Honors Calculus Curriculum Maps

Unit of Study: Prerequisites for Calculus
Unit of Study: Limits and Continuity
Unit of Study: Differentiation
Unit of Study: Applications of Derivatives
Unit of Study: The Definite Integral
Unit of Study: Differential Equations and Mathematical Modeling
Unit of Study: Applications of Definite Integrals

Grade: 12 **Subject:** Honors Calculus	**Unit of Study: Prerequisites for Calculus**
Big Idea/Rationale	Big Idea: Review Functions and Their Graphs – the essential building blocks of Calculus Rationale: Important mathematical concepts will be used to investigate mathematical ideas, to support analytic work and solve problems numerically and graphically throughout Calculus
Enduring Understanding (Mastery Objective)	• Functions are one of the major tools used for describing the real world in mathematical terms • Trigonometric, exponential, logarithmic, logistic and polynomial functions can approximate many of these real world situations and help us to describe their behaviors.
Essential Questions (Instructional Objective)	• Which previous math skills are essential for the understanding of Calculus? • How does the math that you previously studied relate to the math that you are going to be studying? • How are graphical, numerical, analytical and verbal forms of functions related?
Content (Subject Matter)	*Students will know….* **Key Words –** Increments, Regression analysis, Dependent/Independent Variables, Domain, Range, Interval Notation, Even/Odd Functions, Piecewise Functions, Composite Functions, Polynomial Functions, Exponential Functions, Compound Interest, Logarithmic Functions, One-To-One, Inverse Functions, Change of Base Formula, Trigonometric Functions, Inverse Trigonometric Functions *Students will be able to….* • Use previous mathematical knowledge to solve problems essential in the study of Calculus • Find, graph and compare mathematical models for different data sets
Skills/ Benchmarks (Standards)	Students taking this course have the option to earn college credit from Fairleigh Dickinson University as part of the Middle College Program. Course content is established by this program. For more information, please visit http://view.fdu.edu/default.aspx?id=2748 .
Materials and Resources	• Calculus: Graphical, Numerical, Algebraic (Finney, Demana, Waits, Kennedy) Textbook and Resource Materials • Graphing Calculators • Document Camera • Calculus of a Single Variable (Larson, Hostetler, Edwards)

Notes	

Grade: 12 **Subject:** Honors Calculus	**Unit of Study: Limits and Continuity**
Big Idea/Rationale	Big Idea: Defining Limits and Their Relationship to Calculus Rationale: The key mathematical idea that makes calculus work is the concept of a limit. Limits allow us to “straighten” curved lines and enable us to use algebra and geometry to solve real world problems. One of the uses of limits is to test continuous functions which arise frequently in scientific work because they model a wide range of natural behavior.
Enduring Understanding (Mastery Objective)	• Limits can be used to describe continuity, the derivative, and the integral: the ideas giving the foundation of calculus. • Limits can be used to describe the behavior of functions in absolute value for large numbers. • Continuous functions are used to describe how a body moves through space and how the speed of a chemical reaction changes with time. • The tangent line determines the direction of a body’s motion at every point along its path.
Essential Questions (Instructional Objective)	• What is a limit? • How do you find a limit with a table? A graph? Analytically? • What does indeterminate form mean? Which techniques can we use to solve these types of problems? • When does a limit not exist? • What is the difference between a one-sided and a two-sided limit? • Why can we use the Sandwich Theorem to find limits of certain functions? • How can limits be used to identify vertical and horizontal asymptotes? • How can limits be used to determine if a function is continuous? • Can all discontinuities be removed to create a continuous extension of the function? • What does the Intermediate Value Theorem allow us to find for continuous functions? • What is the difference between average and instantaneous rates of change? • How can limits be used to find the slope of a tangent line at a given point on a curve? • Why can we use our knowledge of average and instantaneous rates of change to find the slopes of secant and tangent lines to a curve? • What is the normal line to a curve at a point?

