

Unit 1: Functions and Graphs  
Unit 2: Polynomial, Power, and Rational Functions  
Unit 3: Exponential, Logistic, and Logarithmic Functions  
Unit 4: Trigonometric Functions  
Unit 5: Analytic Trigonometry  
Unit 6: Applications of Trigonometry  
Unit 7: Systems, Sequences, and Series  
Unit 8: An Introduction to Limits

<b>Grade: 11-12</b> <b>Subject: Honors</b> <b>Precalculus</b>	<b>Unit 1: Functions and Graphs</b>
<b>Big Idea/Rationale</b>	<p><b>Big Idea:</b> Function – Algebraically, Numerically and Graphically</p> <p><b>Rationale:</b> Strong skills are essential for success in Pre-Calculus and Calculus. Reviewing key mathematical processes and learning new skills will help on the path to future success. By studying functions numerically, graphically, and algebraically, along with their properties, students are able to use different models and methods that help them to visualize, analyze and understand data.</p>
<b>Enduring Understanding (Mastery Objective)</b>	<ul style="list-style-type: none"> <li>• Pre-Calculus connects the many pieces of mathematics from you past learning and is fundamental in the study of mathematics and science.</li> <li>• Functions and graphs form the basis for understanding the mathematics and applications seen in both the work place and in college coursework.</li> <li>• Knowing The Twelve Basic Functions and their properties will enable the student to work with various combinations and transformations that appear in calculus and other course work.</li> </ul>
<b>Essential Questions (Instructional Objective)</b>	<ul style="list-style-type: none"> <li>• How can we use numerical, algebraic and graphical models to understand and interpret functions?</li> <li>• Which of these models lead us to the best depiction of the function?</li> <li>• How can we avoid drawing incorrect conclusions when using these models?</li> <li>• How can we build new functions by combining and transforming the 12 Basic Functions in a variety of ways?</li> <li>• How can we use appropriate basic functions to model real world problems, for example, in physics and psychology?</li> </ul>
<b>Content (Subject Matter)</b>	<p><i>Students will know...</i></p> <p><b>Key Terms</b> – mathematical model, numerical model, algebraic model, graphical model, zero factor property, root(solution), zero, x-intercept, hidden behavior, domain, range, independent variable, dependent variable, implied domain, relevant domain, removable discontinuity, jump discontinuity, infinite discontinuity, continuous, discontinuous, increasing, decreasing, constant, lower bound, upper bound, bounded on an interval, absolute and relative extrema, symmetry, odd/even functions, horizontal/vertical/slant asymptotes, end behavior, twelve basic functions, piecewise functions, sum, difference, product and quotient of functions, composition of functions, decomposition of functions, relation, explicit/implicit function, parametric equations, inverse relations and functions, horizontal line test, graphical transformations, conversion factors</p> <p><i>Students will be able to:</i></p>

	<ul style="list-style-type: none"> <li>• Represent functions numerically, algebraically, and graphically.</li> <li>• Determine the domain and range for functions.</li> <li>• Analyze the 12 Basic Functions graphically and algebraically by determining domains and ranges, and analyzing function characteristics, such as, extreme values, odd and even functions, boundedness, asymptotes, symmetry, continuity, and end behavior.</li> <li>• Graph piecewise functions.</li> <li>• Compose and decompose functions.</li> <li>• Define functions implicitly.</li> <li>• Define functions and relations parametrically.</li> <li>• Identify inverses graphically and algebraically.</li> <li>• Model real world problems numerically, algebraically, and graphically.</li> <li>• Apply basic transformations of functions using algebraic and graphical methods.</li> <li>• Produce specific functions to model data from formulas, graphs, and verbal descriptions.</li> </ul>
<b>Skills/ Benchmarks (Standards)</b>	<p><b>HSN-Q.A.1</b> Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.</p> <p><b>HSN-Q.A.2</b> Define appropriate quantities for the purpose of descriptive modeling.</p> <p><b>HSF-IF.C.9</b> Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p> <p><b>HSF-BF.A.1c</b> (+) Compose functions. <i>For example, if <math>T(y)</math> is the temperature in the atmosphere as a function of height, and <math>h(t)</math> is the height of a weather balloon as a function of time, then <math>T(h(t))</math> is the temperature at the location of the weather balloon as a function of time.</i></p> <p><b>HSF-BF.B.4a</b> Solve an equation of the form <math>f(x) = c</math> for a simple function <math>f</math> that has an inverse and write an expression for the inverse. <i>For example, <math>f(x) = 2x^3</math> or <math>f(x) = (x+1)/(x-1)</math> for <math>x \neq 1</math>.</i></p> <p><b>HSF-BF.B.4b</b> (+) Verify by composition that one function is the inverse of another.</p> <p><b>HSF-BF.B.4c</b> (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.</p> <p><b>HSS-ID.B.6a</b> Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</p>
<b>Materials and Resources</b>	<p><b>Textbook:</b></p> <ul style="list-style-type: none"> <li>• Title: Precalculus, Graphical, Numerical, Algebraic</li> <li>Authors: Demana, Waits, Foley, Kennedy</li> <li>Published by: Addison Wesley 2007</li> </ul>

