AP Calculus Summer Work Packet

In this packet, you should find:

- A letter from me, discussing expectations for next year.
- Two letters from previous AP students, giving you advice for success in AP Calculus
- A 5 page packet of summer work.

- For the summer work packet, you will find instructions for each section of problems. These are types of problems you are expected to be able to work out on your own. I will not re-teach these topics next year. The purpose of this packet is to keep you thinking about math over the summer and to reinforce the concepts that are expected to be prior knowledge. If you have questions about any of the problems, consult your notes or internet. You may also email me at **a2hughes@randolph.k12.nc.us**.
- **ALL work should be shown as you work through these problems. You should attach any extra paper you use. I do not expect you to fit all of your work on the worksheets.**
- **You may NOT use a calculator on any part of this packet. Part of your AP exam is calculator inactive and a portion of each test we take will be calculator inactive, so you need this practice.**
- **This work is due the first day of class. There will be NO excuses and NO late work accepted.**
- **You will receive a classwork grade for correctness and another for completion of this assignment.**

-This is a college level course and you will be treated accordingly.

See you in August!

Future AP Calculus Students,

Congratulations as you are preparing to enter the realm of Calculus; I can promise this will be unlike any other math courses you have taken. It will be challenging and frustrating at times but will be a rewarding experience if you give it your all. For many of you, math is second nature. However, I warn you, you may struggle and WILL be forced to STUDY and DO HOMEWORK in order to be successful in this class.

Before you begin your journey, I do need to make sure we are clear on a few things. First, I will treat you as much like college students as is possible in a high school setting. Meaning, I will not remind you tirelessly of due dates or missing assignments, nor will I watch over you to make sure you complete your assignments. You are responsible for staying on track and keeping up with the fast pace of the class. Second, missing class can put you behind very quickly. If you must miss class, it is your responsibility to get with someone prior to the next class to get the notes and assignments missed. Then, it is still you responsibility to see me if you need extra help to catch up.

I will expect much from you in this class, however, most the effort you put forth will be of the utmost importance. As I said, you may get frustrated but you cannot give up. You must do work outside of this class to be successful. I expect you to THINK and come up with reasonable solutions. Saying "I don't know" will not be accepted in this class.

I say all of this to give you an idea of what I will expect from you and what you should expect from yourself over the next year. This class can be very fun and rewarding if you put in 100%. I will be there helping you every step of the way, but you must take the steps yourself.

Enjoy your summer! I will see you in August ☺

Annie Hughes ☺

Dear future AP Calculus students,

Congratulations on making the decision to take AP Calculus! This upcoming year is not going to be easy. For some, it will be a whole new type of learning. It is going to be difficult but through hard work, determination, and faith in yourself, you will succeed in this class.

Some of you have really never had to study or had trouble with math, but I can tell you calculus is a whole new type of learning. You have to teach your brain to process calculus, not just simple math and algebra anymore. There will be times when you become loaded down with so many other things that you will not want to do your homework for calculus, but it is a necessity. Mrs. Hughes gives you homework for your benefit, and to help you understand the concept more. If you do not do your homework, how will you know if you fully understand what is going on. Work hard in class and at home, and always do your homework.

One thing that you will definitely need for this class is determination. There will be times when you have no idea what is going on and will get frustrated. Ask questions and just keep pushing through. Mrs. Hughes will always be there for questions and to explain it in any way she can. Get help from your classmates, make sure to ask questions and keep going.

Over the next year you will need to have some faith in yourself. I know there are some of you who believe that this class is going to be too much to handle. I promise you it is nothing that you cannot take. Believe in yourself and know that you will get through it, if you choose to put in the hard work.

For the AP exam at the end of the year, you need to spend as much time as you can working on your own. Rework problems and ask about concepts that you do not understand as well as the others. Mrs. Hughes will get you prepared but there is only so far she can go. Once it passes that point it becomes your responsibility to step up to the plate.

This year will be difficult but there is no doubt in my mind that you can handle it. I have never been very strong in math, but calculus turned out to be better than expected. It is essential that you ask question, keep going, do your homework, and always give the class your 110%.

I wish you all a very successful year!

