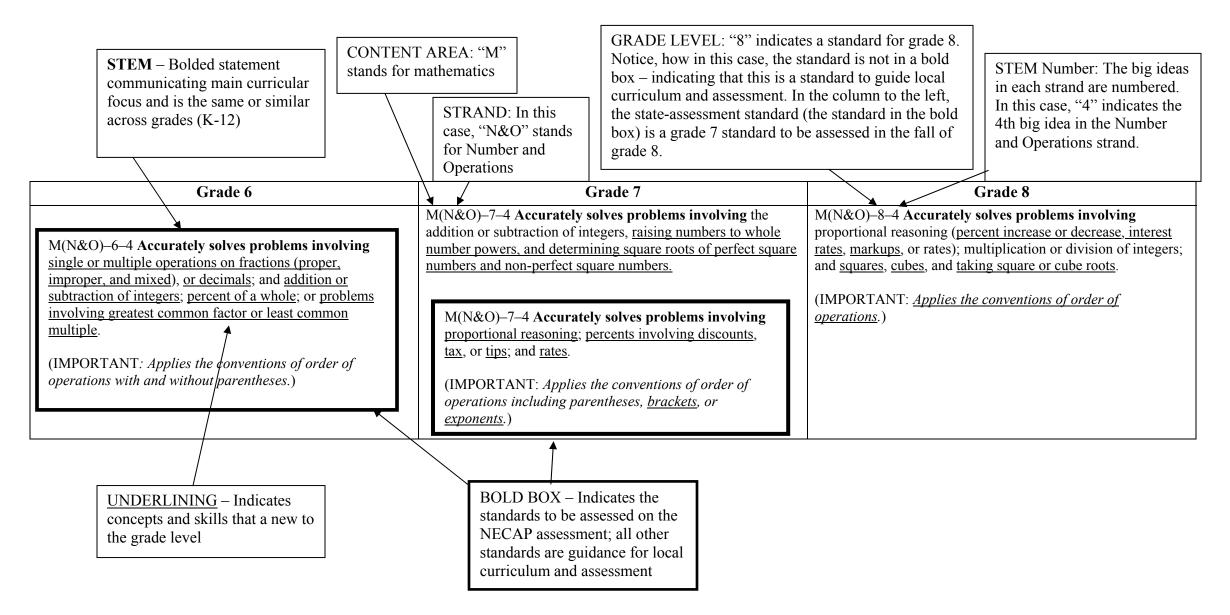
#### About the Mathematics GLEs

This set of New Hampshire Grade-Level Expectations (GLEs) for K-8 includes expectations that will be assessed on the state-level assessment and others that will be a local curriculum and assessment option (i.e., guidance for local curriculum and assessment but not assessed on the state assessment). Those GLEs that will be assessed on the state assessment are boxed with bold outlines. Furthermore, these assessment GLEs are common across NH, RI, and VT and will be assessed through the New England Common Assessment Program (NECAP). The assessment GLEs in this document can be interpreted as describing expectations for the end of the grade identified or the beginning of the next grade. The local content GLEs were developed in collaboration with the Rhode Island Department of Education. As you review the DRAFT New Hampshire GLEs in Mathematics the following are important to understand.

- 1) The GLEs are organized into four content strands (Number and Operations; Functions and Algebra; Data, Statistics, and Probability; Geometry and Measurement) and two process strands (Problem-Solving, Reasoning, and Proof; Communications, Representations, and Connections).
- 2) Since it is crucial that process standards (problem-solving, reasoning, proof, communication, connections, and representations) are not seen separate from content standards, the process standards have been imbedded throughout the content strands (e.g., M(F&A)-4-1 Identifies and <u>extends to specific cases</u> a variety of patterns (linear and <u>nonlinear</u>) represented in models, tables or sequences; and <u>writes a rule in words or<sup>se</sup> symbols to find the next case.</u>). This mirrors classroom instruction as in most classes, as students are learning content knowledge, instruction is also focusing on improving their abilities in problem solving, reasoning, and communication; furthermore, students are looking for and making appropriate connections, and they are able to understand and use multiple representations of mathematical ideas. Since it is crucial that students are strong in both content and process knowledge, we have included two local process strands following the content strands. These process strands are in addition to the process standards that are embedded in the content standards and are included to help guide local curriculum, assessment, and instruction. The process standards have been separated by grade-spans (K-2, 3-5, 6-8, and 9-12). The K-2, 3-5, and 6-8 spans have been included in this document. Each span should be thought of as building upon the skills and concepts in the previous span.
- 3) Each GLE includes a **bolded** statement called the "stem." Each "stem" is the same or similar across the grades for a given GLE, and is meant to communicate the main curricular and instructional focus of the GLE across the grades.
- 4) The unbolded text within a GLE indicates the proficiencies for that given grade-level.
- 5) The underlined text indicates the concepts and skills that are new to a given grade. (Note: Sometimes nothing is underlined within a GLE. In these situations examine other GLEs across the strand to identify the differences or check the Depth of Knowledge table in the test specifications, http://www.ed.state.nh.us/Education/doe/organization/curriculum/NECAP/GLEs.htm.)
- 6) Each GLE is coded for the content strand, grade level, and the GLE "stem" number (e.g., M(F&A)–7–3: The "M" stands for mathematics, the "F&A" stands for the Functions and Algebra strand, the "7" indicates a standard for grade 7 to be assessed in the fall of grade 8, and the "3" stands for stem 3 or the 3<sup>rd</sup> big idea in the Functions and Algebra strand). See the diagram on page 2.
- 7) Unless otherwise specified the number parameters defined in the Number and Operations strand for a particular grade-level apply to all GLEs at that grade-level.
- 8) All the concepts and skills identified at a given grade level that are boxed with bold outlines are "fair game" for state assessment purposes. However, conjunctions in this document have specific meaning. The conjunction "and" separates parts of a GLE that will be assessed every year (to the extent possible), while the conjunction "or" separates parts of the GLE that may be assessed each year, but will be more likely to be assessed over several years. In some situations "or<sup>se</sup>" is used. While students will have choices on strategies they use or methods to communicate their thinking throughout the assessment, there are special cases that the NECAP states thought it was necessary to communicate to the test developer that students should not be required to use a specific method (e.g., "...writes in words or<sup>se</sup> symbols...").

#### **Reading the Mathematics GLEs**



Number and Operations		
Purpose: Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts. Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study. Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation to apply to a given problem situation they necounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and their properties will provide a good foundation for the study of algebra.		
Grade K	Grade 1	Grade 2
M(N&O)–K–1 Demonstrates conceptual understanding of	M(N&O)–1–1 Demonstrates conceptual understanding of	Grade 2
rational numbers with respect to:	rational numbers with respect to:	M(N&O)–2–1 Demonstrates conceptual
whole numbers from 0 to 12 through investigations that apply the concepts of equivalency in composing or	whole numbers from <u>0 to 100 using place value</u> , by applying the concepts of equivalency in composing or	understanding of rational numbers with respect to: whole numbers from <u>0 to 199</u> using place value, by
decomposing numbers using models, explanations, or other	decomposing numbers; and <u>in expanded notation</u> using	applying the concepts of equivalency in composing or
representations; and positive fractional numbers (1/2) as	models, explanations, or other representations; and	decomposing numbers (e.g., $34 = 17 + 17$ ; $34 = 29 + 17$
"fair share" (i.e., equal sized parts or sets) using models,	positive fractional numbers (benchmark fractions: <i>a</i> /2, <u><i>a</i>/3</u> ,	5); and in expanded notation (e.g., $141 = 1$ hundred $+ 4$
explanations, or other representations.	or $a/4$ , where a is a whole number greater than 0 and less	tens + 1 one or $141 = 100 + 40 + 1$ ) using models,
	than or equal to the denominator) as a part to whole	explanations, or other representations; and positive
	relationship in area models where the denominator is equal	<b>fractional numbers</b> (benchmark fractions: $a/2$ , $a/3$ , or
	to the number of parts in the whole using models, explanations, or other representations.	a/4, where a is a whole number greater than 0 and less
	explanations, of other representations.	than or equal to the denominator) as a part to whole relationship in area and set models where the
		denominator is equal to the number of parts in the
		whole using models, explanations, or other
		representations.

Number and Operations Continued		
Grade K	Grade 1	Grade 2
M(N&O)–K–2 <b>Demonstrates understanding of the relative</b> <b>magnitude of numbers</b> from 0 to 20 <b>through investigations</b> that demonstrate one-to-one correspondence; that compare whole numbers to each other or to benchmark whole numbers (5, 10); that demonstrate an understanding of the relation of inequality when comparing whole numbers by using "1 more" or "1 less"; that connect numbers orally and written as numerals to the quantities that they represent using models, representations, or number lines.	M(N&O)-1-2 <b>Demonstrates understanding of the relative</b> <b>magnitude of numbers</b> from 0 to 100 by ordering whole numbers; by comparing whole numbers to each other or to benchmark whole numbers (5, 10, <u>25</u> , <u>50</u> , <u>75</u> , <u>100</u> ); by demonstrating an understanding of the relation of inequality when comparing whole numbers by using "1 more", "1 less", " <u>5 more</u> ", " <u>5 less</u> ", " <u>10 more</u> ", " <u>10 less</u> "; and by connecting <u>number words</u> (from 0 to 20) and numerals (from 0 to 100) to the quantities and <u>positions</u> that they represent using investigations, models, representations, or number lines.	M(N&O)–2–2 <b>Demonstrates understanding of the</b> <b>relative magnitude of numbers</b> from <u>0 to 199</u> by ordering whole numbers; by comparing whole numbers to each other or to benchmark whole numbers (10, 25, 50, 75, 100, <u>125</u> , <u>150</u> , or <u>175</u> ); by demonstrating an understanding of the relation of inequality when comparing whole numbers by using "1 more", "1 less", "10 more", "10 less", " <u>100 more</u> ", or " <u>100 less</u> "; or by connecting number words and numerals to the quantities they represent using models, number lines, or explanations.
M(N&O)–K–3 Demonstrates conceptual understanding of mathematical operations through investigations involving addition and subtraction of whole numbers (from 0 to 10) by solving problems involving joining actions, separating actions, part-part whole relationships, and comparison situations; and addition of multiple one-digit whole numbers. (See Appendix A.)	M(N&O)–1–3 <b>Demonstrates conceptual understanding of</b> <b>mathematical operations involving</b> addition and subtraction of whole numbers ( <u>from 0 to 30</u> ) by solving problems involving joining actions, separating actions, part-part whole relationships, and comparison situations; and addition of multiple one-digit whole numbers. (See Appendix A.)	M(N&O)–2–3 <b>Demonstrates conceptual understanding</b> <b>of mathematical operations involving</b> addition and subtraction of whole numbers by solving problems involving joining actions, separating actions, part-part whole relationships, and comparison situations; and addition of multiple one-digit whole numbers. (See Appendix A.)
M(N&O)–K–4 No standard at this grade M(N&O)–K–5 <b>Demonstrates understanding of monetary</b> value through investigation involving knowing the names and values for coins (penny, nickel and dime).	M(N&O)-1-4 No standard at this grade M(N&O)-1-5 <b>Demonstrates understanding of monetary</b> <b>value by</b> knowing the names and values for coins (penny, nickel, dime, and <u>quarter</u> ); and <u>by adding collections of like</u> <u>coins together to a sum no greater than \$1.00</u> .	M(N&O)–2–4 No standard at this grade M(N&O)–2–5 <b>Demonstrates understanding of</b> <b>monetary value</b> by adding coins together to a value no greater than \$1.99 and representing the result in dollar notation; making change from \$1.00 or less, or recognizing equivalent coin representations of the same value (values up to \$1.99).