	<ul style="list-style-type: none"><li>• <b>Resource Manual</b>-Chapter 1</li><li>• <b>Precalculus Express</b> Power Point Slides</li><li>• <b>Technology:</b> Graphing Calculators-TI 83 Plus</li></ul>
<b>Notes</b>	

<b>Grade: 11-12</b> <b>Subject: Honors</b> <b>PreCalculus</b>	<b>Unit 2: Polynomial, Power and Rational Functions</b>
<b>Big Idea/Rationale</b>	<p><b>Big Idea:</b> Families of Functions</p> <p><b>Rationale:</b> Polynomial, power and rational function families are used extensively to model behavior in the social, behavioral and natural sciences. Modeling with these functions can provide approximations to more complicated functions that students will see in calculus.</p>
<b>Enduring Understanding (Mastery Objective)</b>	<ul style="list-style-type: none"> <li>• Algebraic methods can be used for finding real number and complex number solutions for polynomial and rational functions.</li> <li>• The graphical behavior of these functions is directly related to the function.</li> <li>• All rates are ratios no matter how they are expressed.</li> <li>• Power functions specify the proportional relationships of geometry, chemistry and physics.</li> <li>• The characteristics of polynomial functions and their representations are useful in solving real-world problems.</li> <li>• Rational Equations often result in extraneous solutions.</li> <li>• Methods of solving polynomial and rational equations can be extended to inequalities.</li> <li>• Sign charts show us the real zeroes displayed as the locations of potential sign changes and the factors displayed with their sign value in each corresponding interval allowing us to draw an accurate sketch of a polynomial or rational inequality.</li> </ul>
<b>Essential Questions (Instructional Objective)</b>	<ul style="list-style-type: none"> <li>• What is the definition of a polynomial function?</li> <li>• What is meant by constant rate of change?</li> <li>• How is speed different from velocity?</li> <li>• How do we use a quadratic function to model vertical free-fall motion?</li> <li>• How do we use a linear function to model vertical velocity?</li> <li>• What are applications of power functions?</li> <li>• How can we predict the end behavior of a polynomial function by examining the equation?</li> <li>• How do we transform and graph rational functions?</li> <li>• How do we solve rational equations in one variable?</li> <li>• How do we solve polynomial inequalities?</li> <li>• How do we solve rational inequalities in one variable?</li> </ul>
<b>Content (Subject Matter)</b>	<p><i>Students will know...</i></p> <p><b>Key Terms</b> – polynomial function, leading coefficient, average rate of change, initial value, constant term, correlation, regression analysis, vertical</p>

free fall, acceleration due to gravity, power function, constant of variation, direct variation, inverse variation, leading coefficient test for polynomial end behavior, zeros of odd and even multiplicity, x and y intercepts, vertical asymptotes and end behavior asymptotes of a rational function, remainder and factor theorems, synthetic and long division, fundamental theorem of algebra, linear factorization theorem, zeros of complex conjugates, extraneous solutions, sign chart,

*Students will be able to...*

- Model real world problems using Linear and Quadratic Functions
- Find the average rate of change
- Model Free-Fall Motion
- Analyze Power Functions
- Model Power Functions using Direct and Inverse Variation
- Graph Power Functions
- Analyze Monomial Functions
- Analyze and graph Polynomial Functions
- Determine the end behavior of Polynomial Functions
- Find the Zeros of Polynomial Functions
- Sketch the graph of a Polynomial Function
- Analyze Graphs of Rational Functions
- Determine limits, asymptotes, x and y intercepts of Rational Functions
- Sketch the graph of a Rational Function
- Solve Rational Equations
- Finding a Minimum Perimeter given the area
- Solve polynomial, rational and other inequalities using a Sign Chart

**Skills/ Benchmarks  
(Standards)**

**HSN-Q.A.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**HSN-Q.A.2** Define appropriate quantities for the purpose of descriptive modeling.

**HSN-CN.A.1** Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.

**HSN-CN.A.2** Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

**HSN-CN.C.7** Solve quadratic equations with real coefficients that have complex solutions.

**HSN-CN.C.8** (+) Extend polynomial identities to the complex numbers. *For example, rewrite  $x^2 + 4$  as  $(x + 2i)(x - 2i)$ .*