Best Wishes,

Sophie E. Farlow

Sophie Farlow

Future Calculus Students,

There are three things you need to understand about this class. One, you will likely struggle with at least one concept (but probably more). Two, you must put in some actual effort (yes, this means studying and *gasp* doing your homework). Three, it's okay to ask for help.

Calculus is a class unlike any you have ever taken before. It is some weird hybrid of every math you have ever taken, physics, and an occasional piece of math-magic. Don't get lost in it all though. It's easy to be intimidated by the mass amount of integrals, derivatives, and chain rule inceptions- but if you truly put in an effort you will succeed and you will be fine.

Now, the effort you must put into this class is greater than the effort you have likely ever put into any other class. Ever. If you want to get an A, if you want to get a 5 on your exam, if you want to be prepared for college- study! Do your homework even when it's not required, because it will only benefit you in the long run. Calculus is not always straightforward, in fact, it hardly ever is. You won't get many "plug and chug" problems, you'll have to apply concepts to problems you have never seen the like of before. This will take some thinking, and occasionally some internal crying, but don't give up. Something that helped me in here was to get to school by 7:30 and work through homework problems in class, with Mrs. Hughes there to help.

One thing you can never be is scared to ask questions. There were several occasions in class when Mrs. Hughes was working out a problem on the board and I just asked, "Can you do all of that again? I don't get any of it." Nobody in class ever laughed, and Mrs. Hughes patiently explained it in a way I understood. If you fake understanding of a concept in this class you will be in hot water later on. Every concept builds upon the one before it, so if you don't have a solid understanding of something- ask a question!

I have no regrets about taking Calculus. I want to go into an engineering field, so it was the smartest choice for me. Even to those of you who don't want to pursue a STEM career though, Calculus is beneficial. If you put in an effort you will learn study habits, work ethic, and critical thinking skills to benefit you in every aspect of life. Don't fall to senioritis (or early onset senioritis for you juniors). If you stick it out in here, you will be thanking yourself at the end of the year. I know I am.

-Regan Kibby

A. Functions

The lifeblood of precalculus is functions. A **function** is a set of points (x,y) such that for every x, there is one and only one y. In short, in a function, the x-values cannot repeat while the y-values can. In AB Calculus, all of your graphs will come from functions.

The notation for functions is either "$y=$" or "$f(x)=$". In the $f(x)$ notation, we are stating a rule to find y given a value of x.

1. If $f(x)=x^2-5x+8$, find a) $f(-6)$ b) $f\left(\frac{3}{2}\right)$ c) $\frac{f(x+h)-f(x)}{h}$

a) $f(-6)=(-6)^2-5(-6)+8$
$36+30+8$
74

b) $f\left(\frac{3}{2}\right)=\left(\frac{3}{2}\right)^2-5\left(\frac{3}{2}\right)+8$
$\frac{9}{4}-\frac{15}{2}+8$
$\frac{11}{4}$

c) $\frac{f(x+h)-f(x)}{h}$
$\frac{(x+h)^2-5(x+h)+8-(x^2-5x+8)}{h}$
$\frac{x^2+2xh+h^2-5x-5h+8-x^2+5x-8}{5}$
$\frac{h^2+2xh-5h}{h}=\frac{h(h+2x-5)}{h}=h+2x-5$

Functions do not always use the variable x. In calculus, other variables are used liberally.

2. If $A(r)=\pi r^2$, find a) $A(3)$ b) $A(2s)$ c) $A(r+1)-A(r)$

$A(3)=9\pi$

$A(2s)=\pi(2s)^2=4\pi s^2$

$A(r+1)-A(r)=\pi(r+1)^2-\pi r^2$
$\pi(2r+1)$

One concept that comes up in AP calculus is **composition of functions**. The format of a composition of functions is: plug a value into one function, determine an answer, and plug that answer into a second function.

