

Properties of Integer Exponents

Name: _____

Prerequisite: Evaluate Numerical Exponential Expressions

Study the example problem showing how to write and evaluate expressions with exponents. Then solve problems 1–9.

Example

Jacob decides to save money for a new tablet. He will save \$3 the first week and then triple the amount he has saved each week for 5 weeks. Write and evaluate an exponential expression to find how much money Jacob will have in his savings in Week 5.

Represent the problem with repeated multiplication and exponential expressions.

Week 1	Week 2	Week 3	Week 4	Week 5
$3 = 3^1$	$3 \cdot 3 = 3^2$	$3 \cdot 3 \cdot 3 = 3^3$	$3 \cdot 3 \cdot 3 \cdot 3 = 3^4$	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5$

Week 5 expression: 3^5

Evaluate the expression: $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$

Jacob will have \$243 in his savings in Week 5.

- 1 Look at the table. How many times greater is the amount in Jacob's savings in Week 3 than in Week 2?

- 2 How much will Jacob have in his account in Week 3?

- 3 Jacob thinks that 3^5 is $5 \cdot 5 \cdot 5$, or 125. Explain what Jacob is doing wrong.

- 4 Margo's dad offers to give her 5¢ on Sunday. Then for each day of the week, he offers to give her 5 times the amount from the previous day. How much will he give her on Saturday? Write an expression to show how much Margo's dad gives her on Saturday.

Vocabulary

base the number being used as a factor in an exponential expression.

5 is the base. $\rightarrow 5^3$

exponent the number that shows how many times a base is used as a factor.

$5^3 \leftarrow 3$ is the exponent.

Solve.

- 5 Is 2^4 equal to $2 \cdot 4$? Explain.

- 6 A bacterium cell splits into 2 cells every hour. Write and evaluate an exponential expression to find how many cells there will be in 6 hours. Then use your answer to help you find the number of hours it will take for there to be 1,024 cells.

Show your work.

Solution: _____

- 7 The population of California is about 39 million. Is this greater than or less than 10^7 ? Explain.

- 8 Write each of the numbers 1, 8, 27, 64, and 125 as a base raised to the third power.

$$\begin{array}{lll} 1 = \square^3 & 8 = \square^3 & 27 = \square^3 \\ 64 = \square^3 & 125 = \square^3 & \end{array}$$

- 9 The exponential expression 2^8 has a value of 256. Write two other exponential expressions that have a value of 256. Explain how you got your answers. (Begin by writing out 2^8 as the product of 2s.)

Products of Powers

Study the example problems showing how to find the power of a power and the products of powers with the same exponent. Then solve problems 1–10.

Example

Same Base and Same Exponent

Simplify: $(8^2)^3$

One Way:

$$\begin{aligned}(8^2)^3 &= 8^2 \cdot 8^2 \cdot 8^2 \\ &= 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \\ &= 8^6\end{aligned}$$

Another Way:

$$\begin{aligned}(8^2)^3 &= 8^2 \cdot 8^2 \cdot 8^2 \\ &= 8^{2 \cdot 3} \quad \text{Multiply the exponents.} \\ &= 8^6\end{aligned}$$

Different Base and Same Exponent

Simplify: $(2^4)(5^4)$

One Way:

$$\begin{aligned}(2^4)(5^4) &= (2 \cdot 2 \cdot 2 \cdot 2)(5 \cdot 5 \cdot 5 \cdot 5) \\ &= (2 \cdot 5)(2 \cdot 5)(2 \cdot 5)(2 \cdot 5) \\ &= 10 \cdot 10 \cdot 10 \cdot 10 \\ &= 10^4\end{aligned}$$

Another Way:

$$\begin{aligned}(2^4)(5^4) &= (2 \cdot 5)^4 \quad \text{Multiply the bases.} \\ &= 10^4\end{aligned}$$

- 1** The expression $(8^2)^3$ in the example problem is a product of powers. What are the powers being multiplied? What are the powers being multiplied in the expression $(2^4)(5^4)$?

- 2** Simplify: $(7^5)^6$. Write your answer using an exponent.

- 3** Simplify: $(6^3)(9^3)$. Write your answer using an exponent.

- 4** Is the statement $(3^5)^4 = (3^4)^5$ true? Explain your reasoning.



Solve.

- 5 Simplify: $(7^5)(4^5)$. Write your answer using an exponent.

- 6 Explain in words how to simplify: $(153^2)^7$.

- 7 Is the statement $(10^5)(4^5) = 14^5$ true? Explain your reasoning.

