Transformations and Congruence

Name: _____
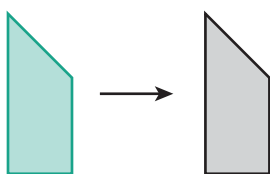
Recognize Translations, Reflections, and Rotations

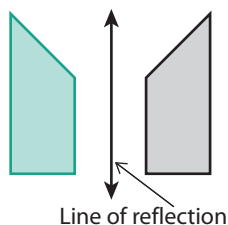**Study the example showing three different transformations of a figure. Then solve problems 1–5.**

Example

Transformations change the location or size of a figure. Three types of transformations are shown below.
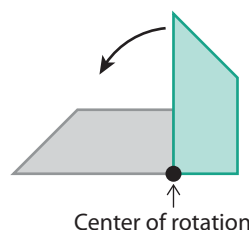
Translation   Reflection   Rotation

Line of reflection   Center of rotation

Every point of the figure moves the same distance and in the same direction.

The figure is flipped across a line of reflection.

The figure is turned clockwise or counterclockwise around a center of rotation.

1 Look at the figures in the example. Describe what happens to the size, shape, and location of a figure when it is translated, reflected, or rotated.

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2 The original green figure in the example has two right angles and one pair of parallel lines. Do the translation, reflection, and rotation produce images that have those same properties? Explain why or why not.

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Vocabulary

translation a transformation that moves each point of a figure the same distance and in the same direction.
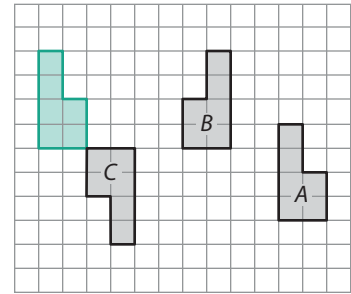
reflection a transformation that flips a figure across a line of reflection.

rotation a transformation that turns a figure around a fixed point called the center of rotation.

Solve.

3 Tell which of the gray figures, *A*, *B*, or *C*, appears to be a translation of the green figure. Explain your reasoning.

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4 Quadrilateral *WXYZ* is a reflection of quadrilateral *ABCD*.

The lengths of the sides and the measures of the angles of quadrilateral *ABCD* are given below.
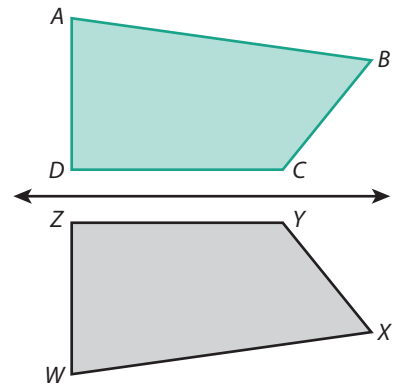
$AB = 4$ cm $BC = 2$ cm $CD = 3$ cm $DA = 2$ cm
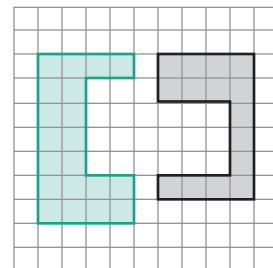
$m\angle A = 82°$ $m\angle B = 59°$ $m\angle C = 129°$ $m\angle D = 90°$

Predict the lengths of the sides and measures of the angles in quadrilateral *WXYZ*. Explain your reasoning.

$WX = $ _____ $XY = $ _____ $YZ = $ _____ $ZW = $ _____

$m\angle W = $ _____ $m\angle X = $_____ $m\angle Y = $ _____ $m\angle Z = $ _____

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5 Troy says that the green figure is the result of a single transformation or combination of a series of transformations on the gray figure. Is Troy correct? Explain.
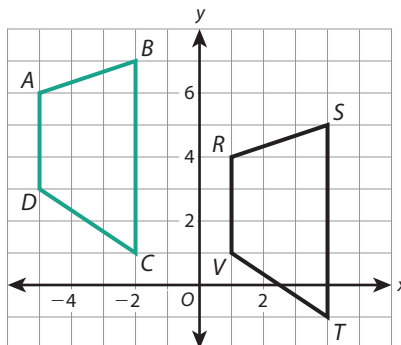
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Name: _____

Determine Whether Two Shapes Are Congruent

Study the example problem showing how to determine whether a shape and its image are congruent. Then solve problems 1–6.

