Solutions of Linear Equations

Name: ______________________

Prerequisite: Solve Equations with Rational Coefficients

Study the example problem showing how to solve an equation with rational coefficients. Then solve problems 1–6.

Example

Solve the equation: $6p = \frac{1}{2}(4p + 8)$.

Here is one way you can solve the given equation.

$6p = \frac{1}{2}(4p + 8)$	
$6p = 2p + 4$	**Step 1:** Apply the distributive property.
$6p - 2p = 2p + 4 - 2p$	**Step 2:** Subtract $2p$ from both sides.
$4p = 4$	**Step 3:** Simplify.
$\frac{4p}{4} = \frac{4}{4}$	**Step 4:** Divide both sides by 4.
$p = 1$	**Step 5:** Simplify.

1 Explain how the distributive property was applied in Step 1.

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2 A different Step 1 for the example is shown below.

$6p = \frac{1}{2}(4p + 8)$

$2 \cdot 6p = 2 \cdot \frac{1}{2}(4p + 8)$ **Step 1:** Multiply both sides by 2.

Explain why both sides of the equation were multiplied by 2. What is the resulting equation?

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3 What are the coefficients of p in the equation you wrote in problem 2? Are they rational numbers?

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Vocabulary

rational number a number that can be expressed as the quotient of two integers.

$3 = \frac{3}{1}$ $\quad 0.5 = \frac{5}{10}$

coefficient the number multiplied by the variable in a variable term.

10 is the coefficient in the term $10x$.

Solve.

4 Show two different ways to solve $\frac{2}{3}(3x + 6) = 4x + 3$.

Show your work.

Solution: ______________________________

5 Koby's solution for $4(x - 3) = \frac{1}{2}x + 2$ is shown at the right. Did he solve the equation correctly? Explain why or why not.

$$4(x - 3) = \frac{1}{2}x + 2$$
$$4x - 12 = \frac{1}{2}x + 2$$
$$2 \cdot (4x - 12) = 2 \cdot \left(\frac{1}{2}x + 2\right)$$
$$8x - 12 = x + 2$$
$$8x = x + 14$$
$$\frac{7x}{7} = \frac{14}{7}$$
$$x = 2$$

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6 In the equation $3(x - 1) = 2x + c$, for what value of c will $x = 3$? Explain your reasoning.

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Name: ______________________

Determining the Number of Solutions of an Equation

Study the example showing how to identify the number of solutions an equation has. Then solve problems 1–7.

Example

How many solutions does the equation $2(x + 2) + 1 = 2x - 3 + 6$ have?

Simplify the equation.

$$2(x + 2) + 1 = 2x - 3 + 6$$

$$2x + 4 + 1 = 2x + 3$$

$$2x + 5 = 2x + 3$$

The variable terms on each side of the simplified equation are the same but the constants are different, so the equation has no solution.

Number of Solutions

- An equation has *infinitely many solutions* when you simplify and the variable terms and constants are the same on each side, as in $2x + 5 = 2x + 5$ or $4 = 4$.
- An equation has *no solution* when you simplify and the variable terms on each side are the same but the constants are different, as in $2x + 5 = 2x + 3$ or $4 = 2$.
- An equation has *one solution* when the variable terms on each side are different, as in $3x + 1 = 2x - 5$.

1 Suppose the right side of the equation in the example problem is $2x - 3 + 8$. How many solutions would the equation have? Explain.

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2 Suppose the right side of the equation in the example problem is $3x - 3 + 6$. How many solutions would the equation have? Explain.

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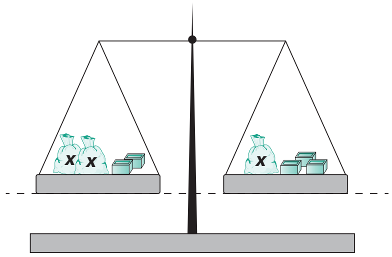
3 Look at the model at the right. Does it represent an equation that has one solution, no solutions, or infinitely many solutions? Explain how you know.



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Solve.

