Solve Systems of Equations Algebraically

Name: ______________________

Prerequisite: Find the Number of Solutions of a System

Study the example showing a system of linear equations with no solution. Then solve problems 1–6.

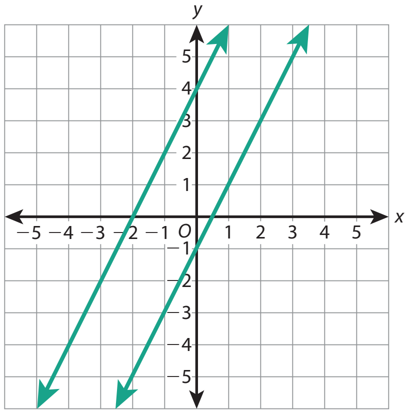
Example

Graph of the system of equations below to show that it has no solutions.

$y = 2x + 4$
$y = 2x - 1$

The lines do not intersect, so there is no ordered pair that will satisfy both equations.

The system of equations has no solution.



1 What type of lines are shown in the graph of the system of equations in the example?

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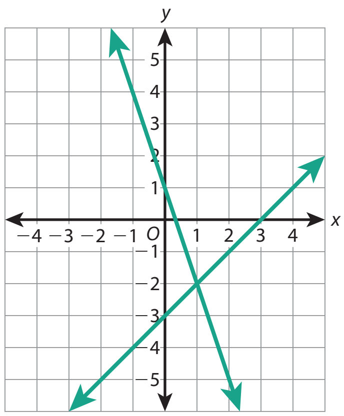
2 The graph at the right shows this system of equations.

$y = x - 3$
$y = -3x + 1$

What is the solution? Explain how you can verify that it is a solution to both equations.



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Vocabulary

system of linear equations a set of two or more related linear equations that share the same variables.

$y = 3x - 2$
$y = x + 1$

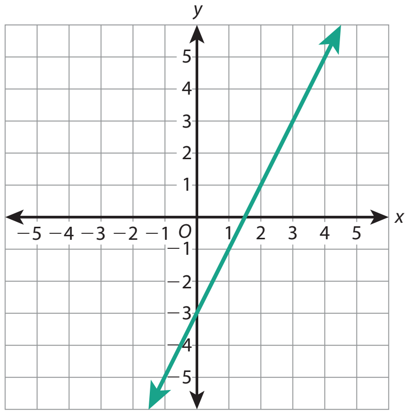
Solve.

3 The graph at the right shows this system of equations.

$y = 2x - 3$

$y = \frac{1}{2}(4x - 6)$

Explain what the graph tells you about the two equations that form the system and about the number of solutions the system has.



4 Describe two ways in which you can determine how many solutions a system of equations has.

5 How many solutions does this system of equations have? Explain how you know.

$y + 2x = 5$

$y = 3x + 5$

6 Jason and Sylvie worked together to write a system of equations that has no solution. Jason wrote the equation $y = 4x + 2$. Sylvie wrote the second equation. Write a possible equation that Sylvie might have written. Explain your reasoning.

Name: ______________________________

Use Substitution to Solve Systems of Equations

Study the example showing how to use substitution to solve a system of equations. Then solve problems 1–6.

Example

Solve the system of equations.

$y = x - 3$ $\qquad$ $y + 2x = 3$

Notice that the first equation tells you that $y = x - 3$, so substitute $x - 3$ for y in the second equation and solve for x.

$$\begin{aligned} \mathbf{y} + 2x &= 3 \\ \mathbf{(x - 3)} + 2x &= 3 \\ 3x - 3 &= 3 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

Now that you know the value of x, you can find the value of y. Substitute 2 for x in either equation and solve for y.

$$\begin{aligned} y &= x - 3 \\ y &= \mathbf{2} - 3 \\ y &= -1 \end{aligned}$$

The solution is $(2, -1)$.

1 Substitute the value of x in the example into the second equation, $y + 2x = 3$. What value do you get for y? Is it the same solution as in the example problem?

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2 The solution in the example is $(2, -1)$. Explain what the graph of the system looks like.

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3 Look at the system of equations below. Describe how you can use substitution to find the solution. Then find the solution.

$y - 3x = 4$ $\qquad$ $y = x - 4$

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Solve.

4 Use substitution to solve the system of equations.

$y + x = 3$

$y = 1.5x + 1$

Show your work.

Solution: ____________________________________

5 The system of equations at the right shows x by itself on the left side of one equation. Solve the system by substituting for x instead of for y.

$x = -y - 2$

$0.5x + y = 1$

Show your work.