3. If $f(x)=x^2-x+1$ and $g(x)=2x-1$, a) find $f(g(-1))$ b) find $g(f(-1))$ c) show that $f(g(x))\neq g(f(x))$

$g(-1)=2(-1)-1=-3$
$f(-3)=9+3+1=13$

$f(-1)=1+1+1=3$
$g(3)=2(3)-1=5$

$f(g(x))=f(2x-1)=(2x-1)^2-(2x-1)+1$
$=4x^2-4x+1-2x+1+1=4x^2-6x+3$
$g(f(x))=g(x^2-x+1)=2(x^2-x+1)-1$
$=2x^2-2x+1$

Finally, expect to use **piecewise functions**. A piecewise function gives different rules, based on the value of x.

4. If $f(x)=\begin{cases}x^2-3, & x\geq 0\\ 2x+1, & x<0\end{cases}$, find a) $f(5)$ b) $f(2)-f(-1)$ c) $f(f(1))$

$f(5)=25-3=22$

$f(2)-f(-1)=1-(-1)=2$

$f(1)=-2,\ f(-2)=-3$

A. Function Assignment

- If $f(x) = 4x - x^2$, find

1. $f(4) - f(-4)$

2. $\sqrt{f\left(\frac{3}{2}\right)}$

3. $\dfrac{f(x+h) - f(x-h)}{2h}$

- If $V(r) = \frac{4}{3}\pi r^3$, find

4. $V\left(\frac{3}{4}\right)$

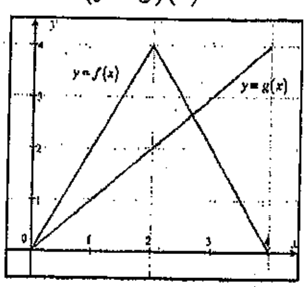
5. $V(r+1) - V(r-1)$

6. $\dfrac{V(2r)}{V(r)}$

- If $f(x)$ and $g(x)$ are given in the graph below, find

7. $(f-g)(3)$

8. $f(g(3))$



- If $f(x) = x^2 - 5x + 3$ and $g(x) = 1 - 2x$, find

9. $f(g(x))$

- If $f(x) = \begin{cases} \sqrt{x+2} - 2, & x \geq 2 \\ x^2 - 1, & 0 \leq x < 2 \\ -x, & x < 0 \end{cases}$, find

10. $f(0) - f(2)$

11. $\sqrt{5 - f(-4)}$

12. $f(f(3))$

B. Domain and Range

First, since questions in calculus usually ask about behavior of functions in intervals, understand that intervals can be written with a description in terms of <, ≤, >, ≥ or by using **interval notation.**

Description	Interval notation	Description	Interval notation	Description	Interval notation
$x>a$	(a,∞)	$x\le a$	$(-\infty,a]$	$a\le x<b$	$[a,b)$
$x\ge a$	$[a,\infty)$	$a<x<b$	(a,b) - open interval	$a<x\le b$	$(a,b]$
$x<a$	$(-\infty,a)$	$a\le x\le b$	$[a,b]$ - closed interval	All real numbers	$(-\infty,\infty)$

If a solution is in one interval or the other, interval notation will use the connector $\cup$. So $x\le 2$ or $x>6$ would be written $(-\infty,2]\cup(6,\infty)$ in interval notation. Solutions in intervals are usually written in the easiest way to define it. For instance, saying that $x<0$ or $x>0$ or $(-\infty,0)\cup(0,\infty)$ is best expressed as $x\ne 0$.

The **domain of a function** is the set of allowable x-values. The domain of a function f is $(-\infty,\infty)$ except for values of x which create a zero in the denominator, an even root of a negative number or a logarithm of a non-positive number. The domain of $a^{p(x)}$ where a is a positive constant and $p(x)$ is a polynomial is $(-\infty,\infty)$

• Find the domain of the following functions using interval notation:

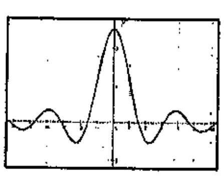
1. $f(x)=x^2-4x+4$ $\boxed{(-\infty,\infty)}$
2. $y=\dfrac{6}{x-6}$ $\boxed{x\ne 6}$
3. $y=\dfrac{2x}{x^2-2x-3}$ $\boxed{x\ne -1, x\ne 3}$
4. $y=\sqrt{x+5}$ $\boxed{[-5,\infty)}$
5. $y=\sqrt[3]{x+5}$ $\boxed{(-\infty,\infty)}$
6. $y=\dfrac{x^2+4x+6}{\sqrt{2x+4}}$ $\boxed{(-2,\infty)}$