Example

Polygon *ABCD* is translated 2 units down and 6 units to the right. Are polygons *ABCD* and *RSTV* congruent?

Because polygon *RSTV* is the image of polygon *ABCD* after a translation, each of its sides is congruent to the corresponding side of polygon *ABCD*, and each of its angles is congruent to the corresponding angle of polygon *ABCD*.

$\angle A \cong \angle R$ $\angle B \cong \angle S$ $\angle C \cong \angle T$ $\angle D \cong \angle V$

$\overline{AB} \cong \overline{RS}$ $\overline{BC} \cong \overline{ST}$ $\overline{CD} \cong \overline{TV}$ $\overline{DA} \cong \overline{VR}$

All of the corresponding sides and corresponding angles are congruent, so the polygons are congruent.

1 The example shows that $\angle A$ is congruent to $\angle R$. What does it mean to say that angles are congruent?

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2 Suppose you reflect polygon *ABCD* across the *y*-axis. Would the image be congruent to polygon *ABCD*? Explain.

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3 In the example, the length of $\overline{BC}$ in polygon *ABCD* is 6 units. Without measuring or counting, tell which side in polygon *RSTV* has a length of 6 units. Explain how you know.

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Vocabulary

congruent polygons
polygons with exactly the same size and shape. The symbol $\cong$ is read "is congruent to."

$\triangle ABC \cong \triangle DEF$
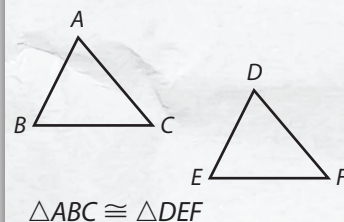
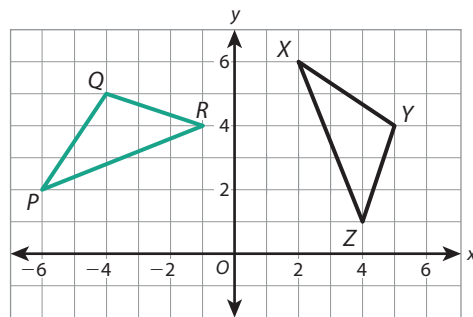**Solve.**

4. Triangle *PQR* is rotated 90° clockwise about the origin. The diagram shows the triangle and its image, △*XYZ*. Complete the congruence statements.

$\overline{PQ} \cong$ _____ $\overline{QR} \cong$ _____ $\overline{RP} \cong$ _____

$\angle P \cong$ _____ $\angle Q \cong$ _____ $\angle R \cong$ _____
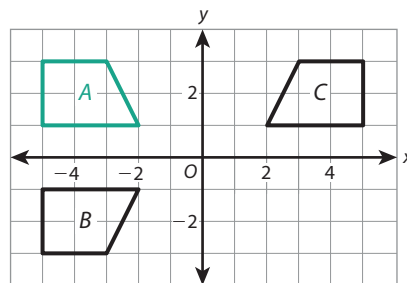

5. Sketch the image of △*FGH* after a translation 2 units down and 5 units to the left. Label the vertices of the image *K*, *L*, and *M*. Then complete the congruence statements below.

$\overline{GH} \cong$ _____   _____ $\cong \overline{MK}$

_____ $\cong \angle K$ $\angle H \cong$ _____
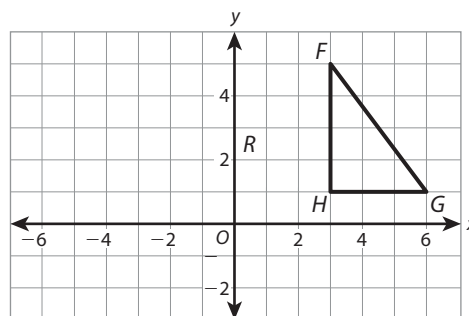
6. Polygon *B* is a reflection of polygon *A* across the *x*-axis. Polygon *C* is a rotation of polygon *B* about the origin. Is polygon *C* congruent to polygon *A*? Explain why or why not.

