ACTIVITY 20 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 20-1

1. Write *arithmetic, geometric,* or *neither* for each sequence. If arithmetic, state the common difference. If geometric, state the common ratio.
   a. 4, 12, 36, 108, 324, . . .
   b. 1, 2, 6, 24, 120, . . .
   c. 4, 9, 14, 19, 24, . . .
   d. 35, −30, 25, −20, 15, . . .
2. Find the indicated term of each geometric series.
   a. $a_1 = 1, r = -3; a_{10}$
   b. $a_1 = 3072, r = \frac{1}{4}; a_8$
3. If a_n is a geometric sequence, express the quotient of $\frac{a_7}{a_4}$ in terms of r.
4. The first three terms of a geometric series are $\frac{1}{81}, \frac{1}{27}, \frac{1}{9}, \ldots$ What is a_6?
   A. $\frac{3}{81}$
   B. 3
   C. $\frac{364}{81}$
   D. 9
5. Determine the first three terms of a geometric sequence with a common ratio of 2 and defined as follows:
   $x - 1, x + 6, 3x + 4$
6. Determine whether each sequence is geometric. If it is a geometric sequence, state the common ratio.
   a. $x, x^2, x^4, \ldots$
   b. $(x + 3), (x + 3)^2, (x + 3)^3, \ldots$
   c. $3^x, 3^{x+1}, 3^{x+2}, \ldots$
   d. $x^2, (2x)^2, (3x)^2, \ldots$
7. If $a_3 = \frac{9}{32}$ and $a_5 = \frac{81}{512}$, find a_1 and r.
8. The 5 in the expression $a_n = 4(5)^{n-1}$ represents which part of the expression?
   A. n
   B. a_1
   C. r
   D. S_n
9. A ball is dropped from a height of 24 feet. The ball bounces to 85% of its previous height with each bounce. Write an expression and solve to find how high (to the nearest tenth of a foot) the ball bounces on the sixth bounce.
10. Write the recursive formula for each sequence.
   a. 4, 2, 1, 0.5, . . .
   b. 2, 6, 18, 54, 120, . . .
   c. $\frac{4}{5}, \frac{4}{25}, \frac{4}{125}, \ldots$
   d. $-45, 5, -\frac{5}{9}, \ldots$
11. Write the explicit formula for each sequence.
   a. 4, 2, 1, 0.5, . . .
   b. 2, 6, 18, 54, 120, . . .
   c. $\frac{4}{5}, \frac{4}{25}, \frac{4}{125}, \ldots$
   d. $-45, 5, -\frac{5}{9}, \ldots$

Lesson 20-2

12. Find the indicated partial sum of each geometric series.
   a. $5 + 2 + \frac{4}{5} + \frac{8}{25} + \ldots; S_7$
   b. $\frac{1}{8} + \frac{1}{4} + \frac{1}{2} + \ldots; S_{15}$
13. For the geometric series 2.9 + 3.77 + 4.90 + 6.37 + . . . , do the following:
   a. Find S_9 (to the nearest hundredth).
   b. How many more terms have to be added in order for the sum to be greater than 200?
14. George and Martha had two children by 1776, and each child had two children. If this pattern continued to the 12th generation, how many descendants do George and Martha have?
15. A finite geometric series is defined as 0.6 + 0.84 + 1.12 + 1.65 + . . . + 17.36. How many terms are in the series?
   A. $n = 5$
   B. $n = 8$
   C. $n = 10$
   D. $n = 11$
16. Evaluate $\sum_{j=1}^{6} 3(2)^j$

ACTIVITY 20 Continued

ACTIVITY PRACTICE

1. **a.** geometric; $r = 3$
   b. neither
   c. arithmetic; $d = 5$
   d. neither
2. **a.** $a_{10} = -19{,}683$
   b. $a_8 = \frac{3}{16}$, or 0.1875
3. r^3
4. B
5. $x = 8$; first three terms are 7, 14, 28
6. **a.** not geometric
   b. yes; $r = (x + 3)$
   c. yes; $r = 3$
   d. not geometric
7. $a_1 = \frac{1}{2}$ and $r = \frac{3}{4}$
8. C
9. $a_6 = 20.4(0.85)^{6-1} = 9.1$ ft
10. **a.** $a_n = \frac{1}{2}a_{n-1}$
   b. $a_n = 3a_{n-1}$
   c. $a_n = \frac{1}{5}a_{n-1}$
   d. $a_n = -\frac{1}{9}a_{n-1}$
11. **a.** $a_n = 4\left(\frac{1}{2}\right)^{n-1}$
   b. $a_n = 2(3)^{n-1}$
   c. $a_n = \frac{4}{5}\left(\frac{1}{5}\right)^{n-1}$
   d. $a_n = 45\left(-\frac{1}{9}\right)^{n-1}$
12. **a.** $S_7 = \frac{25{,}999}{3125} = 8.31968$
   b. $S_{15} = 4095.875$
13. **a.** $S_9 = 92.84$
   b. 3 more terms; $S_{12} = 215.55$
14. George and Martha are the first generation; they have 4,096 descendants at the 12th generation.
15. D
16. 378

