Go sit with your 3 o’clock appointment and complete Explore.

2.5 Perimeter and Area on the Coordinate Plane

Essential Questions: How do you find the perimeter and area of polygons in the coordinate plane?

Explore

Finding Perimeters of Figures on the Coordinate Plane

Recall that the perimeter of a polygon is the sum of the lengths of the polygon’s sides. You can use the Distance Formula to find perimeters of polygons in a coordinate plane.

Find the perimeter of each polygon with vertices A(-1, 1), B(1, 2), C(2, 1), and D(-1, -1). Round to the nearest tenth.

- Plot the points. Then use a straightedge to draw the segments that are determined by the points.
- Are there any sides for which you do not need to use the Distance Formula? Explain, and give their length(s).

- Use the Distance Formula to find the remaining side lengths. Round your answers to the nearest tenth.

Find the sum of the side lengths.

Explain

6. Explain how you can find the perimeter of a rectangle by checking that your answer is reasonable.

Explains

Finding Areas of Figures on the Coordinate Plane

You can use area formulas together with the Distance Formula to determine areas of figures such as triangles, rectangles, and parallelograms.

- Find the area of each figure.

- Step 1: Find the coordinates of the vertices of \( \triangle ABC \).

- \( A(-4, -4), B(-2, 1), C(3, 3) \)

- Step 2: Choose a base for which you can easily find the height of the triangle.

- Use \( \triangle ABC \) as the base. A segment from the opposite vertex, \( \overrightarrow{BC} \), appears to be perpendicular to the base \( \overrightarrow{BC} \).

- Use slopes to check:

- \( \text{slope of } \overrightarrow{BC} = \frac{-1 - 3}{-2 - 3} = 1 \)

- \( \text{slope of } \overrightarrow{AB} = \frac{1 - (-4)}{1 - (-4)} = -1 \)

- The product of the slopes is \( 1 \cdot -1 = -1 \). \( \overrightarrow{AB} \) is perpendicular to \( \overrightarrow{BC} \), so \( \overrightarrow{AB} \) is the height for the base \( \overrightarrow{BC} \).

- Find the length of the base and the height.

- \( \text{base } \overrightarrow{BC} = \sqrt{(-2 - 3)^2 + (1 - 3)^2} = \sqrt{25 + 4} = \sqrt{29} \)

- \( \text{height } \overrightarrow{AB} = 5 \)

- Step 3: Determine the area of \( \triangle ABC \).

- Area = \( \frac{1}{2} \) \( \text{base} \times \text{height} \) = \( \frac{1}{2} \times \sqrt{29} \times 5 \approx 30 \) square units
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Step 1: Find the coordinates of the vertices of \( \triangle ABC \):
\( A(-2, 0), B(4, 0), C(2, 3) \)

Step 2: \( \triangle ABC \) appears to be a right triangle. Use slopes to check that adjacent sides are perpendicular:
- slope of \( AB: \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 0}{4 - (-2)} = 0 \)
- slope of \( BC: \frac{3 - 0}{2 - 4} = -\frac{3}{2} \)
- slope of \( AC: \frac{3 - 0}{2 - (-2)} = \frac{3}{4} \)

No \( \triangle ABC \) is a __________.

Step 3: Find the area of \( \triangle ABC \):
- \( \text{base} = \sqrt{(-2-4)^2 + (0-0)^2} = \sqrt{36} = 6 \)
- \( \text{height} = \sqrt{(2-0)^2 + (3-0)^2} = \sqrt{13} \)
- Area of \( \triangle ABC \) = \( \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 6 \times \sqrt{13} = 3\sqrt{13} \) square units

Review:

2. Is it possible to use another side of \( \triangle ABC \) as the base? If so, what length represents the height of the triangle?

3. Discussion: In Part 1B, why was it necessary to find the slopes of the sides?

New Test:

4. Find the area of quadrilateral \( \text{ABCD} \) with vertices \( A(-4, -2), B(2, 1), C(5, 1), D(-4, 1) \).

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### Finding Areas of Composite Figures

A composite figure is made up of simple shapes, such as triangles, rectangles, and parallelograms. To find the area of a composite figure, add the areas of the simple shapes and then use the Area Addition Postulate. You can use the Area Addition Postulate to find the area of a composite figure.

