Lesson 15

Understand Systems of Equations

Prerequisite: How can you find out whether an equation has infinitely many solutions?



Study the example showing how to tell if an equation has infinitely many solutions. Then solve problems 1–8.

Example

Does the equation 3(x + 5) - 10 = 3 + 3x + 2 have infinitely many solutions?

When you solve a linear equation and the equation simplifies to have the same variable terms and constants on both sides, the linear equation has infinitely many solutions.

3(x + 5) - 10 = 3 + 3x + 2 3x + 15 - 10 = 5 + 3x3x + 5 = 5 + 3x

You can see from 3x + 5 = 5 + 3x that both sides of the equation are equal, or you can further simplify to get 5 = 5. Either way, the equation 3(x + 5) - 10 = 3 + 3x + 2 has infinitely many solutions.

1 Substitute 4 for *x* in the equation in the example and evaluate both sides of the equation. What is the result? Is it a true statement?

- 2 Choose any value for x and substitute it in the equation in the example. Evaluate both sides of the equation. Is the result a true statement?
- 3 How many solutions does the equation 3(x + 2) = 2x + 14 - x have? Explain.

Vocabulary

infinitely many an

unlimited amount; for example, a linear equation has infinitely many solutions when the equation is true no matter what value is substituted for the variable.

157

Solve.

Use the equations to solve problems 4–5.						
Equation 1: $-4x - 5 = 4x + 11$						
Equation 2: $4x - 4 + x = 5x - 4$ Equation 3: $x + 3 + 2x = 3x + 2$						
						Which equation has infinitely many solutions? Explain.
Which equation has no solution? Explain.						
Chelsea wrote the equation $cx + d = 4(0.5x - 5)$. What values can she use for c and d so that the equation has infinitely many solutions? Explain your reasoning.						
Suppose that you use 3 for c and -20 for d in problem 6. How many solutions would the equation have? Explain.						
The equation $3(x + 2) + 4x = 6x + 8 + x$ has no solution. Ricco correctly says that he can change one number in the equation to make an equation that has infinitely many solutions. What number should he change? What number should he use instead? Explain.						

Name:

Determining the Number of Solutions of a System of Equations

Study the example showing how to determine the number of solutions for a system of equations. Then solve problems 1–6.

Example

You can compare the slopes and *y*-intercepts of a system of equations to predict how many solutions the system has.

System	Slopes	y-intercepts	Number of Solutions
y = 2x - 1 $y = x + 1$	2 and 1 Different	-1 and 1 Different	One solution
y = 4x + 3 $y = 2x + 3$	4 and 2 Different	3 and 3 Same	One solution
y = 3x - 2 $y = 3x + 3$	3 and 3 Same	-2 and 3 Different	No solution
y = 2x - 3 $y = 2x - 3$	2 and 2 Same	−3 and −3 Same	Infinitely many solutions

Look at the slopes and the number of solutions for the first two systems of equations in the example. What do the systems have in common?

2 Use the coordinate grid showing a graph of the third system of equations in the example. Why does the system have no solution?





159

Solve.

Use these equations to solve problems 3–5. Equation 1: $y = 2x + 3$ Equation 2: $y = 2x - 3$ Equation 3: $2y = 4x + 6$					
4	Form a system of equations with Equations 1 and 3. Without graphing, explain how you can tell how many solutions the system has.				
5	Tonya says that a system of equations formed by Equations 2 and 3 will have the same number of solutions as a system formed by Equations 1 and 2. Is she correct? Use your answers to problems 3 and 4 to help you explain your reasoning.				
6	The system of equations shown below has no solution. Change one number in one of the equations so that the system has one solution. Graph your new system on the coordinate grid to support your answer. y = 2x - 1 $y = 2x + 1$				

and the

Reason and Write

Study the example problem. Underline two parts that you think make it a particularly good answer and a helpful example.

Example

Look at the three systems of equations shown below.

System 1	System 2	System 3
y = 2x + 5	y = 4x - 1	y - 3x = 2
$y = \frac{1}{2}(4x + 10)$	3y = 3x + 6	y = 3x - 5

Choose one of the systems. Without graphing, explain how you can use the slopes and *y*-intercepts of the equations to predict how many solutions the system of equations has. Then predict the number of solutions.

Justify your prediction by graphing the system of equations. Explain how your graph supports your prediction.

Show your work. Use words, graphs, and numbers to explain your answer.

I chose System 3. I solved for y in the first equation, y - 3x = 2, to get y = 3x + 2. The equations have the same slope, 3, and different y-intercepts, 2 and -5. When the equations in a system have the same slopes and different y-intercepts, there is no solution. I predict that the system of equations has no solution.





Where does the example . . .

- answer all parts of the problem?
- use words to explain?
- use a graph to explain?
- use numbers to explain?



Solve the problem. Use what you learned from the model.

Consider the equation 3y - 6x = 9.

Write another equation that forms a system of equations with the equation above that has either one solution or infinitely many solutions.

Without graphing, explain how you know that your system has one solution or infinitely many solutions.

Justify your equation by graphing the system of equations. Explain how your graph supports your choice of equation.

Show your work. Use words,

models, and numbers to explain your answer.

5	
4	
3	
2	
-5 - 4 - 3 - 2 - 1 0 -1 -2 -3 -4 -5	1 2 3 4 5

Where did you ...

- answer all parts of the problem?
- use words to explain?
- use a graph to explain?
- use numbers to explain?

162 Lesson 15 Understand Systems of Equations