**HSN-CN.C.9** (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

**HSA-APR.B.2** Know and apply the Remainder Theorem: For a polynomial  $p(x)$

	<p>and a number <math>a</math>, the remainder on division by <math>x - a</math> is <math>p(a)</math>, so <math>p(a) = 0</math> if and only if <math>(x - a)</math> is a factor of <math>p(x)</math>.</p> <p><b>HSA-APR.D.6</b> Rewrite simple rational expressions in different forms; write <math>\frac{a(x)}{b(x)}</math> in the form <math>q(x) + \frac{r(x)}{b(x)}</math>, where <math>a(x)</math>, <math>b(x)</math>, <math>q(x)</math>, and <math>r(x)</math> are polynomials with the degree of <math>r(x)</math> less than the degree of <math>b(x)</math>, using inspection, long division, or, for the more complicated examples, a computer algebra system.</p> <p><b>HSA-APR.D.7 (+)</b> Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.</p> <p><b>HSA-REI.A.2</b> Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.</p> <p><b>HSS-ID.B.6a</b> Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</p>
<p><b>Materials and Resources</b></p>	<ul style="list-style-type: none"> <li>• <b>Textbook:</b> Title: Precalculus, Graphical, Numerical, Algebraic Authors: Demana, Waits, Foley, Kennedy Published by: Addison Wesley 2007</li> <li>• <b>Resource Manual-Chapter 2</b></li> <li>• <b>Precalculus Express Power Point Slides</b></li> <li>• <b>Technology:</b> Graphing Calculators-TI 83 Plus</li> </ul>
<p><b>Notes</b></p>	<p>Appendix A.2 Factoring Polynomials</p>

<b>Grade: 11-12</b> <b>Subject: Honors</b> <b>Precalculus</b>	<b>Unit 3: Exponential, Logistic, and Logarithmic Functions</b>
<b>Big Idea/Rationale</b>	<p><b>Big Idea:</b> Transcendental Functions</p> <p><b>Rationale:</b> Transcendental Functions go beyond their algebraic cousins and can be used to model a variety of real world applications such as: unrestricted/restricted population growth, the decay of radioactive substances, the spread of rumors and diseases, the Richter Scale of earthquake intensity, the pH scale and the decibel measurement of sound.</p>
<b>Enduring Understanding (Mastery Objective)</b>	<ul style="list-style-type: none"> <li>• Exponential, logarithmic and logistic functions are transcendental functions that have wide spread applications.</li> <li>• Exponential and logarithmic functions are inverses of each other and their graphs are reflections of each other.</li> <li>• Exponential and logarithmic functions can be used to model behavior which will enable us to predict the future and rediscover the past.</li> <li>• Algebraic and numerical methods can be used for finding real number solutions for Exponential, Logistic, and Logarithmic Functions which can be related to the graphical behavior of these functions.</li> </ul>
<b>Essential Questions (Instructional Objective)</b>	<ul style="list-style-type: none"> <li>• What are the properties of Exponential, Logistic, and Logarithmic Functions and their graphs?</li> <li>• How can an exponential function represent a real world scenario?</li> <li>• What is a logarithm?</li> <li>• How can the properties of logarithms be used to solve equations?</li> <li>• How do you solve Exponential, Logistic, and Logarithmic equations algebraically? Graphically?</li> <li>• How do we use these equations to model and solve real world situations as enumerated above?</li> <li>• Why is the number <math>e</math> important?</li> </ul>
<b>Content (Subject Matter)</b>	<p><i>Students will know...</i></p> <p><b>Key Terms</b> – Algebraic functions, transcendental functions, exponential function, base, exponential growth function, growth factor, exponential decay function, decay function, the natural number <math>e</math>, logistic growth function, limit to growth constant, logistic decay function, logistic function, exponential population model, exponential growth and decay models, radioactive decay, half-life, maximum sustainable population, logarithmic function, common logarithm, natural logarithm, decibels, properties of logarithms, change of base formula, Newton’s Law of Cooling, compound interest, annual percentage yield.</p>

	<p><i>Students will be able to....</i></p> <ul style="list-style-type: none"> <li>• Identify, graph and model using Exponential and Logistic functions with appropriate bases including <math>e</math></li> <li>• Identify the Constant Percentage Rate, apply exponential model to bacteria growth and radioactive decay</li> <li>• Apply logistic models to predict spread of disease, restricted population growth and spread of rumors.</li> <li>• Convert between exponential and logarithmic form</li> <li>• Evaluate expressions containing logs base <math>b</math>, common logs base 10 and natural logs base <math>e</math></li> <li>• Graph Logarithmic Functions of any base and analyze the graphs.</li> <li>• Apply the properties of logarithms to solve exponential and logarithmic equations algebraically and solve application problems using these equations.</li> <li>• Solve business and finance applications related to compound interest.</li> </ul>
<p><b>Skills/ Benchmarks (Standards)</b></p>	<p><b>A.SSE.B.3.C</b> Use the properties of exponents to transform expressions for exponential functions. <i>For example the expression <math>1.15^t</math> can be rewritten as <math>(1.15^{1/12})^{12t} \approx 1.012^{12t}</math> to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</i></p> <p><b>A.CED.A.1</b> Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></p> <p><b>F.IF.C.7.E</b> Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p> <p><b>F.IF.C.8.B</b> Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as <math>y = (1.02)^t</math>, <math>y = (0.97)^t</math>, <math>y = (1.01)12^t</math>, <math>y = (1.2)^t/10</math>, and classify them as representing exponential growth or decay.</p> <p><b>F.BF.B.3</b> Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</p> <p><b>F.BF.B.5 (+)</b> Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.</p> <p><b>F.LE.A.1</b> Distinguish between situations that can be modeled with linear functions and with exponential functions.</p> <ol style="list-style-type: none"> <li>a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.</li> <li>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</li> </ol>