Number and Operations Continued		
Grade K	Grade 1	Grade 2
M(N&O)–K–6 <b>Mentally adds and subtracts whole numbers</b> by naming the number that is one more or one less than the original number. (IMPORTANT: <i>The intent of this GLE is to embed mental</i> <i>arithmetic throughout the instructional program, not to teach it</i> <i>as a separate unit.</i> )	M(N&O)-1-6 Mentally adds and subtracts whole numbers by naming the number that is one or <u>two</u> more or less than the original number; and adds and subtracts whole number facts to ten (e.g., $5 + 3 = 8$ ; $8 - 5 = 3$ ). (IMPORTANT: <i>The intent of this GLE is to embed mental</i> <i>arithmetic throughout the instructional program, not to teach it</i> <i>as a separate unit.</i> )	M(N&O)-2-6 Mentally adds and subtracts whole number facts to a sum of 20; names the number that is 10 more or less than the original number, and mentally adds and subtracts two- digit multiplies of ten (e.g., $60 + 80$ , $90 - 30$ ). (IMPORTANT: <i>The intent of this GLE is to embed mental</i> <i>arithmetic throughout the instructional program, not to teach it</i> <i>as a separate unit.</i> )
M(N&O)–K–7 <b>Makes estimates</b> of the number of objects in a set (up to 20) by making and revising estimates as objects are counted (e.g., A student estimates the number of pennies in a jar as 20. Then the student counts the first 10 and makes another estimate based on those that have been counted and those that remain in the jar.). (IMPORTANT: <i>Estimation should be imbedded instructionally</i> <i>throughout all strands</i> .)	<ul> <li>M(N&amp;O)-1-7 Makes estimates of the number of objects in a set (<u>up to 30</u>) and revises estimates as objects are counted (e.g., A student estimates the number of pennies in a jar as 28. Then the student counts the first 10 and makes another estimate based on those that have been counted and those that remain in the jar.).</li> <li>(IMPORTANT: <i>Estimation should be imbedded instructionally throughout all strands</i>.)</li> </ul>	M(N&O)–2–7 <b>Makes estimates</b> of the number of objects in a set ( <u>up to 50</u> ) by selecting an appropriate method of estimation. (IMPORTANT: <i>The intent of this GLE is to embed estimation throughout the instructional program, not to teach it as a separate unit.</i> )
	M(N&O)-1-8 Applies properties of numbers (odd, even, composition, and decomposition [e.g., 5 is the same as 2 + 3]) and field properties (commutative and identity for addition) to solve problems and to simplify computations involving whole numbers.	M(N&O)–2–8 <b>Applies properties of numbers</b> (odd and even) and <b>field properties</b> (commutative for addition, identity for addition, and <u>associative for addition</u> ) <b>to solve problems and</b> <b>to simplify computations involving whole numbers</b> .

Number and Operations Continued		
Grade 3	Grade 4	Grade 5
M(N&O)-3-1 Demonstrates conceptual understanding of rational numbers with respect to: whole numbers from 0 to 999 through equivalency, composition, decomposition, or place value using models, explanations, or other representations; and positive fractional numbers (benchmark fractions: $a/2$ , a/3, $a/4$ , $a/6$ , or $a/8$ , where a is a whole number greater than 0 and less than or equal to the denominator) as a part to whole relationship in area and set models where the number of parts in the whole is equal to the denominator; and <u>decimals (within a context of money)</u> as a part of 100 using models, explanations, or other representations.	M(N&O)-4-1 Demonstrates conceptual understanding of rational numbers with respect to: whole numbers from 0 to 999,999 through equivalency, composition, decomposition, or place value using models, explanations, or other representations; and positive fractional numbers (benchmark fractions: $a/2$ , $a/3$ , $a/4$ , $a/5$ , $a/6$ , $a/8$ , or a/10, where a is a whole number greater than 0 and less than or equal to the denominator) as a part to whole relationship in area, set, or <u>linear models</u> where the number of parts in the whole are equal to, and a <u>multiple or factor of the denominator</u> ; and <u>decimals as hundredths</u> within the context of money, or tenths <u>within the context of metric measurements</u> (e.g., 2.3 cm) using models, explanations, or other representations.	M(N&O)-5-1 Demonstrates conceptual understanding of rational numbers with respect to: whole numbers from <u>0 to 9,999,999</u> through equivalency, composition, decomposition, or place value using models, explanations, or other representations; and positive fractional numbers (proper, mixed number, and improper) (halves, fourths, eighths, thirds, sixths, <u>twelfths</u> , fifths, or <u>powers of ten</u> (10, 100, 1000)), decimals (to thousandths), or <u>benchmark percents</u> (10%, 25%, 50%, 75% or 100%) as a part to whole relationship in area, set, or linear models using models, explanations, or other representations.
M(N&O)–3–2 Demonstrates understanding of the relative magnitude of numbers from 0 to 999 by ordering whole numbers; by comparing whole numbers to benchmark whole numbers (100, 250, 500, 750); or by comparing whole numbers to each other; and comparing or identifying equivalent positive fractional numbers $(a/2, a/3, a/4$ where <i>a</i> is a whole number greater than 0 and less than or equal to the denominator) using models, number lines, or explanations.	M(N&O)–4–2 <b>Demonstrates understanding of the</b> <b>relative magnitude of numbers</b> from <u>0 to 999,999</u> by ordering or comparing whole numbers; and ordering, comparing, or identifying equivalent proper positive <u>fractional numbers</u> ; <u>or decimals</u> using models, number lines, or explanations.	M(N&O)–5–2 Demonstrates understanding of the relative magnitude of numbers by ordering, comparing, or identifying equivalent positive <u>fractional</u> <u>numbers</u> , decimals, or <u>benchmark percents within</u> <u>number formats (fractions to fractions, decimals to decimals, or percents to percents); or integers in context using models or number lines.</u>

Number and Operations Continued		
Grade 3	Grade 4	Grade 5
M(N&O)–3–3 <b>Demonstrates conceptual</b> <b>understanding of mathematical operations</b> <u>by</u> <u>describing or illustrating the inverse relationship</u> <u>between addition and subtraction of whole numbers;</u> and <u>the relationship between repeated addition and</u> multiplication using models, number lines, or	M(N&O)-4-3 <b>Demonstrates conceptual</b> <b>understanding of mathematical operations</b> by describing or illustrating <u>the relationship between</u> <u>repeated subtraction and division (no remainders); the</u> <u>inverse relationship between multiplication and division</u> <u>of whole numbers; or the addition or subtraction of</u>	M(N&O)-5-3 <b>Demonstrates conceptual understanding of</b> <b>mathematical operations</b> by <u>adding and subtracting decimals</u> and positive proper fractions with unlike denominators. M(N&O)-5-3 <b>Demonstrates conceptual</b> <b>understanding of mathematical operations</b> by
explanations.	positive fractional numbers with like denominators using models, number lines, or explanations.	describing or illustrating the <u>meaning of a remainder with</u> respect to division of whole numbers using models, explanations, or <u>solving problems</u> .
M(N&O)–3–4 Accurately solves problems involving addition and subtraction with regrouping; the concept of multiplication; and addition or subtraction of decimals (in the context of money).	M(N&O)-4-4 Accurately solves problems involving multiple operations on whole numbers or the use of the properties of factors and multiples; and addition or subtraction of <u>decimals and positive proper fractions</u> with like denominators. ( <u>Multiplication limited to 2</u> digits by 2 digits, and division limited to 1 digit <u>divisors</u> .)	M(N&O)–5–4 Accurately solves problems involving multiple operations on whole numbers or the use of the properties of factors, multiples, <u>prime</u> , or <u>composite</u> <u>numbers</u> ; and addition or subtraction of <u>fractions (proper)</u> and decimals to the hundredths place. (Division of whole <u>numbers by up to a two-digit divisor</u> .)
M(N&O)–3–5 No standard at this grade	(IMPORTANT: <u>Applies the conventions of order of</u> <u>operations where the left to right computations are</u> <u>modified only by the use of parentheses</u> .) M(N&O)-4-5 No standard at this grade	(IMPORTANT: Applies the conventions of order of operations <u>with and without parentheses</u> .) M(N&O)–5–5 No standard at this grade

Number and Operations continued		
Grade 3	Grade 4	Grade 5
M(N&O)-3-6 Mentally adds and subtracts whole number facts through 20; adds two-digit and one-digit whole numbers; adds combinations of two-digit and three-digit whole numbers that are multiples of ten (e.g., $60 + 50$ , $300 + 400$ , $320 + 90$ ); subtracts a one-digit whole number from a two-digit whole number (e.g., $37 - 5$ ); and subtracts two-digit whole numbers that are multiples of ten and three-digit whole numbers that are multiples one hundred (e.g., $50 - 20$ , $500 - 200$ ). (IMPORTANT: <i>The intent of this GLE is to embed mental</i> <i>arithmetic throughout the instructional program, not to teach it</i>	M(N&O)-4-6 Mentally adds and subtracts whole number facts through 20; <u>multiplies whole number facts to a product of</u> <u>100</u> , and <u>calculates related division facts</u> ; adds <u>two-digit whole</u> <u>numbers</u> , combinations of two-digit and 3-digit whole numbers that are multiples of ten, and <u>4-digit whole numbers that are</u> <u>multiples of 100</u> (limited to two addends) (e.g., 67 + 24; 320 + 430; 320 + 90; 1,300 + 1,400); and subtracts a one-digit whole number from a two-digit whole number (e.g., 67 - 9); and <u>subtracts combinations of two-digit and three-digit whole</u> <u>numbers that are multiples of ten (e.g., 50 - 20, 230 - 80, 520 - 200)</u> .	M(N&O)-5-6 Mentally <u>calculates</u> change back from \$1.00, <u>\$5.00, and \$10.00</u> ; calculates multiplication and related division facts to a product of 144; multiplies a two-digit whole number by a one-digit whole number (e.g., 45 x 5), two-digit whole numbers that are multiples of ten (e.g., 50 x 60), a three- digit whole number that is a multiple of 100 by a two- or three- digit number which is a multiple of 10 or 100, respectively (e.g., 400 x 50, 400 x 600); and divides 3- and 4-digit multiples of powers of ten by their compatible factors (e.g., 360 $\div$ 6; 360 $\div$ 60; 3600 $\div$ 6; 3600 $\div$ 60; 3600 $\div$ 600; 360 $\div$ 12; 360 $\div$ 120; 3600 $\div$ 12; 3600 $\div$ 120; 3600 $\div$ 1200).
as a separate unit.)	(IMPORTANT: The intent of this GLE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.)	(IMPORTANT: The intent of this GLE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.)
M(N&O)-3-7 <u>Makes estimates in a given situation by</u> identifying when estimation is appropriate, selecting the appropriate method of estimation, and evaluating the reasonableness of solutions appropriate to grade level GLEs across content strands. (IMPORTANT: <i>The intent of this GLE is to embed estimation</i> <i>throughout the instructional program, not to teach it as a</i> <i>separate unit.</i> )	M(N&O)-4-7 <b>Makes estimates</b> in a given situation by identifying when estimation is appropriate, selecting the appropriate method of estimation, and evaluating the reasonableness of solutions appropriate <u>to grade level GLEs</u> across content strands. (IMPORTANT: <i>The intent of this GLE is to embed estimation</i> <i>throughout the instructional program, not to teach it as a</i> <i>separate unit.</i> )	M(N&O)-5-7 <b>Makes estimates</b> in a given situation by identifying when estimation is appropriate, selecting the appropriate method of estimation, <u>determining the level of</u> <u>accuracy needed given the situation, analyzing the effect of the</u> <u>estimation method on the accuracy of results</u> , and evaluating the reasonableness of solutions appropriate <u>to grade level GLEs</u> across content strands. (IMPORTANT: <i>The intent of this GLE is to embed estimation</i>
M(N&O)-3-8 Applies properties of numbers (odd, even, and multiplicative property of zero for single-digit whole numbers [ $6 \times 0 = 0$ ]) and field properties (commutative for addition, associative for addition, <u>identity for multiplication</u> , and <u>commutative for multiplication for single-digit whole numbers</u> [e.g., $3 \times 4 = 4 \times 3$ ]) to solve problems and to simplify computations involving whole numbers.	M(N&O)-4-8 Applies properties of numbers (odd, even, <u>multiplicative property of zero</u> , and <u>remainders</u> ) and <b>field</b> properties ( <u>commutative</u> , <u>associative</u> , and identity) to solve problems and to simplify computations.	<pre>(Init Ortificity The intent of this OfEL is to embed estimation throughout the instructional program, not to teach it as a separate unit.) M(N&amp;O)-5-8 Applies properties of numbers (odd, even, and <u>divisibility</u>) and field properties (commutative, associative, identity, and <u>distributive</u>) to solve problems and to simplify computations.</pre>