The **range of a function** is the set of allowable y-values. Finding the range of functions algebraically isn't as easy (it really is a calculus problem), but visually, it is the [lowest possible y-value, highest possible y-value]. Finding the range of some functions are fairly simple to find if you realize that the range of $y=x^2$ is $[0,\infty)$ as any positive number squared is positive. Also the range of $y=\sqrt{x}$ is also positive as the domain is $[0,\infty)$ and the square root of any positive number is positive. The range of $y=a^x$ where a is a positive constant is $(0,\infty)$ as constants to powers must be positive.

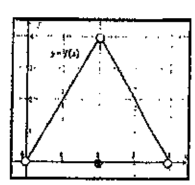
• Find the range of the following functions using interval notation:

7. $y=1-x^2$ $\boxed{(-\infty,1]}$
8. $y=\dfrac{1}{x^2}$ $\boxed{(0,\infty)}$
9. $y=\sqrt{x-8}+2$ $\boxed{[2,\infty)}$

• Find the domain and range of the following functions using interval notation.

10. 

Domain: $(-\infty,\infty)$
Range: $[-0.5,2.5]$

11. 

Domain: $(0,4)$
Range: $[0,4)$

B. Domain and Range Assignment

- Find the domain of the following functions using interval notation:

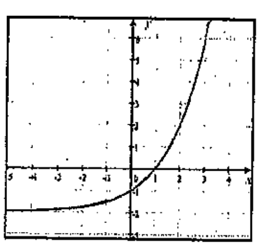
1. $f(x)=3$
2. $y=x^3-x^2+x$
3. $y=\dfrac{x^3-x^2+x}{x}$
4. $y=\dfrac{x-4}{x^2-16}$
5. $f(x)=\dfrac{1}{4x^2-4x-3}$
6. $y=\sqrt{2x-9}$
7. $f(t)=\sqrt{t^3+1}$
8. $f(x)=\sqrt[5]{x^2-x-2}$
9. $y=5^{x^2-4x-2}$
10. $y=\log(x-10)$
11. $y=\dfrac{\sqrt{2x+14}}{x^2-49}$
12. $y=\dfrac{\sqrt{5-x}}{\log x}$

Find the range of the following functions.

13. $y=x^4+x^2-1$
14. $y=100^x$
15. $y=\sqrt{x^2+1}+1$

Find the domain and range of the following functions using interval notation.

16.



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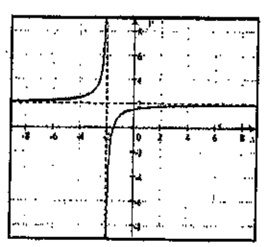

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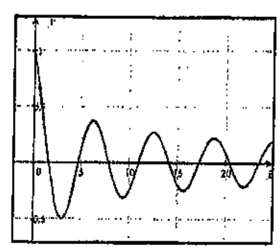

G. Linear Functions

Probably the most important concept from precalculus that is required for differential calculus is that of linear functions. The formulas you need to know backwards and forwards are:

Slope: Given two points (x_1, y_1) and (x_2, y_2), the slope of the line passing through the points can be written as: $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

Slope intercept form: the equation of a line with slope m and y-intercept b is given by $y = mx + b$.

Point-slope form: the equation of a line passing through the points (x_1, y_1) and slope m is given by $y - y_1 = m(x - x_1)$. While you might have preferred the simplicity of the $y = mx + b$ form in your algebra course, the $y - y_1 = m(x - x_1)$ form is far more useful in calculus.

Intercept form: the equation of a line with x-intercept a and y-intercept b is given by $\frac{x}{a} + \frac{y}{b} = 1$.

General form: $Ax + By + C = 0$ where A, B and C are integers. While your algebra teacher might have required your changing the equation $y - 1 = 2(x - 5)$ to general form $2x - y - 9 = 0$, you will find that on the AP calculus test, it is sufficient to leave equations for a lines in point-slope form and it is recommended not to waste time changing it unless you are specifically told to do so.