- 8 What is the value of x in the equation $(5^x)^5 = 5^{35}$? Explain.

- 9 Without evaluating the expressions, tell which is greater, $(4^4)(5^4)$ or $(2^5)(10^5)$. Explain your reasoning.

- 10 Nicholas says that $(2^6)(2^6)$ equals 2^{12} and also equals 4^6 . Do you agree? Explain your reasoning.

Zero and Negative Exponents

Study the example problems showing how to simplify expressions with zero and negative exponents. Then solve problems 1–12.

Example

Zero Exponent

Simplify: 217^0

In general, $n^0 = 1$, where $n \neq 0$.

So, $217^0 = 1$.

Negative Exponent

Simplify: 15^{-2}

In general, $n^{-a} = \frac{1}{n^a}$, where $n \neq 0$.

So, $15^{-2} = \frac{1}{15^2}$.

- 1 Simplify: 100^0 _____
- 2 Write $(-32)^{-2}$ as an expression without a negative exponent. _____
- 3 Write $\frac{1}{7^6}$ as an expression with a negative exponent. _____
- 4 Compare 8^0 and 8^{-2} . Which is greater? Explain your reasoning.

- 5 Simplify $(12^0)(12^6)$. Write your answer using an exponent. Explain how you found your answer.

- 6 What is the value of x in the equation $(-35)^x = 1$? Explain.

- 7 Write an expression using exponents that is equivalent to each of the following expressions.
 - a. $5^{-3} =$ _____
 - b. $(-16)^{-2} =$ _____
 - c. $\frac{1}{8^4} =$ _____



Solve.

- 8 Simplify the expression 52^{-5} . Then write it as repeated multiplication.

- 9 Write an exponential expression that is equivalent to $(4^{14})^{-2}$.

- 10 Is 6^{-2} positive or negative? Explain.

- 11 Write 9^0 , 9^3 , and 9^{-2} in order from least to greatest.

Show your work.

Solution: _____

- 12 Lizbeth says that $(-12)^{-3}$ equals a positive number because the product of two negative integers is a positive integer. Do you agree? Use what you know about exponential expressions to explain.

Simplify Expressions with Exponents

Solve the problems.

- 1 Look at the equations below. Tell whether each equation is *True* or *False*.

- | | | |
|--|-------------------------------|--------------------------------|
| a. $3^5 \cdot 3^7 = 3^{35}$ | <input type="checkbox"/> True | <input type="checkbox"/> False |
| b. $(6^3 \cdot 3^3)^2 = 18^6$ | <input type="checkbox"/> True | <input type="checkbox"/> False |
| c. $7^{-6} \cdot \frac{1}{7^4} = \frac{1}{7^{10}}$ | <input type="checkbox"/> True | <input type="checkbox"/> False |
| d. $4^4 \cdot 4^2 = 4^6$ | <input type="checkbox"/> True | <input type="checkbox"/> False |
| e. $\frac{13^{-4}}{13^4} = 13^0$ | <input type="checkbox"/> True | <input type="checkbox"/> False |
| f. $(2^3 \cdot 8^3)^0 = 16^9$ | <input type="checkbox"/> True | <input type="checkbox"/> False |

You may have to apply more than one rule when working with exponents.



- 2 Tyler simplified the expression $5^4 \cdot 5^{-9}$. All of his work except his answer is shown below.

$$\begin{aligned} 5^4 \cdot 5^{-9} &= 5^{4+(-9)} \\ &= 5^{-5} \\ &= ? \end{aligned}$$

Which expression is the correct answer for Tyler's work?

- A 5^5
 B $\frac{1}{5^5}$
 C $\frac{1}{5^{-5}}$
 D 5

Remember what you know about negative exponents.



- 3 Complete the table.

Expression	$10^4 \cdot 10^{-2}$	$5^4 \cdot 7^4$	$(2^7 \cdot 4^7)^3$
Simplified Expression			

Are the bases equal?
 Are the exponents equal?



Solve.

- 4** Simplify: $\frac{32^{-1}}{32^6}$. Write your answer with a positive exponent.

Show your work.

The expression is a quotient of powers.



Solution: _____

- 5** Write 9^6 as a power with a base of 3.

What are the factors of 9?



- 6** Which expression is equivalent to $(3^4 \cdot 5^4)^{-3}$?

- A** $\frac{1}{15^5}$
- B** 15^{-48}
- C** $\frac{1}{15^{12}}$
- D** 15^5

Tania chose **B** as the correct answer. How did she get that answer?

Remember the order of operations. Simplify the expression within the parentheses first.