	<p><b>F.LE.A.2</b> Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</p> <p><b>F.LE.A.3</b> Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</p> <p><b>F.LE.A.4</b> For exponential models, express as a logarithm the solution to <math>ab^{ct} = d</math> where <math>a</math>, <math>c</math>, and <math>d</math> are numbers and the base <math>b</math> is 2, 10, or <math>e</math>; evaluate the logarithm using technology.</p> <p><b>F.LE.B.5</b> Interpret the parameters in a linear or exponential function in terms of a context.</p>
<p><b>Materials and Resources</b></p>	<ul style="list-style-type: none"> <li>• <b>Textbook:</b> Title: Precalculus, Graphical, Numerical, Algebraic Authors: Demana, Waits, Foley, Kennedy Published by: Addison Wesley 2007</li> <li>• <b>Resource Manual</b>-Chapter 3</li> <li>• <b>Precalculus Express</b> Power Point Slides</li> <li>• <b>Technology:</b> Graphing Calculators-TI 83 Plus</li> </ul>
<p><b>Notes</b></p>	

<b>Grade: 11-12</b> <b>Subject: Honors</b> <b>Precalculus</b>	<b>Unit 4: Trigonometric Functions</b>
<b>Big Idea/Rationale</b>	<p><b>Big Idea:</b> The Study of Trigonometry Functions – Old and New</p> <p><b>Rationale:</b> Students began their study of trigonometry in previous math courses with right triangles. They will extend their knowledge to understand how this information can help them solve a variety of real world problems. The study of calculus has made the knowledge of trigonometric functions more important than ever. Sinusoidal equations are especially useful in modeling periodic behavior in real world applications including average monthly temperatures, daylight hours, tides, sales forecasts, and blood pressure. These functions can help us predict future behavior.</p>
<b>Enduring Understanding (Mastery Objective)</b>	<ul style="list-style-type: none"> <li>• Angles are the domain elements of the trigonometric functions.</li> <li>• The radian allows mathematicians a link between an angle’s measure and the length of its radius.</li> <li>• Real world data can be modeled with a sinusoidal function.</li> <li>• There exists a definite and distinct bridge between the fields of trigonometry and physics.</li> <li>• Right triangles can be used to model and solve real world problems.</li> <li>• Members within a family of functions, including the trigonometric functions, have common characteristics.</li> </ul>
<b>Essential Questions (Instructional Objective)</b>	<ul style="list-style-type: none"> <li>• What is a radian and how is it related to a degree?</li> <li>• How are linear and angular velocities similar? Different?</li> <li>• Describe the difference between an angle with a positive measure and an angle with a negative measure.</li> <li>• What is a quick way to verify coterminal angles?</li> <li>• What are the reciprocal ratios of sine, cosine, and tangent?</li> <li>• Explain the process of evaluating a trigonometric function using reference angles and the unity circle.</li> <li>• How do the values on the unit circle correlate to the rectangular graph of a trigonometric function?</li> <li>• Function composition extends our ability to model periodic phenomena like heartbeats and sound waves.</li> <li>• Why is it necessary to restrict the domain in order to discuss inverse trigonometric functions?</li> <li>• What does it mean to find the exact value of a trigonometric function?</li> <li>• Why are trigonometric functions periodic?</li> <li>• What is the effect of A, B, h, and k on the graph of a Sine or Cosine curve using the equations  <math display="block">y = A \sin B(x - h) + k</math> <math display="block">y = A \cos B(x - h) + k</math> ? </li> <li>• How do we apply the trig ratios to solve for the parts of a right triangle?</li> </ul>