Parallel lines Two distinct lines are parallel if they have the same slope: $m_1 = m_2$.

Normal lines: Two lines are normal (perpendicular) if their slopes are negative reciprocals: $m_1 \cdot m_2 = -1$.

Horizontal lines have slope zero. **Vertical lines** have no slope (slope is undefined).

1. Find the equation of the line in slope-intercept form, with the given slope, passing through the given point.

   a. $m = -4, (1,2)$

   $y - 2 = -4(x - 1) \Rightarrow y = -4x + 6$

   b. $m = \frac{2}{3}, (-5,1)$

   $y - 1 = \frac{2}{3}(x - 5) \Rightarrow y = \frac{2x}{3} - \frac{7}{3}$

   c. $m = 0, \left(-\frac{1}{2}, \frac{3}{4}\right)$

   $y = -\frac{3}{4}$

2. Find the equation of the line in slope-intercept form, passing through the following points.

   a. $(4,5)$ and $(-2,-1)$

   $m = \frac{5+1}{4+2} = 1$
   $y - 5 = x - 4 \Rightarrow y = x + 1$

   b. $(0,-3)$ and $(-5,3)$

   $m = \frac{3+3}{-5-0} = \frac{-6}{5}$
   $y + 3 = \frac{-6}{5}x \Rightarrow y = \frac{-6}{5}x - 3$

   c. $\left(\frac{3}{4}, -1\right)$ and $\left(1, \frac{1}{2}\right)$

   $m = \left(\frac{\frac{1}{2}+1}{1-\frac{3}{4}}\right)\left(\frac{4}{4}\right) = \frac{2+4}{4-3} = 6$
   $y - \frac{1}{2} = 6(x - 1) \Rightarrow y = 6x - \frac{11}{2}$

3. Write equations of the line through the given point a) parallel and b) normal to the given line.

   a. $(4,7)$, $4x - 2y = 1$

   $y = 2x - \frac{1}{2} \Rightarrow m = 2$
   a) $y - 7 = 2(x - 4)$ b) $y - 7 = \frac{-1}{2}(x - 4)$

   b. $\left(\frac{2}{3}, 1\right)$, $x + 5y = 2$

   $y = \frac{-1}{5}x + 2 \Rightarrow m = \frac{-1}{5}$
   a) $y - 1 = \frac{-1}{5}\left(x - \frac{2}{3}\right)$ b) $y - 1 = 5\left(x - \frac{2}{3}\right)$

G. Linear Functions - Assignment

1. Find the equation of the line in slope-intercept form, with the given slope, passing through the given point.

   a. $m=-7, (-3,-7)$ b. $m=\frac{-1}{2}, (2,-8)$ c. $m=\frac{2}{3}, \left(-6,\frac{1}{3}\right)$

2. Find the equation of the line in slope-intercept form, passing through the following points.

   a. $(-3,6)$ and $(-1,2)$ b. $(-7,1)$ and $(3,-4)$ c. $\left(-2,\frac{2}{3}\right)$ and $\left(\frac{1}{2},1\right)$

3. Write equations of the line through the given point a) parallel and b) normal to the given line.

   a. $(5,-3)$, $x+y=4$ b. $(-6,2)$, $5x+2y=7$ c. $(-3,-4)$, $y=-2$

4. Find an equation of the line containing $(4,-2)$ and parallel to the line containing $(-1,4)$ and $(2,3)$. Put your answer in general form.

