Section 9-1: Inverse Variation (p478)

- Recall: Direct Variation (DV) "y varies directly with x" "x & y vary directly"
  - Linear
  - Equation: y = kx multiply
  - Constant of variation = k
    - k = y/x
- Inverse Variation (IV) "y varies inversely with x" "x & y vary inversely"
  - exponential
  - Equation: y = k/x divide
  - Constant of variation (k) = xy
- To tell whether the relationship among a set of numbers is direct, inverse, or neither, plot the points. Direct variation is a linear relationship and inverse variation is an exponential relationship. Now try to find the constant of variation (k). If the value for k is the same for each of the points, then there is variation (whether it is direct or inverse).
- Combined Variation:

Example of Combined Variation

** Remember: Direct Variation (y = kx) and
Inverse Variation (y = k/x)

Combined Variation	Equation Form
y varies directly with the square of x	$y = kx^2$
y varies inversely with the cube of x	$y = k/x^3$
z varies jointly with x and y	$z = kxy$
z varies jointly with x and y and inversely with w	$z = kxy/w$
z zaries jointly with x and inversely with the product of w multiply and y	$z = \frac{kx}{wy}$

** z varies jointly with x and y is the same as z varies directly with the product of x and y

1 EXAMPLE Modeling Inverse Variation

Suppose that x and y vary inversely, and $x = 3$ when $y = -5$.

Write the function that models the inverse variation.

$y = \frac{k}{x}$ Find $k \rightarrow$ $k = xy$

$k = 3(-5) = -15$

$$\boxed{y = \frac{-15}{x}}$$

CU#1) IV $x = 0.3$ when $y = 1.4$

$y = \frac{0.42}{x}$

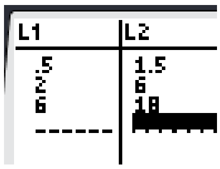
2 EXAMPLE Identifying Direct and Inverse Variations

Is the relationship between the variables in each table a direct variation, an inverse variation, or neither? Write functions to model the direct and inverse variations.

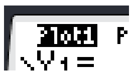
a.

x	0.5	2	6
y	1.5	6	18

Enter in STAT



Turn on Plot 1



Looks Linear

Probably DV

DV $k = y/x$

x	0.5	2	6
y	1.5	6	18

$k = \frac{y}{x} = \frac{1.5}{.5} = \frac{6}{2} = \frac{18}{6}$

$3 = 3 = 3$

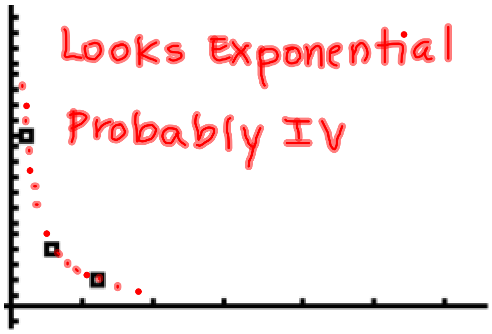
DV $k = 3$

$y = 3x$ $(y = kx)$

b.

x	0.2	0.6	1.2
y	12	4	2

L1	L2
.2	12
.6	4
1.2	2



Looks Exponential

Probably IV

IV $k = xy$

x	0.2	0.6	1.2
y	12	4	2

$k = xy = 0.2(12) = 0.6(4) = 1.2(2)$

$2.4 = 2.4 = 2.4$

IV $k = 2.4$

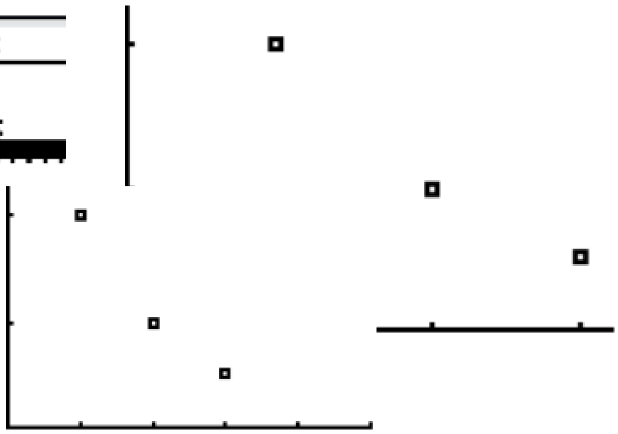
$$y = \frac{2.4}{x}$$

c.