	<ul style="list-style-type: none"> <li>• How do we extend the trig functions to solve problems involving periodic behavior and real numbers?</li> <li>• How do we transform graphs of sine and cosine functions?</li> <li>• How do we relate the concept of inverse function to trig functions?</li> <li>• How do we solve a trig equation?</li> </ul>
<p><b>Content (Subject Matter)</b></p>	<p><i>Students will know...</i></p> <p><b>Key Terms</b> - radian, circular arc, central angle, length of an arc, sector, area of a circular sector, angular displacement, angular velocity, dimensional analysis, linear velocity, right triangle trigonometric ratios, trigonometric functions of special angles, unit circle, vertex, initial side, terminal side, standard position, degree, minutes, seconds, quadrantal angle, coterminal angle, reference angle, cofunction, angle of elevation, angle of depression, periodic function, period, amplitude, phase shift, frequency, compound functions, sinusoidal function, inverse trigonometric functions</p> <p><i>Students will be able to:</i></p> <ul style="list-style-type: none"> <li>• Convert between degree measures to degrees, minutes, seconds and vice versa.</li> <li>• Convert between degrees and radians.</li> <li>• Determine arc length using the appropriate formula.</li> <li>• Use the relationships in the 2 special right triangles to evaluate the 6 trigonometric functions.</li> <li>• Recognize 4 common calculator errors when evaluating trigonometric function.</li> <li>• Evaluate the trigonometric functions of any angle.</li> <li>• Use reference triangles to find trigonometric functions of real numbers.</li> <li>• Find the amplitude, the period, the phase shift, and the vertical shift for trigonometric functions</li> <li>• Sketch transformations of graphs of sine, cosine tangent, secant, cosecant, and cotangent and analyze their characteristics.</li> <li>• Model periodic behavior with sinusoids.</li> <li>• Write trigonometric equations to model a given situation</li> <li>• Use graphs of inverse sine, cosine and tangent functions with their respective domains to solve equations.</li> <li>• Solve applications of right triangles.</li> </ul>
<p><b>Skills/ Benchmarks (Standards)</b></p>	<p><b>HSN-Q.A.1</b> Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.</p> <p><b>HSN-Q.A.2</b> Define appropriate quantities for the purpose of descriptive modeling.</p>

**HSF-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.\**

**HSF-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.\**

**HSF-BF.B.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

**HSF-BF.B.4d (+)** Produce an invertible function from a non-invertible function by restricting the domain.

**HSF-TF.A.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

**HSF-TF.A.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

**HSF-TF.A.3 (+)** Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $x$ ,  $\pi + x$ , and  $2\pi - x$  in terms of their values for  $x$ , where  $x$  is any real number.

**HSF-TF.A.4 (+)** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

**HSF-TF.B.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.\*

**HSF-TF.B.6 (+)** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

**Materials and Resources**

- **Textbook:**  
Title: Precalculus, Graphical, Numerical, Algebraic  
Authors: Demana, Waits, Foley, Kennedy  
Published by: Addison Wesley 2007
- **Resource Manual**-Chapter 4
- **Precalculus Express** Power Point Slides
- **Technology:** Graphing Calculators-TI 83 Plus

<b>Notes</b>	Omit Section 4.6
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<b>Grade: 11-12</b> <b>Subject: Honors</b> <b>Precalculus</b>	<b>Unit 5: Analytic Trigonometry</b>
<b>Big Idea/Rationale</b>	<p><b>Big Idea:</b> Solving Puzzles – Connection among the Trigonometric Functions</p> <p><b>Rationale:</b> Identities provide insight into the way mathematical proofs are constructed and they continue their usefulness in the study of calculus. Emphasis is placed on theory and proof as well as exploring where the properties of these special functions lead us.</p>
<b>Enduring Understanding (Mastery Objective)</b>	<ul style="list-style-type: none"> <li>• Identities are important when working with trigonometric functions in calculus.</li> <li>• Proving identities gives insight into the way mathematical proofs are constructed.</li> <li>• Equivalent expressions can be written in a variety of formats.</li> <li>• Trigonometry can be used to measure oblique triangles as well as right triangles.</li> <li>• Sinusoidal functions have an infinite number of solutions that are the same interval apart.</li> </ul>
<b>Essential Questions (Instructional Objective)</b>	<ul style="list-style-type: none"> <li>• What does it mean to prove an identity?</li> <li>• What strategies can be used to prove a trigonometric identity?</li> <li>• How can a trigonometric identity enable a trigonometric equation to be solved?</li> <li>• How do we represent multiple solutions on a given domain?</li> <li>• How can we use identities to simplify the process involved in solving an equation?</li> <li>• How can you use the sum and difference identities to find values for the secant, cosecant, and cotangent functions of a sum or difference?</li> <li>• Why do many trigonometric equations have infinitely many solutions?</li> <li>• When and how do we use the Law of Sines and Cosines to solve a real world problem?</li> </ul>
<b>Content (Subject Matter)</b>	<p><i>Students will know...</i></p> <p><b>Key Terms</b> - identity, trigonometric identities, reciprocal identities, quotient identities, cofunction identities, odd/even identities, Pythagorean identities, symmetry identities, opposite-angle identities, sum and difference identities, double angle identities, power reducing identities, half-angle identities, principal values, Law of Sines, Law of Cosines, Heron’s Formula, triangle area formulas given SAS or SSS.</p> <p><i>Students will be able to...</i></p>

