Learning Targets:

- Solve quadratic equations by factoring.
- Interpret solutions of a quadratic equation.
- Create quadratic equations from solutions.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Paraphrasing, Think-Pair-Share, Create Representations, Quickwrite

To solve a quadratic equation $ax^2 + bx + c = 0$ by factoring, the equation must be in factored form to use the Zero Product Property.

Example A

Solve $x^2 + 5x - 14 = 0$ by factoring.

	Original equation	$x^2 + 5x - 14 = 0$
Step 1:	Factor the left side.	$(x + 7)(x - 2) = 0$
Step 2:	Apply the Zero Product Property.	$x + 7 = 0$ or $x - 2 = 0$
Step 3:	Solve each equation for x.	

Solution: $x = -7$ or $x = 2$

Try These A

a. Solve $3x^2 - 17x + 10 = 0$ and check by substitution.

$3x^2-17x+10=0$	Original equation
$(3x-2)(x-5)=0$	Factor the left side.
$3x-2=0$ $x-5=0$	Apply the Zero Product Property.
$x=2/3$ or $x=5$	Solve each equation for x.

Solve each equation by factoring. Show your work.

b. $12x^2 - 7x - 10 = 0$

$x=5/4$
$x=-2/3$

c. $x^2 + 8x - 9 = 0$

$x=-9$
$x=1$

d. $4x^2 + 12x + 9 = 0$

$x=\frac{-3}{2}$

e. $18x^2 - 98 = 0$

$x=\pm 7/3$

f. $x^2 + 6x = -8$

$x=-2$
$x=-4$

g. $5x^2 + 2x = 3$

$x=-1$
$x=3/5$

My Notes

MATH TIP

The *Zero Product Property* states that if $a \cdot b = 0$, then either $a = 0$ or $b = 0$.

b) $x^2-7x-120=0$
$(x-15)(x+8)=0$
$\left(x-\frac{15}{12}\right)\left(x+\frac{8}{12}\right)=$
$\left(x-\frac{5}{4}\right)(x+2/3)=0$
$(4x-5)(3x+2)\ 0$
$x=5/4$ $x=-2/3$

c) $(x+9)(x-1)=0$
$x=-9$ $x=1$

d) $(2x+3)^2=0$

e) $18x^2-98=0$
$18x^2=98$
$x^2=98/18$

MATH TIP

You can check your solutions by substituting the values into the original equation.

$x^2=\frac{49}{9}$ $x=\pm\frac{7}{3}$

f) $x^2+6x+8=0$
$(x+2)(x+4)=0$

g) $5x^2+2x-3=0$
$x^2+2x-15=0$
$\left(x+\frac{5}{5}\right)\left(x-\frac{3}{5}\right)=0$
$(x+1)(5x+3)=0$

My Notes

$2L + 2W = 160$ perim.
$L \cdot W = 1600$ area
$W = \frac{1600}{L}$
$2L + 2\left(\frac{1600}{L}\right) = 160$
$2L + \frac{3200}{L} = 160$

MATH TIP

It is often easier to factor a quadratic equation if the coefficient of the x^2-term is positive. If necessary, you can multiply both sides of the equation by -1 to make the coefficient positive.

$2L^2 + 3200 = 160l$
$l^2 - 80l + 1600 = 0$

In the previous lesson, you were asked to determine the dimensions of a rectangle with an area of 525 ft^2 that can be enclosed by 100 ft of fencing. You wrote the quadratic equation $l^2 - 50l + 525 = 0$ to model this situation, where l is the length of the rectangle in feet.

$2L + 2W = 100$
$L \cdot W = 525$

1. a. Solve the quadratic equation by factoring.

$(l-35)(l-15) = 0$
$l = 35 \quad l = 15$

b. What do the solutions of the equation represent in this situation?

The rectangle could have a length of 35 feet or a length of 15 feet.

c. What are the dimensions of a rectangle with an area of 525 ft^2 that can be enclosed by 100 ft of fencing?

35×15 or 15×35

$2(35) + 2(15) = 100$

d. Reason quantitatively. Explain why your answer to part c is reasonable.

$A = 35 \times 15 = 525$
$P = 2(35) + 2(15) = 100$
The dimensions give the correct area and perimeter.