Solution: ____________________________________

6 Fina wants to use substitution to solve the system of equations at the right. Explain what she needs to do first before using substitution. Then solve the system of equations.

$2x - y = 3$

$-1.5x + 3y = -18$

Name: ____________________

Use Elimination to Solve Systems of Equations

Study the example problem showing how to use elimination to solve a system of equations. Then solve problems 1–7.

Example

Solve the system of equations.

$-x + 3y = 1 \qquad 2x - 5y = -3$

Look for a way for one of the variables to have opposite coefficients in the system.
You can multiply the first equation by 2 so that the coefficients of x in the system are 2 and -2.

Multiply $-x + 3y = 1$ by 2 to get $-2x + 6y = 2$. Then rewrite the system and add the like terms.

$$\begin{array}{r} -2x + 6y = 2 \\ 2x - 5y = -3 \\ \hline y = -1 \end{array}$$

Now find the value of x by substituting the value of y into either equation. For example, you can substitute -1 for y in the first equation and solve for x.

$$\begin{aligned} -x + 3y &= 1 \\ -x + 3(-1) &= 1 \\ -x - 3 &= 1 \\ -x &= 4 \\ x &= -4 \end{aligned}$$

The solution is $(-4, -1)$.

1 Substitute -1 for y into the second equation from the example. Do you still get $x = -4$?

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2 One student began to solve the example problem by multiplying the second equation by 0.5. Would that work? Explain.

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3 Which equation would you multiply to get opposite coeffiecients for one of the variables in this system? What number would you multiply that equation by? What would the new equation be?

$$\begin{aligned} 3x + 5y &= 1 \\ -2x + y &= 2 \end{aligned}$$

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Solve.

4 Use elimination to solve the system of equations. Check your solution.

$4x + 3y = 6$

$2x - y = -2$

Show your work.

Solution: ______________________________

5 Becca says that she can use elimination to solve the system of equations at the right if she multiplies either of the equations by -1. Do you agree with Becca? Explain.

$3x + 2y = -1$
$3x + 4y = -5$

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6 Use elimination to find the solution of the system in problem 5.

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7 The value of x in the system of equations at the right is -2. What is the value of a? Use elimination to help you find the value.

$ax + 4y = 6$
$2x - 4y = -16$

Show your work.

Solution: ______________________________

Name: ____________________

Solve Systems of Equations Algebraically

Solve the problems.

1 Solve the system of equations.

$2y = 6x + 10 \qquad y = 2x + 4$

Show your work.

Solution: ____________________

2 Which ordered pair is the solution to this system of equations?

$y = x - 4 \qquad x - 3y = 10$

A (1, 3)

B (1, −3)

C (−3, 1)

D (−3, −1)

Zara chose **C** as the correct answer. How did she get that answer?

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3 Blaine is trying to solve the following system of equations.

$5x + y = -2 \qquad -10x - 2y = 4$

Which method would not help Blaine solve the system?

A Multiply the first equation by 0.5 and solve by elimination.

B Multiply the second equation by 0.5 and solve by elimination.

C Solve the first equation for y and solve by substitution.

D Solve the second equation for y and solve by substitution.

Solve.

4 Consider this system of equations:

$2x - 3y = 1$
$9y = 6x - 3$

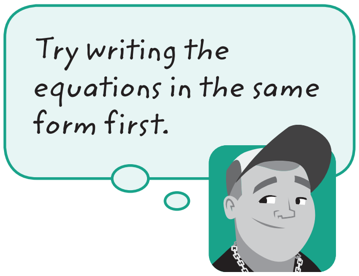
Tell whether each statement is *True* or *False*.

a. The system has infinitely many solutions. ☐ True ☐ False

b. The system has exactly one solution. ☐ True ☐ False

c. (5, 3) is a solution of the system. ☐ True ☐ False

d. The equations in the system have the same slope and the same y-intercept. ☐ True ☐ False



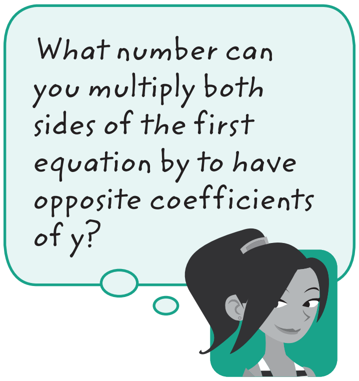
5 Consider this system of equations:

$5x + y = -2$
$2x - 2y = 4$

Part A

Solve the system of equations algebraically.

Show your work.



Solution: ____________________

Part B

Graph the system of equations to check your solution.