4 Consider the equation $cx - d = 2x + 4$.

a. Replace c in the equation with 2. For what value of d would the equation have infinitely many solutions? Explain.

b. Replace d in the equation with 2. For what value of c would the equation have no solution? Explain.

5 Evelyn says that the equation $3(x - 3) + 5 = 3x + 1 + 4$ has infinitely many solutions because the variable terms on each side are the same. Do you agree with Evelyn? Explain why or why not.

6 Explain why the equation $5(2x + 1) - 2 = 6x + 5$ has only one solution. Then find the solution.

7 Write an equation that has one solution, an equation that has no solution, and an equation that has infinitely many solutions. Each equation should have one variable term on each side and a total of four terms.

One solution: ______

No solution: ______

Infinitely many solutions: ______

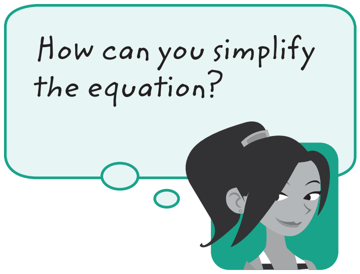
Name: ____________________

Solutions of Linear Equations

Solve the problems.

1 How many solutions does the equation $4x + 3 - 1 = 2(2x + 2) - 2x$ have? Explain your reasoning.

Show your work.



Solution: ____

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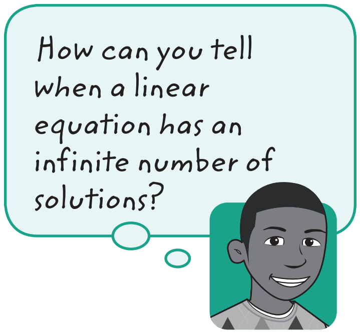
2 For which value of c will the equation $2x - 5 = 2x - c$ have an infinite number of solutions? Select all that apply.

A 3

B 4

C 5

D 6



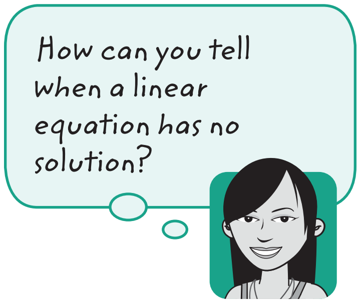
3 For which value of c will the equation $3x + c = 3x + 2$ have no solution? Select all that apply.

A 2

B 3

C 4

D 5



Charles chose **A** as the correct answer. How did he get that answer?

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Solve.

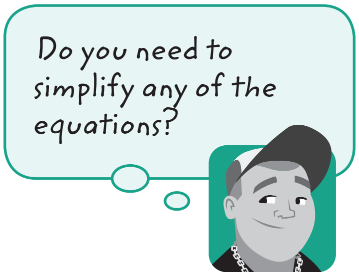
4 Tell whether each statement is *True* or *False*.

a. $3t + 7 = 3t + 6$ has no solution. ☐ True ☐ False

b. $5x - 6 = 3x - 6 + 2x$ has one solution. ☐ True ☐ False

c. $5 + 5b - 3 = 2 + 5b$ has infinitely many solutions. ☐ True ☐ False

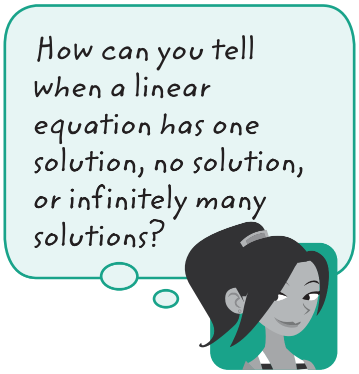
d. When an equation has one solution, the variable terms on both sides are the same. ☐ True ☐ False



5 Nyoko wrote these two equations.

Equation 1: $6x - 5 + 2x = 4(2x - 1) - 1$

Equation 2: $3x + 7 = bx + 7$



Part A

Nyoko says that Equation 1 has one solution. Do you agree with her? Explain your reasoning.

Show your work.

Solution: ______________________________

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Part B

Can Nyoko find a value for b in Equation 2 so that the equation has no solutions? Explain your reasoning.

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