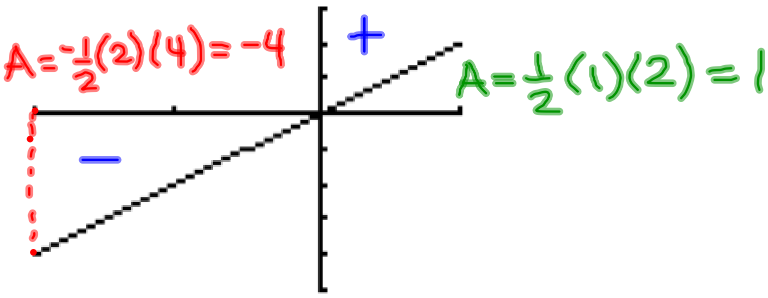
p265 Section 4.3: Reimann Sums and Definite Integrals

Power Point Notes:

Example 2: Evaluating a Definite Integral

$$\int_{-2}^{1} 2x\,dx = \frac{2x^2}{2}\Big]_{-2}^{1} = x^2\Big]_{-2}^{1} = (1)^2 - (-2)^2 = 1 - 4 = -3$$

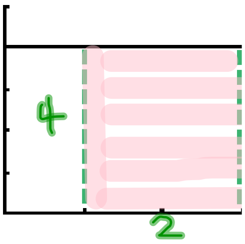
b a



Example 3: Areas of Common Geometric Figures

Sketch the region corresponding to each definite integral. Then calculate each integral using a geometric formula

a. $\int_1^3 4dx$

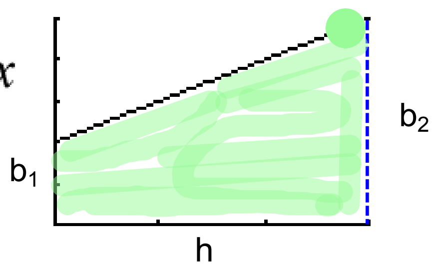


What is this shape? Rectangle

What is the formula to find the area? A = lw

Find the area of this shape: A = 2*4 = 8

b. $\int_0^3 (x+2)dx$

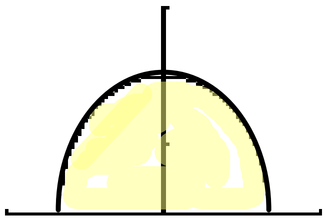


What is this shape? Trapezoid

What is the formula to find the area? $A = (1/2)h(b_1 + b_2)$

Find the area of this shape: A = (1/2)(3)(2 + 5) = 21/2 = 10.5

c. $\int_{-2}^2 \sqrt{4-x^2}\,dx$



What is this shape? Semicircle r = 2

What is the formula to find the area? $A = (1/2)\pi r^2$

Find the area of this shape: $A = (1/2)\pi(2)^2 = 2\pi$

Example 4: Evaluating Definite Integrals

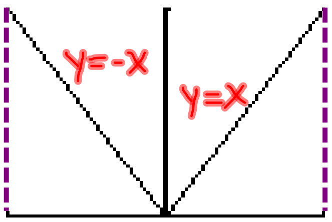
a. $\int_{\pi}^{\pi} \sin x dx = 0$

b. This is the same at the integral in Example 3b except the upper and lower limit have been interchanged. Remember the integral in Example 3b was equal to 21/2

$$\int_{3}^{0} (x+2)dx = -\int_{0}^{3} (x+2)dx = -\frac{21}{2}$$

Example 5: Using the Additive Interval Property

$$\int_{-1}^{1} |x|\, dx = \int_{-1}^{0} -x dx + \int_{0}^{1} x dx = \frac{1}{2} + \frac{1}{2} = 1$$



What are the shapes? Triangles

Can you calculate the area of each shape?

Example 6: Evaluation of a Definite Integral
Evaluate the integral using each of the following values

$$\int_1^3 x^2 dx = \frac{26}{3}, \quad \int_1^3 x dx = 4, \quad \int_1^3 dx = 2$$

$$\int_1^3 (-x^2 + 4x - 3) dx = -\int_1^3 x^2 dx + 4\int_1^3 x dx - 3\int_1^3 dx$$

$$= -\left(\frac{26}{3}\right) + 4(4) - 3(2) = \frac{4}{3}$$

Evaluate the definite integral

#3. $\int_4^{10} 6dx = 6x\Big]_4^{10} = 6(10) - 6(4) = 60 - 24 = 36$

#5. $\int_{-1}^1 x^3 dx = \frac{x^4}{4}\Big]_{-1}^1 = \frac{(1)^4}{4} - \frac{(-1)^4}{4} = \frac{1}{4} - \frac{1}{4} = 0$

#7. $\int_1^2 (x^2 + 1) dx = \left(\frac{x^3}{3} + x\right)\Big]_1^2 = \left(\frac{2^3}{3} + 2\right) - \left(\frac{1^3}{3} + 1\right) = \frac{8}{3} + 2 - \frac{1}{3} - 1 = \frac{10}{3}$

Express the limit as a definite integral on the interval [a, b], where c_i is any point in the *i*th subinterval

Limit	Interval
#9. $\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} (3c_i + 10)\Delta x_i$	[-1, 5]

$$= \int_{-1}^{5} (3x + 10)dx$$

```
fnInt((3X+10),X,
-1,5)
                96
```

Limit	Interval
#11. $\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} \sqrt{c_i^{\,2} + 4}\Delta x_i$	[0, 3]

$$= \int_{0}^{3} \sqrt{x^2 + 4}dx$$

```
fnInt(√(X²+4),X,
0,3)
       7.797853348
```