5. Find k if the lines $3x-5y=9$ and $2x+ky=11$ are a) parallel and b) perpendicular.

H. Solving Quadratic Equations

Solving quadratics in the form of $ax^2+bx+c=0$ usually show up on the AP exam in the form of expressions that can easily be factored. But occasionally, you will be required to use the quadratic formula. When you have a quadratic equation, factor it, set each factor equal to zero and solve. If the quadratic equation doesn't factor or if factoring is too time-consuming, use the quadratic formula:

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$. The **discriminant** b^2-4ac will tell you how many solutions the quadratic has:

$$b^2-4ac\begin{cases}>0,\ 2\text{ real solutions (if a perfect square, the solutions are rational)}\\ =0,\ 1\text{ real solution}\\ <0,\ 0\text{ real solutions (or 2 imaginary solutions, but AP calculus does not deal with imaginaries)}\end{cases}$$

1. Solve for x.

a. $x^2+3x+2=0$

$(x+2)(x+1)=0$

$x=-2, x=-1$

b. $x^2-10x+25=0$

$(x-5)^2=0$

$x=5$

c. $x^2-64=0$

$(x-8)(x+8)=0$

$x=8, x=-8$

d. $2x^2+9x=18$

$(2x-3)(x+6)=0$

$x=\dfrac{3}{2}, x=-6$

e. $12x^2+23x=-10$

$(4x+5)(3x+2)=0$

$x=-\dfrac{5}{4}, x=-\dfrac{2}{3}$

f. $48x-64x^2=9$

$(8x-3)^2=0$

$x=\dfrac{3}{8}$

g. $x^2+5x=2$

$x=\dfrac{-5\pm\sqrt{25+8}}{2}$

$x=\dfrac{-5\pm\sqrt{33}}{2}$

h. $8x-3x^2=2$

$x=\dfrac{8\pm\sqrt{64-24}}{6}$

$x=\dfrac{8\pm2\sqrt{10}}{6}=\dfrac{4\pm\sqrt{10}}{3}$

i. $6x^2+5x+3=0$

$x=\dfrac{-5\pm\sqrt{25-72}}{12}=\dfrac{-5\pm\sqrt{-47}}{12}$

No real solutions

j. $x^3-3x^2+3x-9=0$

$x^2(x-3)-3(x-3)=0$

$(x-3)(x^2-3)=0$

$x=3,\ x=\pm\sqrt{3}$

k. $\dfrac{x}{3}-\dfrac{5}{2}=\dfrac{-3}{x}$

$6x\left(\dfrac{x}{3}-\dfrac{5}{2}=\dfrac{-3}{x}\right)$

$2x^2-15x+18=0$

$(2x-3)(x-6)=0$

$x=\dfrac{3}{2}, x=6$

l. $x^4-7x^2-8=0$

$(x^2-8)(x^2+1)=0$

$x=\pm\sqrt{8}=\pm2\sqrt{2}$

2. If $y=5x^2-3x+k$, for what values of k will the quadratic have two real solutions?

$(-3)^2-4(5)k>0\Rightarrow 9-20k>0\Rightarrow k<\dfrac{9}{20}$

H. Solving Quadratic Equations Assignment

1. Solve for x.

a. $x^2+7x-18=0$

b. $x^2+x+\frac{1}{4}=0$

c. $2x^2-72=0$

d. $12x^2-5x=2$

e. $20x^2-56x+15=0$

f. $81x^2+72x+16=0$

g. $x^2+10x=7$

h. $3x-4x^2=-5$

i. $7x^2-7x+2=0$

j. $x+\frac{1}{x}=\frac{17}{4}$

k. $x^3-5x^2+5x-25=0$

l. $2x^4-15x^3+18x^2=0$

2. If $y=x^2+kx-k$, for what values of k will the quadratic have two real solutions?

3. Find the domain of $y=\frac{2x-1}{6x^2-5x-6}$.

J. Negative and Fractional Exponents

In calculus, you will be required to perform algebraic manipulations with **negative exponents** as well as **fractional exponents**. You should know the definition of a negative exponent: $x^{-n} = \frac{1}{x^n}, x \neq 0$. Note that negative powers do not make expressions negative; they create fractions. Typically expressions in multiple-choice answers are written with positive exponents and students are required to eliminate negative exponents. Fractional exponents create roots. The definition of $x^{1/2} = \sqrt{x}$ and $x^{a/b} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$.

As a reminder: when we multiply, we add exponents: $\left(x^a\right)\left(x^b\right) = x^{a+b}$.

