Solving for Half-Life

Sodium-24 has a half-life of 15 hours. How much sodium-24 will remain in an 18.0g sample after 60 hours?

$$\#\text{of half lives} = \frac{t_{total}}{t_{1/2}} = \frac{60\text{hrs}}{15\text{hrs}} = 4$$

$$\frac{18.0g}{2^4} = \frac{18.0g}{16} = 1.125g$$

$= 1.13g$ sig figs

Practice Problems

7. Manganese-56 is a beta emitter with a half-life of 2.6 h. What is the mass of manganese-56 in a 1.0-mg sample of the isotope at the end of 10.4 h?

$$\#\tfrac{1}{2}\text{ life} = \frac{t_{tot}}{t_{1/2}} = \frac{10.4\text{hrs}}{2.6\text{hrs}} = 4$$

$$\frac{1.0mg}{2^4} = \frac{1.0}{16} = .0625mg$$

$= .063\ mg$

38. A patient is administered 20 mg of iodine-131. How much of this isotope will remain in the body after 40 days if the half-life for iodine-131 is 8 days?

$$\#\tfrac{1}{2}\text{ lives} = \frac{t_{tot}}{t_{1/2}} = \frac{40\text{ days}}{8\text{ days}} = 5$$

$$\frac{20mg}{2^5} = .63mg$$

$= .6mg$

only 1 sig fig

But what if $t_{1/2}$ doesn't divide evenly into $t_{total?}$

More complicated half-life problems can be solved using the integrated rate law equation.

$$- kt = \ln[A]_t - \ln[A]_o$$

or can be rewritten for Chemistry

$$\ln\left(\frac{[A]}{[A]_o}\right) = k\left(\frac{t}{t_{1/2}}\right)$$

ln = natural logarithm "log base e" which is the inverse of e^x
$[A]$ = amount of material left
$[A]_o$= original amount of material
t = total time
$t_{1/2}$ = half-life
k = rate constant which is -0.693 for first order rate reactions

Phosphorus - 32 has a half-life of 14.28 days. How many grams of a 45.0 gram sample will remain after 35.0 days?

Step 1: Determine what you know and what you don't know.

$$\ln\left(\frac{[A]}{[A]_o}\right) = k\left(\frac{t}{t_{1/2}}\right)$$

$[A]$ = ?
$[A]_o$ = 45.0 grams
t = 35.0 days
$t_{1/2}$ = 14.28 days
k = -0.693

Solving for Half-Life

Phosphorus - 32 has a half-life of 14.28 days. How many grams of a 45.0 gram sample will remain after 35.0 days?

Step 1: Determine what you know and what you don't know.

$$\ln\left(\frac{[A]}{[A]_o}\right) = k\left(\frac{t}{t_{1/2}}\right)$$

$$\ln\left(\frac{?\ \text{grams}}{45.0\ \text{grams}}\right) = -0.693\left(\frac{35.0\ \text{days}}{14.28\ \text{days}}\right)$$

$$\ln\left(\frac{[A]}{45.0\ g}\right) = -1.70$$

$$\frac{[A]}{45.0\ g} = e^{(-1.70)}$$ have to do the inverse of the natural log, e^x

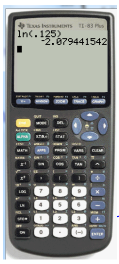
$$\frac{[A]}{45.0\ g} = .183$$

$$[A] = 45 \times .183$$

$$[A] = 8.24\ g \text{ left after 35.0 days}$$

After 42 days a 2.0g sample of phosphorus-32 contains only a 0.25g of isotope. What is the half-life of phosphorus-32?

$$\ln\left(\frac{[A]}{[A]_o}\right) = k\left(\frac{t}{t_{1/2}}\right)$$



$$\ln\left(\frac{.25g}{2.0g}\right) = K\left(\frac{42\ \text{days}}{t_{1/2}}\right)$$

$$\ln(.125) = K\left(\frac{42}{t_{1/2}}\right)$$

$$-2.079 = -.693\left(\frac{42}{t_{1/2}}\right)$$

$$-2.079\, t_{1/2} = (-.693)(42)$$

$$\frac{-2.079\, t_{1/2}}{-2.079} = \frac{29.106}{-2.079}$$

$$t_{1/2} = 14\ \text{days}$$

The half - life of radon-222 is 3.823 days. What was the original mass if 0.050g remains after 7.646 days.

Solving for Half-Life

After 42 days a 2.0g sample of phosphorus-32 contains only a 0.25g of isotope. What is the half-life of phosphorus-32?

$$\ln\left(\frac{[A]}{[A]_o}\right) = k\left(\frac{t}{t_{1/2}}\right)$$

$$\ln\left(\frac{.25g}{2.0g}\right) = k\left(\frac{42 days}{t_{1/2}}\right)$$

$$-2.079(x) = -.693\left(\frac{42 day}{t_{1/2}}\right)$$

$$\frac{-2.079}{-.693}x = \frac{}{-.693}$$

$$3x = \frac{42}{t_{1/2}}$$

$$x = \frac{42}{3}$$

$$= 14 \text{ days}$$

Some other students solved this way.... just wanted to share

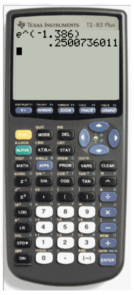
The half - life of radon-222 is 3.823 days. What was the original mass if 0.050g remains after 7.646 days.

$$\ln\left(\frac{[A]}{[A]_o}\right) = k\left(\frac{t}{t_{1/2}}\right)$$

$$\ln\left(\frac{.050g}{[A]_o}\right) = -.693\left(\frac{7.646 days}{3.823 days}\right)$$

$$\ln\left(\frac{.050g}{[A]_o}\right) = -1.386$$

$$= e^x \quad e^{(-1.386)}$$



$$\frac{.050}{[A]_o} = .25007$$

$$[A]_o = \frac{.050}{.25007}$$

$$= .2g$$ was the original mass

You can always solve the other way too if you do not have a scientific calc.

$$\# \text{ of } \tfrac{1}{2} \text{ lives} = \frac{t_{tot}}{t_{1/2}} = \frac{7.646}{3.823} = 2$$

$$(.050g)(2^2) = .2g$$

$$\text{or } (.050g)(8) = .2g$$