Number and Operations Continued		
Grade 6	Grade 7	Grade 8
M(N&O)–6–1 <b>Demonstrates conceptual</b> understanding of rational numbers with respect to	M(N&O)-7-1 <b>Demonstrates conceptual understanding of</b> <b>rational numbers with respect to</b> square roots of perfect squares, rates, and proportional reasoning.	M(N&O)-8-1 <b>Demonstrates conceptual understanding of</b> <b>rational numbers with respect to</b> absolute values, perfect square and cube roots, and percents as a way of describing change (percent increase and decrease) <b>using explanations</b> ,
<u>ratios (comparison of two whole numbers by division</u> <u><math>a/b</math>, <math>a : b</math>, and <math>a \div b</math>, where <math>b \ne 0</math>); and rates (e.g., <u><math>a</math></u> <u>out of b, 25%</u>) using models, explanations, or other representations.</u>	M(N&O)-7-1 <b>Demonstrates conceptual</b> <b>understanding of rational numbers with respect to</b> <b>percents</b> as a means of comparing the same or different parts of the whole when the wholes vary in magnitude (e.g., 8 girls in a classroom of 16 students compared to 8 girls in a classroom of 20 students, or 20% of 400 compared to 50% of 100); and percents as a way of expressing multiples of a number (e.g., 200% of 50) <b>using models, explanations, or other representations</b> .	models, or other representations.
M(N&O)-6-2 <b>Demonstrates understanding of the</b> relative magnitude of numbers by ordering or comparing numbers with whole number bases and whole number exponents (e.g., $3^3$ , $4^3$ ), integers, or rational numbers within and across number formats (fractions, decimals, or whole number percents from 1–100) using number lines or equality and inequality symbols.	M(N&O)–7–2 <b>Demonstrates understanding of the</b> <b>relative magnitude of numbers</b> by ordering, comparing, or identifying equivalent rational numbers <u>across number</u> <u>formats</u> , numbers with whole number bases and whole number exponents (e.g., 3 <sup>3</sup> , 4 <sup>3</sup> ), integers, <u>absolute values</u> , or <u>numbers represented in scientific notation</u> using number lines or equality and inequality symbols.	M(N&O)-8-2 <b>Demonstrates understanding of the relative</b> <b>magnitude of numbers</b> by ordering or comparing rational numbers, <u>common irrational numbers</u> (e.g., $\sqrt{2}$ , $\pi$ ), numbers with whole number or <u>fractional bases</u> and whole number exponents, <u>square roots</u> , absolute values, integers, or numbers represented in scientific notation using number lines or equality and inequality symbols.
M(N&O)–6–3 <b>Demonstrates conceptual understanding of</b> <b>mathematical operations</b> by adding and subtracting <u>positive</u> <u>fractions</u> and <u>integers</u> ; and <u>multiplying and dividing fractions</u> <u>and decimals</u> .	M(N&O)-7-3 <b>Demonstrates conceptual understanding of</b> <b>operations</b> <u>with integers and whole number exponents</u> (where the base is a whole number) using models, diagrams, or explanations.	M(N&O)–8–3 No standard at this grade
M(N&O)-6-3 <b>Demonstrates conceptual understanding</b> of mathematical operations by <u>describing or illustrating</u> the meaning of a power by representing the relationship between the base (whole number) and the exponent (whole number) (e.g., $3^3$ , $4^3$ ); and the effect on the magnitude of a whole number when multiplying or dividing it by a whole number, decimal, or fraction.		

Number and Operations Continued		
Grade 6	Grade 7	Grade 8
M(N&O)-6-4 Accurately solves problems involving single or multiple operations on fractions (proper, improper, and mixed), or decimals; and addition or subtraction of integers; percent of a whole; or problems involving greatest common factor or least common multiple. (IMPORTANT: Applies the conventions of order of operations with and without parentheses.)	M(N&O)-7-4 Accurately solves problems involving the addition or subtraction of integers, <u>raising numbers to whole number powers</u> , and determining square roots of perfect square numbers and non-perfect square numbers. M(N&O)-7-4 Accurately solves problems involving proportional reasoning; percents involving discounts, <u>tax</u> , or <u>tips</u> ; and <u>rates</u> . (IMPORTANT: <i>Applies the conventions of order of operations including parentheses</i> , <u>brackets</u> , or <u>exponents</u> .)	M(N&O)–8–4 Accurately solves problems involving proportional reasoning (percent increase or decrease, interest rates, markups, or rates); multiplication or division of integers; and squares, cubes, and taking square or cube roots. (IMPORTANT: <u>Applies the conventions of order of</u> <u>operations</u> .)
$\frac{M(N\&O)-6-5 \text{ No standard at this grade}}{M(N\&O)-6-6 \text{ Uses a variety of mental computation}}$ strategies to solve problems (e.g., using compatible numbers, applying properties of operations, using mental imagery, using patterns) and to determine the reasonableness of answers; and mentally calculates change back from \$5.00, \$10.00, \$20.00, \$50.00, and \$100.00; multiplies a two-digit whole number by a one-digit number whole number (e.g., 45 x 5), two-digit whole numbers that are multiples of ten (e.g., 50 x 60), a three-digit number which is a multiple of 100 by a two- or three-digit number which is a multiple of 10 or 100, respectively (e.g., 400 x 50, 400 x 600); divides 3- and 4-digit multiples of powers of ten by their compatible factors (e.g., 360 ÷ 6; 360 ÷ 60; 3600 ÷ 6; 3600 ÷ 60; 3600 ÷ 60; 3600 ÷ 12; 3600 ÷ 120; 3600 ÷ 120); and determines the part of a whole number using benchmark percents (1%, 10%, 25%, 50%, and 75%).	M(N&O)-7-6 Uses a variety of mental computation strategies to solve problems (e.g., using compatible numbers, applying properties of operations, using mental imagery, using patterns) and to determine the reasonableness of answers; and mentally calculates benchmark perfect squares and related square roots (e.g., $1^2$ , $2^2$ $12^2$ , $15^2$ , $20^2$ , $25^2$ , $100^2$ , $1000^2$ ); determines the part of a number using benchmark percents and related fractions (1%, 10%, 25%, $33\frac{1}{3}$ %, 50%, $66\frac{2}{3}$ %, 75%, and 100%) (e.g., 25% of 16; $33\frac{1}{3}$ % of 330). (IMPORTANT: The intent of this GSE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.)	M(N&O)–8–5 No standard at this grade M(N&O)–8–6 Uses a variety of mental computation strategies to solve problems (e.g., using compatible numbers, applying properties of operations, using mental imagery, using patterns) and to determine the reasonableness of answers; and mentally calculates benchmark perfect squares and related square roots (e.g., $1^2$ , $2^2$ ,, $12^2$ , $15^2$ , $20^2$ , $25^2$ , $100^2$ , $1000^2$ ); determines the part of a number using benchmark percents and related fractions (1%, 10%, 25%, $33\frac{1}{3}$ %, 50%, $66\frac{2}{3}$ %, 75%, and 100%) (e.g., 25% of 16; $33\frac{1}{3}$ % of 330). (IMPORTANT: <i>The intent of this GSE is to embed mental</i> <i>arithmetic throughout the instructional program, not to teach it</i> <i>as a separate unit.</i> )

Number and Operations continued		
Grade 6	Grade 7	Grade 8 (Local Option)
M(N&O)–6–7 <b>Makes estimates</b> in a given situation by identifying when estimation is appropriate, selecting the appropriate method of estimation, determining the level of accuracy needed given the situation, analyzing the effect of the estimation method on the accuracy of results, and evaluating the reasonableness of solutions appropriate to grade level GLEs across content strands. (IMPORTANT: <i>The intent of this GLE is to embed estimation</i> <i>throughout the instructional program, not to teach it as a</i> <i>separate unit.</i> )	M(N&O)-7-7 <b>Makes estimates</b> in a given situation ( <u>including</u> <u>tips, discounts, and tax</u> ) by identifying when estimation is appropriate, selecting the appropriate method of estimation, determining the level of accuracy needed given the situation, analyzing the effect of the estimation method on the accuracy of results, and evaluating the reasonableness of solutions appropriate <u>to grade level GLEs</u> across content strands. (IMPORTANT: <i>The intent of this GLE is to embed estimation throughout the instructional program, not to teach it as a separate unit.</i> )	<ul> <li>M(N&amp;O)-8-7 Makes estimates in a given situation (including tips, discounts, tax, and the value of a non-perfect square root as between two whole numbers) by identifying when estimation is appropriate, selecting the appropriate method of estimation; determining the level of accuracy needed given the situation; analyzing the effect of the estimation method on the accuracy of results; and evaluating the reasonableness of solutions appropriate to grade level GLEs across content strands.</li> <li>(IMPORTANT: <i>The intent of this GLE is to embed estimation throughout the instructional program, not to teach it as a separate unit.</i>)</li> </ul>
M(N&O)-6-8 Applies properties of numbers (odd, even, remainders, divisibility, and <u>prime factorization</u> ) and field properties (commutative, associative, identity [including the <u>multiplicative property of one, e.g., <math>1 = 2/2</math> and <math>2/2 \ge 3/4 = 6/8</math>, so <math>3/4 = 6/8</math>], distributive, and <u>additive inverses</u>) to solve problems and to simplify computations.</u>	M(N&O)-7-8 <b>Applies properties of numbers</b> (odd, even, remainders, divisibility, and prime factorization) and <b>field properties</b> (commutative, associative, identity, distributive, <u>inverses</u> ) <b>to solve problems and to simplify computations</b> , and <u>demonstrates conceptual understanding of field</u> <b>properties</b> as they apply to subsets of the real numbers (e.g., the set of whole numbers does not have additive inverses, the set of integers does not have multiplicative inverses).	M(N&O)-8-8 <b>Applies properties of numbers</b> (odd, even, remainders, divisibility, and <u>prime factorization</u> ) and <b>field</b> <b>properties</b> (commutative, associative, identity [including the <u>multiplicative property of one, e.g., <math>2^0 \ge 2^3 = 2^{0+3} = 2^3</math>, so <math>2^0 =</math> <u>1</u>], distributive, inverses) <b>to solve problems and to simplify</b> <b>computations</b>, and <b>demonstrates conceptual understanding</b> <b>of field properties</b> as they apply to subsets of real numbers when addition and multiplication are not defined in the <u>traditional ways (e.g., If <math>a\Delta b = a + b - 1</math>, is <math>\Delta</math> a commutative <u>operation?</u>)</u></u>