x	1	2	3
y	2	1	0.5

L1	L2
1	2
2	1
3	.5

$k = xy : 1(2) = 2(1) \neq 3(.5)$



Even though the points look exponential, this is NOT IV because the k's are not the same

Neither

3 EXAMPLE Real-World Connection

"x & y vary inversely"

x y

Zoology Heart rates and life spans of most mammals are inversely related. Use the data to write a function that models this inverse variation. Use your function to estimate the average life span of a cat with a heart rate of 126 beats/min.

$k = xy$

```
634*1576800
       999691200
158*6307200
       996537600
76*13140000
       998640000
63*15768000
       993384000
```

$k \approx 1{,}000{,}000{,}000$

Heart Rate and Life Span

Mammal	Heart rate (beats/min) x	Life span (min) y
Mouse	634	1,576,800
Rabbit	158	6,307,200
Lion	76	13,140,000
Horse	63	15,768,000

Source: *The Handy Science Answer Book*

$$y = \frac{1{,}000{,}000{,}000}{x}$$

Cat $x = 126$

$$y = \frac{1{,}000{,}000{,}000}{126}$$

$y = 8{,}000{,}000$ min

```
1000000000/126
       7936507.937

8000000/60
       133333.3333
Ans/24
       5555.555556
Ans/365
       15.22070015
```

↳ years

Suppose that x and y vary inversely. Write a function that models each inverse variation and find y when $x = 10$.

13. $x = 20$ when $y = 5$

ⓐ Find $k = xy$

$k = 20(5) = 100$

$\boxed{y = \frac{100}{x}}$

ⓑ Find y $x = 10$

$y = \frac{100}{x} = \frac{100}{10} = \boxed{10}$

Write the function that models each relationship. Find z when $x = 4$ and $y = 9$.

25. z varies jointly with x and y. When $x = 2$ and $y = 3$, $z = 60$.

use to find k

ⓐ Find k

$z = kxy$

$60 = k(2)(3)$

$10 = k$

$\boxed{z = 10xy}$

ⓑ Find z $x = 4$ $y = 9$

$z = 10xy$

$z = 10(4)(9) = \boxed{360}$

26. z varies directly with the square of x and inversely with y. When $x = 2$ and $y = 4$, $z = 3$.

use to find k

ⓐ Find k

$z = \frac{kx^2}{y}$

$3 = \frac{k\cancel{(2)^2}}{\cancel{4}}$

$3 = k$

$\boxed{z = \frac{3x^2}{y}}$

Find z $x = 4$ $y = 9$

$z = \frac{3x^2}{y} = \frac{\cancel{3}(4)^2}{\cancel{9}\,3} = \boxed{\frac{16}{3}}$

Describe the combined variation that is modeled by each formula.

16. $A = \boxed{\pi} r^2 \rightarrow DV$ $k = \text{constant} = \pi$

A varies directly with the square of r

23. $\ell = \dfrac{V \rightarrow DV}{wh \rightarrow IV}$ $k = 1$

ℓ varies directly w/ V & inversely w/ the product of w & h

4 EXAMPLE Real-World Connection

Physics Newton's Law of Universal Gravitation is modeled by the formula $F = \frac{Gm_1m_2}{d^2}$. F is the gravitational force between two objects with masses m_1 and m_2, and d is the distance between the objects. G is the gravitational constant. Describe Newton's law as a combined variation.

$$F = \frac{\boxed{G} m_1 m_2 \rightarrow JV}{d^2 \rightarrow IV} \qquad k = G$$

F varies jointly w/ m_1 & m_2 & inversely with the square of d

From Enrichment WS

1. $z = \dfrac{ky^2}{x}$ Find k x=2 y=4 z=16

$16 = \dfrac{k(4)^2}{2}$

$2 = k$

Find z x=2 y=1

$z = \dfrac{2y^2}{x} = \dfrac{2(1)^2}{2} = 1 \rightarrow A$