2. A park has two rectangular tennis courts side by side. Combined, the courts have a perimeter of 160 yd and an area of 1600 yd^2.

a. Write a quadratic equation that can be used to find l, the length of the court in yards.

$L^2 - 80L + 1600 = y$

b. Construct viable arguments. Explain why you need to write the equation in the form $al^2 + bl + c = 0$ before you can solve it by factoring. You need to apply zero product property, can only do this when equal to zero

c. Solve the quadratic equation by factoring, and interpret the solution.

$(L-40)(L-40) = 0$

The length of the court is $L = 40$ 40 yds.

d. Explain why the quadratic equation has only one distinct solution.

Double root, only one answer makes the statement true.

3. The equation $2x^2 + 9x - 3 = 0$ cannot be solved by factoring. Explain why this is true. $x^2+9x-6=0$

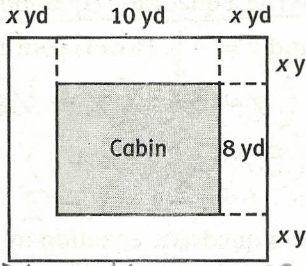
There is no factor pair of -6 and has a sum of 9.

Check Your Understanding

4. Explain how to use factoring to solve the equation $2x^2 + 5x = 3$.

5. Critique the reasoning of others. A student incorrectly states that the solution of the equation $x^2 + 2x - 35 = 0$ is $x = -5$ or $x = 7$. Describe the student's error, and solve the equation correctly.

6. Fence Me In has been asked to install a fence around a cabin. The cabin has a length of 10 yd and a width of 8 yd. There will be a space x yd wide between the cabin and the fence on all sides, as shown in the diagram. The area to be enclosed by the fence is 224 yd².



a. Write a quadratic equation that can be used to determine the value of x.

b. Solve the equation by factoring.

c. Interpret the solutions.

you know area.

$A = L \cdot W$

$A=(2x+10)(2x+8)=A$

$4x^2+36x+80=224$ → (b)

First get to one side

(4) $2x^2+5x-3=0$

$x^2+5x-6=0$

$(x+6)(x-1)=0$

$(x+3)(x-\frac{1}{2})=0$

$x=-3 \quad x=\frac{1}{2}$

Use zero product property.

(5) $(x+7)(x-5)=0$

$x=-7 \quad x=5$

They used the constant terms or factored wrong.

$4x^2+36x-144=0$

$x^2+9x-36=0$

$(x+12)(x-3)=0$

$x=\cancel{-12} \quad x=3$

cannot have negative length.

If you know the solutions to a quadratic equation, then you can write the equation.

Example B

Write a quadratic equation in ***standard form*** with the solutions $x = 4$ and $x = -5$.

Step 1: Write linear equations that correspond to the solutions. $x - 4 = 0$ or $x + 5 = 0$

Step 2: Write the linear expressions as factors. $(x - 4)$ and $(x + 5)$

Step 3: Multiply the factors to write the equation in factored form. $(x - 4)(x + 5) = 0$

Step 4: Multiply the binomials and write the equation in standard form. $x^2 + x - 20 = 0$

Solution: $x^2 + x - 20 = 0$ is a quadratic equation with solutions $x = 4$ and $x = -5$.

MATH TERMS

The **standard form of a quadratic equation** is $ax^2 + bx + c = 0$, where $a \neq 0$.

Try These B

a. Write a quadratic equation in standard form with the solutions $x = -1$ and $x = -7$. Go backwards

$(x+1)=0$ $(x+7)=0$	Write linear equations that correspond to the solutions.
$(x+1), (x+7)$	Write the linear expressions as factors.
$(x+1)(x+7)=0$	Multiply the factors to write the equation in factored form.
$x^2+8x+7=0$	Multiply the binomials and write the equation in standard form.

MATH TIP

To avoid fractions as coefficients, multiply the coefficients by the LCD.

b. Write a quadratic equation in standard form whose solutions are $x = \frac{2}{5}$ and $x = -\frac{1}{2}$. How is your result different from those in Example B?