	Geometry and Measurement	
develop their reasoning and justification skills. We live in a three-dimensional world. To interpret, under depends, in part, on the development of spatial abilities. Spatial Measurement is the process of assigning a numerical val and analyze their world. It also provides an opportunity to make mature insights into the essential role of measurement as a link b	erstand, and appreciate that world, students need to develop an und skills include making and interpreting drawings, forming mental ir ue to an attribute of an object. The study of measurement provides connections within mathematics and between mathematics and oth etween the abstractness of mathematics and the concreteness of the	lerstanding of space. In addition, success in mathematics mages, and visualizing changes. s students with techniques and tools they will need to describe her curricular areas. High school students must develop more e real-world.
Grade K	stigate, experiment, and explore geometric properties using both teo Grade 1	Grade 2
M(G&M)–K–1 Uses properties, attributes, composition, or decomposition to sort or classify polygons (triangles, squares, rectangles, rhombi, trapezoids, and hexagons) or objects by using one non-measurable or measurable attribute; and recognizes, names, and builds polygons and circles in the environment. M(G&M)–K–2 No standard at this grade M(G&M)–K–3 No standard at this grade	<ul> <li>M(G&amp;M)-1-1 Uses properties, attributes, composition, or decomposition to sort or classify polygons (triangles, squares, rectangles, rhombi, trapezoids, and hexagons) or objects by <u>a</u> combination of two non-measurable or measurable attributes; and recognizes, names, builds, and <u>draws</u> polygons and circles in the environment.</li> <li>M(G&amp;M)-1-2 No standard at this grade</li> <li>M(G&amp;M)-1-3 Given an example of a three-dimensional geometric shape (rectangular prisms, cylinders, or spheres) finds examples of objects in the environment that are of the same geometric shape (e.g., show a wooden cylinder and students identify common objects of the same shape).</li> </ul>	M(G&M)–2–1 Uses properties, attributes, composition, or decomposition to sort or classify polygons or objects by <u>a combination of two or more</u> non-measurable or measurable attributes. M(G&M)–2–2 No standard at this grade M(G&M)–2–3 No standard at this grade
M(G&M)–K–4 No standard at this grade	M(G&M)–1–4 <b>Demonstrates conceptual understanding of</b> <b>congruency</b> by making mirror images and creating shapes that have line symmetry.	M(G&M)–2–4 <b>Demonstrates conceptual understanding of</b> <b>congruency</b> by <u>composing and decomposing two-dimensional</u> <u>objects using models or explanations (e.g., using triangular</u> <u>pattern blocks to construct a figure congruent to the hexagonal</u> <u>pattern block)</u> ; and <u>uses line symmetry to demonstrate</u> <u>congruent parts within a shape</u> .
M(G&M)–K–5 No standard at this grade M(G&M)–K–6 No standard at this grade	M(G&M)-1-5 No standard at this grade M(G&M)-1-6 Demonstrates conceptual understanding of the length/height of a two-dimensional object using non- standard units (e.g. comparing objects to trains of small cubes, using iterations of a small unit to measure an object).	M(G&M)–2–5 No standard at this grade M(G&M)–2–6 <b>Demonstrates conceptual</b> <b>understanding of perimeter and area</b> by using models or manipulatives to surround and cover polygons.

Geometry and Measurement continued		
Grade K	Grade 1	Grade 2
<ul> <li>M(G&amp;M)-K-7 Demonstrates conceptual understanding of measurable attributes using comparative language to describe and compare attributes of objects (length [longer, shorter], height [taller, shorter], weight [heavier, lighter], temperature [warmer, cooler], and capacity [more, less]); and compares objects visually and with direct comparison.</li> <li>M(G&amp;M)-K-8 Determines elapsed and accrued time as it relates to calendar patterns (days of the week, yesterday, today, and tomorrow), the sequence of events in a day; and identifies a clock and calendar as measurement tools (days of week,</li> </ul>	M(G&M)-1-7 <b>Demonstrates conceptual understanding of</b> <b>measurable attributes</b> using comparative language to describe and compare attributes of objects (length [longer, shorter], height [taller, shorter], weight [heavier, lighter], temperature [warmer, cooler], and capacity [more, less]); compares objects visually, with direct comparison, and <u>using non-standard units</u> . M(G&M)-1-8 <b>Determines elapsed and accrued time</b> as it relates to calendar patterns (days of the week, <u>months of the</u> <u>year</u> ), the sequence of events in a day; and <u>recognizes an hour</u> and "on the ½ hour".	M(G&M)–2–7 Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems across the content strands. M(G&M)–2–8 No standard at this grade
<ul> <li>months of the year).</li> <li>M(G&amp;M)–K–9 Demonstrates understanding of spatial relationships using location and position by using positional words to locate and describe where an object is found in the environment.</li> </ul>	M(G&M)–1–9 <b>Demonstrates understanding of spatial</b> <b>relationships using location and position</b> by using positional words (e.g., close by, on the right, underneath, above, beyond) to describe one location in reference to another <u>on a map</u> , <u>in a</u> <u>diagram</u> , and in the environment.	M(G&M)–2–9 <b>Demonstrates understanding of spatial</b> <b>relationships using location and position</b> by using positional language <u>in two- and three- dimensional situations to describe</u> <u>and interpret relative positions (e.g., above the surface of the</u> <u>desk, below the triangle on the paper)</u> ; and <u>creates and</u> <u>interprets simple maps and names locations on simple</u> <u>coordinate grids</u> .

Geometry and Measurement continued		
Grade 3	Grade 4	Grade 5
M(G&M)–3–1 <u>Uses properties or attributes of angles</u> (number of angles) or sides (number of sides or length of sides) or composition or decomposition of shapes to identify, describe, or distinguish among triangles, squares, rectangles, rhombi, trapezoids, hexagons, or circles.	M(G&M)-4-1 Uses properties or attributes of angles (number of angles) or sides (number of sides, length of sides, <u>parallelism</u> , or <u>perpendicularity</u> ) to identify, describe, or distinguish among triangles, squares, rectangles, rhombi, trapezoids, hexagons, or <u>octagons</u> ; or <u>classify angles relative to 90°</u> as more than, less than, or equal to.	M(G&M)–5–1 Uses properties or attributes of angles (right, acute, or obtuse) or sides (number of congruent sides, parallelism, or perpendicularity) to identify, describe, <u>classify</u> , or distinguish among <u>different types</u> of triangles (right, acute, obtuse, equiangular, or equilateral) or <u>quadrilaterals</u> (rectangles, squares, <u>rhombi</u> , trapezoids, or parallelograms).
M(G&M)–3–2 No standard at this grade	M(G&M)–4–2 No standard at this grade	M(G&M)–5–2 No standard at this grade
M(G&M)–3–3 No standard at this grade	M(G&M)–4–3 Uses properties or attributes (shape of bases or number of lateral faces) to identify, compare, or describe three-dimensional shapes (rectangular prisms, triangular prisms, cylinders, or spheres).	M(G&M)–5–3 Uses properties or attributes (shape of bases, number of lateral faces, or <u>number of bases</u> ) to identify, compare, or describe three-dimensional shapes (rectangular prisms, triangular prisms, cylinders, spheres, <u>pyramids</u> , or <u>cones</u> ).
M(G&M)–3–4 Demonstrates conceptual understanding of		M(G&M)–5–4 No standard at this grade
<b>congruency</b> by <u>matching congruent figures using reflections</u> , translations, and rotations (flips, slides, and turns) (e.g., recognizing when pentominoes are reflections, translations and rotations of each other); composing and decomposing two- and three-dimensional objects using models or explanations (e.g., Given a cube, students use blocks to construct a congruent cube.); and by using line symmetry to demonstrate congruent parts within a shape.	M(G&M)–4–4 <b>Demonstrates conceptual</b> <b>understanding of congruency</b> by matching congruent figures using reflections, translations, or rotations (flips, slides, or turns), or as the result of composing or decomposing shapes using models or explanations.	
M(G&M)–3–5 <b>Demonstrates conceptual understanding of</b> <b>similarity</b> by identifying similar shapes.	M(G&M)–4–5 <b>Demonstrates conceptual</b> <b>understanding of similarity</b> by applying scales on maps, or applying characteristics of similar figures (same shape but not necessarily the same size) to identify similar figures, or to solve problems involving similar figures. Describes relationships using models or <sup>sc</sup> explanations.	M(G&M)–5–5 <b>Demonstrates conceptual understanding of</b> <u>similarity by describing the proportional effect on the linear</u> <u>dimensions of triangles and rectangles when scaling up or</u> <u>down while preserving angle measures</u> , or by solving related problems (including applying scales on maps). Describes effects using models or <sup>sc</sup> explanations.

Geometry and Measurement continued		
Grade 3	Grade 4	Grade 5
M(G&M)–3–6 <b>Demonstrates conceptual understanding</b> <b>of perimeter of</b> polygons, and <b>the area of</b> rectangles <u>on</u> <u>grids</u> using a variety of models or manipulatives. <u>Expresses</u> <u>all measures using appropriate units</u> .	M(G&M)–4–6 <b>Demonstrates conceptual</b> <b>understanding of perimeter of</b> polygons, and <b>the area</b> <b>of</b> rectangles, <u>polygons or irregular shapes on grids</u> using a variety of models, manipulatives, or <u>formulas</u> . Expresses all measures using appropriate units.	M(G&M)–5–6 <b>Demonstrates conceptual</b> <b>understanding of perimeter</b> of polygons, <b>and the area</b> <b>of</b> rectangles or <u>right triangles</u> through models, manipulatives, or formulas, the area of polygons or irregular figures on grids, <b>and</b> <u>volume of rectangular</u> <u>prisms (cubes)</u> using a variety of models, manipulatives, or formulas. Expresses all measures using appropriate units.
M(G&M)-3-7 Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems across the content strands. Benchmarks in Appendix B.	M(G&M)-4-7 Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems across the content strands. Benchmarks in Appendix B.	M(G&M)–5–7 Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems across the content strands. Benchmarks in Appendix B.
M(G&M)–3–8 No standard at this grade	M(G&M)–4–8 No standard at this grade	M(G&M)–5–8 No standard at this grade
M(G&M)-3-9 <b>Demonstrates understanding of spatial</b> relationships using location and position by interpreting and giving directions from one location to another (e.g., classroom to the gym, from school to home) using positional words; and between locations on a map or coordinate grid (first quadrant) using positional words or compass directions.	M(G&M)-4-9 <b>Demonstrates understanding of spatial</b> relationships using location and position by interpreting and giving directions between locations on a map or coordinate grid (first quadrant); <u>plotting points in the first quadrant in context</u> (e.g., games, mapping); and finding the horizontal and vertical distances between points on a coordinate grid in the first quadrant.	M(G&M)–5–9 <b>Demonstrates understanding of spatial</b> relationships using location and position by interpreting and giving directions between locations on a map or coordinate grid (all four quadrants); plotting points in four quadrants in context (e.g., games, mapping, identifying the vertices of polygons as they are reflected, rotated, and translated); and determining horizontal and vertical distances between points on a coordinate grid in the first quadrant.
M(G&M)-3-10 <b>Demonstrates conceptual understanding of</b> <b>spatial reasoning and visualization</b> by copying, comparing, and drawing models of triangles, squares, rectangles, rhombi, trapezoids, hexagons, and circles; and builds models of rectangular prisms from three-dimensional representations.	M(G&M)-4-10 <b>Demonstrates conceptual understanding of</b> <b>spatial reasoning and visualization</b> by copying, comparing, and drawing models of triangles, squares, rectangles, rhombi, trapezoids, hexagons, <u>octagons</u> , and circles; and builds models of rectangular prisms from <u>two-</u> or three-dimensional representations.	M(G&M)–5–10 <b>Demonstrates conceptual understanding of</b> <b>spatial reasoning and visualization</b> by building models of rectangular and <u>triangular prisms</u> , <u>cones</u> , <u>cylinders</u> , and <u>pyramids</u> from two- or three-dimensional representations.