**Area Addition Postulate**
The area of a region is equal to the sum of the areas of its nonoverlapping parts.

**Example 2** Find the area of each figure:

- **Quadrilateral:** \( \text{ABCD} \) can be divided into a rectangle and two triangles, each with base \( b \) and height \( h \):
  - area of rectangle \( ABCD \): \( A = bh = (6)(3) = 18 \) square units
  - area of \( \triangle ABC \): \( A = \frac{1}{2}bh = \frac{1}{2}(6)(3) = 9 \) square units
  - area of \( \triangle ADC \): \( A = \frac{1}{2}bh = \frac{1}{2}(3)(3) = 4.5 \) square units
  - area of \( \triangle CDB \): \( A = \frac{1}{2}bh = \frac{1}{2}(4)(3) = 6 \) square units
  - Total area of \( \text{ABCD} \): \( 18 + 9 + 4.5 + 6 = 37.5 \) square units
Now, put all of your perimeter and area skills together to solve this cost problem. *Groups of 3*
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2. Analyze Information

- Identify the important information:
  
  - The vertices are (0, 1), (6, 1), (6, 5), (4, 6), (4, -5), (0, -1), (2).
  
  - The cost of turf is \$ per square yard.
  
  - The cost of the ornamental stone is \$ per yard.

2. Formulate a Plan

- Draw the garden on graph paper.
- Add up the \( \) of the similar figures.
- Find the cost of turf by \( \) the total area by the cost per square yard.
- Find the perimeter of the garden by adding the \( \) of the sides.
- Find the cost of the border by \( \) the perimeter by the cost per yard.
- Total cost is by adding the \( \).

3. Solve

- Divide the garden into smaller figures.
  
  The garden can be divided into two rectangles (C, D, and E, and F, G, H, and I).

- Find the areas of each smaller figure.

  \[
  \text{area of } BCD: \quad A = \text{area of square} = \text{side}^2.
  \]

  \[
  \text{area of kite } ABHI: \quad A = \frac{1}{2} \times \text{base} \times \text{height}.
  \]

  \[
  \text{area of parallelogram } EFGH: \quad A = \text{base} \times \text{height}.
  \]

- Find the total area of the garden and the cost of turf.

  \[
  \text{area of garden: } A = \text{sum of areas} = \text{total area} = \text{cost of turf}.
  \]

- Find the perimeter of the garden.

  \[
  \text{perimeter of garden: } P = \text{sum of sides} = \text{perimeter}.
  \]

- Find the cost of the stone for the border.

  \[
  \text{cost of stone: } C = \text{length} \times \text{cost per yard} = \text{total cost}.
  \]
2.5 Perimeter and Area

- Excellent! I think you are ready to try some on your own.

2.5 Homework.doc

- A designer is making a model in the shape of the letter "L." Each unit on the coordinate grid represents 1 inch, and the model is to be cut from a 1-inch square of metal. How much metal is wanted to make each model? Write your answer in a divided:

- Elaborate

- 10. Observe: If two polygons have approximately the same area, do they have approximately the same perimeter? Draw a picture to justify your answer:

- Essential Question Check-In: What formulas might you need to solve problems involving the perimeter and area of triangles and quadrilaterals in the coordinate plane?
Find the perimeter of the figure with the given vertices. Round to the nearest tenth.

3. (–5, 1), (4, 5), (1, 2), and (7, 2)

Find the area of each figure.

7. Find the area of each figure by addition.

8. Find the area of each figure by subtraction.
15. Drawing units $5 \times 5$ per grid, and each unit on the grid represents 1 cm. Each plan is plotted from an 8 cm square of the grid. The cost of the alloy is $500/300$, but $100/500$ can be recovered on-waste parts of the alloy. What is the net cost of alloy for each component?

16. What is the area of each triangle?

17. What is the area of each rectangle?

18. What is the area of each square?

19. What is the area of each parallelogram?

20. What is the area of each trapezoid?
16. Explain the firm's actions. Vendell is trying to prove that \( ABCD \) is a parallelogram and to find its area. Identify and correct the errors. (Note: A rhombus is a quadrilateral with four congruent sides.)

\[
\begin{align*}
AD &= \sqrt{(2 - (-2))^2 + (0 - 2)^2} = \sqrt{16 + 4} = \sqrt{20} = 2 \\
AB &= \sqrt{(2 -- 2)^2 + (2 - 0)^2} = \sqrt{4 + 4} = \sqrt{8} = 2 \\
CD &= \sqrt{(-2)^2 + (2 - 0)^2} = \sqrt{4 + 4} = \sqrt{8} = 2 \\
BC &= \sqrt{(2 -- 2)^2 + (0 - 2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2
\end{align*}
\]

So \( AB = BC \) in \( ABCD \), and therefore \( ABCD \) is a parallelogram.

area of \( ABCD \) = \( AB \times h \) and \( AB = 2 \), so \( h = \frac{15}{2} = 7.5 \).

19. Communicate Mathematical Ideas: Using the figure, prove that the area of a kite is half the product of its diagonals. (Do not make numerical calculations.)

20. Justify Reasoning: Use the Trapezoid Midsegment Theorem to show that the area of a trapezoid is the product of its midsegment and its height.