Lesson 14

Equivalent Linear Expressions

Writing Equivalent Expressions

Study the example showing how to write equivalent expressions. Then solve problems 1–9.

Example

Leroy plants a flower garden that is 4 m long and more than 3 m wide. The expression 4(3 + x) represents the area of the garden. Write an expression that is equivalent to 4(3 + x).

You can use tiles to model four groups of 3 + x.

You can reorder and regroup the tiles.

There are (3 + 3 + 3 + 3) + (x + x + x + x) tiles.

1 How do the expressions 4(3 + x) and (3 + 3 + 3 + 3) + (x + x + x + x) compare to one another?

2 Simplify the expression 4 \cdot 3 + 4 \cdot x.

3 What property can you apply to write 4(3 + x) as the equivalent expression in problem 2?

4 Write two expressions that are equivalent to 2(x + 4).

Vocabulary

equivalent expressions
expressions that have the same value.

x + 2 + 1 + x and 2x + 3 are equivalent

distributive property
allows you to distribute a factor over the terms in a sum without changing the overall value.

2(3 + 7) = 2 \cdot 3 + 2 \cdot 7
Solve.

5 Use the distributive property to write an expression that is equivalent to \(7(3x - 4)\).

6 Use the distributive property to write an expression that is equivalent to \(10 + 15x\).

7 Use the distributive property to find an expression that is equivalent to \(27x - 42x + 12\).

8 Evaluate \(27x - 42x + 12\) and the equivalent expression that you wrote in problem 7 for \(x = 2\). What do you notice about the value of the two expressions?

*Show your work.*

Solution: _______________________________

9 Four students were asked to write an expression that is equivalent to \(8x - 20\). The students’ names and the expressions they wrote are shown in the table.

<table>
<thead>
<tr>
<th>Student</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ali</td>
<td>(-4(2x + 5))</td>
</tr>
<tr>
<td>Craig</td>
<td>(4(2x - 5))</td>
</tr>
<tr>
<td>Ester</td>
<td>(-2(-4x + 10))</td>
</tr>
<tr>
<td>Lisa</td>
<td>(2(4x - 10))</td>
</tr>
</tbody>
</table>

a. Which student(s) wrote correct expressions?

b. For each incorrect expression, explain what the student did to get his or her answer.
Equivalent Expressions for the Perimeter of a Square

Study the example problem showing how to write equivalent expressions for the perimeter of a square. Then solve problems 1–8.

Example

Tanya plans to make a square deck. She is not sure what size she is going to make the deck, but she knows it needs to be more than 5 feet long. She represents the deck as a square in which each side is \( d + 5 \) feet long. How can she write an expression for the perimeter of the deck?

Tanya can add the four equal side lengths to find the perimeter:

\[
\text{Perimeter} = (d + 5) + (d + 5) + (d + 5) + (d + 5)
\]

1. Write \((d + 5) + (d + 5) + (d + 5) + (d + 5)\) as an equivalent expression by grouping the like terms.

2. Write an expression for the perimeter by multiplying the number of sides by the length of a side.

\[
\text{Perimeter} = \underline{\quad}
\]

3. Simplify each expression for the perimeter of the deck that you wrote in problems 1 and 2. What do you notice?

4. The perimeter of an equilateral triangle is given as \(15x + 30\). Write two different expressions to represent the perimeter. Use factoring to write one of the expressions.

Vocabulary

**like terms** terms in an expression that have the same variable raised to the same power.

Constants are like terms.

- \( x \) and \(-4x\)
- \( 1 \) and \(1.5\)
- \( x^2 \) and \( 8x^2 \)
Solve.

5 Write four different expressions for the perimeter of a pentagon whose sides are all $s - 2$ units long.

6 Kin made a picture frame for a square picture with sides that are $5f - 4$ inches long. Framing the picture adds 2 inches to the length of each side. Use the distributive property to write two expressions for the perimeter of the framed picture.

Show your work.

Solution:

7 The expression $9x + 6$ represents the cost for three friends to go to the movies. Write $9x + 6$ as a product. Then tell how many friends went to the movies and what expression represents the cost of a movie ticket.

Solution:

8 A square playground is surrounded by a sidewalk on all sides. The sidewalk is $2n + 3$ yards long on each side of the park. The sidewalk is 0.5 yard wide. What is the perimeter of the playground? Write two equivalent expressions for the perimeter of the playground.