	<ul style="list-style-type: none"> <li>• Use the fundamental identities to simplify trigonometric expressions.</li> <li>• Derive the Pythagorean Identities.</li> <li>• Use identities to solve trigonometric equations.</li> <li>• Determine whether an equation is an identity.</li> <li>• Prove identities analytically.</li> <li>• Apply the identities for the sine, cosine, and tangent of a difference or sum.</li> <li>• Apply the double-angle, power-reducing and half-angle identities.</li> <li>• Apply the Law of Sines, Law of Cosines and Triangle Area Formulas to solve geometric and real world problems.</li> </ul>
<b>Skills/ Benchmarks (Standards)</b>	<p><b>HSN-Q.A.2</b> Define appropriate quantities for the purpose of descriptive modeling.</p> <p><b>HSF-IF.B.4</b> For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*</i></p> <p><b>HSF-TF.B.7</b> (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*</p> <p><b>HSF-TF.C.8</b> Prove the Pythagorean identity <math>\sin^2(\theta) + \cos^2(\theta) = 1</math> and use it to find <math>\sin(\theta)</math>, <math>\cos(\theta)</math>, or <math>\tan(\theta)</math> given <math>\sin(\theta)</math>, <math>\cos(\theta)</math>, or <math>\tan(\theta)</math> and the quadrant of the angle.</p> <p><b>HSF-TF.C.9</b> (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.</p> <p><b>HSG-SRT.D.9</b> (+) Derive the formula <math>A = 1/2 ab \sin(C)</math> for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.</p> <p><b>HSG-SRT.D.11</b> (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).</p>
<b>Materials and Resources</b>	<ul style="list-style-type: none"> <li>• <b>Textbook:</b> Title: Precalculus, Graphical, Numerical, Algebraic Authors: Demana, Waits, Foley, Kennedy Published by: Addison Wesley 2007</li> <li>• <b>Resource Manual</b>-Chapter 5</li> <li>• <a href="http://www.regentsprep.org">www.regentsprep.org</a></li> <li>• <b>Precalculus Express</b> Power Point Slides</li> <li>• <b>Technology:</b> Graphing Calculators-TI 83 Plus</li> </ul>

<b>Notes</b>	

<b>Grade: 11-12</b> <b>Subject: Honors</b> <b>Precalculus</b>	<b>Unit 6: Applications of Trigonometry</b>
<b>Big Idea/Rationale</b>	<p><b>Big Idea:</b> Vectors and Parametric Equations</p> <p><b>Rationale:</b> Vector methods are used extensively in physics, engineering, and applied mathematics. Parametric equations are used to study and simulate motion. These topics are important in many real-world applications, such as calculating the effect of the wind on an airplane's path.</p>
<b>Enduring Understanding (Mastery Objective)</b>	<ul style="list-style-type: none"> <li>• Two numbers are all that is needed to represent both the magnitude of an object's velocity and its direction of motion.</li> <li>• Vectors are used to model magnitude and direction of moving objects in 2 dimensions.</li> <li>• The quick way to associate arrows with the vectors they represent is to use the Head Minus Tail (HMT) rule.</li> <li>• Vector addition and scalar multiplication are easily represented geometrically and have immediate application to many real-world problems.</li> <li>• Vectors help determine the net effect of several forces acting on an object as well as the work done by a force acting on an object.</li> <li>• The dot product of two vectors is a real number.</li> <li>• Use parametric equations to model motion and as an alternative to modeling with functions</li> <li>• Demonstrate the connection between modeling with functions and modeling with parametric equations.</li> </ul>
<b>Essential Questions (Instructional Objective)</b>	<ul style="list-style-type: none"> <li>• How can a vector be used to determine force and direction of a moving object?</li> <li>• What factors determine whether two vectors are equal?</li> <li>• Can a vector have more than one direction angle?</li> <li>• What is the range of possible values of the magnitude of a vector?</li> <li>• How do you find the angle between two vectors?</li> <li>• How do you eliminate the parameter from a pair of parametric equations to show the connection to a familiar function?</li> <li>• How do you use a graphing calculator to graph a pair of parametric equations to simulate motion?</li> </ul>
<b>Content (Subject Matter)</b>	<p><i>Students will know...</i></p> <p><b>Key Terms</b> – directed line segment, arrow, position vector, two-dimensional vector, component form, components, standard representation, magnitude, direction, zero vector, initial point, terminal point, equivalent vectors, Head Minus Tail Rule, scalars, vector addition, scalar multiplication, resultant, tail to head representation, unit vector, standard unit vector, linear combination, horizontal and vertical components,</p>

direction angle, resolving the vector, velocity, speed, dot product, inner product, angle between two vectors, orthogonal vectors, vector projection, force, work

*Students will be able to...*

- Find the component form, magnitude and direction angle of a vector.
- Perform Basic operations with vectors.
- Calculate dot products and projections of vectors.
- Apply vectors to problems involving Velocity, Work, and Force.
- Define and graph equations parametrically.
- Simulate motion and solve problems using a graphing calculator.

**Skills/ Benchmarks  
(Standards)**

**HSN-Q.A.2** Define appropriate quantities for the purpose of descriptive modeling.

**HSN-CN.A.3** (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

**HSN-CN.B.4** (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

**HSN-CN.B.5** (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,  $(-1 + \sqrt{3}i)^3 = 8$  because  $(-1 + \sqrt{3}i)$  has modulus 2 and argument  $120^\circ$ .*

**HSN-CN.B.6** (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

**HSN-VM.A.1** (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g.,  $\mathbf{v}$ ,  $|\mathbf{v}|$ ,  $\|\mathbf{v}\|$ ,  $v$ ).

