it is.

(b) Explain how the equivalence of the two functions in (a) shows that the natural logarithm function is *not* bounded above (even though it *appears* to be bounded above in Figure 1.42).

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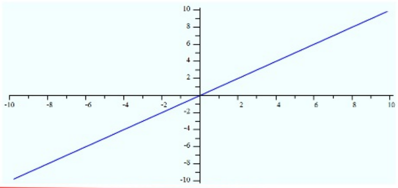

a) $f(x) = x$

b) The natural log undoes e^x and we know e^x goes on for infinity x's, so natural log will keep giving us results getting bigger as it approaches ∞.

Ex: $\ln(10) \approx 2.3$ $\ln(1{,}000) \approx 6.9$ $\ln(1{,}000{,}000) \approx 13.8$

Ex:9 p. 108

67. Pepperoni Pizzas For a statistics project, a student counted the number of pepperoni slices on pizzas of various sizes at a local pizzeria, compiling the following table:

Table 1.10

Type of Pizza	Radius	Pepperoni Count
Personal	4″	12
Medium	6″	27
Large	7″	37
Extra large	8″	48

(a) Explain why the pepperoni count (P) ought to be proportional to the square of the radius (r).

(b) Assuming that $P = k \cdot r^2$, use the data pair $(4, 12)$ to find the value of k.

(c) Does the algebraic model fit the rest of the data well?

(d) Some pizza places have charts showing their kitchen staff how much of each topping should be put on each size of pizza. Do you think this pizzeria uses such a chart? Explain.

Ex:9 p. 108

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a) *The shape of pizza is a circle and the toppings should be proportional to the area.*

b) $12 = k * 4^2$ $12 = 16k$ $k = 0.75$

c) $0.75(6)^2 = 27$ $0.75(7)^2 = 36.75 \approx 37$ $0.75(8)^2 = 48$

Yes, it fits very well.

d) Yes, the data fits a quadratic model too perfectly. If it was off, we wouldn't think there was a chart. If repeated observations produced the same results, there would be little doubt. We would be pretty sure there was a chart.