AP Calculus AB Syllabus

Introduction

Our study of calculus, the mathematics of motion and change, is divided into two major branches – differential and integral calculus. Differential calculus allows us to calculate instantaneous rates of change, to find the slope of a curve, and to calculate velocities and accelerations of moving bodies. We use integral calculus to find the total distance traveled by those moving bodies, to find the area of an irregular region, and to find the volume of solids of revolution and solids with known cross sections.

This course covers all topics included in the *Calculus AB Course Description*. The aim is to provide students with a conceptual understanding of the basics of calculus supported by the skills, methods, and technology necessary to apply those concepts. The main sections of this course are functions and limits, derivatives and applications of derivatives, definite integrals, indefinite integrals, and some applications of integrals.

Classes in AP Calculus follow a variety of formats. Some topics are introduced in a lecture and discussion format, with the students taking notes from the overhead, supported by guided practice with problems similar to the homework. Others are introduced via an exploration, with the students working on a task in small groups. This is usually followed by group presentations, in verbal form, of the students' conclusions and methods.

Problems are presented and solved in four distinct ways: analytically, numerically, graphically, and verbally, with emphasis on the connections between these representations. Students are encouraged to present their solutions to the class to reinforce their skills in verbal and written communication.

The TI-83 calculator is used extensively. Most students have their own calculator and are reasonably proficient with it before they take AP Calculus. A set of TI-83 Plus calculators is available for anyone who needs it. The emphasis is on using the calculator to support an analytic solution graphically, to make a conjecture which can be confirmed analytically, or to interpret results. In addition, students are encouraged to develop an understanding of when calculator use is or is not appropriate. Students will be able to use the calculator to graph a function in any window, solve an equation, determine the value of a derivative at a point, and determine the value of a definite integral.

Course Outline

Unit 1: Functions, Limits, and Continuity - Time approximately 3 weeks

Functions:

Students review basic functions and graphs over the summer, completing several assignments from Chapter 1 of the textbook. When they return in September, we briefly review function notation, domain and range, odd and even functions, piecewise functions, inverse functions, and basic trig functions. Logarithmic and exponential functions are studied more fully when their derivatives are first encountered. Special emphasis is given to recognizing and graphing ten basic functions. We return to these topics as needed throughout the year.

Limits:

Limits at a point:

Intuitive definition, properties of limits - one-sided and two-sided

Calculate limits algebraically and estimate graphically and numerically

Limits involving infinity

End behavior, horizontal and vertical asymptotes.

Understand asymptotic behavior graphically and relate to the basic functions studied in the first unit.

Continuity:

Continuous functions - Conditions for continuity at a point – conceptual and analytic

Discontinuities – removable, jump, infinite

Intermediate Value Theorem for Continuous functions

Unit 2: Derivatives - Time approximately 6 weeks.

Definition of the Derivative

Average and instantaneous rates of change

Tangent and normal lines to a curve at a given point

Limit definition, different forms of the difference quotient,

Derivative as slope of the curve and instantaneous rate of change

Relationships between the graphs of f and $f\,¢$

Differentiability and local linearity

Numeric derivatives in the graphing calculator

Differentiability and continuity

Computation of Derivatives

Algebraic functions and the Chain Rule

Trigonometric functions and inverse functions

Logarithmic and exponential functions
Implicit differentiation

Velocity and Other Rates of Change

Includes motion along a line and modeling with parametric graphs
Approximate rate of change from a table of data or a graph

Unit 3: Applications of Derivatives - Time approximately 5 weeks.

Extreme values –

Local and global extrema
Determine extrema using analytic and/or graphical methods
The Extreme Value Theorem

Using the derivative

Increasing and decreasing behavior of a function and relationship to the sign of its derivative.
The Mean Value Theorem and its physical consequences.

Analysis of graphs using the first and second derivatives

First derivative test for extrema
Concavity – definition using f' and test for concavity using f''
Point of inflection
Second derivative test for extrema
Interpret graphs of f, f' and f''
Using f' and f'' to graph the function.

Modeling and Optimization

Related rates of change

Linearization and differentials

Local linear approximation and relationship to the tangent line.
Estimating change with differentials

Unit 4: Definite Integrals - Time approximately 5 weeks.

Approximating Areas

Rectangular approximation, Riemann sums

Definite Integral

Definite integral and area

Evaluate integrals on a graphing calculator

Definite Integrals and Antiderivatives

Properties of definite integrals

Average value of a function

Fundamental Theorem of Calculus – Part 1 and Part 2

Includes analysis of accumulation functions

Trapezoidal Rule

Unit 5: Differential Equations and Mathematical Modeling -Time approximately 3 weeks.

Antiderivatives and initial value problems

Indefinite integrals

Slope Fields

Integration using u-substitution

Separable Differential Equations

Exponential growth and decay

Unit 6 – Applications of Definite Integrals - Time approximately 3 weeks.

Integral as net change

Summing rates of change including distance traveled

Areas in the plane

Area between curves

Intersecting curves and curves with changing boundaries

Integrating with respect to *y*

Volumes

Volumes of solids with known cross sections
Volumes of solids of revolution

Unit 7 – AP Review – Time approximately three weeks

1998 and 2003 released exams
Multiple choice practice – D & S Marketing book, various AP review books.
Free Response Questions from previous years.

Unit 8 - After the Exam

Integration by parts
Research paper – topics from history of mathematics

Assessments

Assessments are of several types. Some quizzes and tests are in a non-AP format, allowing an assessment of the students' basic understanding and skills. Others are all AP-style multiple choice questions, taken from released exams or the D&S Marketing books. Free response questions are used throughout the year. These are graded, when possible, using the released grading rubrics. Discussion of the grading, especially the type of justification needed, always follows. The midterm is a "mini AP exam" of multiple choice and free response questions with the same time and calculator constraints as the real exam.

Students practice with two released exams – the 1998 released exam is done over Spring Break and the 2003 exam is completed in class during the last two weeks before the actual exam in May.

Sample Activities

- **Analysis of graphs:** f, f', f'' activity: Students are given a set of cards that include graphs of f, f', f'' and written descriptions of f, f', f''. They work in groups to match each f, f', f'' set both graphically and verbally. The students will then present their conclusions to the class in verbal form.

- **Accumulation Functions:** An exploration, taken from a workshop taught by Lin McMullin, uses several sets of simulated data to introduce the concept of accumulating a rate of change to estimate the total change. Students work in groups to estimate the total change in each scenario and then present their conclusions to the class. Since they use different methods to estimate the change, their group presentations usually provoke interesting discussions with each group justifying its response.

- **Slope fields:** A differential equation is put on the board. Each student is given one or two points for which they calculate the value of dy/dx. Each student then graphs his or her segment on a transparency of a coordinate grid so the completed slope field is revealed.

- **Free Response Questions:** Students will write answers to numerous Free Response Questions during the year and particularly during the review period preceding the Exam. Emphasis will be given to reviewing the mathematics of calculus and to presenting results and justifying answers in complete sentences.

Textbook and Resources:

Finney, Ross L., Franklin D. Demana, Bert K. Waits, and Daniel Kennedy. *Calculus—Graphical, Numerical, Algebraic.* Upper Saddle River: Prentice Hall, 2003

Lederman, David. *Multiple-Choice and Free Response Questions in Preparation for the AP Calculus (AB) Examination*. D & S Marketing, 1999, 7th edition.

McMullin, Lin. *Teaching AP Calculus*, D&S Marketing Systems, Inc. Brooklyn, NY. 2002 (Pre-publication edition, AP Workshop 2002)