**HSN-VM.A.2** (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

**HSN-VM.A.3** (+) Solve problems involving velocity and other quantities that can be represented by vectors.

**HSN-VM.B.4a** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

**HSN-VM.B.4b** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

**HSN-VM.B.4c** Understand vector subtraction  $\mathbf{v} - \mathbf{w}$  as  $\mathbf{v} + (-\mathbf{w})$ , where  $-\mathbf{w}$  is the additive inverse of  $\mathbf{w}$ , with the same magnitude as  $\mathbf{w}$  and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

**HSN-VM.B.5a** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-

	<p>wise, e.g., as <math>c(v_x, v_y) = (cv_x, cv_y)</math>.</p> <p><b>HSN-VM.B.5b</b> Compute the magnitude of a scalar multiple <math>c\mathbf{v}</math> using <math>\ c\mathbf{v}\  =  c \mathbf{v}</math>. Compute the direction of <math>c\mathbf{v}</math> knowing that when <math> c \mathbf{v} \neq 0</math>, the direction of <math>c\mathbf{v}</math> is either along <math>\mathbf{v}</math> (for <math>c &gt; 0</math>) or against <math>\mathbf{v}</math> (for <math>c &lt; 0</math>).</p>
<p><b>Materials and Resources</b></p>	<ul style="list-style-type: none"> <li>• <b>Textbook:</b>  Title: Precalculus, Graphical, Numerical, Algebraic  Authors: Demana, Waits, Foley, Kennedy  Published by: Addison Wesley 2007</li> <li>• <b>Resource Manual</b>-Chapter 6</li> <li>• <a href="http://www.regentsprep.org">www.regentsprep.org</a></li> <li>• <b>Precalculus Express</b> Power Point Slides</li> <li>• <b>Technology:</b> Graphing Calculators-TI 83 Plus</li> </ul>
<p><b>Notes</b></p>	<p>Section 6.4 to 6.6 will be completed if time allow.</p>

<b>Grade: 11-12</b> <b>Subject: Honors</b> <b>Precalculus</b>	<b>Unit 7: Systems, Sequences and Series</b>
<b>Big Idea/Rationale</b>	<p><b>Big Idea:</b> Key Concept in Calculus</p> <p><b>Rationale:</b> Many application of mathematics in science, engineering, business and other areas involve the use of systems. Infinite sequence and series are at the heart of integral calculus. Decomposition of a rational function is used in calculus to integration rational functions analytically.</p>
<b>Enduring Understanding (Mastery Objective)</b>	<ul style="list-style-type: none"> <li>• Investigate techniques commonly used to solve nonlinear systems and their applications.</li> <li>• Partial fraction decomposition is used to simplify rational functions in the integration process in calculus and can be used to guide the sketch of the graph of a rational function.</li> <li>• Write explicit and recursive formulas for a variety of different types of sequences</li> <li>• Display and evaluate terms of an infinite series using sigma notation</li> <li>• Determine when a infinite geometric series converges</li> <li>• Sum an infinite geometric series</li> </ul>
<b>Essential Questions (Instructional Objective)</b>	<ul style="list-style-type: none"> <li>• How many solutions will a nonlinear system have?</li> <li>• Should we round intermediate results when solving a system? Why or why not?</li> <li>• What method of solving a system should be used when algebraic techniques fail?</li> <li>• What are the steps used in decomposing a rational functions with distinct linear factors?</li> <li>• How does the partial fraction decomposition change if you have repeated linear factors? A non-repeated quadratic factor? A repeated quadratic factor?</li> <li>• How do you express a sequence explicitly and recursively?</li> <li>• How do you represent a sequence using sigma notation?</li> <li>• When does an infinite geometric sequence converge?</li> <li>• How do you find the sum of an infinite geometric series?</li> </ul>
<b>Content (Subject Matter)</b>	<p><i>Students will know...</i></p> <p><b>Key Terms</b> – nonlinear system, partial fraction, partial fraction decomposition, repeated linear factors, non-repeated/repeated quadratic functions, finite sequence, infinite sequence, <math>k^{\text{th}}</math> term, converges, diverges, limit of the sequence, recursive, arithmetic sequence, geometric sequence, common difference, common ratio, Fibonacci sequence, summation notation, index of summation, sigma, infinite series, partial sums, Formula for Sum of a Finite Arithmetic Sequence, Formula for Sum of a Finite Geometric Sequence Sum, Formula for Sum of an Infinite Geometric Series.</p>