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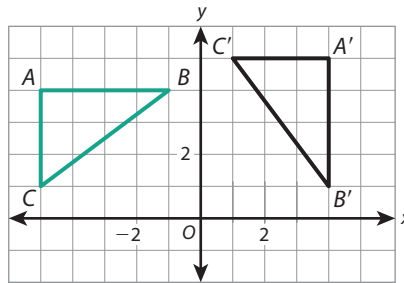
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Compare Coordinates

Study the example problem showing how to describe a transformation. Then solve problems 1–7.

Example

$\triangle ABC$ was transformed to produce a congruent triangle, $\triangle A'B'C'$. What transformation produced $\triangle A'B'C'$?

Compare the corresponding vertices in $\triangle ABC$ and $\triangle A'B'C'$.

$\triangle ABC$ (Original)	$\triangle A'B'C'$ (Image)
$A(-5, 4)$	$A'(4, 5)$
$B(-1, 4)$	$B'(4, 1)$
$C(-5, 1)$	$C'(1, 5)$

The x-coordinates in the image are the y-coordinates in the original figure. The y-coordinates in the image are the opposites of the x-coordinates in the original figure. The transformation was a 90° clockwise rotation about the origin.
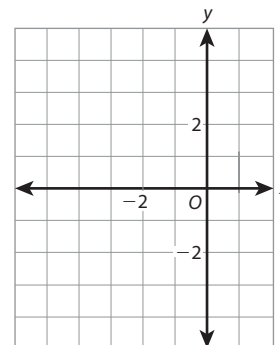
1 Suppose the vertices of the original figure in the example were $A(-6, 6)$, $B(-2, 5)$, and $C(-6, 2)$. What would be the vertices of the image after a 90° clockwise rotation about the origin?

$A'($ _____ $)$　　　　$B'($ _____ $)$　　　　$C'($ _____ $)$

2 On the coordinate plane at the right, sketch $\triangle ABC$ from the example above. Then sketch the triangle with the following vertices.

$L(-5, -4)$　　　　$M(-1, -4)$　　　　$N(-5, -1)$

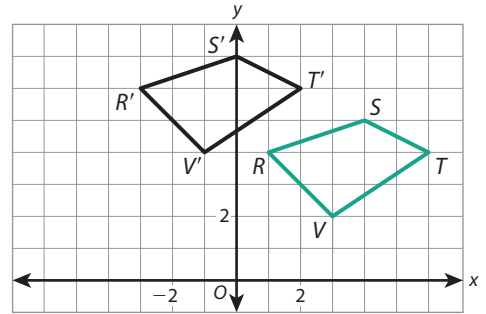Is $\triangle ABC$ congruent to $\triangle LMN$? Explain how you know.

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3 Compare the corresponding vertices in problem 2 and identify the transformation that produced $\triangle LMN$ from $\triangle ABC$.

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Solve.

Use polygon *RSTV* and its congruent image polygon *R′S′T′V′* for problems 4–6.

4 Describe the transformation that maps Polygon *RSTV* to Polygon *R′S′T′V′*.

_____

5 Write the coordinates of the vertices of the original polygon and its image. Then compare the corresponding vertices in the original polygon and its image.

R(_____)   *S*(_____)   *T*(_____)   *V*(_____)

R′(_____)   *S′*(_____)   *T′*(_____)   *V′*(_____)

_____

_____

6 How is the comparison of the corresponding vertices related to your description of the translation?

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7 The coordinates of the vertices of △*XYZ* and its image after a transformation are shown below.

△*XYZ*: *X*(3, 4), *Y*(3, 1), *Z*(1, 1)

△*X′Y′Z′*: *X′*(−3, 4), *Y′*(−3, 1), *Z′*(−1, 1)

a. Describe the difference in the *x*-values and the *y*-values of the corresponding vertices. What transformation produced △*X′Y′Z′*?