Content (Subject Matter)	*Students will know….* • **Key Terms** – secant line, tangent line, average speed, instantaneous speed, limit, polynomial function, rational function, limit, one-sided limit, Sandwich Theorem (Squeeze Theorem), indeterminate form, horizontal asymptote, vertical asymptote, end behavior model, continuity, discontinuity (removable and nonremovable), jump discontinuity, infinite discontinuity, oscillating discontinuity, continuous extension, greatest integer function, Intermediate Value Theorem, existence theorems *Students will be able to …..* • Calculate average and instantaneous speed. • Define and calculate limits for function values and apply the properties of limits. • Use the Sandwich Theorem to find certain limits indirectly. • Find and verify end behavior models for various functions. • Calculate limits as $x \to \pm\infty$ to identify vertical asymptotes. • Calculate infinite limits as $x \to a$ to identify horizontal asymptotes. • Identify the interval upon which a given function is continuous and understand the meaning of a continuous function. • Remove removable discontinuities by extending or modifying a function. • Apply the Intermediate Value Theorem and the properties of algebraic combinations and composites of continuous functions. • Apply directly the definition of the slope of a curve in order to calculate slope. • Find the equations of the tangent line and normal line to a curve at a given point. • Find the average and instantaneous rates of change of a function.
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Grade: 12 **Subject:** Honors Calculus	**Unit of Study: Differentiation**
Big Idea/Rationale	Big Idea: Differential Calculus – The Study of Rates of Change of Functions Rationale: The discovery of derivative enabled mathematicians to unlock the secrets of planetary motion and gravitational attraction which would be impossible without the knowledge of derivatives and how they work.
Enduring Understanding (Mastery Objective)	• The derivative gives the value of the slope of the tangent line to a curve at a point. • Graphs of differentiable functions can be approximated by their tangent lines at points where the derivative exists. • Rules for differentiations help us find derivatives of functions analytically more efficiently. • Derivatives give the rates at which things change in the world. • The derivatives of sines and cosines play a key role in describing periodic change including simple harmonic motion. • The Chain Rule allows us to solve real world examples of composite functions such as normal daily maximum temperature for a given location. • Implicit differentiation helps us to find the derivatives of the union of functions (the Eight Curve, The cissoids of Diocles, the Devil's Curse, etc.) that are not defined or written explicitly as a function of a single variable. • The relationship between the graph of a function and its inverse allows us to see the relationship between their derivatives. • Logarithmic differentiation is a powerful tool that aids in solving complex non-logarithmic functions.
Essential Questions (Instructional Objective)	• What is a derivative? • Define a derivative using the formal and the alternate definitions using limits. • How can you use the definition of derivatives to determine algebraic derivatives of functions? • Why does a derivative describe the slope of the tangent line to a graph? • What is the connection between derivatives and the graph of a function? • How can a left-handed and right-handed derivative exist at a point but the derivative not exist there? • When does the derivative at a point fail to exist? • How can local linearity be used to confirm differentiability of a function? • Does differentiability imply continuity? Does continuity imply differentiability? • Why use differentiation rules to find derivatives? • How are derivatives used to describe instantaneous rates of change? • What is the difference between displacement, average velocity and