$(x-2/5)(x+1/2)=0 \rightarrow x^2 + 1/10x - 1/5 = 0$

$(5x-2)(2x+1)=0$ coefficients may be fractions or a may not be 1

$10x^2+1x-2=0$

Write a quadratic equation in standard form with integer coefficients for each pair of solutions. Show your work.

c. $x = \frac{2}{3}, x = 2$

$(x-2/3)(x-2)=0$

$(3x-2)(x-2)=0$

$3x^2-8x+4=0$

d. $x = -\frac{3}{2}, x = \frac{5}{2}$

$(x+3/2)(x-5/2)=0$

$(2x+3)(2x-5)=0$

$4x^2-4x-15=0$

Check Your Understanding

7. Write the equation $3x^2 - 6x = 10x + 12$ in standard form.
8. Explain how you could write the equation $x^2 - \frac{7}{6}x + \frac{1}{3} = 0$ with integer values of the coefficients and constants.
9. **Reason quantitatively.** Is there more than one quadratic equation whose solutions are $x = -3$ and $x = -1$? Explain.
10. How could you write a quadratic equation in standard form whose only solution is $x = 4$?

7) $3x^2-16x-12=0$

8) Multiply by LCD $6x^2-7x+2=0$
Both sides of the equation

9) Yes $(x+3)(x+1) = x^2+4x+3=0$
Multiply by any # other than 1 will still give you this solution.

10) $(x^2-8x+16)=0$
$(x-4)^2=0$

LESSON 7-3 PRACTICE

Solve each quadratic equation by factoring.

11. $2x^2 - 11x + 5 = 0$

12. $x^2 + 2x = 15$

13. $3x^2 + x - 4 = 0$

14. $6x^2 - 13x - 5 = 0$

Write a quadratic equation in standard form with integer coefficients for which the given numbers are solutions.

15. $x = 2$ and $x = -5$

16. $x = -\frac{2}{3}$ and $x = -5$

17. $x = \frac{3}{5}$ and $x = 3$

18. $x = -\frac{1}{2}$ and $x = \frac{3}{4}$

19. Model with mathematics. The manager of Fence Me In is trying to determine the best selling price for a particular type of gate latch. The function $p(s) = -4s^2 + 400s - 8400$ models the yearly profit the company will make from the latches when the selling price is s dollars.

a. Write a quadratic equation that can be used to determine the selling price that would result in a yearly profit of $1600.

b. Write the quadratic equation in standard form so that the coefficient of s^2 is 1.

c. Solve the quadratic equation by factoring, and interpret the solution(s).

d. Explain how you could check your answer to part c.

11 $x^2 - 11x + 10 = 0$
$(x-10)(x-1) = 0$
$(x - 10/2)(x - 1/2) = 0$
$(x-5)(2x-1) = 0$
$x = 5 \quad x = 1/2$

12 $x^2 + 2x - 15 = 0$
$(x+5)(x-3) = 0$
$x = -5 \quad x = 3$

13 $x^2 + x - 12 = 0$
$(x+4)(x-3) = 0$
$(x + 4/3)(x-1) = 0$
$(3x+4)(x-1) = 0$
$x = -4/3 \quad x = 1$

14 $x^2 - 13x - 30 = 0$
$(x-15)(x+2) = 0$
$(x - 15/6)(x + 2/6) = 0$
$x = 5/2 \quad x = -1/3$

15 $(x-2)(x+5) = 0$
$x^2 + 3x - 10 = 0$

16 $(x + 2/3)(x+5) = 0$
$(3x+2)(x+5) = 0$
$3x^2 + 17x + 10 = 0$

CONNECT TO ECONOMICS

The selling price of an item has an effect on how many of the items are sold. The number of items that are sold, in turn, has an effect on the amount of profit a company makes by selling the item.

17 $(x - 3/5)(x-3) = 0$
$(5x-3)(x-3) = 0$
$5x^2 - 18x + 9 = 0$

18 $(x + 1/2)(x - 3/4) = 0$
$(2x+1)(4x-3) = 0$
$8x^2 - 2x - 3 = 0$

19 a $1600 = -4s^2 + 400s - 8400$
$\frac{1}{4}(4s^2 - 400s + 10000 = 0)$
b $s^2 - 100s + 2500 = 0$
c $(s-50)(s-50) = 0$ The selling price that will result in a yearly profit of $1600 is $50.
$s = \$50$
d $p(50) = -4(50)^2 + 400(50) - 8400 = 1600$