When we divide, we subtract exponents: $\frac{x^a}{x^b} = x^{a-b}, x \neq 0$

When we raise powers, we multiply exponents: $\left(x^a\right)^b = x^{ab}$

In your algebra course, leaving an answer with a radical in the denominator was probably not allowed. You had to rationalize the denominator: $\frac{1}{\sqrt{x}}$ changed to $\left(\frac{1}{\sqrt{x}}\right)\left(\frac{\sqrt{x}}{\sqrt{x}}\right) = \frac{\sqrt{x}}{x}$. In calculus, you will find that it is not necessary to rationalize and it is recommended that you not take the time to do so.

• Simplify and write with positive exponents. Note: # 12 involves complex fractions, covered in section K.

1. $-8x^{-2}$

$\boxed{\frac{-8}{x^2}}$

2. $\left(-5x^3\right)^{-2}$

$\boxed{(-5)^{-2}x^{-6} = \frac{1}{(-5)^2x^6} = \frac{1}{25x^6}}$

3. $\left(\frac{-3}{x^4}\right)^{-2}$

$\boxed{\frac{(-3)^{-2}}{\left(x^4\right)^{-2}} = \frac{1}{(-3)^2x^{-8}} = \frac{x^8}{9}}$

4. $\left(36x^{10}\right)^{1/2}$

$\boxed{6x^5}$

5. $\left(27x^3\right)^{-2/3}$

$\boxed{\frac{1}{\left(27x^3\right)^{2/3}} = \frac{1}{9x^2}}$

6. $\left(16x^{-2}\right)^{3/4}$

$\boxed{16^{3/4}x^{-4/3} = \frac{8}{x^{4/3}}}$

7. $\left(x^{1/2} - x\right)^{-2}$

$\boxed{\frac{1}{\left(x^{1/2} - x\right)^2} = \frac{1}{x - 2x^{3/2} + x^2}}$

8. $\left(4x^2 - 12x + 9\right)^{-1/2}$

$\boxed{\frac{1}{\left[(2x-3)^2\right]^{1/2}} = \frac{1}{2x-3}}$

9. $\left(x^{1/3}\right)\left(\frac{1}{2}x^{-1/2}\right) + \left(x^{1/2} + 1\right)\left(\frac{1}{3}x^{-1/3}\right)$

$\boxed{\frac{x^{1/3}}{2x^{1/2}} + \frac{x^{1/2}+1}{3x^{1/3}} = \frac{1}{2x^{1/6}} + \frac{x^{1/2}+1}{3x^{1/3}}}$

10. $\frac{-2}{3}(8x)^{-5/3}(8)$

$\boxed{\frac{-16}{3(8x)^{5/3}} = \frac{-16}{3(32)x^{5/3}} = -\frac{1}{6x^{5/3}}}$

11. $\frac{(x+4)^{1/2}}{(x-4)^{-1/2}}$

$\boxed{(x+4)^{1/2}(x-4)^{1/2} = \left(x^2 - 16\right)^{1/2}}$

12. $\left(x^{-1} + y^{-1}\right)^{-1}$

$\boxed{\left(\frac{1}{\frac{1}{x} + \frac{1}{y}}\right)\left(\frac{xy}{xy}\right) = \frac{xy}{y+x}}$

J. Negative and Fractional Exponents - Assignment

Simplify and write with positive exponents.

1. -12^2x^{-5}

2. $\left(-12x^5\right)^{-2}$

3. $\left(4x^{-1}\right)^{-1}$

4. $\left(\frac{-4}{x^4}\right)^{-3}$

5. $\left(\frac{5x^3}{y^2}\right)^{-3}$

6. $\left(x^3-1\right)^{-2}$

7. $\left(121x^8\right)^{1/2}$

8. $\left(8x^2\right)^{-4/3}$

9. $\left(-32x^{-5}\right)^{-3/5}$

10. $(x+y)^{-2}$

11. $\left(x^3+3x^2+3x+1\right)^{-2/3}$

12. $x\left(x^{1/2}-x\right)^{-2}$

13. $\frac{1}{4}\left(16x^2\right)^{-3/4}(32x)$

14. $\frac{\left(x^2-1\right)^{-1/2}}{\left(x^2+1\right)^{1/2}}$

15. $\left(x^{-2}+2^{-2}\right)^{-1}$