3.3 Logarithmic Functions

Lesson Objectives
1. Graph exponential functions including y-intercept and horizontal asymptote.
2. Translate and reflect exponential functions.
3. Graph logarithmic functions including x-intercept and vertical asymptote.
4. Find inverse function pairs of exponential and logarithmic functions.

3.3 LOGARITHMIC FUNCTIONS AND THEIR GRAPHS

Reversing Logarithmic Expressions
A logarithmic function is simply an inverse of an exponential function. The following definition relates the two functions:

\[ \log_b y = x \quad \text{if and only if} \quad b^x = y \]

Example 1: Evaluate each expression without a calculator.

a) \( \log_2 125 \)  
   b) \( \log_7 1 \)  
   c) \( \log_5 81 \)  
   d) \( \log_8 \sqrt{2} \)
Logs & PH of Rainwater

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Two Special Logarithms

A logarithm with base 10 is called a _______ logarithm and is written _______.

A logarithm with base e is called a _______ logarithm and is written _______.

Example 2: Evaluate each expression without a calculator.

a) \( \log \sqrt{10} \)  
b) \( \ln 2 \)

A Consequence of the Inverse Properties of Logarithms:

\( \sqrt[10]{M} = M \) and \( \log_a (a^2) = 2 \)

Example 3: Evaluate each logarithmic expression without a calculator.

a) \( \log_2 8 \)  
b) \( \log 10^2 \)

c) \( e^2 \)  
d) \( \log_e e^3 \)
An Interview of Decibels:

What is a decibel (dB)?

The intensity of sound is measured in a unit called the decibel (dB). It describes the relative intensity of a sound based on a logarithmic scale containing values ranging from 0 to 194.

What does a zero value on the scale represent?

This is the weakest sound audible to humans. The relationship among the values on the decibel scale is not linear, so you might think, but they fit a logarithmic curve. For instance, a sound with 50 dB is not twice as intense as a sound with 25 dB.

How are decibels calculated?

Decibels are calculated using a formula that’s not as complex as you might imagine. Let L represent loudness and I₀ = 10⁻¹² which is barely audible:

\[ L = 10\log\frac{I}{I_0} \]

Now, you be a Decibel expert!

Find the loudness in decibels of a sound that has an intensity of 10⁻¹.
Now, you be a Decibel expert!

Find the loudness in decibels of a sound that has an intensity of $10^{-6}$.

\[
L = 10 \log \frac{I}{I_0}
\]

\[
L = 10 \log \frac{10^{-6}}{10^{-12}}
\]

\[
L = 60 \text{ dB}
\]

One more example... Find the intensity of a sound that measures 90 dB.

\[
L = 10 \log \frac{I}{I_0}
\]

\[
90 = 10 \log \frac{x}{10^{-12}}
\]

\[
x = \frac{10^{90}}{10^{-12}}
\]

\[
10^{92} = x
\]
### Graphing Logarithmic Functions

1) Complete the table.
2) Plot the points.
3) Sketch the graph.

<table>
<thead>
<tr>
<th>log₂(x)</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

### Translating Logarithmic Graphs

Drag the red star to translate the graph of \( y = \log_3(x) \) one unit down and two units to the right.

What is the equation of the translated graph?

What is the \( x \)-intercept? vertical asymptote?

The function is \( y = \log_3(x - 2) - 1 \).

The \( x \)-intercept is (5, 0).

The vertical asymptote is the line \( x = 2 \).

### Reflecting Logarithmic Graphs

How is the graph of \( f(x) = \log_2(x) \) transformed to create the red graph?

Click the graph for a hint.

Write the equation for the red graph in two different ways.
### Practice

Find the equation of the graph.

Use the magnifying glass to reveal key information on the graph.

- **V.A.** $x = -2$
- **x-int** $(2, 0)$

**Pull for Answer**

V.A. of $x = -2$ implies equation looks like $y = A + \log B (x + 2)$.  

$(1, -1)$ gives $-1 = A + \log B (1)$.  So, $A = -1$.

$(2, 0)$ gives $0 = -1 + \log B (2 + 2)$.  So, $B = 4$.  

So, the equation is $y = -1 + \log 4 (x + 2)$.

### Inverse Function Pairs

Pass the magnifying glass over the graph to find the inverse.  

Find the equations for both functions.

**Exponential graph:**

- **H.A.** of $x = 0$ implies equation looks like $y = B x + A$.

1. $(1, -1)$ gives $1 = B - 1 + A$.  So, $A = 0$.

2. $(0, 3)$ gives $3 = B 0 + 1$ So, $B = 3$.  So, $y = 3 x + 1$.

**Logarithmic equation.**

1. Switch $x$ and $y$ to get $x = 3 y + 1$.

2. Solve for $y$ to get $y = -1 + \log 3 (x)$.

**Pull for Answer**
Now, describe the transformations on the following graphs:

a) \( y = \log_{0.5}(x-3)+1 \)

b) \( y = \ln(x-3) \)

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The 3 W's

What did we learn?
Why did we learn it?
What do you think is next?

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End of Lesson
Lesson Objectives

Teacher's Notes

Lesson Notes

Grades: 9 through 12

These activities may be used to review, reinforce, or enrich your study of exponential and logarithmic functions.

Prior Knowledge:
Students should know about translation, reflection, intercepts, and asymptotes. They should also be familiar with exponential, logarithmic, and inverse functions.

Students graph, translate, and reflect both exponential and logarithmic functions. They are asked to find the equations of these graphs. Finally, they are asked to find the equations of inverse pairs.