Geometry and Measurement continued		
Grade 6	Grade 7	Grade 8
M(G&M)–6–1 Uses properties or attributes of angles (right, acute, or obtuse) or sides (number of congruent sides, parallelism, or perpendicularity) to identify, describe, classify, or distinguish among different types of triangles (right, acute, obtuse, equiangular, <u>scalene</u> , <u>isosceles</u> , or equilateral) or quadrilaterals (rectangles, squares, rhombi, trapezoids, or parallelograms).	M(G&M)–7–1 <u>Uses properties of angle relationships</u> resulting from two or three intersecting lines (adjacent angles, vertical angles, straight angles, or angle relationships formed by two non-parallel lines cut by a transversal), or two parallel lines cut by a transversal to solve problems.	M(G&M)–8–1 No standard at this grade
M(G&M)–6–2 No standard at this grade	M(G&M)–7–2 <b>Applies theorems or relationships</b> (triangle inequality or sum of the measures of interior angles of regular polygons) to solve problems.	M(G&M)–8–2 <u>Applies the Pythagorean Theorem to find a</u> missing side of a right triangle, or in problem solving <u>situations</u> .
M(G&M)–6–3 Uses properties or attributes (shape of bases, number of lateral faces, number of bases, <u>number of edges</u> , or <u>number of vertices</u> ) to identify, compare, or describe three-dimensional shapes (rectangular prisms, triangular prisms, cylinders, spheres, pyramids, or cones).	M(G&M)–7–3 No standard at this grade	M(G&M)–8–3 No standard at this grade
M(G&M)–6–4 <b>Demonstrates conceptual understanding of</b> <b>congruency</b> by <u>predicting and describing the transformational</u> <u>steps (reflections, translations, and rotations) needed to show</u> <u>congruence (including the degree of rotation) and as the result</u> <u>of composing and decomposing two- and three-dimensional</u> <u>objects using models or explanations;</u> and using line and rotational symmetry to demonstrate congruent parts within a shape.	M(G&M)–7–4 <u>Applies the concepts of congruency by</u> <u>solving problems on a coordinate plane involving</u> <u>reflections, translations, or rotations</u> .	M(G&M)–8–4 No standard at this grade

Geometry and Measurement continued		
Grade 6	Grade 7	Grade 8
M(G&M)–6–5 <b>Demonstrates conceptual</b> <b>understanding of similarity</b> by describing the proportional effect on the linear dimensions of <u>polygons</u> <u>or circles</u> when scaling up or down while preserving the angles of polygons, or by solving related problems (including applying scales on maps). Describes effects using models or <sup>sc</sup> explanations.	M(G&M)–7–5 <u>Applies concepts of similarity by</u> <u>solving problems</u> involving scaling up or down and their impact on angle measures, linear dimensions and <u>areas of polygons, and circles when the linear</u> <u>dimensions are multiplied by a constant factor</u> . Describes effects using models or <sup>sc</sup> explanations.	M(G&M)–8–5 <b>Applies concepts of similarity</b> <u>to determine the</u> <u>impact of scaling on the volume or surface area of three-</u> <u>dimensional figures when linear dimensions are multiplied by a</u> <u>constant factor</u> ; to <u>determine the length of sides of similar</u> <u>triangles</u> , or to <u>solve problems involving growth and rate</u> .
M(G&M)–6–6 <b>Demonstrates conceptual</b> <b>understanding of perimeter</b> of polygons, <b>the area of</b> <u>quadrilaterals</u> or <u>triangles</u> , and <u>the volume of rectangular</u> <u>prisms</u> by using models, formulas, or by <u>solving</u> <u>problems</u> ; and <u>demonstrates understanding of the</u> <u>relationships of circle measures (radius to diameter and</u> <u>diameter to circumference) by solving related problems</u> . Expresses all measures using appropriate units.	M(G&M)-7-6 Demonstrates conceptual understanding of the area of <u>circles</u> or <u>the area or</u> perimeter of composite figures (quadrilaterals, triangles, or parts of circles), and the <u>surface area of rectangular</u> prisms, or <u>volume of rectangular prisms</u> , triangular prisms, or cylinders using models, formulas, or by solving related problems. Expresses all measures using appropriate units.	M(G&M)–8–6 Demonstrates conceptual understanding of surface area or volume by solving problems involving surface area and volume of rectangular prisms, triangular prisms, cylinders, pyramids, or cones. Expresses all measures using appropriate units.
M(G&M)–6–7 Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems across the content strands.	M(G&M)–7–7 No standard at this grade	M(G&M)–8–7 No standard at this grade
Benchmarks in Appendix B.		

Geometry and Measurement continued		
Grade 6	Grade 7	Grade 8
M(G&M)–6–8 No standard at this grade	M(G&M)–7–8 No standard at this grade	M(G&M)–8–8 No standard at this grade
M(G&M)–6–9 No standard at this grade	M(G&M)–7–9 No standard at this grade	M(G&M)–8–9 No standard at this grade
M(G&M)–6–10 No standard at this grade	M(G&M)–7–10 Demonstrates conceptual understanding of	M(G&M)–8–10 No standard at this grade
	spatial reasoning and visualization by sketching three-	
	dimensional solids; and draws nets of rectangular and	
	triangular prisms, cylinders, and pyramids and uses the nets as	
	a technique for finding surface area.	

	Functions and Algebra	
physical models, tables, graphs). In later years students progress	matics is communicated. Students in Kindergarten begin to explor to more abstract representations. The study of patterns is one of the evels should recognize, describe, and generalize patterns and build portant tools that students can use throughout their lives	he central themes of algebraic thinking and leads to an
Grade K	Grade 1	Grade 2
M(F&A)–K–1 I Identifies and extends to specific cases a variety of patterns (sequences of shapes, sounds, movement, colors, and letters) by extending the pattern to the next one, two or three elements, or by translating AB patterns across formats (e.g., an abb pattern can be represented as snap, clap, clap or red, yellow, yellow) or by identifying number patterns in the environment. M(F&A)–K–2 No standard at this grade	M(F&A)-1-1 <b>Identifies and extends to specific cases a</b> <b>variety of patterns</b> (repeating and growing [numeric and non- numeric]) represented in models, tables, or sequences by extending the pattern to the next one, two, or <u>three elements</u> , <u>by</u> finding a missing element (e.g., 2, 4, 6,, 10), or by translating repeating patterns across formats (e.g., an abb pattern can be represented as snap, clap, clap; or red, yellow, yellow; or 1,2,2). M(F&A)-1-2 No standard at this grade	M(F&A)-2-1 Identifies and extends to specific cases a variety of patterns (linear and non-numeric) represented in models, tables, or sequences by extending the pattern to the next element, or finding a missing element (e.g., 2, 4, 6,, 10). M(F&A)-2-2 No standard at this grade
M(F&A)–K–3 No standard at this grade	M(F&A)–1–3 No standard at this grade	M(F&A)–2–3 No standard at this grade
M(F&A)–K–4 No standard at this grade	M(F&A)-1-4 Demonstrates conceptual understanding of equality by <u>finding the value that will make an open sentence</u> <u>true (e.g., 2+ =7) using models, verbal explanations, or</u> written equations. (limited to one operation and limited to use addition or subtraction)	M(F&A)-2-4 <b>Demonstrates conceptual</b> <b>understanding of equality</b> by finding the value that will make an open sentence true (e.g., $2 + = 7$ ). (limited to one operation and limited to use addition or subtraction)

Functions and Algebra continued		
Grade 3	Grade 4	Grade 5
M(F&A)-3-1 <b>Identifies and extends to specific cases a variety of patterns</b> (linear and non-numeric) represented in models, tables, or sequences by extending the pattern to the next one, <u>two</u> , or <u>three elements</u> , or finding missing <u>elements</u> .	M(F&A)-4-1 <b>Identifies and <u>extends to specific cases</u></b> <b>a variety of patterns</b> (linear and <u>nonlinear</u> ) represented in models, tables or sequences; and <u>writes</u> <u>a rule in words or<sup>se</sup> symbols to find the next case</u> .	M(F&A)–5–1 <b>Identifies and extends to specific cases</b> <b>a variety of patterns</b> (linear and nonlinear) represented in models, tables, sequences, or <u>in problem situations</u> ; and writes a rule in words or <sup>sc</sup> symbols <u>for finding</u> <u>specific cases of a linear relationship</u> .
M(F&A)–3–2 No standard at this grade	M(F&A)-4-2 <b>Demonstrates conceptual understanding of</b> <b>linear relationships</b> ( $y = kx$ ) as a constant rate of change by identifying, describing, or comparing situations that represent constant rates of change.	M(F&A)-5-2 <b>Demonstrates conceptual understanding of</b> <b>linear relationships</b> ( $y = kx$ ) as a constant rate of change by identifying, describing, or comparing situations that represent constant rates of change (e.g., tell a story given a line graph about a trip).
M(F&A)–3–3 No standard at this grade	M(F&A)-4-3 <b>Demonstrates conceptual</b> <b>understanding of algebraic expressions</b> by using letters or symbols to represent unknown quantities to write simple linear algebraic expressions involving any one of the four operations; or by evaluating simple linear algebraic expressions using whole numbers.	M(F&A)–5–3 <b>Demonstrates conceptual</b> <b>understanding of algebraic expressions</b> by using letters to represent unknown quantities to write <u>linear</u> <u>algebraic expressions</u> involving <u>any two</u> of the four operations; or by evaluating <u>linear algebraic expressions</u> using whole numbers.
M(F&A)-3-4 <b>Demonstrates conceptual</b> <b>understanding of equality</b> by showing equivalence between two expressions using models or different representations of the expressions; or by finding the value that will make an open sentence true (e.g., 2+ =7). (limited to one operation and limited to use addition, subtraction, or <u>multiplication</u> )	M(F&A)-4-4 <b>Demonstrates conceptual</b> <b>understanding of equality</b> by showing equivalence between two expressions using models or different representations of the expressions, by <u>simplifying</u> <u>numerical expressions where left to right computations</u> may be modified only by the use of parentheses [e.g., 14 $-(2 \times 5)$ ] (expressions consistent with the parameters of M(F&A)-4-3), and by solving one-step linear equations of the form $ax = c, x \pm b = c$ , where a, b, and c are whole numbers with $a \neq 0$ .	M(F&A)-5-4 <b>Demonstrates conceptual understanding</b> of equality by showing equivalence between two expressions using models or different representations of the expressions (expressions consistent with the parameters of M(F&A)-5-3), by solving one-step linear equations of the form $ax = c$ , $x \pm b = c$ , or $x/a = c$ , where $a$ , $b$ , and $c$ are whole numbers with $a \neq 0$ ; or by determining which values of a replacement set make the equation (multi-step of the form $ax \pm b = c$ where $a$ , $b$ , and $c$ are whole numbers with $a \neq 0$ ) a true statement (e.g., $2x + 3 =$ $11$ , $\{x: x = 2, 3, 4, 5\}$ ).

Functions and Algebra continued		
Grade 6	Grade 7	Grade 8
M(F&A)–6–1 Identifies and extends to specific cases a variety of patterns (linear and nonlinear) represented in models, tables, sequences, graphs, or in problem situations; or writes a rule in words or symbols for finding specific cases of a linear relationship; or writes a rule in words or <sup>sc</sup> symbols for finding specific cases of a nonlinear relationship; and writes an expression or <sup>sc</sup> equation using words or <sup>sc</sup> symbols to express the generalization of a linear relationship (e.g., twice the term number plus 1 or <sup>sc</sup> $2n + 1$ ).	M(F&A)–7–1 <b>Identifies and extends to specific cases a</b> <b>variety of patterns</b> (linear and nonlinear) represented in models, tables, sequences, graphs, or in problem situations; <b>and generalizes</b> a linear relationship using words and symbols; generalizes a linear relationship to find a specific case; or writes an expression or <sup>sc</sup> equation using words or <sup>sc</sup> symbols to express the <b>generalization</b> of a nonlinear relationship.	M(F&A)–8–1 Identifies and extends to specific cases a variety of patterns (linear and nonlinear) represented in models, tables, sequences, graphs, or in problem situations; and generalizes a linear relationship (non-recursive explicit equation); generalizes a linear relationship to find a specific case; generalizes a nonlinear relationship using words or <sup>sc</sup> symbols; or generalizes a common nonlinear relationship to find a specific case.
M(F&A)-6-2 Demonstrates conceptual understanding of linear relationships ( $y = kx$ ; $y = mx + b$ ) as a constant rate of change by constructing or interpreting graphs of real occurrences and describing the slope of linear relationships (faster, slower, greater, or smaller) in a variety of problem situations; and describes how change in the value of one variable relates to change in the value of a second variable in problem situations with constant rates of change.	M(F&A)-7-2 Demonstrates conceptual understanding of linear relationships ( $y = kx$ ; $y = mx + b$ ) as a constant rate of change by solving problems involving the relationship between slope and rate of change, by describing the meaning of slope in concrete situations, or informally determining the slope of a line from a table or graph; and distinguishes between constant and varying rates of change in concrete situations represented in tables or graphs; or describes how change in the value of one variable relates to change in the value of a second variable in problem situations with constant rates of change.	M(F&A)-8-2 Demonstrates conceptual understanding of linear relationships ( $y = kx$ ; $y = mx + b$ ) as a constant rate of change by solving problems involving the relationship between slope and rate of change; <u>informally and formally determining</u> <u>slopes and intercepts represented in graphs, tables, or problem</u> <u>situations; or describing the meaning of slope and intercept in</u> <u>context; and distinguishes between linear relationships</u> (constant rates of change) and nonlinear relationships (varying rates of change) represented in tables, graphs, equations, or problem situations; or describes how change in the value of one variable relates to change in the value of a second variable in problem situations with constant and varying rates of change.