Now look at #13 - 21 in your book (p272) and set up a definite integral that would yield the area of the region on each graph

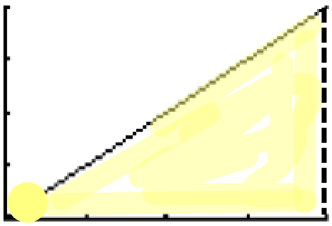
Sketch the region whose area is given by the definite integral.
Then use a geometric formula to evaluate the integral (a > 0, r > 0)

#23. $\int_0^3 4dx$

Find the Area of a rectangle (A = lw)

A = 3*4 = 12

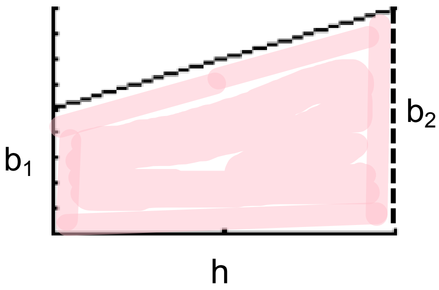
#25. $\int_0^4 xdx$



Find the Area of a Triangle
(A = (1/2)bh)

A = (1/2)(4)(4) = 8

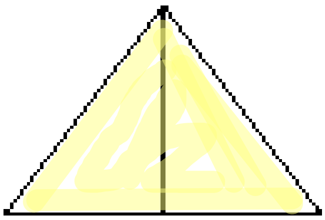
#27. $\int_0^2 (2x+5)dx$



Find the Area of a Trapezoid
A = (1/2)h(b_1 + b_2)

A = (1/2)2(5 + 9) = (1/2)(2)(14) = 14

#29. $\int_{-1}^{1}(1-|x|)\,dx$

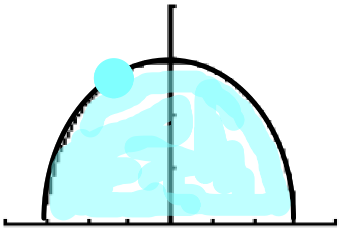


Find the Area of the Triangle

A = (1/2)bh

A = (1/2)(2)(1) = 1

#31. $\int_{-3}^{3}\sqrt{9-x^2}\,dx$



Find the Area of the Semicircle (r = 3)

A = (1/2)πr^2

A = (1/2)π(3)^2 = 9π/2

#41. Use the given information to find each of the following:

$$\int_0^5 f(x)dx = 10 \qquad \int_5^7 f(x)dx = 3$$

(a) $\int_0^7 f(x)dx = \int_0^5 f(x)dx + \int_5^7 f(x)dx = 10 + 3 = 13$

(b) $\int_5^0 f(x)dx = -\int_0^5 f(x)dx = -10$

(c) $\int_5^5 f(x)dx = 0$

(d) $\int_0^5 3f(x)dx = 3\int_0^5 f(x)dx = 3(10) = 30$

#45. $\int_2^6 f(x)dx = 10, \quad \int_2^6 g(x)dx = -2$

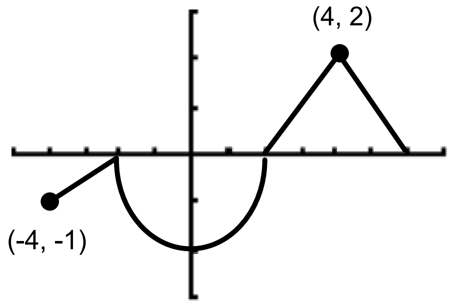
(a) $\int_2^6 [f(x) + g(x)]dx = 10 + (-2) = 8$

(b) $\int_2^6 [g(x) - f(x)]dx = -2 - 10 = -12$

(c) $\int_2^6 2g(x)dx = 2\int_2^6 g(x)dx = 2(-2) = -4$

(d) $\int_2^6 3f(x)dx = 3\int_2^6 f(x)dx = 3(10) = 30$

#45. The graph of f consists of line segments and a semicircle, as shown in the figure. Evaluate each definite integral by using geometric formulas.



(a) $\int_0^2 f(x)dx$ Area of a Quarter Circle (below x-axis - negative)

$$A = -\frac{1}{4}\pi r^2 = -\frac{1}{4}\pi(2)^2 = -\pi$$

(b) $\int_2^6 f(x)dx$ Area of Triangle

$$A = \frac{1}{2}bh = \frac{1}{2}(4)(2) = 4$$

(c) $\int_{-4}^2 f(x)dx$ Area of Triangle and Semicircle (below x-axis - negative)

$$A = -\frac{1}{2}bh - \frac{1}{2}\pi r^2 = -\frac{1}{2}(2)(1) - \frac{1}{2}\pi(2)^2 = -1 - 2\pi$$

(d) $\int_{-4}^6 f(x)dx$ Area below x-axis (from [-4, 2]) + Area above x-axis (from [2, 6])
Area from part (b) + part (c)

$$= -1 - 2\pi + 4 = 3 - 2\pi$$

(e) $\int_{-4}^6 |f(x)|\,dx = \int_{-4}^2 |f(x)|dx + \int_2^6 |f(x)|\,dx = |-1 - 2\pi| + |4| = 5 + 2\pi$

(f) $\int_{-4}^6 [f(x) + 2] = \int_{-4}^6 f(x) + 2x\Big]_{-4}^6$

$$= (3 - 2\pi) + (2*6 - 2*(-4)) = 3 - 2\pi + 20 = 23 - 2\pi$$

HW p272 #4 - 10 (even), 24, 28, 42, 44

Attachments

Section 4.3 - Reimann Sums and Definite Integrals.pptx