	instantaneous velocity? • How can derivatives be used to solve problems in everyday life? Velocity, speed and acceleration? Marginal costs and marginal revenue? Simple harmonic motion? • How is the Chain Rule used to compute the derivative of a composite function? • What is an explicitly defined function? Implicitly? • When is implicit differentiation necessary? • How do you find the derivative of an inverse function? • How can logarithmic differentiation be used to simplify the derivative process?
Content (Subject Matter)	*Students will know….* **Key Terms/Ideas** - derivative, differentiable function, differentiation, derivative notation, derivative graphs, differentiability, difference quotient, locally linear, composite function, numerical derivative, jerk, Intermediate Value Theorem for Derivatives *Students will be able to ….* • Calculate slopes and derivatives using the definition of a derivative. • Graph a function from the graph of a derivative and graph a derivative from the graph of a function. • Graph the derivative of a function given numerically with data. • Find where a function is not differentiable and distinguish between corners, cusps, discontinuities, vertical and horizontal tangents. • Analyze differentiable functions using a graphing calculator to explore local linearity. • Approximate derivatives numerically and graphically. • Use the rules of differentiation to calculate derivatives, including second and higher order derivatives. • Use derivatives to analyze straight-line motion and solve other problems involving rates of change. • Use the rules of differentiating the six basic trigonometric functions. • Differentiate composite functions using the Chain Rule. • Find derivatives using implicit differentiation • Find the derivatives using the Power Rule for Rational Powers of x. • Calculate derivatives of functions involving the inverse trigonometric functions. • Calculate derivatives of exponential and logarithmic functions.
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Grade: 12 **Subject:** Honors Calculus	**Unit of Study: Applications of Derivatives**
Big Idea/Rationale	Big Idea: Derivatives – How they help us understand the behavior of a function Rationale: Differentiation is used to maximize or minimize values of real world functions not only in science but in business and economic situations.
Enduring Understanding (Mastery Objective)	• The derivative has both theoretical and real life applications. • Derivatives provide useful information about the behavior of a function and its graph including its absolute maximum/minimum, relative maximum/minimum, where it is increasing/decreasing, its concavity and point(s) of inflection. • Understanding the rate of change of a function allows you to predict future behavior.
Essential Questions (Instructional Objective)	• How can derivatives be used to solve real world problems? • How is the derivative presented graphically, numerically, and analytically? • What is a critical number and what can they tell us? • How is the derivative of a function *f'* related to the function *f?* • What are the similarities and differences between *f'* and *f* graphically? • What is the second derivative and how is it related to *f* and *f'?* • What does a derivative tell us about a function? • How can the derivative be used to solve optimization and related rate problems? • How do rates of change relate in real-life situations?
Content (Subject Matter)	*Students will know….* **Key Terms:** Absolute (Global) Extrema, Relative (Local) Extrema, Critical Point, Mean Value Theorem, Increasing and Decreasing Functions, antiderivative, antidifferentiation, First Derivative Test, Second Derivative Test, Concavity, Point of Inflection, Optimization, Maximum Profit, Minimizing Average Cost, Linear Approximation, Locally Linear, Linearization, Newton's Method, Differentials, Absolute, Relative and Percentage Change, Related Rates *Students will be able to…* • Determine the local or global extreme values of a function. • Understand and Apply the Mean Value Theorem. • Find the intervals on which a function is increasing or decreasing. • Use the 1^{st} and 2^{nd} Derivative Tests to determine the local extreme values of a function. • Determine the concavity of a function and locate the points of inflection by analyzing the 2^{nd} derivative. • Graph *f* using information about *f'*.

	• Solve application problems involving finding minimum or maximum values of functions. • Find the linearization and use Newton's method to approximate the zeros of a function. • Estimate the change in a function using differentials. • Solve related rate problems.
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Grade: 12 **Subject:** Honors Calculus	**Unit of Study: The Definite Integral**
Big Idea/Rationale	Big Idea: Integration – Accumulation and Antidifferentiation Rationale: The integral allows mathematicians to describe how the instantaneous changes could accumulate over an interval to produce the function. Integral calculus is the second main branch of calculus.
Enduring Understanding (Mastery Objective)	• Estimating with finite sums sets the foundation for understanding integral calculus. • The definite integral is the basis of integral calculus, just as the derivative is the basis of differential calculus. • The Fundamental Theorem of Calculus is the connection between derivatives and definite integrals. • Definite integrals can be found using rectangular and trapezoidal numerical approximations.
Essential Questions (Instructional Objective)	• What is an integral? • What is the connection between derivatives and integrals? • How can we estimate the area under a curve using geometric shapes? • What is the relationship between limits and areas under curves? • How are the properties of definite integrals related to the Riemann Sum? • How are definite integral and area related? • How would we define the average value of an arbitrary function f over a closed interval? • What is the Fundamental Theorem of Calculus?
Content (Subject Matter)	*Students will know…* **Key Terms:** Rectangular Approximation Method (RAM), LRAM, RRAM, MRAM, Riemann Sums, Sigma Notation, Definite Integral, Summation Notation, Integration Notation, Average Value of a Function, Mean Value Theorem for Definite Integrals, Antidifferentiation, Antiderivatives, Fundamental Theorem of Calculus, Trapezoidal Approximation *Students will be able to….* • Approximate the area under the graph of a nonnegative continuous function by using the rectangle approximation methods • Interpret the area under a graph as a net accumulation of a rate of change • Express the area under a curve as a definite integral and as a limit of Riemann sums • Compute the area under a curve using a numerical integration procedure • Apply rules for definite integrals and find the average value of a function over a closed interval • Apply the Fundamental Theorem of Calculus

	• Understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus • Approximate the definite integral by using the Trapezoidal Rule
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Grade: 12 **Subject:** Honors Calculus	**Unit of Study: Differential Equations and Mathematical Modeling**
Big Idea/Rationale	Big Idea: Differential Calculus Rationale: Differential Calculus allows us to deduce information about a function from one of its known values and its rate of change.
Enduring Understanding (Mastery Objective)	• Differential equations allow us to find initial values. • Differential calculus can be used to make conclusions about the behavior of a function. • Differential equations allow mathematicians to solve problems involving the study of moving objects. • Calculus allows us to predict the future position of planets from its present position and velocity.
Essential Questions (Instructional Objective)	• How can we use differential equations to model real world problems? • How can we approximate the solution to a differential equation graphically? • How can differential equations be used to predict the behavior of a function? • How can differential equations be analyzed analytically, graphically and numerically to make predictions? • What is a slope field and how can it be used to solve initial value problems?
Content (Subject Matter)	*Students will know….* **Key Terms –** Differential equation, slope field, Euler's Method, Indefinite Integral, Properties of Indefinite Integrals, Power Formulas, Trigonometric Formulas, Exponential and Logarithmic Formula, U-Substitution Method, Integration By Parts, Initial Value Problem, Tabular Integration, Separation of Variables, Law of Exponential Change, Half-Life, Compound Interest, Newton's Law of Cooling, Partial Fractions, Logistic Differential Equation, Logistic Growth Model, Carrying Capacity *Students will be able to….* • Construct antiderivatives using the Fundamental Theorem of Calculus • Find antiderivatives of polynomials, exponentials and selected trigonometric functions of *kx*, as well as linear combinations of these functions • Solve initial value problems • Construct slope fields with and without technology and interpret slope fields as visualizations of differential equations • Compute indefinite and definite integrals using the substitution method • Solve a differential equation using separation of variables • Use integration by parts to evaluate indefinite and definite integrals • Use tabular integration or the method of solving for the unknown integral in order to evaluate integrals that require repeated use of integration by parts • Solve problems involving exponential growth and decay in a variety of

	applications • Solve problems involving exponential or logistic population growth • Use Euler's method to find approximate solutions to differential equations with initial values
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Grade: 12 **Subject:** Honors Calculus	**Unit of Study: Applications of Definite Integrals**
Big Idea/Rationale	Big Idea: Area and Volume Rationale: Definite integrals can be used to answer real world problems involving areas between curves and the volumes of solids of revolution by fitting an integrable function to the situation and then finding an antiderivative for it.
Enduring Understanding (Mastery Objective)	• The integral is a tool that can be used to calculate net change and total accumulation. • Integration techniques allow us to compute areas of complex regions of the plane. • The volume of a variety of three dimensional solids can be obtained using integration.
Essential Questions (Instructional Objective)	• What is the difference between displacement and total distance traveled? • How can integration be used to find the area between two curves? • How can integrals be applied to finding a generated volume? • How can integrals be used to find volumes of complex figures? • What are the practical applications of finding the volumes of complex figures? • How does the graph effect the way that area/volume is determined?
Content (Subject Matter)	*Students will know…* **Key Terms –** Displacement, Total Distance Traveled, Consumption Over Time, Work, Area Between Curves, Area Enclosed By Intersecting Curves, Volume of a Solid with known Cross Sectional Area, Disk Method, Washer Method *Students will be able to….* • Solve problems in which a rate in integrated to find the net change over time in a variety of applications. • Use integration to calculate areas of regions in a plane. • Describe integration as an accumulation process. • Use integration (by disks or washers) to calculate volumes of solids. • Find the volume of a solid with known cross sections.
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	• Document Camera • Calculus of a Single Variable (Larson, Hostetler, Edwards)
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