Functions and Algebra continued		
Grade 6	Grade 7	Grade 8
M(F&A)–6–3 <b>Demonstrates conceptual understanding</b> of algebraic expressions by using letters to represent unknown quantities to write linear algebraic expressions involving <u>any of the four operations and consistent with</u> <u>order of operations expected at this grade level</u> ; or by evaluating linear algebraic expressions ( <u>including those</u> <u>with more than one variable</u> ); or by <u>evaluating an</u> <u>expression within an equation (e.g., determine the value of</u> <u>y when <math>x = 4</math> given <math>y = 3x - 2</math>).</u>	M(F&A)-7-3 <b>Demonstrates conceptual</b> <b>understanding of algebraic expressions</b> by using letters to represent unknown quantities to write <u>algebraic</u> <u>expressions (including those with whole number</u> <u>exponents or more than one variable</u> ); or by evaluating <u>algebraic expressions</u> (including those with <u>whole</u> <u>number exponents</u> or more than one variable); or by evaluating an expression within an equation (e.g., determine the value of y when $x = 4$ given $y = 5x^3 - 2$ ).	M(F&A)–8–3 <b>Demonstrates conceptual understanding of</b> <b>algebraic expressions</b> by evaluating and <u>simplifying algebraic</u> <u>expressions (including those with square roots, whole number</u> <u>exponents, or rational numbers)</u> ; or by evaluating an expression within an equation (e.g., determine the value of y when $x = 4$ given $y = 7\sqrt{x} + 2x$ ).
M(F&A)-6-4 <b>Demonstrates conceptual understanding</b> of equality by showing equivalence between two expressions using models or different representations of the expressions (expressions consistent with the parameters of M(F&A)-6-3), solving multi-step linear equations of the form $ax \pm b = c$ , where a, b, and c are whole numbers with $a \neq 0$ .	M(F&A)-7-4 <b>Demonstrates conceptual</b> <b>understanding of equality</b> by showing equivalence between two expressions ( <u>expressions consistent with</u> the parameters of the left- and right-hand sides of the equations being solved at this grade level) using models or different representations of the expressions, solving multi-step linear equations of the form $ax \pm b = c$ with $a \neq 0$ , $ax \pm b = cx \pm d$ with $a, c \neq 0$ , and $(x/a) \pm b = c$ with $a \neq 0$ , where $a, b, c$ and $d$ are whole numbers; or by translating a problem-solving situation into an equation consistent with the parameters of the type of equations being solved for this grade level.	M(F&A)–8–4 <b>Demonstrates conceptual understanding of</b> <b>equality</b> by showing equivalence between two expressions (expressions consistent with the parameters of the left- and right-hand sides of the equations being solved at this grade level) using models or different representations of the expressions, solving formulas for a variable requiring one transformation (e.g., $d = rt$ ; $d/r = t$ ); by solving multi-step linear equations with integer coefficients; by showing that two expressions are or are not equivalent by applying commutative, associative, or distributive properties, order of operations, or substitution; and by informally solving problems involving systems of linear equations in a context.

	Data, Statistics, and Probability	
Statistical instruction should be carried out in a spirit of investig	interpreting and analyzing the information to make decisions and pation and exploration so students can answer and formulate question certainty. Technology should be used as a tool throughout the inv	ons about data. Probability should be studied in familiar contexts
Grade K	Grade 1	Grade 2
M(DSP)–K–1 Interprets a given representation created by the class (models and tally charts) to answer questions related to the data, or to analyze the data to formulate conclusions using words, diagrams, or verbal/scribed responses to express answers. (IMPORTANT: <i>Analyzes data consistent with concepts and</i> <i>skills in M(DSP)–K–2.</i> ) M(DSP)–K–2 Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using more, less, or equal (e.g., Have there been more, less, or the same number of cloudy days compared to sunny days this week?).	<ul> <li>M(DSP)-1-1 Interprets a given representation created by the class (models, tally charts, <u>pictographs with one-to-one correspondence</u>, and <u>tables</u>) to answer questions related to the data, or to analyze the data to formulate conclusions using words, diagrams, or verbal/scribed responses to express answers.</li> <li>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M(DSP)-1-2</i>.)</li> <li>M(DSP)-1-2 Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using more, less, or equal.</li> </ul>	M(DSP)–2–1 Interprets a given representation (pictographs with one-to-one correspondence, <u>line plots</u> , tally charts, or tables) to answer questions related to the data, or to analyze the data to formulate conclusions. (IMPORTANT: <i>Analyzes data consistent with concepts</i> <i>and skills in M(DSP)–2–2.</i> ) M(DSP)–2–2 Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using more, less, or equal.
M(DSP)–K–3 No standard at this grade	M(DSP)–1–3 No standard at this grade	M(DSP)–2–3 No standard at this grade
M(DSP)–K–4 No standard at this grade	M(DSP)–1–4 No standard at this grade	M(DSP)–2–4 Uses counting techniques to solve problems involving combinations using a variety of strategies (e.g., student diagrams, organized lists, tables, tree diagrams, or <sup>sc</sup> others); (e.g., How many ways can you make 50 cents using nickels, dimes, and quarters?)

Data, Statistics, and Probability continued		
Grade K	Grade 1	Grade 2
M(DSP)–K–5 No standard at this grade	M(DSP)-1-5 For a probability event in which the sample space may or may not contain equally likely outcomes, groups use experiments to describes the likelihood or chance of an event (using "more likely," "less likely", or "equally likely").	M(DSP)–2–5 For a probability event in which the sample space may or may not contain equally likely outcomes, uses experiments to describe the likelihood or chance of an event using "more likely," "less likely," "equally likely," <u>certain</u> or impossible.
M(DSP)–K–6 No standard at this grade	M(DSP)–1–6 No standard at this grade	<ul> <li>M(DSP)-2-6 In response to a teacher or student generated question or hypothesis, groups decide the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the question or hypothesis being tested, and when appropriate makes predictions.</li> <li>(IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)-2-2.)</li> </ul>

Data, Statistics, and Probability continued		
Grade 3	Grade 4	Grade 5
M(DSP)–3–1 <b>Interprets a given representation</b> (line plots, tally charts, tables, or <u>bar graphs</u> ) to answer questions related to the data, to analyze the data to formulate conclusions, or to <u>make predictions</u> . (IMPORTANT: <i>Analyzes data consistent with concepts and skills in M(DSP)–3–2</i> .)	<ul> <li>M(DSP)-4-1 Interprets a given representation (line plots, tables, bar graphs, <u>pictographs</u>, or <u>circle graphs</u>) to answer questions related to the data, to analyze the data to formulate or <u>justify</u> conclusions, to make predictions, or to <u>solve problems</u>.</li> <li>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M(DSP)-4-2</i>.)</li> </ul>	<ul> <li>M(DSP)–5–1 Interprets a given representation (tables, bar graphs, circle graphs, or <u>line graphs</u>) to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.</li> <li>(IMPORTANT: <i>Analyzes data consistent with concepts</i> <i>and skills in M(DSP)–5–2.</i>)</li> </ul>
		II
M(DSP)-3-2 Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using most frequent (mode), least frequent, largest, or smallest. M(DSP)-3-3 Organizes and displays data using tables, tally	M(DSP)-4-2 Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using measures of central tendency (median or mode), or range. M(DSP)-4-3 Organizes and displays data using tables, line	M(DSP)-5-2 Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using measures of central tendency (mean, median, or mode) or range to <u>analyze situations</u> , or to <u>solve problems</u> . M(DSP)-5-3 Organizes and displays data using tables, bar
charts, and bar graphs, to answer questions related to the data, to analyze the data to formulate conclusions, to make predictions, or to solve problems.	<u>plots</u> , bar graphs, and <u>pictographs</u> to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.	graphs, or <u>line graphs</u> to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.
M(DSP)-3-3 Identifies or describes representations or elements of representations that best display a given set of data or situation, consistent with the representations required in M(DSP)-3-1. (IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)-3-2.)	(IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)–4–2.)	M(DSP)-5-3 Identifies or describes representations or elements of representations that best display a given set of data or situation, consistent with the representations required in <u>M(DSP)-5-1</u> . (IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)-5-2.)

Data, Statistics, and Probability continued		
Grade 3	Grade 4	Grade 5
M(DSP)–3–4 Uses counting techniques to solve problems involving combinations and <u>simple permutations</u> using a variety of strategies (e.g., student diagrams, organized lists, tables, tree diagrams, or <sup>se</sup> others).	M(DSP)-4-4 Uses counting techniques to solve problems in context involving combinations or simple permutations (e.g., Given a map – Determine the number of paths from point A to point B.) using a variety of strategies (e.g., organized lists, tables, tree diagrams, or <sup>sc</sup> others).	M(DSP)–5–4 No standard at this grade
M(DSP)–3–5 For a probability event in which the sample space may or may not contain equally likely outcomes, predicts the likelihood of an event using "more likely," "less likely," "equally likely," certain, or impossible and tests the prediction through experiments; and determines if a game is fair.	M(DSP)-4-5 For a probability event in which the sample space may or may not contain equally likely outcomes, predicts the likelihood of an event <u>as a part to whole</u> relationship (e.g., two out of five, zero out of five, five out of five) and tests the prediction through experiments; and determines if a game is fair.	M(DSP)-5-5 For a probability event in which the sample space may or may not contain equally likely outcomes, predicts the likelihood of an event as a <u>fraction</u> and tests the prediction through experiments; and determines if a game is fair.
M(DSP)–3–5 For a probability event in which the sample space may or may not contain equally likely outcomes, determines the likelihood of the occurrence of an event (using "more likely", "less likely", or "equally likely").	M(DSP)-4-5 For a probability event in which the sample space may or may not contain equally likely outcomes, determines the <u>theoretical probability of an</u> <u>event and expresses the result as part to whole (e.g., two out of five)</u> .	M(DSP)-5-5 For a probability event in which the sample space may or may not contain equally likely outcomes, determines the <u>experimental</u> or theoretical probability of an event and <u>expresses the result as a</u> <u>fraction</u> .
M(DSP)–3–6 In response to a teacher or student generated question or hypothesis, groups decide the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the question or hypothesis being tested, and when appropriate makes predictions. (IMPORTANT: <i>Analyzes data consistent with concepts and</i> <i>skills in M(DSP)–3–2.</i> )	M(DSP)-4-6 In response to a teacher or student generated question or hypothesis, groups decide the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the question or hypothesis being tested, and when appropriate makes predictions; and <u>asks new questions and makes connections to</u> real world situations. (IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)-4-2.)	<ul> <li>M(DSP)-5-6 In response to a teacher or student generated question or hypothesis decides the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the question or hypothesis being tested, and when appropriate makes predictions; and asks new questions and makes connections to real world situations.</li> <li>(IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)-5-2.)</li> </ul>