	<p><i>Students will be able to:</i></p> <ul style="list-style-type: none"> <li>• Recognize systems of nonlinear equations in two variables.</li> <li>• Solve nonlinear systems by substitution, elimination or graphing.</li> <li>• Decompose rational expressions into partial fractions.</li> <li>• Distinguish between arithmetic, geometric and sequences that are neither of those.</li> <li>• Express arithmetic and geometric sequences explicitly and recursively Find limits of convergent sequences</li> <li>• Use sigma notation to express a variety of series</li> <li>• Find the sum of finite arithmetic and geometric series</li> <li>• Find the limit of partial sums</li> <li>• Find the sum infinite geometric series</li> </ul>
<p><b>Skills/ Benchmarks (Standards)</b></p>	<p><b>HSN-Q.A.2</b> Define appropriate quantities for the purpose of descriptive modeling.</p> <p><b>HSA-SSE.B.4</b> Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. <i>For example, calculate mortgage payments.</i> ★</p> <p><b>HSF-IF.A.3</b> Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. <i>For example, the Fibonacci sequence is defined recursively by <math>f(0) = f(1) = 1</math>, <math>f(n+1) = f(n) + f(n-1)</math> for <math>n \geq 1</math>.</i></p> <p><b>HSF-BF.A.1a</b> Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p><b>HSF-BF.A.2</b> Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*</p> <p><b>HSF-LE.A.2</b> Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</p>
<p><b>Materials and Resources</b></p>	<ul style="list-style-type: none"> <li>• <b>Textbook:</b> Title: Precalculus, Graphical, Numerical, Algebraic Authors: Demana, Waits, Foley, Kennedy Published by: Addison Wesley 2007</li> <li>• <b>Resource Manual</b>-Chapter 9</li> <li>• <a href="http://www.regentsprep.org">www.regentsprep.org</a></li> <li>• <b>Precalculus Express</b> Power Point Slides</li> <li>• <b>Technology:</b> Graphing Calculators-TI 83 Plus</li> </ul>

<b>Notes</b>	

<b>Grade: 11-12</b> <b>Subject: Honors</b> <b>Precalculus</b>	<b>Unit 8: An Introduction to Limits</b>
<b>Big Idea/Rationale</b>	<p>Big Idea: Limits – The Bridge to Calculus</p> <p>Rationale: Limits are a fundamental concept in the understanding of calculus. The limit concept distinguishes calculus from the other branches of mathematics such as algebra and trigonometry. Limits allow us to model problems that help us better understand the world around us.</p>
<b>Enduring Understanding (Mastery Objective)</b>	<ul style="list-style-type: none"> <li>• Limits can be either an exact value, a quantity that is approached but not reached, or not exist.</li> <li>• There are a variety of mathematical techniques that can be used to determine a limit.</li> <li>• To determine the behavior of a function as the function approaches a real number, it is necessary to analyze the behavior of the function as it approaches the real number from the left and from the right.</li> </ul>
<b>Essential Questions (Instructional Objective)</b>	<ul style="list-style-type: none"> <li>• What is a limit?</li> <li>• How do you determine a limit with a table? A graph? Analytically?</li> <li>• What does indeterminate form mean? Which techniques can we use to solve these types of problems?</li> <li>• What algebraic methods can be applied to determine a limit algebraically?</li> <li>• Under what circumstances does a limit not exist?</li> <li>• What is the difference between a one-sided and a two-sided limit?</li> </ul>
<b>Content (Subject Matter)</b>	<p><i>Students will know...</i></p> <p><b>Key Terms</b> – Limit, one-sided limit, two-sided limit, right-hand limit, left hand limit, limits as <math>x</math> approaches positive/negative infinity</p> <p><i>Students will be able to...</i></p> <ul style="list-style-type: none"> <li>• Understand limit notation.</li> <li>• Find limits using a table of values.</li> <li>• Find limits using a graph.</li> <li>• Find limits using direct substitution.</li> <li>• Determine when a limit does not exist</li> <li>• Find the limit of a constant function.</li> <li>• Find the limit of the identity function.</li> <li>• Find limits using properties of limits.</li> <li>• Find limits of fractional expressions in which direct substitution fails.</li> <li>• Find one-sided limits and use them to determine if a limit exists.</li> </ul>

	<ul style="list-style-type: none"> <li>• Find limits at infinity</li> </ul>
<b>Skills/ Benchmarks (Standards)</b>	The concept of “limits” goes beyond the scope of the Common Core State Standards. However, the skills learned in this unit are necessary for a study of Calculus so this unit is included in this course.
<b>Materials and Resources</b>	<ul style="list-style-type: none"> <li>• <b>Textbooks:</b>  Title: Graphical Approach to Precalculus with Limits, 3<sup>rd</sup> edition  Authors: Hornsby, Lial, Rockwold  Published by: Addison Wesley   Title: Precalculus  Author: Blitzer  Published by: Prentice Hall 2004   Title: Precalculus, Graphical, Numerical, Algebraic  Authors: Demana, Waits, Foley, Kennedy  Published by: Addison Wesley 2007</li> <li>• <b>Technology:</b> Graphing Calculators-TI 83/84 Plus</li> </ul>
<b>Notes</b>	