Show your work.

Solution:
Finding Equivalent Expressions

Study the example problem showing how to determine whether or not expressions are equivalent. Then solve problems 1–7.

Example

Carl and Felipa are trying to write an expression that is equivalent to the expression \(6 - 4(3 - 6x) + 12x\). Which student wrote an equivalent expression?

<table>
<thead>
<tr>
<th>Carl</th>
<th>Felipa</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6 - 4(3 - 6x) + 12x)</td>
<td>(6 - 4(3 - 6x) + 12x)</td>
</tr>
<tr>
<td>(2(3 - 6x) + 12x)</td>
<td>(6 - 12 + 24x + 12x)</td>
</tr>
<tr>
<td>(6 - 6x + 12x)</td>
<td>(-6 + 36x)</td>
</tr>
<tr>
<td>(6 + 6x)</td>
<td></td>
</tr>
</tbody>
</table>

Felipa wrote an equivalent expression. She correctly multiplied \(-4(3 - 6x)\) before adding or subtracting. She then combined like terms correctly. Carl did not write an equivalent expression.

1 Explain what Carl did incorrectly.

__________________________________________________________________________

2 Is \(5 + 3(1 - x)\) equivalent to \(8 - 8x\)?

*Show your work.*

\[\text{Solution: }\]

3 Is \(2(6 - 3x) + x\) equivalent to \(2(3x) + x\)?

*Show your work.*

\[\text{Solution: }\]
Solve.

4 Is $1 + 4(3x - 10) - 12x$ equivalent to $-9$?

Show your work.

Solution: ____________________________

5 Use substitution to show that $9 + 6(10 - 7x)$ is equivalent to $69 - 42x$.

Show your work.

Solution: ____________________________

6 Is $\frac{1}{8} - 10\left(\frac{3}{4} - \frac{3}{8}x\right) + \frac{5}{8}x$ equivalent to $-\frac{1}{8}(59 - 35x)$? Explain your answer.

______________________________

______________________________

7 If $z$ is a positive integer, does $4 + 3(2z - 5)$ represent a number that is greater than, less than, or equal to $2(3z - 4)$?

Show your work.

Solution: ____________________________
Lesson 14

Equivalent Linear Expressions

Solve the problems.

1 Which expression is equivalent to $3n + 2(1 - 4n)$?
   A $2 - n$
   B $2 - 5n$
   C $2 + 11n$
   D $2 - 11n$

   Samuel chose A as the correct answer. How did he get that answer?

2 The width of a rectangle is represented by $5 + 2y$. The length is twice as long as the width. What is the perimeter of the rectangle? Select all correct expressions.
   A $4(5 + 2y)$
   B $6(5 + 2y)$
   C $20 + 8y$
   D $30 + 12y$

3 The length of one side of a square field is represented by the expression $3 - 7x$.
   a. Write an expression for the perimeter of the field expressed as a sum.

   b. What is an expression for the perimeter of the field expressed as a product?

   c. Use the distributive property to write the product in part (b) as an equivalent expression.
Tell whether each statement is True or False.

4 a. \(2(3 - 4y) = 6 - 4y\)  
\[\square \text{True} \quad \square \text{False}\]

b. \(3 + 5(9 + 2n) = 48 + 5n\)  
\[\square \text{True} \quad \square \text{False}\]

c. \(9 - 3(y - 2) = 3 - 3y\)  
\[\square \text{True} \quad \square \text{False}\]

d. \(3(6x - 2) - 7 = 18x - 13\)  
\[\square \text{True} \quad \square \text{False}\]

5 The expression \(24y + 36\) represents the cost of a dozen eggs. Use factoring to write an expression that is equivalent to \(24y + 36\). Then write an expression for the cost of one egg.

\[\text{Expression for one egg: } \frac{24y + 36}{12}\]

6 Roberto examines several geometric figures. The length of each side of each figure is \((2d - 7)\) feet. Using the following table, write the number of sides and two different expressions for the perimeter of each figure.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Sides</th>
<th>Perimeter Expression 1</th>
<th>Perimeter Expression 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Octagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hexagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Square</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7 Use the distributive property to find an expression that is equivalent to \(2x(x + 7) - (3x + 1)\).

Show your work.

Solution: ______________________________