Data, Statistics, and Probability continued			
Grade 6	Grade 7	Grade 8	
<ul> <li>M(DSP)–6–1 Interprets a given representation (circle graphs, line graphs, or <u>stem-and-leaf plots</u>) to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.</li> <li>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M(DSP)–6–2.</i>)</li> </ul>	<ul> <li>M(DSP)-7-1 Interprets a given representation (circle graphs, scatter plots that represent discrete linear relationships, or histograms) to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.</li> <li>(IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)-7-2.)</li> </ul>	<ul> <li>M(DSP)-8-1 Interprets a given representation (line graphs, scatter plots, histograms, or box-and-whisker plots) to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.</li> <li>(IMPORTANT: <i>Analyzes data consistent with concepts and skills in M(DSP)</i>-8-2.)</li> </ul>	
M(DSP)–6–2 Analyzes patterns, trends or distributions in data in a variety of contexts by determining or using measures of central tendency (mean, median, or mode) or <u>dispersion (range)</u> to analyze situations, or to solve problems.	M(DSP)-7-2 Analyzes patterns, trends, or distributions in data in a variety of contexts <u>by</u> <u>solving problems</u> using measures of central tendency (mean, median, or mode), dispersion (range or variation), or <u>outliers</u> to analyze situations to <u>determine their effect</u> on mean, median, or mode; and evaluates the sample from which the statistics were developed (bias).	M(DSP)-8-2 Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, <u>quartile values</u> , or <u>estimated line of best fit</u> to analyze situations, or to solve problems; and evaluates the sample from which the statistics were developed (bias, <u>random</u> , or <u>non-random</u> ).	
M(DSP)–6–3 <b>Organizes and displays data</b> using tables, line graphs, or <u>stem-and-leaf plots</u> to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.	M(DSP)–7–3 <b>Organizes and displays data</b> using tables, line graphs, <u>scatter plots</u> , and <u>circle graphs</u> to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.	M(DSP)–8–3 <b>Organizes and displays data using</b> scatter plots to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems; or identifies representations or elements of	
(IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)–6–2.)	M(DSP)-7-3 Identifies or describes representations or elements of representations that best display a given set of data or situation, consistent with the representations required in <u>M(DSP)-7-1</u> . (IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)-6-2.)	representations that best display a given set of data or situation, consistent with the representations required in <u>M(DSP)–8–1</u> . (IMPORTANT: <i>Analyzes data consistent with concepts and skills in M(DSP)–8–2</i> .)	

Data, Statistics, and Probability continued		
Grade 6	Grade 7	Grade 8
M(DSP)–6–4 Uses counting techniques to solve problems in context involving combinations or simple permutations using a variety of strategies (e.g., organized lists, tables, tree diagrams, models, <u>Fundamental Counting</u> <u>Principle</u> , or <sup>sc</sup> others).	M(DSP)–7–4 Uses counting techniques to solve problems in context involving combinations or <u>permutations</u> (e.g., How many different ways can eight students place first, second, and third in a race?) using a variety of strategies (e.g., organized lists, tables, tree diagrams, models, Fundamental Counting Principle, or <sup>sc</sup> others).	M(DSP)–8–4 Uses counting techniques to solve problems in context involving combinations or permutations using a variety of strategies (e.g., organized lists, tables, tree diagrams, models, Fundamental Counting Principle, or <sup>sc</sup> others).
M(DSP)–6–5 For a probability event in which the sample space may or may not contain equally likely outcomes, predicts the theoretical probability of an event and tests the prediction through experiments and <u>simulations</u> ; and <u>designs</u> fair games.	M(DSP)–7–5 For a probability event in which the sample space may or may not contain equally likely outcomes, predicts the theoretical probability of an event and tests the prediction through experiments and simulations; and <u>compares</u> and contrasts theoretical and experimental probabilities.	M(DSP)–8–5 For a probability event in which the sample space may or may not contain equally likely outcomes, determines the experimental or theoretical probability of an event in a problem-solving situation; and predicts the theoretical probability of an event and tests the prediction through experiments and simulations; and compares and
M(DSP)–6–5 For a probability event in which the sample space may or may not contain equally likely outcomes, determines the experimental or theoretical probability of an <u>event in a problem-solving situation</u> .	M(DSP)–7–5 For a probability event in which the sample space may or may not contain equally likely outcomes, determines the experimental or theoretical probability of an event in a problem-solving situation.	contrasts theoretical and experimental probabilities.
M(DSP)–6–6 In response to a teacher or student generated question or hypothesis decides the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the question or hypothesis being tested, and when appropriate makes predictions; and asks new questions and makes connections to real world situations.	M(DSP)–7–6 In response to a teacher or student generated question or hypothesis decides the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the question or hypothesis being tested while <u>considering the limitations that</u> <u>could affect interpretations;</u> and when appropriate makes predictions; and asks new questions and makes connections to real world situations.	M(DSP)–8–6 In response to a teacher or student generated question or hypothesis decides the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the question or hypothesis being tested while <u>considering the limitations that</u> <u>could affect interpretations;</u> and when appropriate makes predictions; and asks new questions and makes connections to real world situations.
(IMPORTANT: Analyzes data consistent with concepts and skills in $M(DSP)-6-2$ .)	(IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)–7–2.)	(IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)–8–2.)

Since it is crucial that process standards (problem-solving, reasoning, proof, communication, connections, and representations) are not seen separate from content standards the process standards have been imbedded throughout the Number and Operations, Geometry and Measurement, Functions and Algebra, and Data, Statistics, and Probability strands (e.g., M(F&A)-4-1 **Identifies and** <u>extends to specific cases</u> a variety of patterns (linear and <u>nonlinear</u>) represented in models, tables or sequences; and <u>writes a rule in words or<sup>sc</sup> symbols to find the next case</u>.). This mirrors classroom instruction as in most classes, as students are learning content knowledge, instruction is also focusing on improving their abilities in problem solving, reasoning, and communication; furthermore, students are looking for and making appropriate connections, and they are able to understand and use multiple representations of mathematical ideas. Since it is crucial that students are strong in both content and process knowledge, we have included two local process strands. These process strands are in addition to the process standards that are embedded in the content standards and are included to help guide local curriculum, assessment, and instruction. The process standards have been separated by grade-spans (K-2, 3-5, 6-8, and 9-12). Each span should be thought of as building upon the skills and concepts in the previous span.

Problem Solving, Reasoning, and Proof			
Purpose: Problem solving should serve as the organizing feature of the mathematics curriculum as well as other areas of study and be applied to everyday activities. Thus, problem solving			
approaches should be used to investigate and understand new mathematical content, with students working sometimes independently and sometimes in groups. Students should have many experiences in posing and solving problems from their world, from data that are meaningful to them, and from mathematical investigations. Students should build a positive disposition toward			
problem solving, including the confidence needed to explore unit		igations. Students should build a positive disposition toward	
K-2	$\frac{3-5}{3}$	6 - 8	
M(PRP)-2-1 Students will use problem-solving strategies to	M(PRP)-5-1 Students will use problem-solving strategies to	M(PRP)-8-1 Students will use problem-solving strategies to	
investigate and understand increasingly complex	investigate and understand increasingly complex	investigate and understand increasingly complex	
mathematical content and be able to:	mathematical content and be able to:	mathematical content and be able to:	
<ul> <li>Formulate and solve multi-step problems from everyday and mathematical situations.</li> <li>Solve problems using a variety of strategies (e.g., working backwards, looking for patterns and relationships; guess and check; making tables, charts, or organized lists; solving a simpler version of a problem, drawing a diagram; or creating a model)</li> <li>Verify and interpret results with respect to the original problem.</li> <li>Determine if the solution of a problem is reasonable.</li> <li>Solve problems using manipulatives, graphs, charts, diagrams, and calculators.</li> <li>Demonstrate that a problem may be solved in more than one way.</li> <li>Exhibit confidence in their ability to solve problems independently and in groups.</li> <li>Display increasing perseverance, and persistence in problem solving.</li> </ul>	<ul> <li>Determine the reasonableness of solutions to real-world problems.</li> <li>Generalize solutions and apply strategies to new problem situations.</li> <li>Add to the repertoire of problem-solving strategies (e.g., looking for similar problems) and use those strategies in more sophisticated ways.</li> <li>Solve problems with multiple solutions, recognize when a problem has no solution, and recognize problems where more information is needed.</li> <li>Translate results of a computation into solutions that fit the real-world problem (e.g., when a computation shows that one needs 3.2 gallons of paint to paint a room, how much paint do you buy?).</li> </ul>	<ul> <li>Use problem-solving strategies appropriately and effectively for a given situation.</li> <li>Determine, collect and organize the relevant information needed to solve real-world problems.</li> <li>Apply integrated problem-solving strategies to solve problems in the physical, natural, and social sciences and in pure mathematics.</li> <li>Use technology when appropriate to solve problems.</li> <li>Reflect on solutions and the problem-solving process for a given situation and refine strategies as needed.</li> </ul>	

Problem Solving, Reasoning, and Proof continued		
K – 2	3-5	6 - 8
M(PRP)–2–2 Students will use mathematical reasoning and <b>proof</b> and be able to:	M(PRP)-5-2 Students will use mathematical reasoning and <b>proof</b> and be able to:	M(PRP)-8-2 Students will use mathematical reasoning and <b>proof</b> and be able to:
<ul> <li>Use models, known facts, properties, and relationships to explain their thinking.</li> <li>Justify solution processes and answers (e.g., "I chose this method to solve the problem because").</li> <li>Draw conclusions using inductive reasoning.</li> <li>Identify the missing information needed to find a solution to a given story problem.</li> <li>Use patterns and relationships to analyze mathematical situations (e.g., count by fives).</li> </ul>	<ul> <li>Draw conclusions and solve problems using elementary deductive reasoning and reasoning by analogy.</li> <li>Make and defend conjectures and generalizations.</li> <li>Use models, known facts, properties, and relationships to explain thinking and to justify answers and solution processes.</li> <li>Recognize the pervasive use and power of reasoning as a part of mathematics.</li> </ul>	<ul> <li>Draw logical conclusions and make generalizations using deductive and inductive reasoning.</li> <li>Formulate, test, and justify mathematical conjectures and arguments.</li> <li>Construct and determine the validity of a mathematical argument or a solution.</li> <li>Apply mathematical reasoning skills in other disciplines.</li> </ul>

	Communication, Connections, and Representation	ns
Purpose: Reading, writing, talking, listening, and modeling mathematics into their world. Actively exploring, investiga mathematical concepts. Representing ideas and connecting the representa concrete and available for reflection, and they help students understanding of mathematical concepts and relationships a Mathematical topics, ideas, and procedures must	g provide students with the opportunity to develop deeper ma ating, describing, and explaining mathematical ideas promote ations to mathematics lies at the heart of understanding mathe s recognize the common mathematical nature of different situ as they create, compare, and use various representations. be connected to each other and to the students' everyday exp as. Mathematical connections will help students become awar	thematical understanding and to integrate the language of e communication which leads to a greater comprehension of ematics. Representations make mathematical ideas more lations. Students can develop and deepen their eriences, both in and out of school. In particular,
K-2	3-5	6-8
<ul> <li>M(CCR)-2-1 Students will communicate their understanding of mathematics and be able to:</li> <li>Demonstrate mathematical communication through discussion, reading, writing, listening, and responding, individually and in groups.</li> <li>Discuss relationships between everyday language and mathematical language and symbols (e.g., words that mean something different in mathematics and in everyday life).</li> <li>Explain conclusions, thought processes, and strategies in problem-solving situations.</li> <li>Discuss, illustrate, and write about mathematical concepts and relationships.</li> <li>Draw pictures and use objects to illustrate mathematical concepts.</li> </ul>	<ul> <li>M(CCR)-5-1 Students will communicate their understanding of mathematics and be able to:</li> <li>Discuss mathematical ideas and write convincing arguments.</li> <li>Understand, explain, analyze, and evaluate mathematical arguments and conclusions presented by others.</li> <li>Ask clarifying and extending questions related to mathematics they have heard or read about.</li> <li>Understand and appreciate the economy and power of mathematical symbolism and its role in the development of mathematics.</li> <li>Demonstrate an understanding of mathematical concepts and relationships through a variety of methods (e.g., writing, graphing, charts, diagrams, number sentences, or symbols).</li> <li>Use a variety of technologies (e.g., computers, calculators, video, probes) to represent and communicate mathematical ideas.</li> </ul>	<ul> <li>M(CCR)-8-1 Students will communicate their understanding of mathematics and be able to:</li> <li>Articulate ideas clearly and logically in both written and oral form.</li> <li>Present, share, explain, and justify thinking with others and build upon the ideas of others to solve problems.</li> <li>Use mathematical symbols and notation.</li> <li>Formulate questions, conjectures, definitions, and generalizations about data, information, and problem situations.</li> </ul>