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b. Then use this information to find the coordinates of the vertices of △*P′Q′R′* after the same transformation of △*PQR*.

△*PQR*: *P*(−5, 3), *Q*(−1, 2), *R*(−2, −2)

△*P′Q′R′*: *P′*(_____), *Y′*(_____), *Z′*(_____)

Transformations and Congruence

Solve the problems.

**1** Triangle *A* transforms to Triangle *W*.
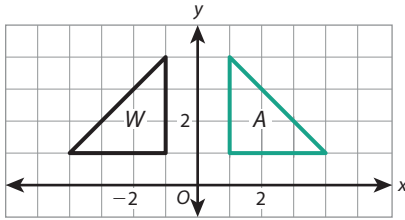
> Is there more than one way in which triangle *A* could have been transformed to produce triangle *W*?

Tell whether each statement is *True* or *False*.

a. The transformation could be a reflection across the *y*-axis. ☐ True ☐ False

b. The transformation could be a reflection across the *x*-axis. ☐ True ☐ False

c. The transformation could be a 90° counterclockwise rotation about the origin. ☐ True ☐ False

d. The transformation could be a translation 5 units to the left. ☐ True ☐ False

2 Mica translated Polygon *C* two units to the right and then reflected the image across the *y*-axis to get Polygon *T*. Sasha used one transformation to transform Polygon *C* to Polygon *T*. Describe the transformation that Sasha used.
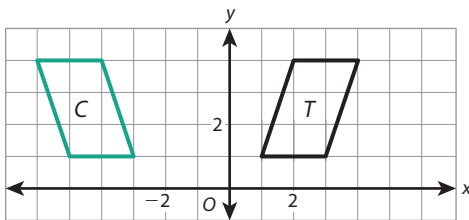
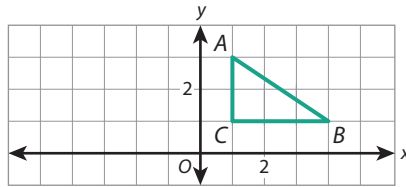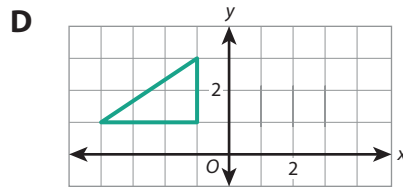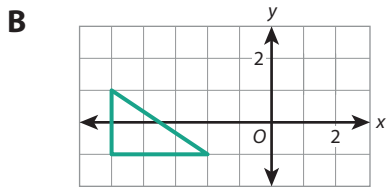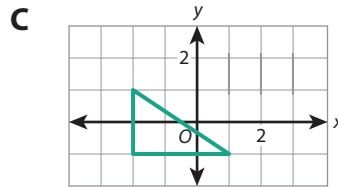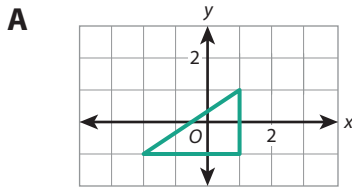> Does Polygon *T* look like a translation, reflection, or rotation of Polygon *C*?

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Solve.

3 Which graph shows the image of △ABC after the following series of transformations?

A translation 6 units left and 2 units down, followed by a reflection over the line x = −2.

> Make a sketch of the transformations before you choose.

A

C

B

D

Tabatha chose **B** as her answer. How did she get that answer?

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4 Rotate Polygon *HJKL* 180° about the origin, reflect it across the *y*-axis, and then reflect it across the *x*-axis. Write the coordinates of the vertices of the image Polygon *H′J′K′L′*. How do the vertices of Polygon *H′J′K′L′* compare to the corresponding vertices of Polygon *HJKL*?
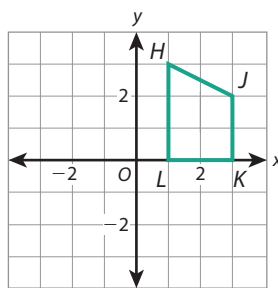
> Use the image from the first transformation as the original figure for the second transformation, and use the image from the second transformation as the original for the third.

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