$x+6$

$a_n = a_1 r^2$

$x+6 = (x-1)(2)$

$x+6 = 2x-2$

$x+8 = 2x$

$8 = x$

ACTIVITY 20 Continued

17. a. \$80; \$5120
b. \$10,230
18. a. $a_n = 3 \cdot 3^{n-1} = 3^n$
b. 81
c. 121
19. a. $S = 48$
b. does not exist since $r = 2 \geq 1$
c. $\frac{7776}{7} \approx 1110.857$
20. $\frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \ldots$
21. $\frac{3}{11}$
22. a. $r = \frac{2}{3}$; series converges
b. $r = -\frac{1}{2}$; series converges
c. $r = 1.5$; series diverges
23. B
24. $r = \frac{1}{4}$
25. B
26. True. Sample answer: The terms in an arithmetic series are added to form partial sums. Since there is a common difference not equal to 0, the partial sum changes at a constant rate as terms are added, so there is no limiting value on the sums.
27. Sample answer: The common ratio can be found between any two terms by determining the number of terms between the given terms and rewriting the ratio as a quantity to that power. Working backward from the first term will give you a value for a_1.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.

17. During a 10-week summer promotion, a baseball team is letting all spectators enter their names in a weekly drawing each time they purchase a game ticket. Once a name is in the drawing, it remains in the drawing unless it is chosen as a winner. Since the number of names in the drawing increases each week, so does the prize money. The first week of the contest the prize amount is \$10, and it doubles each week.
a. What is the prize amount in the fourth week of the contest? In the tenth week?
b. What is the total amount of money given away during the entire promotion?

18. In case of a school closing due to inclement weather, the high school staff has a calling system to make certain that everyone is notified. In the first round of phone calls, the principal calls three staff members. In the second round of calls, each of those three staff members calls three more staff members. The process continues until all of the staff is notified.
a. Write a rule that shows how many staff members are called during the nth round of calls.
b. Find the number of staff members called during the fourth round of calls.
c. If all of the staff has been notified after the fourth round of calls, how many people are on staff at the high school, including the principal?

Lesson 20-3

19. Find the infinite sum if it exists. If it does not exist, tell why.
a. $24 + 12 + 6 + 3 + \ldots$
b. $\frac{1}{12} + \frac{1}{6} + \frac{1}{3} + \frac{2}{3} + \ldots$
c. $1296 - 216 + 36 - 6 + \ldots$

20. Write an expression in terms of a_n that means the same as $\sum_{j=1}^{\infty} 2\left(\frac{1}{3}\right)^j$

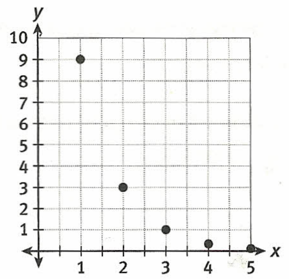
21. Express 0.2727 . . . as a fraction.

22. Use the common ratio to determine if the infinite series converges or diverges.
a. $36 + 24 + 12 + \ldots$
b. $-4 + 2 + (-1) + \ldots$
c. $3 + 4.5 + 6.75 + \ldots$

23. The infinite sum $0.1 + 0.05 + 0.025 + 0.0125 + \ldots$
A. diverges.
B. converges at 0.2.
C. converges at 0.5.
D. converges at 1.0.

24. An infinite geometric series has $a_1 = 3$ and a sum of 4. Find r.

25. The graph depicts which of the following?



A. converging arithmetic series
B. converging geometric series
C. diverging arithmetic series
D. diverging geometric series

26. True or false? No arithmetic series with a common difference that is not equal to zero has an infinite sum. Explain.

MATHEMATICAL PRACTICES
Make Sense of Problems and Persevere in Solving Them

27. Explain how knowing any two terms of a geometric sequence is sufficient for finding the other terms.