Communication, Connections, and Representations continued			
K – 2	3-5	6 - 8	
M(CCR)-2-2 Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:	M(CCR)-5-2 Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:	M(CCR)-8-2 Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:	
<ul> <li>Create and use age level appropriate representations to organize, record, and communicate mathematical ideas (e.g., students should recognize the relationship among seven counters, seven tally marks, and the symbol 7).</li> <li>Select, apply, and translate among mathematical representations to solve problems (e.g., representing fractions with circles, with geoboards, and with pattern blocks).</li> <li>Link different representations.</li> <li>Use representations to model and interpret physical, social, and mathematical phenomena.</li> <li>Use conventional and self-generated (invented) representations and connect them.</li> <li>Realize that any representation is subject to multiple interpretations (e.g., drawings and graphs can be read in a different way).</li> </ul>	<ul> <li>Use physical models and diagrams to represent important mathematical ideas (e.g., multiplication).</li> <li>Use appropriate representations to solve problems or to portray, clarify, or extend a mathematical idea.</li> <li>Recognize equivalent representations of concepts and procedures and translate among them as appropriate (for example, understand how the addition of whole numbers, fractions, and decimals are related).</li> </ul>	<ul> <li>Use models and technology to develop equivalent representations of the same mathematical concept.</li> <li>Use and create representations to solve problems and organize their thoughts and ideas.</li> <li>Convert between representations (e.g., a table of values, an equation, and a graph may all be representations of the same function).</li> </ul>	

Communication, Connections, and Representations continued		
K – 2	3-5	6 - 8
M(CCR)–2–3 Students will recognize, explore, and develop mathematical connections and be able to:	M(CCR)–5–3 Students will recognize, explore, and develop mathematical connections and be able to:	M(CCR)-8-3 Students will recognize, explore, and develop <b>mathematical connections</b> and be able to:
<ul> <li>Link conceptual and procedural knowledge (e.g., they will know that when they "regroup," they are simply changing the representation of the minuend, but not its value).</li> <li>Recognize and use mathematics in other curriculum areas (e.g., science, social studies).</li> <li>Recognize and use mathematics in their daily lives (e.g., graphs, tables, or maps).</li> <li>Identify mathematical situations occurring in literature for children.</li> <li>Identify examples of geometry in nature, art, and architecture.</li> </ul>	<ul> <li>See mathematics as an integrated whole.</li> <li>Recognize relationships among different topics in mathematics.</li> <li>Recognize and use mathematics in other curriculum areas and in their daily lives.</li> <li>Link concepts and procedures.</li> <li>Use mathematical skills, concepts, and applications in other disciplines (e.g., graphs in social studies, patterns in art, or music and geometry in technology education).</li> </ul>	<ul> <li>Connect new mathematical ideas to those already studied and build upon them.</li> <li>Understand that many real-world applications require an understanding of mathematical concepts (e.g., personal finance, running a business, building a house, following a recipe, or sending a rocket to the moon).</li> <li>Explain in oral and written form the relationships between a real-world problem and an appropriate mathematical model.</li> <li>Explain in oral and written form the relationships among various mathematical concepts (e.g., the relationship between exponentiation and multiplication).</li> </ul>

### Appendix A: M(N&O)–2–3

**M(N&O)–2–3 Demonstrates conceptual understanding of mathematical operations involving** addition and subtraction of whole numbers by solving problems involving joining actions, separating actions, part-part whole relationships, and comparison situations. (See Appendix A.)

**Classification of word problems:** Researchers have identified four basic categories of addition and subtraction problems: problems with joining actions, separating actions, part-part whole relationships, and comparison situations. Table 1 contains some examples to illustrate the distinctions in the categories identified by the researchers, but there may be additional ways to express the same actions, relationships, or situations. For example, the following are additional ways to ask questions related to the "Join – Start Unknown" category.

**Example 1:** "I am thinking of a number that when you add 5 to it, the answer is 13. What number am I thinking of?"

**Example 2:** John puts a five-pound toy inside an empty wooden box. The box and toy together weigh 7 pounds. How much does the empty box weigh?

In summary, students should have experiences with problem solving in addition and subtraction across a variety of problem types. It is important to remember that any problem situation that fits the equations given and the category can be asked, not just problems as stated in Table 1.

"Teaching students to add and subtract involves providing students with an opportunity to explore a rich set of problems with different semantic structures and to develop a variety of strategies to quantify, represent, calculate, express, and justify results." (Gutstein, E., Romberg, 1995)

Join	(Result Unknown) Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether?	(Change unknown) Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether?		(Start Unknown) Connie has some marbles. Juan gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?
	5 + 8 = □	5 + 🗆 = 13		$\Box + 5 = 13$
Separate	(Result Unknown) Connie had 13 marbles. She gave 5 marbles to Juan. How many marbles does she have left?	(Change unknown) Connie has 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give Juan?		(Start Unknown) Connie has some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with?
	13 – 5 = □	$13 - \Box = 5$		$\Box - 5 = 8$
Part- Part- Whole	(Whole Unknown) Connie has 5 red marbles and 8 t many marbles does Connie have $5 + 8 = \Box$			es. All the marbles are either blue or red. bles. How many blue marbles does Connie
Compare	(Difference Unknown) Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan? $13-5=\Box$	(Compare Quantity Unknown) Juan has 5 marbles. Connie has 8 more marbles than Juan. How many marbles does Connie have? $5 + 8 = \Box$		(Referent Unknown) Connie has 13 marbles. She has 5 more marbles than Juan. How many marbles does Juan have? $13-5=\Box$

### Table 1: Classification of Word Problems<sup>1</sup>

<sup>&</sup>lt;sup>1</sup> Carpenter, Fennema, Peterson, Chiang, and Loef (1989) cited in Gutstein, E., Romberg, T., *Teaching Children to Add and Subtract*, Journal of Mathematical Behavior, 14, 283-324 (1995).

#### **Appendix B: Measurement Benchmarks**

The following is a list of the measurement benchmarks and equivalences that *can be used* in problems across the content strands at each grade level to address the expectations in M(G&M)-X-7 for the Tri-State New England Assessment<sup>2</sup>.

# M(G&M)–X–7 Uses units of measures appropriately and consistently, and makes conversions within systems when solving problems across the content strands.

The type of measure (e.g., length, time, etc.), the unit (e.g., inches, feet, etc.), the degree of accuracy where appropriate (e.g.,  $\frac{1}{2}$  inch); and equivalences (e.g., 12 inches in a foot) are identified for grades 2 - 8. In addition to measurement benchmarks identified below students will be expected to use the appropriate units when solving problems involving area, volume, surface area, conversions, and rates (e.g., miles per hour, price per pound, pounds per square inch) on the TSNE Assessment<sup>1</sup>.

	Grade 2	Grade 3	Grade 4
Measures			
Length	Unit (accuracy): Inch (to whole inch); Foot (to whole inch); Centimeter (to whole centimeter); Meter (to whole centimeter) Equivalencies: 12 inches in 1 foot; 100 centimeters in 1 meter	Unit (accuracy): Inch (to 1/2 inch); Foot (to whole inch); Centimeter (to whole centimeter); Meter (to whole centimeter) Equivalencies: 12 inches in 1 foot; 100 centimeters in 1 meter	Unit (accuracy): Inch (to 1/4 inch); Foot; Centimeter (to 0.5 centimeter); Meter (to 0.5 centimeter); Yard; Mile (use in scale questions); Kilometer (use in scale questions) Equivalencies: 12 inches in 1 foot; 100 centimeters in 1 meter; 3 feet in 1 yard; 36 inches in 1 yard
Time	Unit (accuracy): Hour (to 15 minute interval) Equivalencies: 60 minutes in 1 hour	Unit (accuracy): Hour (to 5 minute interval); Day; Year Equivalencies: 24 hours in 1 day; 7 days in 1 week; 365 days in 1 year	Unit (accuracy): Hour (to 5 minute interval); Day; Year Equivalencies: 24 hours in 1 day; 7 days in 1 week; 365 days in 1 year; 60 seconds in 1 minute; 60 minutes in 1 hour
Temperature	Unit (accuracy): Degree (to 1 degree)	Unit (accuracy): C° and F° (to 1 degree)	Unit (accuracy): C° and F° (to 1 degree)
Capacity		Units (accuracy): Quart (to whole quart)	Unit (accuracy): Quart (to whole quart)
Mass		Unit (accuracy): Kilogram (to whole kilogram); Gram (to whole gram)	Unit (accuracy): Kilogram (to whole kilogram); Gram (to whole gram)
Weight		Unit (accuracy): Pound (to whole pound)	Unit (accuracy): Pound (to whole pound)

 $<sup>^{2}</sup>$  Contractors will be asked to devise a system to measure the degree to which students use units of measures and make conversions consistently and appropriately when applicable to problems across content strands.

Measures	Grade 5	Grades 6 – 8
Length	Units (accuracy): Inch (to 1/8 inch); Foot; Centimeter (to 0.5 centimeter); Meter (to 0.5 centimeter); Yard; Mile (use in scale questions); Kilometer (use in scale questions) Equivalencies: 12 inches in 1 foot; 100 centimeters in 1 meter; 3 feet in 1 yard; 36 inches in 1 yard; 10 millimeters	Units (accuracy): Inch (to 1/16 inch); Foot; Centimeter (to 1/10 centimeter); Meter (to 1/100 meter); Yard; Mile (use in scale and rate questions); Kilometer (use in scale and rate questions) Equivalencies: 12 inches in 1 foot; 100 centimeters in 1 meter; 3 feet in 1 yard; 36 inches in 1 yard; 10 millimeters
	in 1 centimeter	in 1 centimeter; 1000 millimeters in 1 meter
Time	Unit (accuracy): Hour (to 1 minute); Day; Year Equivalencies: 24 hours in 1 day; 7 days in 1 week; 365 days in 1 year; 60 seconds in 1 minute; 60 minutes in 1 hour	Unit (accuracy): Hour (to 1 minute); Day; Year Equivalencies: 24 hours in 1 day; 7 days in 1 week; 365 days in 1 year; 60 seconds in 1 minute; 60 minutes in 1 hour
Temperature	Unit (accuracy): C° and F° (to 1 degree)	Unit (accuracy): C° and F° (to 1 degree)
Capacity	Unit (accuracy): Quart (to 1 ounce); Gallon; Pint Equivalencies: 32 ounces in 1 quart; 4 quarts in 1 gallon; 2 pints in 1 quart	Unit (accuracy): Quarts (to 1 ounce); Gallon; Pint; Liter Equivalencies: 32 ounces in 1 quart; 4 quarts in 1 gallon; 2 pints in 1 quart; 1000 milliliters in 1 liter
Mass	Unit (accuracy): Kilogram; Gram (to whole gram)	Unit (accuracy): Kilogram; Gram (to 1/10 gram)
Weight	Unit (accuracy): Pound (to 1 ounce) Equivalencies: 16 ounces in 1 pound	Unit (accuracy): Pound (to 1 ounce) Equivalencies: 16 ounces in 1 pound
Angles and Rotation	Unit (accuracy): Degree (to 2 degrees)	Unit (accuracy): Degree (to 2 degrees) Equivalencies: 360° in 1 circle; 90° in 1 right angle

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